

On the Optimal Allocation of Pesticides with Increasing Resistance: The Case of Alfalfa Weevil¹

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The problem of pesticide application under increasing pesticide resistance is explored. A theoretical model is developed to determine optimal pesticide use. This allocation is compared to the laissez-faire solution and to the centralized solution with incomplete information about pesticide resistance. The methodology is then applied in a case study on the Egyptian alfalfa weevil in California.

1. INTRODUCTION

Continued use of chemical pesticides against various agricultural pests (insects, mites, weeds, fungi, etc.) has resulted in increasing resistance to the chemicals by the pests and has created additional problems in pest management (Luck *et al.* [8]). Studies on the optimal allocation of pesticides to control specific insect pests have been made by Regev *et al.* [11] and Shoemaker [14, 15]; however, the problems created by pesticide resistance have had only limited attention (Taylor and Headley [16]; Hueth and Regev [7]). The problem is so general and of such importance that a recent popular article in *Time* magazine (July 12, 1976) reported: "Because of overexposure, insects are becoming more immune to chemical pesticides... As this resistance has developed, farmers have been forced to use ever greater amounts of increasingly expensive pesticides." A vicious cycle has developed in which all by-products (e.g., increased costs, pesticide pollution, secondary pest outbreaks, and wildlife destruction) are negative. The magnitude of externalities caused by pesticide use is difficult to estimate, and additional costs incurred as a result of pesticide

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resistance are unknown. Environmental damage—perhaps some of it irreversible—has rapidly accrued, and susceptibility in the pest population can be considered a resource that is not renewable once depleted. Entomologists and other field scientists have developed some alternative control methods (e.g., plant breeding, biological controls, and cultural practices) to cope with specific pests and, by default, with the development of resistance in the pest population.

In this paper we attempt to develop a general theoretical model for the optimal allocation of pesticides given that resistance can develop in the pest population. The need is emphasized for regional organization or governmental intervention to reduce a portion of the negative externalities created by individual-grower pesticide use and to estimate the developmental investments that should be made in alternative technologies. Special attention is paid to the case of regional centralization with complete ignorance of resistance development. This general model is an outgrowth of empirical work done by Regev, *et al.* [11] and Gutierrez *et al.* [6]. It is constrained to a single-pest, single-crop framework dealing with the case of simple single-gene resistance to a specific pesticide. However, many of the known cases of resistance to pesticides are of this kind (Georghiou [4], Brown [1]).

The conclusions drawn from the model, as presented in Section 2, are twofold: (1) based on the assumption that alternative pest-control techniques exist, an optimal path of current pesticide practices may be found until the economy switches to one of the alternative technologies; and (2) if the central authority conducts its optimal policy only with respect to pest population while ignoring the effects of pesticide resistance, it is then preferable not to intervene by increasing pesticide use.

An empirical study dealing with alfalfa and the Egyptian alfalfa weevil is discussed in Section 3 to complement the theoretical analysis.

2. THE MODEL

A simplified continuous time model of the economic problems associated with the development of pesticide resistance in pest populations is presented in this section. The pest-management system considered here is assumed to be a region closed to external pest migration. Further, it is assumed that a central decision body, possessing perfect information on the pest-plant control system, seeks to maximize the present value of profits for a given crop. All productive factors, with the exception of pest control, are assumed to be at optimal levels and unaffected by the pesticide application which is the control variable in this case.

The model is formulated as follows. Let $n(t)$ denote the total pest population at time t . The dynamics of the system are described by

$$\frac{1}{n(t)} \frac{dn(t)}{dt} = \frac{\dot{n}(t)}{n(t)} = f[n(t)]k[x(t), w(t)]. \quad (1)$$

That is, the rate of growth of the pest population is equal to the natural rate of growth $f[n(t)]$ times the proportion $k[x(t), w(t)]$ of pests that survive a pesticide application $x(t)$. Assuming that there is a maximum population level \bar{n} that can be sustained within the region and excluding the possibility of total extinction implies $\underline{n} \leq n(t) \leq \bar{n}$ for $\underline{n} > 0$.

The level of resistance $w(t)$ is measured as the proportion of pest population immune to a particular pesticide or group of pesticides. Once resistance appears, it changes over time as a result of pesticide application according to the assumed relationship

$$\dot{w}(t)/w(t) = g[x(t), w(t)]. \quad (2)$$

The functions $f(\cdot)$, $k(\cdot)$, and $g(\cdot)$ are assumed to be differentiable and to possess the properties

$$\frac{\partial f[n(t)]}{\partial n(t)} = f_n[n(t)] < 0, \quad \lim_{n \rightarrow \bar{n}} f(n) = 0.$$

The assumption of the negative impact of increasing population density on the natural rate of growth is explained by the competition of the pests for resources and is embodied in the above equation. Further, it is assumed²

$$\begin{aligned} 0 \leq k[x(t), w(t)] \leq 1, \quad \lim_{x \rightarrow \infty} k[x(t), w(t)] = w^2(t) \quad \text{for } w(t) < 1. \\ \lim_{x \rightarrow 0} k[x(t), w(t)] = 1, \quad k_x(x, w) < 0, \quad k_{xx}(x, w) > 0 \\ k_{ww}(x, w) > 0 \\ \lim_{w \rightarrow 1} k_x(x, w) = 0, \quad k_w(x, w) \geq 0; \end{aligned}$$

that is, as a result of increasing resistance, the pesticide effectiveness in controlling population decreases. And, finally, it is assumed that

$$\begin{aligned} g(x, w) \geq 0, \quad \lim_{x \rightarrow 0} g(x, w) = 0, \quad \lim_{w \rightarrow 1} g(x, w) = 0, \\ g_x(x, w) > 0 \quad \text{for } x > 0, \quad \lim_{w \rightarrow 1} g_x(x, w) = 0, \\ g_w(x, w) > 0, \quad \lim_{x \rightarrow 0} g_w(x, w) = 0, \quad g_{ww} > 0. \end{aligned}$$

Note that these properties imply that resistance does not decrease over time once it has appeared. This assumption is somewhat subjective since entomologists have observed that in some cases resistant pests are less able to survive in the absence of pesticides than are the nonresistant ones (Georghiou [4]). Thus, in some cases, the resistance level might decrease in the absence of pesticides, but the rate of decrease is slow relative to its increase once pesticide use is resumed. In the long run, it seems likely that one can reasonably disregard the short-run decrease of resistance and consider the biological process irreversible. If we were to relax this assumption, the optimal pest-control policy might change drastically. First, a steady state of resistance can be obtained since susceptibility now becomes a renewable resource. Heuristically it can be argued that a cycling of pesticides is called for, that is, one kind of pesticide is used for some time, then is replaced by another when resistance to the first pesticide crosses some threshold. Meanwhile, resistance to the first

²Subscripted variables (but not t) denote partial derivatives, and t is omitted for notational convenience whenever it is possible to do so without ambiguity.

pesticide decreases to a point where it can be used again. This policy can be devised only if no cross-resistance develops. It is not certain which is the case in reality, but there are strong indications for cross-resistance and rapid development of it so that our model seems quite close to reality.

By considering the population and resistance dynamics described by (1) and (2), it is evident that, under the assumptions, there is not a nontrivial steady state—that is, a state (n, w) not on the boundaries $(\underline{n}, 0)$, $(\underline{n}, 1)$, $(\bar{n}, 0)$, and $(\bar{n}, 1)$ such that $\dot{n} = 0$, $\dot{w} = 0$, and $\dot{x} = 0$.

Since $g_x(x, w) > 0$ for $x > 0$ and $w < 1$, a steady state of the resistance level ($\dot{w} = 0$) is obtained only when $x = 0$. In that case, however, the pest population increases unless $n = \bar{n}$. On the other hand, assume the existence of a steady state in population density ($\dot{n} = 0$, $\dot{x} = 0$ for $x > 0$); then the resistance level increases unless $w = 0$.

When other inputs are disregarded for simplicity, the profit function generated through the use of pesticides is defined by

$$\Pi[x(t), n(t)] = R[n(t)] - C[x(t)] \quad (3)$$

with the property $(\Pi_n \leq 0)$ of nonpositivity expressing the negative effects on yield of increasing pest population. Without loss of generality, one assumes that $C[x(t)] = cx(t)$ where c is the unit cost of pesticide.

The current pest-control technology x is maintained until some future time T when a readily available alternative technology will be used. This technology α could be either an alternative set of chemical pesticides, biological control, or any mixture of the two; pest-resistant plant, changing the crop, or selling the land. The alternative technology α yields, at time T , a net return S^α of

$$S^\alpha[n(T), w(T)] = s[n(T), w(T)] \int_T^\infty e^{-r(t-T)} dt \quad (4)$$

where $s(\cdot)$ is a stream of identical benefits, and r is a fixed discount rate. If we deal with alternative technologies such as changing the crop or selling the land, S^α may be independent of both $n(T)$ and $w(T)$. Since this model focuses mainly on the current technology, the form and behavior of $s(\cdot)$ are not investigated. Note that if the alternative technology does not, in any context, increase pesticide resistance of the pest population, it is the “backstop technology” (Nordhaus [10]) with respect to pesticides. Furthermore, one assumes that $S_w \leq 0$ and $S_n \leq 0$, vanishing derivatives implying that the alternative technology is considered to be a type of biological control.

Optimal Solution

The economic problem is now defined as the choice of quantities of pesticides x_t to be applied and the date of switching to the new technology which maximizes

$$\Phi(n, w, \alpha) = \int_0^T \Pi[x(t), n(t)] e^{-rt} dt + S^\alpha[n(T), w(T)] e^{-rT} \quad (5)$$

subject to (1) and (2) and $x(t) \geq 0$.

Applying the maximum principle of optimal control, the Hamiltonian to be maximized is

$$\mathcal{H} = e^{-rt} \{ \Pi [x(t), n(t)] + \mu(t) \{ n(t) f [n(t)] k [x(t), w(t)] \} + \lambda(t) w(t) g [x(t), w(t)] + \xi(t) x(t) \}. \quad (6)$$

If $x(t)$ is a solution to this problem and if T is the optimal date of switching, then there exist two continuous functions, $\lambda(t)$ and $\mu(t)$, and a function, $\xi(t) \geq 0$, that satisfy the conditions (Bryson and Ho [2, pp. 71–76])

$$\Pi_x + \mu n f k_x + \lambda w g_x + \xi = 0, \quad \xi x = 0, \quad \xi \geq 0 \quad (7a)$$

$$\dot{\mu} = \mu \left(r - \frac{\dot{n}}{n} - n f_n k \right) - \Pi_n \quad (7b)$$

$$\dot{\lambda} = \lambda \left(r - \frac{\dot{w}}{w} - w g_w \right) - \mu n f k_w \quad (7c)$$

$$\dot{w} = w g(x, w) \quad (7d)$$

$$\dot{n} = n [f(n) k(x, w)] \quad (7e)$$

$$[\Pi(x, n) + \mu \dot{n} + \lambda \dot{w} - r S(n, w)]_{t=T} = 0 \quad (7f)$$

$$S_n|_t = \mu(T), \quad S_w|_t = \lambda(T). \quad (7g)$$

Equation (7a) is the usual marginal condition of long-run profit maximization. It states that, for each time period where $x(t) > 0$, the nominal unit cost of pesticides (since $\Pi_x = -c$) must equal the user cost of pesticides (Scott [13]). This user cost is composed of two elements: (1) a measure of the value of marginal effectiveness of a pesticide in controlling the pest population ($\mu n f k_x$), which is expressed in terms of future benefits, and (2) a measure of the value of future losses implied by increasing resistance ($\lambda w g_x$). When dealing with negative characteristics such as increasing pest population and resistance, it may be suggested that the values of λ and μ are nonpositive along the optimal path. For positive pesticide unit cost c and $x(t)$

$$c = \lambda w g_x + \mu n f k_x > 0 \quad (8)$$

which implies that the value of marginal effectiveness of pesticide use must exceed the losses related to increasing resistance. If this condition does not hold for a positive cost c , pesticide use is discontinued— $[x(t) = 0]$ and $c = \xi(t)$ since $k_x(0, w) = 0$ and $g_x(0, w) = 0$. However, pesticide use can be resumed as n increases and $|\mu n f k_x| > |\lambda w g_x|$. Nevertheless, the ultimate level of resistance $w = 1$ is never attained since

$$\lim_{w \rightarrow 1} k_x(x, w) = 0, \quad \lim_{w \rightarrow 1} g_x(x, w) = 0 \quad \text{for } x(t) > 0$$

by assumption. If, on the other hand, one assumes that the real cost of pesticide use increases with time, the incidence of terminating pesticide use increases. The reason for this is that the difference between $\mu n f k_x$ and $\lambda w g_x$ is more likely to decrease as resistance develops. Condition (8) does not guarantee that, once pesticide use is ceased, the alternative technology is adopted. It is admissible that a period without

pesticides will be followed by a period of pesticide use. This condition only regulates the transition between these states. On the other hand, (7f) controls the shift to a new technology since it expresses T as optimal if the value of the optimal Hamiltonian evaluated at T equals the instantaneous future gain $rS(\cdot)$. The value of the Hamiltonian represents the immediate gain $\Pi(x, n)$ minus the implicit costs of pest resistance λwg and pest infestation $\mu n[f(n)k(x, w)]$.

More interesting is the information conveyed by (7f). This equation can be used to determine which technology should immediately replace the existing one. If for all w , $x \geq 0$, and $n \geq n_0$, $\Pi(x, n) \leq rS(n_0, w_0)$, the alternative technology should be immediately adopted, that is, $T = 0$. The proof is trivial since

$$S(n_0, w_0) \geq \int_0^T rS(n_0, w_0)e^{-rt} dt + e^{-rT}S[n(T), w(T)]$$

and

$$\int_0^T rS(n_0, w_0)e^{-rt} dt \geq \int_0^T \Pi(x, n)e^{-rt} dt.$$

Consequently, since \underline{n} is the lower bound of the pest population, the value of the technology that should be immediately adopted is defined by $\Pi(0, \underline{n}) \leq rS(\underline{n}, w)$ for all w . On the other hand, the alternative technology will never be implemented if $\Pi(x, \bar{n}) > rS(\bar{n}, w)$ for $w \rightarrow 1$ and any $x \geq 0$. This is evident since, at the boundary $(\bar{n}, 1)$, $\dot{n} = 0$, and $\dot{w} = 0$, pesticide control is ineffective, and the existing technology for which immediate profit is $\Pi(0, \bar{n})$ will not be replaced. Hence, given the current practices, policymakers can find and exclude all the nonprofitable alternatives, that is, all the α , such that $\Pi(0, \bar{n}) > rS^{\alpha}(\bar{n}, w)$ when $w \rightarrow 1$.

2.1 Comparison with an Incomplete Model

The study of a centralized pest-management model does not always ensure the availability of knowledge concerning pesticide resistance. More frequently, because of lack of information or because of the complexity of the problem, the policymaker ignores the effects of pesticide use on the development of resistance. Although, it can be argued that policymakers in the field can learn about resistance from experience—e.g., Sarhan *et al.* [12] and Carlson [3]—this knowledge may be acquired when resistance has developed to a level that renders a pesticide ineffective.

This type of optimization was developed by Regev *et al.* [11] in the case of alfalfa and the Egyptian alfalfa weevil. The purpose of this section is to compare the results obtained in (7a)–(7g) with the solution to problems that do not account for increasing resistance.

In that case the problem is reduced to maximizing

$$\Phi(n, \alpha) = \int_0^T \Pi[x(t), n(t)]e^{-rt} dt + e^{-rT}S[n(T)] \quad (9)$$

subject to

$$\dot{n}(t) = n(t)f[n(t)]k[x(t), \bar{w}_0]$$

and

$$x(t) \geq 0$$

where \bar{w}_0 is the parameter representing the initial fixed level of resistance that is unknown to the decision-maker. Furthermore, the decision-maker ignores the existence of that effect. For simplicity, we assume that all the functions are equivalent to the functions in the complete model. The conditions for a maximum are simply

$$\Pi_x + \mu n f k_x + \xi(t) = 0, \quad \xi \geq 0, \quad x(t)\xi(t) = 0 \quad (10a)$$

$$\dot{n} = n[f(n)k(x, \bar{w}_0)] \quad (10b)$$

$$\dot{\mu} = \mu(r - (\dot{n}/n) - n f_n k) - \Pi_n \quad (10c)$$

$$\{\Pi(x, n) + \mu n f(n)k(x, \bar{w}_0) - rS\}_{t=T} = 0 \quad (10d)$$

$$\mu(T) = S_n[n(t)]. \quad (10e)$$

Let $(x, n, \mu)_{IC}$ denote the optimal path of the incomplete model and assume that $\Pi_x = -c$; then for time intervals for which $x(t)_{IC} > 0$,

$$c = (\mu n f k_x)_{IC} > 0. \quad (11)$$

Moreover, for identical levels of resistance and pesticides the survival function is equivalent. This implies

$$k_x(\bar{x}, w_0) = k_x(\bar{x}, \bar{w}_0)_{IC}.$$

The comparison between (11) and (8) gives

$$(\mu n f k_x)_{IC} = \mu n f k_x + \lambda w g_x.$$

However, noting that $\lambda w g_x < 0$ for $w < 1$, $t < T$, and $x > 0$ implies

$$(\mu n f k_x)_{IC} < \mu n f k_x.$$

This result suggests that since the value of marginal effectiveness of pesticide is lower in the incomplete model, there will be a tendency to use larger quantities if the implicit costs of pest development μ , is identical in the two models. In general, this cannot be ascertained. If, however, more pesticides are used in the incomplete model, as we expect, resistance will develop at a faster pace than it will in the optimal model, assuming that resistance does not decrease over time once it has risen. One can also determine the value in acquiring complete information about the development of resistance. For identical technologies, this is just the difference between $\phi(n, w, \alpha)$ [Eq. (5)] and $\Phi(n, \alpha)$ [Eq. (9)] evaluated at their respective optimal levels.

2.2 Comparison with a Competitive Solution

A single decision unit faces basically the problem of pest control described above. Since pests are migratory, however, resistance develops in relation to the total

pesticides applied throughout the region. Therefore, the effect of an individual farmer on resistance development and on the level of pest populations is negligible and must be considered to be beyond his control. Because the individual decision-maker does not affect the general level of resistance, nor the rate of infestation, he may tend to disregard them and seek to maximize his immediate net benefits. His decision, however, depends presumably on the resistance level and the infestation rate he observes.

The competitive decision-maker applies pesticides at the level $\hat{x}(t)$ that maximize

$$\Pi[x(t), n(t)] = R[n(t)] - C[x(t)] \quad \text{for every } t \quad (12)$$

subject to

$$n(t) = \hat{n}(t)k[x(t), \hat{w}(t)]$$

where $x(t) \geq 0$, $n(t)$ is the level of pest after pesticide application, and $\hat{n}(t)$, $\hat{w}(t)$ are the exogenous parameters that represent the level of pest population and pest resistance at the beginning of the period t . Moreover, the decision-maker compares the optimal instantaneous profit $\Pi[\hat{x}(t), n(t)]$ with the profit that can result from the alternative, that is, $rS[\hat{n}(t), \hat{w}(t)]$, and decides which process to adopt. If as before $\Pi_x = -c$, the optimal-decision rule for an individual grower is given by

$$\begin{aligned} -c + R_n \hat{n}(t) k_x[\hat{x}(t), \hat{w}(t)] + \xi(t) &= 0 \\ \xi(t) \hat{x}(t) &= 0, \quad \xi(t) \geq 0 \end{aligned} \quad (13)$$

$$\phi(t) = \max\{\Pi[\hat{x}(t), n(t)], rS[\hat{n}(t), \hat{w}(t)]\} \quad (14)$$

where

$$R_n = \frac{\partial R[n(t)]}{\partial n(t)} \leq 0.$$

Equation (13) merely states that for $\hat{x}(t) > 0$ the unit cost of pesticides must equal the marginal revenue of its effectiveness in controlling the pest.

For convenience, denote by $*$, $x^*(t)$, $w^*(t)$, etc., the optimal values of the complete model; by IC the optimal values of the incomplete model; and by $\hat{}$ the optimal values of the competitive model. Now assume that the unit cost and functions are identical. Comparison of (13) with (11) and (8) for $x > 0$ gives

$$c = R_n \hat{n} k_x(\hat{x}, \hat{w}) = (\mu n f k_x)_{IC} = \mu^* n^* f^* k_x^* + \lambda^* w^* g_x^*.$$

Consider now the initial period: $\hat{n} = (n f)_{IC} = n^* f$ and $\hat{w} = w^* = w_0$. Then

$$R_n k_x(\hat{x}, w_0) = (\mu k_x)_{IC}.$$

Moreover, $R_n \geq \mu_{IC}$ since the competitive grower does not account for future damages due to pest population growth but only considers the present period. Hence, at $t = 0$,

$$(k_x)_{IC} \geq k_x(\hat{x}, w_0)$$

implying that at the beginning of the planning period, the quantity of pesticide sprayed by the competitive farmer is smaller than the amount proposed by the centralized authority which ignores pesticide resistance. Since, however, $x_{IC}(t=0) \geq x^*(t=0)$, the comparison between the amount of pesticide used by the competitive grower (\hat{x}) and the quantity suggested by the optimal solution (x^*) is ambiguous.

2.3 Policy Implications

A policy intended to internalize the externalities generated by a competitive system, while ignoring the effects of pesticide resistance, will cause more damage to the environment than it will correct. It was shown above that such a centralized policy will increase pesticide use because its main goal is to control future pest populations. This type of policy should not be endorsed.

On the other hand, if the competitive solution implies a higher level of pesticide application than the level suggested by the optimal solution, susceptibility to pesticide use will decline too rapidly. To remove this externality, two classical approaches are suggested.

1. A tax may be levied per unit of pesticide use. By (8) and (13), a tax rate equal to $\lambda^*w^*g_x^* - \mu^*n^*f^*k_x^* + R_n \hat{n}k_x$ will induce the individual grower to adopt the optimal control policy.

2. Because it is difficult to evaluate the time path of the tax rate, it may be preferable to impose restrictions on pesticides that would force the farmer to use socially optimal amounts. However, serious organizational obstacles may interfere with the implementation of this policy.

3. AN EMPIRICAL STUDY

The case study involves the problem of the Egyptian alfalfa weevil (*Hypera brunneipennis*), a major alfalfa pest in California. In the late 1950s this pest invaded the southwestern United States causing an estimated annual damage of over \$18 million in California alone. While resistance to pesticides has not yet developed in this species in California, it appears imminent as evidenced by the evolution of hexachlor resistance in a closely related species in the USSR (Mardzhangian *et al.* [9]).

3.1 Biological Context

A detailed description of the biological systems underlying the model is presented in an article by Gutierrez *et al.* [6]. The mathematical formulation is based on a similar model (Regev *et al.* [11]) and uses field and laboratory data from a study by Gutierrez *et al.* [5]. Only a brief summary of these papers is given here.

The biology of the pest and the alfalfa crop (i.e., hay) is briefly described. After aestivation (summer dormancy), adult weevils migrate into the alfalfa fields in the autumn and early winter months and oviposit their eggs. The eggs hatch and produce larvae that feed on the leaf tissues of the alfalfa. The larvae that survive the winter and spring frost can cause considerable damage. The larvae develop to the

pupal stage and in the spring become young adults that leave the fields and remain dormant during the summer, returning the following autumn to begin the cycle anew.

Farmers commonly apply chemicals (e.g., Furadan) when feeding damage is observed in order to control adult and larvae pest population. The effect of the pesticide is formulated by a dosage-response function that depends upon the stage of adult or larva and upon the level of pesticide resistance w in the population. In the model, resistance is regulated by a single dominant gene implying that susceptibility to a pesticide differs in homozygous and heterozygous individuals (Gutierrez *et al.* [6]). Resistance levels in the population are measured as the relative frequency of the resistance gene in the population. Changes in gene frequency are determined by pesticide-induced genetic selection following the Hardy-Weinberg law.³ During the aestivation period when adults from many fields mix together, the frequency of the resistant gene depends on the total amount of pesticide applied in the whole region and not upon applications made by single farmers.

3.2 The Optimization Procedure

For computational purposes, the biological model and its economic components were condensed from their original version [5] to the following form. The multiseasonal function to be maximized is defined as

$$\sum_{t=0}^{T-1} \frac{P_1 L_t^1 + P_2 (L_t^2) L_t^2 - c x_t}{(1+r)^t} + \frac{S}{(1+r)^T} \quad (15)$$

where⁴ P_1 is the price of alfalfa produced by the first cut (dollars/ton), P_2 the price of alfalfa from the second cut $P_2(L_t^2) = \gamma_1 + \gamma_2 L_t^2$, L_t^1 the yield of alfalfa from first cut (tons/acre), L_t^2 the yield of alfalfa from second cut (tons/acre), c the unit cost of pesticide (dollars/oz), x_t the pesticide use (oz/acre), r the discount rate, T the planning horizon, and S the present value of the benefits of the alternative [see (4)].

To preserve the analogy with the continuous model presented in Section 2, the set of difference equations describing the interseasonal pest-plant dynamics and changes in pest resistance and pest population is as follows:

$$m_t = n_t k(x_t, w_t) \quad (16)$$

$$L_t^1 = (31.24 - 6.2m_t)0.048 \quad (17a)$$

$$L_t^2 = (44.74 - 4.25m_t)0.048 \quad (17b)$$

$$w_{t+1} = [g(x_t, w_t) + 1] w_t \quad (18)$$

$$n_{t+1} = 0.025 \sum_{i=1} \delta_i (m_t)^i \quad (19)$$

³The resistance equation following that law is given explicitly by (21). It can be shown that this function is convex with respect to w and x for the positive values of these variables.

⁴The values of the parameters were estimated from [5] and [6] and are $P_1 = 80$, $\gamma_1 = 38$, $\gamma_2 = 28$, $c = 0.60$, and $r = 0.05$.

where⁵

$$k(x_t, w_t) = w_t^2 e^{-\alpha_1 x_t} + 2w_t(1 - w_t)e^{-\alpha_2 x_t} + (1 - w_t)^2 e^{-\alpha_3 x_t} \quad (20)$$

is the survivorship function due to pesticide application x , m_t the level of pest population after pesticide use (species/sq ft), n_t the level of pest population before pesticide use, and w_t the level of resistance, measured as the frequency of the resistant gene.

$$w_t [g(x_t, w_t) + 1] = \frac{w_t^2 e^{-\alpha_1 x_t} + w_t(1 - w_t)e^{-\alpha_2 x_t}}{w_t^2 e^{-\alpha_1 x_t} + 2w_t(1 - w_t)e^{-\alpha_2 x_t} + (1 - w_t)^2 e^{-\alpha_3 x_t}} \quad (21)$$

is the resistance development function and δ is the polynomial coefficient of the natural rate of growth of the pest population.

The survivorship rate is a function of the amount of pesticide used and the relative frequency of the resistant gene w_t . Since we assumed that resistance is regulated by a single dominant gene, susceptibility to pesticide application depends on the frequency of the resistant gene in the population. Hence, w_t^2 is the proportion of the resistant homozygous weevils, $(1 - w_t)^2$ the proportion of susceptible homozygous weevils, and $2w_t(1 - w_t)$ the proportion of heterozygous weevils. For each phenotype the kill parameter α_i is different, which expresses its resistance to pesticide (see [6] for the genetic model).

In addition, the frequency of the resistant gene for the next period is determined by the function $g(x, w)$, which expresses the frequency of resistant genotypes in the pest population that survives pesticide application.

The difference equations (18) and (19) combined with (16) are the discrete transformations of the continuous functions (1) and (2). Hence, the problem is to maximize (15) subject to constraints (16)–(19).

By assuming that the value of the alternative technology does not depend on the level of resistance or population, a solution to the problem is obtained by using the nonlinear programming code (GRG) for arbitrary values of T . The optimal switching date is determined by maximizing the multiseasonal objective function with respect to T .

3.3 The results

The optimal solution to the multiseasonal problem for the initial values of $w_0 = 0.01$ and $n_0 = 1$ is presented in Table I. The real rate of interest is set at 0.05 and the value of the alternative technology is independent of the terminal level of pest resistance w_T and of the terminal level of pest population n_T .

The optimal policy consists of a large application of pesticide at the beginning of the planning period. Thereafter, the quantities become relatively small. This policy enables control of the pest population at a low level. As a result, however, resistance develops quite rapidly. Beginning with $n_0 = 1$, it takes seven seasons to reach this

⁵The values of the parameters are $\alpha_1 = 0$, $\alpha_2 = 0.095$, $\alpha_3 = 0.19$, $\delta_1 = 185.7$, $\delta_2 = -121.7$, $\delta_3 = 34.0$, $\delta_4 = -4.237$, and $\delta_5 = 0.194$. Moreover, the constants in (17a) and (17b) transform grams/square feet to tons/acre.

TABLE I
The Optimal Solution for $n_0 = 1$, $w_0 = 0.01$

Season year	Pesticides level (oz./acre)	Pest population (adults/sq ft)	Level of resistance (%)	Yield		Profit for two cuttings (\$/acre)
				First cut (tons/acre)	Second cut (tons/acre)	
1	31.27	1.000	0.010	1.438	2.147	272.77
2	8.28	0.017	0.165	1.497	2.147	286.49
3	8.61	0.023	0.302	1.416	2.146	286.09
4	8.18	0.040	0.495	1.497	2.143	285.64
5	5.00	0.098	0.681	1.475	2.132	284.53
6	8.43	0.334	0.774	1.416	2.095	272.52
7	4.54	1.002	0.884	1.198	1.959	239.79
8	0.00	2.287	0.922	0.753	1.681	178.95
9	7.22	2.277	0.922	0.813	1.718	182.52
10	0.00	2.340	0.959	0.735	1.670	176.72

level again. Since resistance cannot be decreased, the proportion of resistant pests reaches 88% after seven seasons. From that level, the effectiveness of pesticides decreases to the extent that subsequent applications cannot control the spread of pests. The yield of alfalfa from the first cut decreases by approximately 50% as the level of pest population reaches its maximum of 2.3 adults/sq ft. Damage to the second cut occurs indirectly—via the depletion of carbohydrate storage caused by the feeding damage in the first cut—since weevils do not contaminate after the first cut. Therefore, the damage to the second cut yield is only moderate and the decrease in the seasonal profit is relatively small.

Given the present value of the alternative technology S , the optimal switching time is determined by searching the number of seasons T that maximizes the multiseasonal-profit function (15). The values of the optimal-profit function for different values of S are presented in Table II. It appears that for $S < 176/0.05$ the switching time will tend to infinity and for $S < 286.49/0.05$ the present pest-control technology will be immediately abandoned ($T = 0$). Moreover, for $S = 3600$, the optimal T is 10; for $S = 4800$, $T = 7$; and for $S = 5600$, $T = 6$.

The incomplete model, as shown in Section 2.1, is represented by a centralized pest-management model that ignores the effects of pest resistance. The results for

TABLE II
Multiseasonal Profit for $n_0 = 1$, $w_0 = 0.01$ (\$/acre); $r = 0.05$

Season	Profit	$S = 0$	$S = 3600$	$S = 4800$	$S = 5600$
1	272.77	272.77	3701.34	4844.20	5606.11
2	286.49	545.62	3810.92	4899.36	5624.98
3	286.09	805.11	3914.92	4951.53	5642.60
4	285.64	1051.85	4013.58	5000.83	5658.99
5	284.53	1285.93	4106.63	5046.86	5673.68
6	272.52	1499.46	4185.83	5081.29	5678.26
7	239.79	1678.39	4236.84	5089.66	5658.20
8	178.95	1805.57	4242.19	5054.40	5595.87
9	182.52	1929.11	4249.70	5023.23	5538.92
10	176.72	2043.02	4253.11	4989.81	5480.94

TABLE III
Solution for the Incomplete Model for $n_0 = 1, w_0 = 0.01$

Season year	Pesticides level (oz/acre)	Pest population (adults/sq ft)	Level of resistance (%)	Yield		Profit for two cuttings (\$/acre)
				First cut (tons/acre)	Second cut (tons/acre)	
1	26.65	1.000	0.010	1.497	2.146	275.31
2	4.43	0.036	0.112	1.494	2.144	288.10
3	46.95	0.079	0.162	1.499	2.147	263.44
4	0.0	0.010	0.943	1.496	2.145	291.16
5	0.15	0.048	0.943	1.484	2.137	288.97
6	4.14	0.215	0.943	1.432	2.105	277.74
7	0.00	0.840	0.944	1.225	1.976	246.72
8	6.36	2.205	0.962	0.804	1.713	181.91
9	5.45	2.331	0.978	0.752	1.680	175.50
10	5.51	2.276	0.987	0.764	1.688	177.16

this model are obtained by removing the resistance-development function, that is, $g(x_t, w_t)$ from the optimal program. The resulting "optimal" pesticide strategy is then used in a second run that simulates the development of resistance, the true damage to the yield, and the seasonal profit. These results are shown in Table III. First note that, contrary to our expectations, the first-season level of pesticide does not exceed the level suggested by the optimal solution. However, the total quantity of pesticide used during the 10 seasons (99.4 oz/acre) is much higher than the optimal total. Hence, the level of resistance increases rapidly reducing the effectiveness of the pesticide. The difference between the values of the objective functions serves as an indicator for the costs of information regarding resistance. In our empirical example, this difference was very low (\$2/acre). This resulted from the relatively high profit when no pesticides are used. Thus, even if resistance develops rapidly, its importance is minor since alfalfa can survive without pesticides applications.

The competitive grower model is obtained from a succession of one-season optimal problems with given resistance and population levels. The resulting state

TABLE IV
The Competitive Solution for $n_0 = 1, w_0 = 0.01$

Season year	Pesticides level (oz/acre)	Pest population (adults/sq ft)	Level of resistance (%)	Yield		Profit for two cuttings (\$/acre)
				First cut (tons/acre)	Second cut (tons/acre)	
1	15.22	1.000	0.010	1.480	2.136	279.36
2	8.04	0.260	0.041	1.479	2.135	283.52
3	8.44	0.271	0.084	1.477	2.133	282.92
4	9.04	0.299	0.169	1.473	2.131	281.83
5	9.62	0.354	0.325	1.458	2.122	279.03
6	11.31	0.532	0.546	1.414	2.094	270.54
7	12.00	1.018	0.779	1.260	1.997	244.93
8	8.63	2.076	0.917	0.883	1.762	191.24
9	1.94	2.410	0.962	0.723	1.662	173.91
10	0.00	2.246	0.968	0.766	1.689	180.71

variables, w and n , are then used as initial values for the next problem. The solution to the competitive model is presented in Table IV.

If one compares these results with the optimal solution presented in Table I, one sees that at $t = 0$ the level of pesticides of the optimal solution (31.27 oz/acre) is higher than the quantity used by the competitive grower (15.22 oz/acre). Only from the fifth season on will the competitive grower use more pesticides than the centralized decision-maker. Therefore, the development of resistance will be slower but pest population will be at a higher level. If one compares the level of profit in the two cases, one sees that generally the centralized decision-maker is doing better than the competitive grower. For selected values of S the switching time of the competitive grower is obtained by comparing the instantaneous profit with $rS/(1+r)$. Thus, if $S = 3600$ the alternative technology will never be adopted and the competitive farmer will continue to grow alfalfa without use of pesticide. If $S = 4800$ the switch occurs *after* 7 seasons since $191.24 \leq (0.05 \times 4800)/1.05$. If $S = 5600$ the switching time will be after 6 seasons.

4. CONCLUSION

The thrust of the paper is to model the conflicting role of pesticide application on pest population and pest resistance. To devise optimal policies, the decision-maker must compromise these two distinct state variables. Whereas pesticide use decreases the level of pest population, it undoubtedly increases the level of resistance to pesticides. Thus, an optimal steady-state solution is unattainable, which implies that uniform policies cannot be instituted with respect to pesticides use.

The empirical results reflect the relative importance of the two-state variables in the actual problem at hand; they should, however, serve as an example of the feasibility of applying the model. Therefore, sensitivity analysis of the solution to the many parameters of the problem was not executed. Another difficulty lies in the structure of the profit function of our problem. When no pesticides were applied, the profit function received relatively high values, which resulted in small differences in the level of profit whenever alternative policies were suggested. This may not be the case in many other actual problems.

On the other hand, the difficulties not only reside in the formulation and the understanding of the ecological models, but also in the gathering of information concerning the decreasing effectiveness of pesticides. If the centralized decision-maker is unaware of the development of resistance, or if he ignores these effects because of the high costs involved in research, environmental damage can be expected to increase since the proposed strategy uses more pesticides. Therefore, we suggest that integrated pest management policies that ignore, whatsoever, the effects of pest resistance should not propose increasing pesticide applications to control the level of pest population. Furthermore, it appears that in some cases it is preferable not to intervene, rather to leave the competitive state as it is.

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