
MEAN-GINI HEDGING IN FUTURES MARKETS

HAIM SHALIT

INTRODUCTION

Recent years have witnessed the application of the mean-Gini model to the theory of futures markets allowing the derivation of hedging strategies that are compatible with stochastic dominance. The growing interest in this subject requires that both practitioner and theoretician be able to use the model easily and compare its performance to the standard approaches.

First used by Cheung, Kwan, and Yip (1990), the mean-Gini model was shown to provide a marginal improvement over the mean-variance (MV) hedging model. Kolb and Okunev (1992, 1993) use the extended Gini as a measure of dispersion and introduce differentiated risk aversion into the hedging model. Lien and Luo (1993) follow a nonparametric estimation to numerically evaluate mean-extended Gini (MEG) hedging ratios.

In those studies, there is a problem in that the MEG model of futures hedging presented does not relate intuitively to the MV model used by most practitioners. Indeed, because one cannot compare the two approaches statistically, one cannot assess under what conditions a model is superior. None of the articles just cited derives an explicit form for the MEG hedging ratios, thus comparison with the MV model could only be performed empirically.

Acknowledgment: I am grateful to Shlomo Yitzhaki for helpful comments and fruitful discussions and to Ofir Minay who bore the burden of the computations necessary to the article.

■ *Haim Shalit is a Senior Lecturer at Ben-Gurion University of the Negev; e-mail: shalit@bgumail.bgu.ac.il.*

The purpose of this article is to relate the MEG hedge ratio to the basic expected utility-maximizing model of futures hedging. The explicit form for the optimal MEG futures hedging ratio is derived as the coefficient of the instrumental variable regression of the spot price on the futures price where, as a specific instrument, one takes the cumulative probability distribution of futures prices. Furthermore, using Hausman's specification test, one can establish a statistical methodology to compare the MEG and MV hedging ratios and assess under what conditions a model will be superior.

When futures prices are normally distributed, the MEG ratios converge to the MV ratios, since the extended Gini's become the standard deviation divided by a constant. Normality testing is therefore an important facet of the model. Two normality tests are used to examine the equality hypothesis: the standard Shapiro-Wilk test and the D'Agostino procedure based on Gini's mean difference. The model is applied using the metals futures data from New York's Commodity Exchange (COMEX).

THE FUTURES HEDGING MODEL

Assume that a trader expects to sell a known quantity, Q , at the prevailing uncertain spot price, s_1 . The trader has the opportunity to hedge X contracts at the futures price, f_0 , and then settle the contracts at the prevailing futures price f_1 . The profit resulting from the hedge is defined as the return on the hedged portfolio:

$$w = Qs_1 + X(f_0 - f_1) \quad (1)$$

By assuming risk-averse traders who maximize expected utility of profit one obtains the first-order condition for an optimal hedge:

$$E(f_0 - f_1)U'[Qs_1 + X(f_0 - f_1)] = 0 \quad (2)$$

Explicit solutions for the optimal hedge can be found either by specifying the utility function or by restricting the set of probability distributions of spot and futures prices. Under normality of probability distributions or quadraticity of preferences, expected utility maximization is equivalent to MV utility.

According to this specification, Ederington (1979), Rolfo (1980), Kahl (1983), and Anderson and Danthine (1983) show the optimal

hedge to be:

$$X^* = \frac{f_0 - E(f_1)}{2r\sigma_f^2} + \frac{Qcov(s_1, f_1)}{\sigma_f^2} \quad (3)$$

where r is the MV coefficient of risk aversion, and σ_f^2 is the variance of futures prices. The first component of eq. (3), often labeled the purely speculative hedge portion, is proportional to the futures price bias. Practitioners ignore the first component by assuming that futures prices follow a martingale [$E(f_1) = f_0$]. Alternatively, by assuming infinitely risk-averse individuals with $r \rightarrow \infty$, researchers look for the ratio that minimizes the variance of the hedged portfolio. This is basically the second component of eq. (3), labeled the pure hedge, and is proportional to the regression coefficient of spot prices over futures prices. Hence, under MV, if futures prices follow a martingale, the optimal hedge equals the minimum-variance hedge as:

$$X^* = \hat{\beta}Q \quad (4)$$

where $\hat{\beta}$ is the ordinary least-squares (OLS) estimator of the slope β from:

$$s_1 = \alpha + \beta f_1 + \epsilon_1 \quad (5)$$

By restricting the probability distributions of s_1 and f_1 , Benninga et al. (1983, 1984) found an explicit solution to the optimal hedge in eq. (2) that is valid for all risk-averse individuals. Assuming unbiasedness ($f_0 = Ef_1 = Es_1$; futures prices follow a martingale) and regressibility [i.e., ϵ_1 in eq. (5) is uncorrelated with f_1], Benninga et al. demonstrate that the optimal hedge is $X^* = \beta Q$ for *all* concave utilities.

Hence, independence of f_1 and ϵ_1 is a basic requirement for the optimal hedge ratio to equal the minimum variance ratio. It is also the necessary condition to obtain unbiased estimators of β under OLS. Although it is considered by many investigators to be of minor importance, there are substantial reasons to conclude a presence of dependency between f_1 and ϵ_1 , due either to missing additional explanatory variables or to the causality relation of eq. (5). Furthermore, as shown by Milonas (1986), disturbances, ϵ_1 , are believed to be heteroskedastic because as the delivery date approaches volatility increases, implying that the econometric tests based on OLS estimators will be less powerful.

MEAN-EXTENDED GINI HEDGE RATIOS

The objective is to use the mean-extended Gini (MEG) model to develop hedge ratios that will remedy the failures brought about by the interdependence of futures prices and error terms. Indeed, violation of the Gauss-Markov conditions implied by the regression model (5) invalidates the OLS results as the optimal hedge ratio. Furthermore, in this situation, the minimum-variance hedge ratio will be biased.

The MEG alternative is justified for two reasons. The first relates to the necessary and sufficient conditions for stochastic dominance as implied by using the mean and the Gini or the extended Gini as a measure of risk. This ensures that the hedge ratio is included in the second-degree stochastic dominance (SSD) efficient set. The second justification is an econometric one showing that when mean-variance is used and the regressibility condition is violated, the MEG hedge ratio is a consistent estimator for the minimum-variance hedge ratio.

Yitzhaki (1982, 1983) and Shalit and Yitzhaki (1984, 1989) show that the MEG approach is superior to MV because it allows development of capital market equilibrium conditions that satisfy the necessary and sufficient conditions for SSD.

The extended Gini coefficient of w is defined as:

$$\Gamma(\nu) = -\nu \text{cov}\{w, [1 - G(w)]^{\nu-1}\} \quad (6)$$

where G is the cumulative probability distribution of w and ν is the extended Gini parameter associated with risk aversion. For $\nu = 2$, the standard Gini coefficient is:

$$\Gamma(\nu) = 2\text{cov}[w, G(w)] \quad (7)$$

Given two alternatives, w_1 and w_2 , then

$$\begin{aligned} E(w_1) &\geq E(w_2) \\ E(w_1) - \Gamma_1(\nu) &\geq E(w_2) - \Gamma_2(\nu) \end{aligned} \quad (8)$$

are necessary conditions for w_1 to dominate w_2 under SSD. These are also sufficient conditions for SSD if cumulative distributions intersect at most once.¹ Hence, SSD optimal solutions will be obtained by maximizing the mean less the extended Gini for all the alternatives with higher means. By using MV, the SSD condition is not always achieved unless alternatives are normally distributed or the utility is quadratic.

¹Such distributions are, for example, the normal, lognormal, exponential, beta, and gamma distributions.

In futures contracts, the optimal hedge is derived by maximizing the mean less the extended Gini of the hedged portfolio w . Kolb and Okunev (1992, 1993) derived the optimal hedge ratio by minimizing the extended Gini subject to a given mean portfolio. By assuming the martingale condition implying equal means, the minimum MEG hedge is essentially the optimal hedge ratio. The mean less the extended Gini of the hedged portfolio is:

$$E(w) - \Gamma_w(\nu) = QE(s_1) + XE(f_0 - f_1) + \nu \text{cov}\{Qs_1 + X(f_0 - f_1), [1 - G(w)]^{\nu-1}\} \quad (9)$$

The first-order conditions for maximum $E(w) - \Gamma_w(\nu)$ are obtained by its derivatives with respect to X and Q :

$$\begin{aligned} -E(f_1) &= \nu \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\} + Q\nu \frac{\partial \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\}}{\partial X} \\ &\quad - X\nu \frac{\partial \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\}}{\partial X} = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} E(s_1) + \nu \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\} &+ Q\nu \frac{\partial \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\}}{\partial Q} \\ &\quad - X\nu \frac{\partial \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\}}{\partial Q} = 0 \end{aligned} \quad (11)$$

The derivatives are difficult to evaluate explicitly. Hence, Kolb and Okunev (1992, 1993) proposed an iterative process to solve for the hedge ratio. For the same reason, Lien and Luo (1993) use a numerical approach. The concept here is to apply Euler's condition since $\Gamma_w(\nu)$ is linear homogeneous in X and Q [Shalit and Yitzhaki (1984)]:

$$\Gamma_w(\nu) = Q \frac{\partial \Gamma_w(\nu)}{\partial Q} + X \frac{\partial \Gamma_w(\nu)}{\partial X} \quad (12)$$

or

$$\begin{aligned} \Gamma_w(\nu) &= -\nu \left\{ Q \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\} - X \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\} \right. \\ &\quad + Q^2 \frac{\partial \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\}}{\partial Q} - QX \frac{\partial \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\}}{\partial Q} \\ &\quad \left. + XQ \frac{\partial \text{cov}\{s_1, [1 - G(w)]^{\nu-1}\}}{\partial X} - X^2 \frac{\partial \text{cov}\{f_1, [1 - G(w)]^{\nu-1}\}}{\partial X} \right\}. \end{aligned} \quad (12a)$$

From the definition of $\Gamma_w(w)$ given in (9), it follows that the last four elements of eq. (12a) cancel out. Hence, if one multiplies the first-order conditions (10) and (11) by X and Q , respectively, and then sum up the results, it follows that:

$$-XE(f_1) + QE(s_1) - X\nu cov\{f_1, [1 - G(w)]^{\nu-1}\} + Q\nu cov\{s_1, [1 - G(w)]^{\nu-1}\} = 0 \quad (13)$$

As the covariance remains unchanged when subtracting the constant mean, the MEG optimal hedge ratio is obtained as:

$$X^* = Q \frac{cov\{s_1, [1 - G(w)]^{\nu-1}\}}{cov\{f_1, [1 - G(w)]^{\nu-1}\}} \quad (14)$$

To obtain an explicit solution for the hedge ratio, one needs to assume that the cumulative probability distribution of w is similar to the cumulative probability distribution of f_1 in the sense that the rankings are preserved, i.e. $G(w) = G(f_1)$. This is a valid assumption whose justification in empirical work is shown in the next section. Indeed, since $G(\cdot)$ is a ranking function, $G(f_1)$ and ϵ_1 can be independent even though f_1 and ϵ_1 are dependent. Furthermore, when prices are normally distributed, hedge ratios expressed by eq. (14) are reduced to MV hedge ratios (see Appendix). Hence, the optimal hedge becomes:

$$X^* = Q\beta(\nu) \quad (15)$$

where $\beta(\nu)$ is the coefficient of the MEG regression of the spot price over the futures price for a given coefficient of risk aversion expressed by ν :

$$\beta(\nu) = \frac{cov\{s_1, [1 - G(f_1)]^{\nu-1}\}}{cov\{f_1, [1 - G(f_1)]^{\nu-1}\}} \quad (16)$$

HEDGING RATIO ESTIMATION AND TESTING METHODOLOGY

An alternative justification of the MEG approach to futures hedging is more pragmatic and relies on improving the OLS estimation of beta under the MV model. Equation (5) is not regressible and OLS estimates will be biased and inconsistent if the Gauss-Markov conditions are violated in such a way that the error term is correlated with the futures prices, or if the error terms are serially correlated, or if an errors-in-variables model is involved. To remedy these failures, one can use the instrumental variables method (IV) where the investigator chooses

an additional variable that is highly correlated with the independent variable but not with the error term. This method of estimation leads to a consistent and asymptotically normal estimate.² A valid instrumental variable is the cumulative probability for f_1 or a monotonic function for that cumulative probability. This approach was first suggested by Durbin (1954) who constructed an IV by ranking the observations in their ascending order and used the rank as an instrument by which to solve the errors-in-variables problem.

In practice, one uses as IV the computed cumulative probability distribution for f_1 defined as the rank of f_1 divided by the number of observations. The IV estimator for β is:

$$\hat{\beta}^{IV} = \frac{\sum (s_1 - \bar{s}_1) \left(G(f_1) - \frac{1}{2} \right)}{\sum (f_1 - \bar{f}_1) \left(G(f_1) - \frac{1}{2} \right)} = \frac{\text{cov}[s_1, G(f_1)]}{\text{cov}[f_1, G(f_1)]} \quad (17)$$

which is the standard MG beta for $\nu = 2$.

For $\nu > 2$, the MEG β is obtained, and a consistent estimator for the hedge ratio can be obtained even in the event that OLS yields a biased estimator.³

To what extent is the hedge ratio obtained through OLS significantly different from the MEG hedge ratios obtained using the IV procedure for various ν ? As shown by Nair (1936), when random variables are normally distributed the Gini and the extended Gini coefficients converge to the standard deviation divided by $\sqrt{\pi}$. Therefore, following Schechtman and Yitzhaki (1987), one can prove that the various MEG hedge ratios converge to the MV beta obtained using OLS (see Appendix). Thus, it follows that testing for the normality of the distribution of futures prices is a necessary condition for the hedge ratios to all be equal.

To test for normality, the Royston (1982) extension for the Shapiro and Wilk (1965) W test and the D'Agostino (1971) D statistic are used. The Shapiro-Wilk procedure consists of calculating W from order statistics and then using the sampling distribution of W to reject normality. The D'Agostino D statistic compares the standard deviation of a distribution to its Gini's mean difference as:

$$D = \frac{\Gamma_X}{2S_X} \cdot \frac{(n-1)^{3/2}}{n^{3/2}}$$

²See Judge et al. (1985) for a thorough exposition of IV estimation in cases of distributed lags, errors-in-variables, and simultaneous equations models. See also Cragg (1983) who shows that IV yields more efficient estimators in the presence of heteroskedasticity.

³See Carroll, Thistle, and Wei (1992) for this interpretation of MEG betas.

where Γ_X is the sample's Gini calculated with $G(X) = (\text{Rank}(X) - 1)/(n - 1)$ and S_X is the sample's standard deviation. D'Agostino shows that:

$$\frac{\sqrt{n}(D - 0.282095)}{0.029986}$$

is asymptotically distributed as a normal $N(0, 1)$ variable and can serve as an omnibus normality test for large samples. The Shapiro and Wilk W test is more robust for small samples.

The second approach for testing whether the betas differ significantly uses Hausman's (1978) specification error test for non-nested models. This test is designed to examine a hypothesis in terms of model inconsistency, and runs an efficient estimator, such as OLS, against a less efficient but consistent estimator such as IV. Hausman shows that testing the difference between the betas is appropriate in testing the specification of the model. This approach is used to test the equality between the betas and to show to what extent MV hedge ratios can lead to unbiased estimators.

The test determines the statistical significance of the difference in betas expressed by $\hat{q} = \beta(\nu) - \beta_{MV}$. Hausman proves that the variance of \hat{q} is equal to the variance of $\beta(\nu)$ minus the variance of β_{MV} . With $\hat{V}(\hat{q})$ as a consistent estimator of this variance, the following statistic has a χ^2 distribution with 1 degree of freedom:

$$m = \frac{\hat{q}^2}{\hat{V}(\hat{q})} \quad (18)$$

In the case of OLS vs. IV, the variance estimator $\hat{V}(\hat{q})$ is shown to be:

$$\hat{V}(\hat{q}) = \hat{V}(\beta_{MV}) \frac{1 - \rho^2}{\rho^2} \quad (19)$$

where ρ is the correlation coefficient between futures prices and the appropriate rank function $-[1 - G(f_1)]^{\rho-1}$. Using the m statistic, one can establish whether the difference between the MV and MEG hedge ratios is significant, and whether under differentiated risk aversion MEG hedge ratios provide superior econometric results to MV.

EMPIRICAL EVIDENCE

The data consist of all the monthly futures contracts for metals (gold, silver, copper, and aluminum) traded on the New York Commodity

Exchange (COMEX) between January 1977 and December 1990. The number of observations varies from commodity to commodity, with around 40 observations for each of the short-term contracts (286 contracts comprised of current calendar month and next two months), and around 450 observations for the long-term contracts (285 contracts). For aluminum, copper, and silver futures, the long-term contracts are for the odd-numbered months; whereas, for the gold futures, the long-term contracts are for the even-numbered months.

For each contract, the MV and MEG hedging ratios for values of ν from 1.5 to 100 are estimated. The results are voluminous, so individual hedging ratios are shown for selected contracts only. For the 12 monthly gold futures contracts of 1990, the MEG hedging coefficients are presented in Table I.

The values in parentheses are the Hausman m statistics for the specific MEG hedging ratio. For each contract, the maximum Hausman m is also reported to test the difference between the MV and the MEG hedging ratios. A statistically significant difference exists only for the July, October, and December gold contracts of 1990; the Hausman m statistic is greater than 3.84 at the 5% significance level. For each of the previous years, the number of gold contracts where the MEG hedge ratios significantly differ from the MV ratios varies from three in 1977 to nine in 1986. Hence a sizable number of monthly contracts exhibit hedge ratios that are not equal to the standard MV ratio.

In the 12 diagrams of Figure 1, the MEG hedging ratios for the 1990 gold futures contracts are plotted as a function of ν . The MV hedging ratio is drawn as a horizontal line. The diagrams, for each contract, show the variation of betas as well as the difference between those betas and the MV hedging ratio. The MEG betas are clearly quite different from the β_{MV} , but only the Hausman m test can ascertain the significance of the difference.

The normality distribution of futures prices is tested first using the Royston-Shapiro-Wilk procedure. The critical values rejecting normality are estimated for each contract, together with the D'Agostino D statistic. Basically, a contract that fails to pass normality under the W test also fails under the D test, as shown in Table I for the 12 gold contracts of 1990. Normality distribution of futures prices is rejected for all 1990 contracts except those of January and March. Hence for those two contracts, nothing will be gained by using MEG as shown by the value of Hausman's m statistic. The MEG hedging ratios are not significantly different from the MV hedge ratio, leading to the two meaningless intersections in the graphs of these two contracts.

TABLE I
MEG and MV Hedge Ratios for Comex Gold Futures (1977-1990)

Contract	Jan 90	Feb 90	Mar 90	Apr 90	May 90	Jun 90	Jul 90	Aug 90	Sep 90	Oct 90	Nov 90	Dec 90
Observations	39	461	38	461	40	461	41	459	41	459	40	461
Maximal m	1.020117	2.702652	0.937568	1.225624	1.857268	0.88229	5.437038	1.342338	2.149618	8.372931	0.987658	10.65761
Shapiro-Wilk	1.454189	4.235168	0.575029	9.510072	5.828377	7.743485	2.693993	8.33015	5.533217	16.04537	3.642617	16.66584
D'Agostino	-2.1841	-11.2458	0.107156	-15.911	-14.1387	-15.0444	-5.10887	-15.3511	-12.4424	-20.9325	-6.61715	-20.5986
$\beta(MV)$	0.996046	0.921244	0.989836	0.929711	0.994214	0.929362	0.996509	0.924721	0.99775	0.930848	0.987899	0.939289
$\beta(\nu = 1.5)$	0.995313 (0.128)	0.925504 (2.703)	0.989876 (0.003)	0.929443 (0.009)	0.98883 (1.076)	0.927812 (0.335)	1.002277 (1.797)	0.925344 (0.055)	0.994828 (0.908)	0.925338 (3.341)	0.989328 (0.201)	0.931113 (8.039)
$\beta(\nu = 2)$	0.996196 (0.010)	0.924278 (1.315)	0.988921 (0.938)	0.928864 (0.093)	0.988716 (1.431)	0.92742 (0.529)	1.003008 (3.271)	0.925122 (0.022)	0.994255 (1.369)	0.923227 (6.314)	0.990269 (0.739)	0.930137 (10.658)
$\beta(\nu = 4)$	0.997372 (0.965)	0.920025 (0.119)	0.987903 (0.501)	0.926983 (0.647)	0.989385 (1.812)	0.926564 (0.722)	1.00408 (5.437)	0.924068 (0.036)	0.993093 (2.150)	0.920561 (8.373)	0.98965 (0.739)	0.930963 (7.023)
$\beta(\nu = 6)$	0.997625 (0.852)	0.917239 (0.901)	0.988433 (0.137)	0.925958 (0.937)	0.990957 (1.103)	0.926242 (0.678)	1.002831 (3.192)	0.922922 (0.206)	0.993983 (1.341)	0.919673 (8.122)	0.989092 (0.350)	0.932412 (4.003)
$\beta(\nu = 8)$	0.997409 (0.438)	0.915303 (1.570)	0.989242 (0.017)	0.925273 (1.092)	0.992312 (0.454)	0.926165 (0.590)	1.000717 (1.186)	0.92179 (0.455)	0.995265 (0.585)	0.919289 (7.645)	0.988632 (0.176)	0.935846 (2.219)

$\beta(\nu = 10)$	0.997062 (0.184)	0.913866 (2.042)	0.990044 (0.002)	0.924774 (1.181)	0.993321 (0.113)	0.926208 (0.499)	0.99845 (0.217)	0.92077 (0.722)	0.996383 (0.181)	0.919215 (7.042)	0.988631 (0.086)	0.935237 (1.123)
$\beta(\nu = 12)$	0.996726 (0.066)	0.912769 (2.357)	0.990803 (0.029)	0.924416 (1.221)	0.994061 (0.004)	0.926321 (0.415)	0.996316 (0.002)	0.919919 (0.096)	0.997253 (0.025)	0.91938 (6.346)	0.98842 (0.035)	0.936595 (0.461)
$\beta(\nu = 14)$	0.996439 (0.018)	0.911926 (2.552)	0.991525 (0.074)	0.924172 (1.223)	0.994612 (0.025)	0.92648 (0.340)	0.994405 (0.202)	0.919252 (1.142)	0.997904 (0.002)	0.919731 (5.601)	0.988189 (0.009)	0.937921 (0.112)
$\beta(\nu = 16)$	0.996206 (0.003)	0.911280 (2.658)	0.992212 (0.128)	0.924016 (1.199)	0.995029 (0.106)	0.926669 (0.274)	0.992726 (0.594)	0.918758 (1.262)	0.998386 (0.043)	0.92022 (4.852)	0.987942 (0.000)	0.939205 (0.000)
$\beta(\nu = 18)$	0.996017 (0.000)	0.910788 (2.700)	0.992864 (0.185)	0.923928 (1.158)	0.995348 (0.207)	0.926874 (0.219)	0.991262 (1.053)	0.918413 (1.325)	0.99874 (0.108)	0.920807 (4.131)	0.987688 (0.003)	0.940434 (0.071)
$\beta(\nu = 20)$	0.995864 (0.003)	0.910417 (2.695)	0.993478 (0.241)	0.923888 (1.108)	0.995595 (0.305)	0.927088 (0.172)	0.989983 (1.514)	0.918191 (1.342)	0.999 (0.177)	0.92146 (3.462)	0.987435 (0.015)	0.941597 (0.276)
$\beta(\nu = 50)$	0.994839 (0.048)	0.910332 (1.515)	0.998944 (0.648)	0.925591 (0.342)	0.996561 (0.614)	0.930176 (0.013)	0.981107 (4.708)	0.920238 (0.397)	0.999759 (0.544)	0.931887 (0.029)	0.98466 (0.247)	0.951574 (5.226)
$\beta(\nu = 100)$	0.994021 (0.087)	0.913376 (0.515)	1.001825 (0.738)	0.932944 (0.146)	0.996832 (0.531)	0.937364 (0.882)	0.977495 (5.276)	0.927199 (0.086)	0.999808 (0.541)	0.945325 (4.092)	0.982847 (0.371)	0.956636 (7.120)

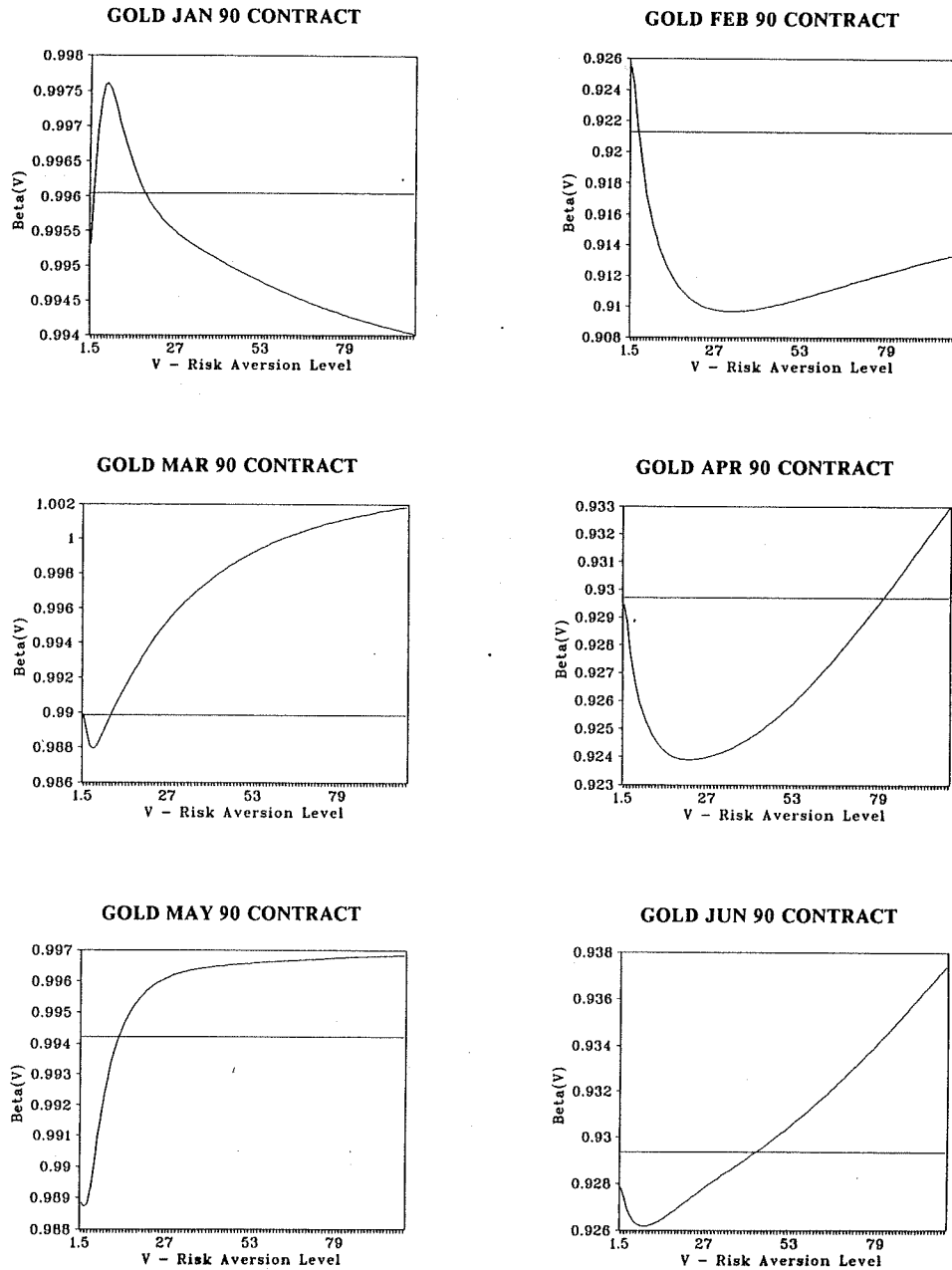


FIGURE 1

MEG and MV hedging ratios for 1990 gold futures contracts as a function of the level of risk aversion ν .

The issue then is to combine the last two tests and ask the question: Can one assess from the test of normality of futures prices whether risk-differentiated hedge ratios are equal to the MV ratio? Moreover, can

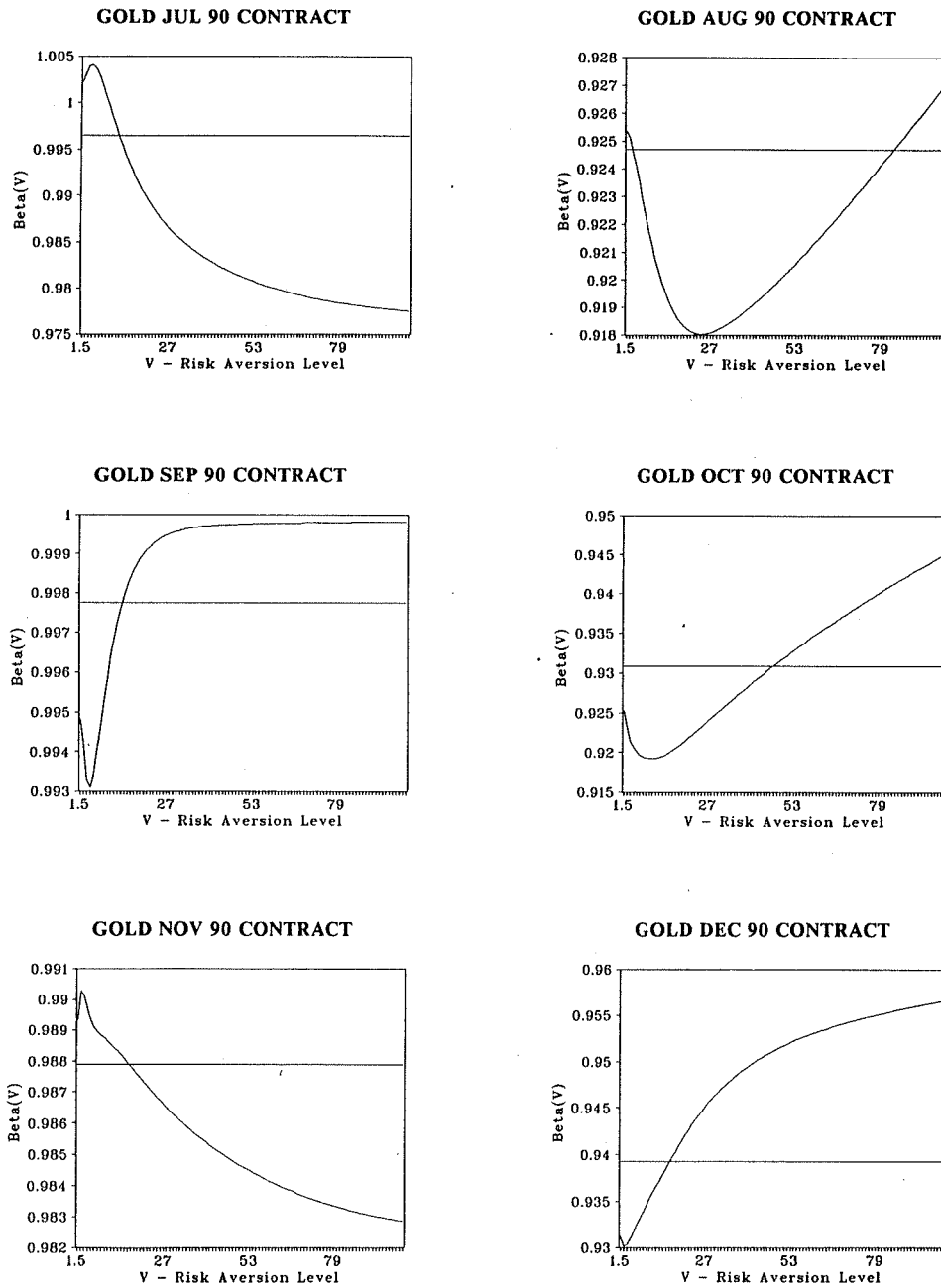


FIGURE 1 (Continued)

one predict that when futures prices are not normally distributed MEG hedge ratios will exhibit sizable differences that must be accounted for when hedging commodities?

The answer is provided in Table II for all four metals at three levels of significance and in Table III for the entire set of data. Table II gives results for the long-term and the short-term contracts separately and together. For each metal, the columns show the number of contracts with at least one MEG ratio that is different from the MV ratio according to the Hausman specification test at the 1%, 5%, and 10% significance levels (using the χ^2 distribution).

For example, in gold futures, 41.3% of all contracts (69 out of 167) have at least one MEG ratio that is different from the MV ratio at the 1% significance level. At the 10% significance level, the figure is 60.48% of all gold contracts. Looking at the short-term and the

TABLE II
Normally and Not Normally Distributed Contracts with Different MEG Hedge Ratios for All Comex Metals Monthly Contracts (1977–1990)

	<i>Significance Level</i>								
	1%			5%			10%		
	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>
Gold									
All contracts									
Not normal	43	52	95	36	66	102	32	76	108
Normal	55	17	72	42	23	65	34	25	59
Total	98	69	167	78	89	167	66	101	167
Long-term contracts									
Not normal	27	51	78	21	60	81	16	66	82
Normal	3	2	5	1	1	2	0	1	1
Total	30	53	83	22	61	83	16	67	83
Short-term contracts									
Not normal	16	1	17	15	6	21	16	10	26
Normal	52	15	67	41	22	63	34	24	58
Total	68	16	84	56	28	84	50	34	84
Silver									
All contracts									
Not normal	37	48	85	25	66	91	24	68	92
Normal	64	19	83	49	28	77	38	38	76
Total	101	67	168	74	94	168	62	106	168
Long-term contracts									
Not normal	25	47	71	14	58	72	13	59	72
Normal	9	3	11	7	5	12	7	5	12
Total	34	50	84	21	63	84	20	64	84
Short-term contracts									
Not normal	12	1	13	11	8	19	11	9	20
Normal	55	16	71	42	23	65	31	33	64
Total	67	17	84	53	31	84	42	42	84

continued

TABLE II (Continued)

	Significance Level								
	1%			5%			10%		
	Equal β	Unequal β	Total	Equal β	Unequal β	Total	Equal β	Unequal β	Total
Copper									
All contracts									
Not normal	39	16	55	32	30	62	28	47	75
Normal	86	15	101	67	27	94	46	35	81
Total	125	31	156	99	57	156	74	82	156
Long-term contracts									
Not normal	34	15	49	24	28	52	18	38	56
Normal	22	7	29	14	12	26	12	10	22
Total	56	22	78	38	40	78	30	48	78
Short-term contracts									
Not normal	5	1	6	8	2	10	10	9	19
Normal	64	8	72	53	15	68	34	25	59
Total	69	9	78	61	17	78	44	34	78
Aluminum									
All contracts									
Not normal	34	7	41	25	21	46	23	26	49
Normal	38	1	39	31	3	34	24	7	31
Total	72	8	80	56	24	80	47	33	80
Long-term contracts									
Not normal	27	5	32	17	15	32	14	20	34
Normal	8	0	8	7	1	8	6	0	6
Total	35	5	40	24	16	40	20	20	40
Short-term contracts									
Not normal	7	2	9	8	6	14	9	6	15
Normal	30	1	32	24	2	26	18	7	25
Total	37	3	40	32	8	40	27	13	40

long-term contracts separately, it is clear the long-term contracts are more apt to have unequal MEG hedge ratios than the short-term contracts. This is a useful finding for commodities hedgers who prefer to use long-term contracts.

Tables II and III show, at various significance levels, the number of contracts that are normally distributed versus those that are not normal according to the Royston-Shapiro-Wilk test. For most of the long-term contracts normality is rejected. For example, at the 5% significance level, 97% of the long-term gold futures are not normally distributed. Short-term contracts are more likely to be normally distributed as shown by the silver, copper, and aluminum futures.

Information in Table II indicates the joint frequency that a specific futures contract will be normally or not normally distributed *and* that MEG hedge ratios will or will not equal the MV ratio. The empirical

TABLE III
 Not Normally Distributed Futures with Different MEG Hedge Ratios
 Total for All Comex Metals Monthly Contracts (1977–1990)

	<i>Significance Level</i>									
	1%			5%			10%			
	<i>All Metals</i>	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>	<i>Equal β</i>	<i>Unequal β</i>	<i>Total</i>
<i>All contracts</i>										
Not normal	153	123	276	118	183	301	107	217	314	
Normal	243	52	295	189	81	270	142	105	247	
Total	396	175	571	307	264	571	249	322	571	
<i>Long-term contracts</i>										
Not normal	113	118	231	76	161	237	61	183	244	
Normal	42	12	54	29	19	48	25	16	41	
Total	155	130	285	105	180	285	86	199	285	
<i>Short-term contracts</i>										
Not normal	40	5	45	42	22	64	46	34	80	
Normal	201	40	241	160	62	222	117	89	206	
Total	241	45	286	202	84	286	163	123	286	

results confirm the hypothesis that if assets are not normally distributed, MEG betas will differ from the MV beta and for normally distributed contracts, the betas will be equal. Indeed, for all the gold contracts, there exists a higher frequency for the unequal betas with not normal distribution (39.52% at the 5% significance level) on one hand and for the equal betas with normal distribution (25.15% at the 5% significance level) than for the two other cells in the matrix. The same effect obtains for most of the copper, silver, and aluminum futures contracts.

Table III totals the results for all the metals. It is shown here that the probability that a contract will not be normal and have different hedge ratios is 32% at the 5% significance level. The probability that a contract is normally distributed and has equal ratios is 33% at the 5% level.

More valuable is information about the conditional probability of unequal hedge ratios, given that futures contracts are not normally distributed. The results are meaningful if the probability is greater than 50%. For the complete metals sample in Table III, given that contracts are not normally distributed, 60.8% (183 of 301) exhibit unequal hedge ratios at the 5% significance level. Where contracts are normally distributed, 70% (189 of 270) have equal hedge ratios.

These results are also distinguishable by the short-term and long-term contracts. The short-term contracts are more likely to be normal,

so equal hedging ratios are more likely. The long-term contracts are more probably not normal, leading to greater incidence of unequal hedge ratios.

CONCLUSION

An unified methodology to consider differentiated risk aversion in futures hedging is provided by using the MEG approach. As the hedging ratio is proved to be the MEG regression coefficient of the spot price over the futures prices, statistical procedures can be established to test the significance of the approach. By combining the normality test of futures contracts with the MEG difference test, the statistical importance of MEG hedging in futures markets is demonstrated, using metals contracts traded on the COMEX as an example.

The MEG methodology appeals to the practitioner because it provides a way to include risk-aversion differentials into hedging. Furthermore, the MEG methodology as an IV estimator can be used whenever OLS fails to provide consistent estimators for MV. The question is how to choose the relevant value of risk aversion ν . The answer has two parts.

First, the analyst establishes whether or not contracts are normally distributed by using a test from the D'Agostino approach. Once normality is rejected, the practitioner will benefit by using a MEG hedge ratio instead of a MV ratio, because the latter is not consistent.

Second, MEG ratios are estimated for several values of ν together with their Hausman m statistic to assess what MEG ratios are significantly different from the MV ratios. From this reduced set of statistically different ratios the practitioner chooses the relevant coefficient of risk aversion.

APPENDIX

The purpose of this Appendix is to show that when prices are normally distributed, the minimum extended Gini hedge ratio becomes the standard minimum variance ratio.

The extended Gini of the hedge portfolio given by eq. (9) is:

$$\Gamma_w(\nu) = -\nu \text{cov}\{Qs_1 + X(f_0 - f_1), [1 - G(w)]^{\nu-1}\} \quad (\text{A1})$$

When f_1 and w are bivariate normal, the conditional expectation of f_1 given w can be expressed as:

$$E(f_1 | w) = \mu_f + \rho_{f,w}(w - \mu_w) \frac{\sigma_f}{\sigma_w} \quad (\text{A2})$$

where ρ is the coefficient of correlation. For s_1 and w being bivariate normal, it becomes:

$$E(s_1 | w) = \mu_s + \rho_{s,w}(w - \mu_w) \frac{\sigma_s}{\sigma_w} \quad (\text{A3})$$

Substituting (A2) and (A3) into (A1) yields:

$$\begin{aligned} \Gamma(\nu) &= -\nu(QE_w\{E_{s|w}[s_1 - \mu_s] \cdot [1 - G_w(w)]^{\nu-1}\} \\ &\quad - XE_w\{E_{f|w}[f_1 - \mu_s] \cdot [1 - G_w(w)]^{\nu-1}\}) \\ &= -\nu(Q\rho_{s,w} \frac{\sigma_s}{\sigma_w} E_w\{[w - \mu_w] \cdot [1 - G(w)]^{\nu-1}\} \\ &\quad - X\rho_{f,w} \frac{\sigma_f}{\sigma_w} E_w\{[w - \mu_w] \cdot [1 - G(w)]^{\nu-1}\}) \quad (\text{A4}) \end{aligned}$$

Let $z = (w - \mu_w)/\sigma_w$ be a standard normal variate and let $F(z)$ be the cumulative distribution of z then $F(z) = G(w)$. Hence:

$$\begin{aligned} \Gamma(\nu) &= Q\rho_{s,w}\sigma_s \text{cov}\{z, [1 - F(z)]^{\nu-1}\} \\ &\quad - X\rho_{f,w}\sigma_f \text{cov}\{z, [1 - F(z)]^{\nu-1}\} \quad (\text{A5}) \end{aligned}$$

According to Nair (1936), the Gini of a normal variate equals its standard deviation divided by $\sqrt{\pi}$. Hence, the extended Gini of z is equal to $1/\sqrt{\pi}$ and the extended Gini of w becomes:

$$\Gamma(\nu) = \frac{1}{\sqrt{\pi}} \left[Q \frac{\text{cov}(s_1, w)}{\sigma_s} - X \frac{\text{cov}(f_1, w)}{\sigma_w} \right] \quad (\text{A6})$$

Setting the derivative of (A6) with respect to X equal to 0 yields the standard MV hedge ratio:

$$X^* = Q \frac{\text{cov}(s_1, f_1)}{\sigma_f^2} \quad (\text{A7})$$

BIBLIOGRAPHY

- Anderson, R., and Danthine, J.-P. (1983, June): "Hedger Diversity in Futures Markets", *Economic Journal*, 93:370-389.
- Benninga, S., Eldor, R., and Zilcha, I. (1983): "Optimal Hedging in the Futures Market under Price Uncertainty," *Economics Letters*, 13:155-159.
- Benninga, S., Eldor, R., and Zilcha I. (1984): "The Optimal Hedge Ratio in Unbiased Futures Markets," *The Journal of Futures Markets*, 4:155-159.
- Carroll, C., Thistle, P., and Wei, K. C. J. (1992, September): "The Robustness of Risk-Return Nonlinearities to the Normality Assumption," *Journal of Financial and Quantitative Analysis*, 27:419-436.

- Cheung, S., Kwan, C., and Yip, P. (1990, February): "The Hedging Effectiveness of Options and Futures: A Mean-Gini Approach," *The Journal of Futures Markets*, 10:61–73.
- Cragg, J. C. (1983, May): "More Efficient Estimation in the Presence of Heteroscedasticity of Unknown Form," *Econometrica*, 51:751–764.
- D'Agostino, R. B. (1971, August): "An Omnibus Test of Normality for Moderate and Large Size Samples," *Biometrika*, 58:341–348.
- Durbin, J. (1954): "Errors in Variables," *International Statistical Review*, 22:23–32.
- Ederington, L. (1979, March): "The Hedging Performance of the New Futures Markets," *Journal of Finance*, 34:157–170.
- Hausman, J. A. (1978, November): "Specification Tests in Econometrics," *Econometrica*, 46:1251–1271.
- Judge, G. G., Griffiths, W. E., Hill, R. C., Lutkepohl, H., and Lee, T-C. (1985): *The Theory and Practice of Econometrics* (2nd ed.). New York: Wiley.
- Kahl, K. H. (1983, August): "Determination of the Recommended Hedging Ratio," *American Journal of Agricultural Economics*, 65:603–605.
- Kolb, R. W., and Okunev, J. (1992, April): "An Empirical Evaluation of the Extended Mean-Gini Coefficient for Futures Hedging," *The Journal of Future Markets*, 12:177–186.
- Kolb, R. W., and Okunev, J. (1993, September): "Utility Maximizing Hedge Ratios in the Extended Mean-Gini Framework," *The Journal of Futures Markets*, 13:597–609.
- Lien, D., and Luo, X. (1993, September): "Estimating the Extended Mean-Gini Coefficient for Futures Hedging," *The Journal of Futures Markets*, 13:665–676.
- Milonas, N. (1986): "Price Variability and the Maturity Effect in Futures Markets," *The Journal of Futures Markets*, 6:443–460.
- Nair, U. S. (1936): "The Standard Error of Gini's Mean Difference," *Biometrika*, 38:428–436.
- Rolfo, J. (1980): "Optimal Hedging under Price and Quantity Uncertainty: The Case of a Cocoa Producer," *Journal of Political Economy*, 88:100–116.
- Royston, J. P. (1982): "An Extension of Shapiro and Wilk's W Test for Normality to Large Samples," *Applied Statistics*, 31:115–124.
- Schechtman, E., and Yitzhaki, S. (1987, January): "A Measure of Association Based on Gini's Mean Difference," *Communications in Statistics, Theory and Methods*, A16:207–231.
- Shalit, H., and Yitzhaki, S. (1984, December): "Mean-Gini, Portfolio Theory and the Pricing of Risky Assets," *Journal of Finance*, 39:1449–1468.
- Shalit, H., and Yitzhaki, S. (1989): "Evaluating the Mean-Gini Approach to Portfolio Selection," *International Journal of Finance*, 1:15–31.
- Shapiro, S. S., and Wilk, M. B. (1965, December): "An Analysis of Variance Test for Normality (Complete Samples)," *Biometrika*, 52:591–611.
- Yitzhaki, S. (1982, March): "Stochastic Dominance, Mean-Variance, and Gini's Mean Difference," *American Economic Review*, 72:178–185.
- Yitzhaki, S. (1983, October): "On an Extension of the Gini Inequality Index," *International Economic Review*, 24:716–628.