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# The Democratic Provision of Public and Private Goods from Exhaustible Resources

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The problem of distributing exhaustible natural resources between consumption goods and environmental amenities through a voting process is analyzed. Assuming that individuals are endowed with an equal share of private goods, the method of majority decision does not always achieve a Pareto-optimal distribution. However, by means of side payments, the intensity of preferences is revealed. The voting procedure then leads to a Pareto-optimal solution which is more prone to environmental amenities than the simple method of majority decision.

## 1. INTRODUCTION

The problem of defining optimal depletion rates for exhaustible resources has become a serious issue in recent times. Some natural resources, while serving as a primary factor in the production of private goods, also provide environmental amenities—a public good. In this case the problem of optimal allocation is thus to decide what portion of these resources is to be available for the production of private goods and what portion is to be left over for the provision of environmental amenities. As an example, consider an economy that possesses reserves of coal and must determine the rate of extraction over time. Assume, furthermore, that the existing extraction technology (strip-mining) results in irreversible environmental degradation. The agents in this economy must then select a rate of extraction that minimizes their utility with ecological arguments incorporated.

The type of problem described above has been considered for the case of a well-defined social welfare function[1] by Vousden [7] and by Lusky [3]. However, individuals place different values on the existing stock of resources independently of the level of utility derived from consumption goods. As an alternative, Neher [4] attempted to define a political scheme of exploitation for the case of a renewable resource. However, resource depletion decisions are often more crucial when dealing with exhaustible resources because of the added problem of irreversibility. Thus, an examination of the question of social choice with regard to optimal depletion of exhaustible resources is deemed desirable.

[1] The Arrowian concept of a social welfare function is used here, i.e., a collective choice rule that specifies orderings for society (reflexivity, transitivity, and completeness of the preference relationship: see Sen [6]).



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In reality, many problems of allocation of natural resources have been solved through a voting procedure.[2] This type of solution usually is not optimal since the set of alternatives on the ballot is constrained by practical limitations. Even if one enlarges the set of alternatives to include all the alternatives with equal distribution of income and deals with single-peaked preferences, the simple method of majority decision (MMD) does not always achieve a Pareto-optimal allocation. In this paper it will be demonstrated that, in an economy of different individuals, a democratic provision of public and private goods from a finite resource is Pareto optimal if vote trading is allowed. Moreover, this solution is more amenity inclined than the MMD choice with equal treatment for all society members. This result, which is shown in a static model, depends on the fact that the stock of natural resources left for environmental amenities is an indivisible public good.[3]

## 2. THE MODEL

Consider a simple economy with $N$ individuals and a finite stock of resources, $X_0$. The resource can either be transformed without cost into a private consumption good $Y$ or left over for environmental amenities $X$ following the rule, $X + Y = X_0$. Moreover, assume that the consumption good is distributed among the individuals such that

$$\sum_{i=1}^{N} y_i = Y,$$

where $y_i$ is the amount of the private good received by individual $i$. Individuals are assumed to have strictly convex preferences for public and private goods and to possess smooth, strictly concave utility functions, $U^i(X, y_i)$, with the following properties:

$$\frac{\partial U^i}{\partial X} \equiv U_X{}^i > 0; \qquad U_{XX}{}^i < 0; \qquad U_{y_i}{}^i > 0; \qquad U^i{}_{y_iy_i} < 0; \qquad i = 1, \ldots, N.$$

If one assumes individuals with identical preferences, the optimal distribution of $X_0$ between $X$ and $y$ is obtained by solving the first-order conditions for utility maximization. This can be done easily, providing a criterion for the distribution of the private good among the individuals. If an individual receives an equal share $(\bar{y})$ of the private good, then the conditions are reduced to

$$\frac{U_X{}^i}{U_y{}^i} = \frac{1}{N} \qquad \text{for all } i = 1, \ldots, N. \tag{1}$$

$$X + N\bar{y} = X_0$$

However, since individuals differ in their appreciation of environmental ameni-

[2] For example, in California citizens were faced with the decision to allow the construction of a dam or to preserve a scenic river (New Melones Dam initiative, 1974).

[3] Samuelson's [5] definition of a collective consumption good is a good "which all enjoy in common in the sense that each individual's consumption of such a good leads to *no substraction* from any other individual's consumption of such a good."

ties, the following definition is required to compare individuals:

DEFINITION. Given an identical bundle of private and public goods $(X, y)$ and two individuals, $i \neq j$, if

$$\frac{U_X{}^i}{U_y{}^i} \geqslant \frac{U_X{}^j}{U_y{}^j},$$

one says that individual $i$ is more *amenity inclined* than $j$ or that individual $j$ is more consumption inclined than $i$.

The marginal rate of substitution, $U_X{}^i/U_y{}^i$, is called the conservation motive; and it permits the ranking of individuals' preferences for identical bundles only. Moreover, since the utility function is strictly concave, individuals can rank in a unique preferential way all the different alternative distributions of $X$ and $y$. Now make the following postulate:

*Each individual receives an equal share of the private good.* This assumption emerges from the following rationality. When society undertakes a public project financed by taxes, an identical tax schedule is usually imposed on all individuals implying equal treatment for all. In the theoretical case of democratic preference functions, elaborated upon by Campbell [2], the domain of collective choice includes only a vector of public activities and a set of parameters affecting *equally* individual private decisions that are left to the initiative of the individual.[4] Moreover, the egalitarian principle is used in the absence of a well-defined social welfare function since society does not possess a valid instrument for determining an unequal distribution of income a priori or defining a tax schedule that will be differential with respect to preferences instead of income.

Let $y$ be the equal share of the private good received by each individual. Since $X + Ny = X_0$, an indirect utility function, $V^i(X)$, can be constructed for each individual. Since $U^i(X, y)$ is strictly concave, $V^i(X)$ is unimodal and concave,[5] thus guaranteeing that individual's preferences for the stock of resources are single-peaked. The problem of choice is reduced to one dimension since, given a value for $X$, $y$ can be instantaneously computed.

It is well known that the MMD leads to a social welfare function provided that the number of individuals, $N$, is an odd number (Arrow [1, p. 78]). Each individual votes for his best choice. The allocation that wins is the median of the first-best choices since it has a simple majority over every other alternative. Within this framework, the MMD outcome was supposed to be Pareto-optimal since one public good was being determined (Zeckhauser and Weinstein [8]). However, because of the equal share principle, Pareto-optimality of the MMD is not always achieved.

Let $(X_M, y_M)$ be the winning outcome; i.e., the number of persons who prefer $(X_M, y_M)$ over every other $(X_k, y_k)$ is larger than the number of individuals who prefer any other alternative over $(X_M, y_M)$. Consider an economy of three individuals labeled A, B, and C and suppose the following preferential ranking

[4] In the California case cited in footnote 2, the social choice embraces only scenic rivers and energy (dams), while the consumption vector is chosen by sovereign consumers who are equally affected by the level of available energy.

[5] For a construction of an indirect utility function for public goods in the case of many commodities, see Zeckhauser and Weinstein [8].

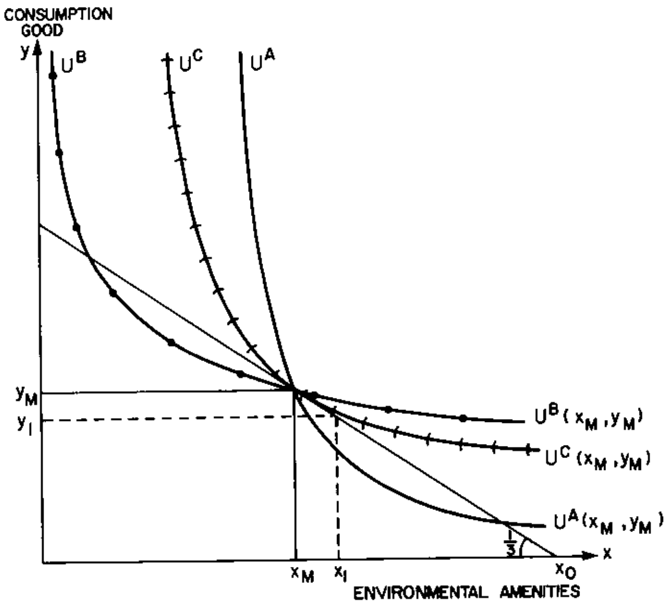


FIGURE 1

for the allocation $(X_M, y_M)$: individual A is more amenity inclined than B and C; individual B is more consumption inclined than A and C; and individual C is fully satisfied with the MMD outcome $(X_M, y_M)$; i.e., the median is C's best choice. Then, given $(X_M, y_M)$ one has

$$\left.\frac{U_X{}^A}{U_y{}^A}\right|_{(X_M, y_M)} > \left.\frac{U_X{}^C}{U_y{}^C}\right|_{(X_M, y_M)} > \left.\frac{U_X{}^B}{U_y{}^B}\right|_{(X_M, y_M)}$$

and

$$\left.\frac{U_X{}^C}{U_y{}^C}\right|_{(X_M, y_M)} = \frac{1}{3}.$$

Since A, B, and C have different preferences for public and private goods, their indifference maps may be depicted as in Fig. 1. By the equal share principle, the budget line is identical for all individuals; hence, the ordinate represents the amount of the private good that each individual can consume. The slope of the budget line is $-1/N$ or, in the example, $-\frac{1}{3}$. Individual A would be better off if the winning allocation were $X_k > X_M$ and $y_k < y_M$, implying that he possesses an "excess" supply of the consumption good.

Now assume the possibility of a subsequent ballot. Individual A may be inclined to trade his excess supply of $y$ for an allocation more prone to environmental amenities, say $(X_1, y_1)$. A needs only to induce one individual to vote for it, say, individual C. The possibility of vote trading exists if the political system allows it. Technically, vote trading is feasible since $y$ is a divisible private good. On the other hand, individual B cannot trade his excess supply of $X$, the public good. Therefore, by compensating individual C from a welfare loss due to a move in the election outcome, individual A may reach a higher level of utility. However, this is not a satisfactory procedure for determining whether the new allocation will be Pareto-optimal since individual B will be worse off at $(X_1, y_1)$ and unable to block the move since $X$ is an indivisible public good. It is then sufficient to show that a potential compensation for individual B exists even if one needs only to bribe individual C to vote for $(X_1, y_1)$.

Let $s_B$ and $s_C$ be the amount of compensation paid to individuals B and C, respectively, by individual A. A proposes to bribe if

$$U^A(X_1, y_1 - s_B - s_C) \geqq U^A(X_M, y_M).$$

Individual C will accept the bribe and will vote for $(X_1, y_1)$ over $(X_M, y_M)$ if

$$U^C(X_1, y_1 + s_C) \geqq U^C(X_M, y_M).$$

Individual B will not be worse off at $(X_1, y_1)$ if

$$U^B(X_1, y_1 + s_B) = U^B(X_M, y_M).$$

One has to find the quadruple $(X_1, y_1, s_B, s_C)$ that maximizes

$$U^A(X_1, y_1 - s_B - s_C)$$

subject to

$$U^B(X_1, y_1 + s_B) = U^B(X_M, y_M),$$
$$U^C(X_1, y_1 + s_C) \geqq U^C(X_M, y_M),$$
$$X_1 + 3y_1 = X_0.$$

Assume that $U^C(X_M, y_M) = U^C(X_M, y_M + s_C)$ since it is sufficient to show that the welfare of A is improved without worsening that of B and C in order to prove the Pareto-optimality of the vote-trading process. The solution $(X_1, y_1, s_B, s_C)$ is obtained by solving

$$\frac{U_X{}^A}{U_y{}^A} + \frac{U_X{}^B}{U_y{}^B} + \frac{U_X{}^C}{U_y{}^C} = 1, \tag{2}$$

$$X_1 + 3y_1 = X_0, \tag{3}$$

$$U^B(X_M, y_M) = U^B(X_1, y_1 + s_B), \tag{4}$$

$$U^C(X_M, y_M) = U^C(X_1, y_1 + s_C). \tag{5}$$

It can be shown that: the simple MMD does not always[6] achieve a Pareto-optimal allocation of public and private goods derived from finite natural resources. That is, the quadruple $(X_1, y_1, s_B, s_C)$ is not Pareto inferior to $(X_M, y_M)$,

*Proof.* Since $U^A$, $U^B$, and $U^C$ are strictly concave[7]

$$U^A(X_1, y_1 - s_B - s_C) - U^A(X_M, y_M) > U_X{}^A[X_1 - X_M] + U_y{}^A[y_1 - y_M - s_B - s_C].$$

By (3)

$$3y_1 = X_0 - X_1 \text{ and } 3y_M = X_0 - X_M;$$

[6] If the sum of marginal rates of substitution at the MMD allocation is equal to one, no improvement can be made since, by definition, the MMD allocation is Pareto-optimal.

[7] The derivates are computed at $(X_1, y_1 - s_B - s_C)$ for $U^A$; $(X_1, y_1 + s_B)$ for $U^B$; and $(X_1, y_1 + s_C)$ for $U^C$.

and, since $U_y{}^A > 0$,

$$\frac{1}{U_y{}^A}[U^A(X_1, y_1 - s_B - s_C) - U^A(X_M, y_M)] > \left[\frac{U_X{}^A}{U_y{}^A} - \frac{1}{3}\right][X_1 - X_M] - s_B - s_C. \quad (6)$$

On the other hand, by (4),

$$U^B(X_1, y_1 + s_B) - U^B(X_M, y_M) = 0;$$

then, by strict concavity of the utility function,

$$0 > \left[\frac{U_X{}^B}{U_y{}^B} - \frac{1}{3}\right][X_1 - X_M] + s_B. \quad (7)$$

Similarly, for individual C:

$$0 > \left[\frac{U_X{}^C}{U_y{}^C} - \frac{1}{3}\right][X_1 - X_M] + s_C. \quad (8)$$

Adding (7) and (8) and reversing the inequality sign gives

$$\left[\frac{2}{3} - \frac{U_X{}^B}{U_y{}^B} - \frac{U_X{}^C}{U_y{}^C}\right][X_1 - X_M] - s_B - s_C > 0.$$

However, by (2),

$$\frac{U_X{}^A}{U_y{}^A} - \frac{1}{3} = \frac{2}{3} - \frac{U_X{}^B}{U_y{}^B} - \frac{U_X{}^C}{U_y{}^C}.$$

Then, by (6),

$$U^A(X_1, y_1 - s_B - s_C) - U^A(X_M, y_M) > 0,$$

$$U^B(X_1, y_1 + s_B) - U^B(X_M, y_M) = 0,$$

and

$$U^C(X_1, y_1 + s_C) - U^C(X_M, y_M) = 0.$$

Q.E.D.

Since the proof hold also if individual B bribes individuals A and C to vote for a more consumption-inclined allocation, it is necessary to determine whether the optimal allocation is more prone to environmental amenities than the MMD solution.

Let

$$\phi^i(X, y) = \left.\frac{U_X{}^i}{U_y{}^i}\right|_{(X,y)} \quad \text{for } i = \text{A, B, and C}.$$

Thus, by assumption,

$$\phi^A(X_M, y_M) > \phi^C(X_M, y_M) > \phi^B(X_M, y_M)$$

and

$$\phi^C(X_M, y_M) = \tfrac{1}{3}.$$

Moreover, if $(X_M, y_M)$ is not Pareto-optimal, the sum of marginal rates of substitution, $\phi^i$, is different from one.

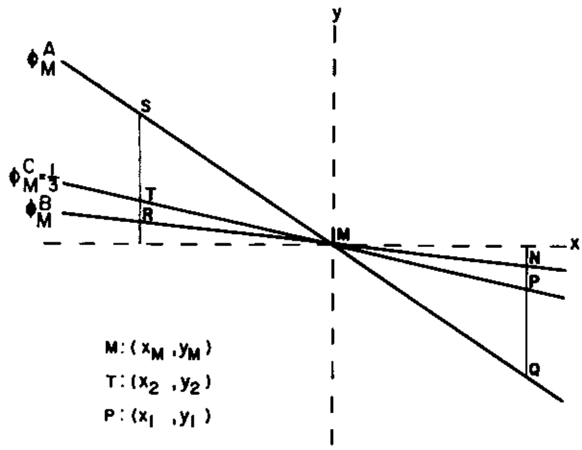


FIG. 2. Neighborhood of $(x_M, y_M)$.

Consider the case where

$$\phi^A(X_M, y_M) + \phi^B(X_M, y_M) + \phi^C(X_M, y_M) > 1.$$

It implies that the mean of marginal rate of substitution exceeds the median's marginal rate of substitution.

Then,

$$\phi^A - \tfrac{1}{3} > \tfrac{1}{3} - \phi^B$$

and

$$\phi^B < \tfrac{1}{3}, \quad \phi^A > \tfrac{1}{3}.$$

In the neighborhood of $(X_M, y_M)$, $\phi^A$, $\phi^B$, and $\phi^C$ are identical to the indifference lines of $U^A$, $U^B$, and $U^C$ passing through $(X_M, y_M)$ (Fig. 2). If individual B wants to bribe individual A to move to the allocation $(X_2, y_2)$ [$T$ at the left of M on $X_0 = X + 3y$ in Fig. 2], B must provide A with at least the amount $(S - T)$ of the private good. However, the amount $(T - R)$ that individual B can spare and still be on utility level $\phi^B(X_M, y_M)$ is smaller than $(S - T)$ since $\phi^A - \frac{1}{3} > \frac{1}{3} - \phi^B$. Only individual A is able to induce a move to the right of M to $P$, for example, since $(P - Q) > (N - P)$.

Individual B cannot use commodity $X$ to bribe individual A since $X$ is an indivisible public good. Therefore, since the above proof also holds for individual B, it means that a negative compensation is paid to individual A. Individual B is asking for a compensation from A in return for a vote favoring a more conservation-oriented allocation. Thus, the optimal move is in favor of the conservationist members of society. One can state:

PROPOSITION. *If preferences are convex and the sum of marginal rates of substitution $\phi^i(X_M, y_M)$ exceeds one, the new MMD allocation $(X_1, y_1)$ will be at least more amenities inclined than the simple MMD solution, i.e., $X_1 \geqslant X_M$, $y_1 \leqslant y_M$.*

On the other hand, if $\phi^A(X_M, y_M) + \phi^B(X_M, y_M) + \phi^C(X_M, y_M) < 1$, individual B can be able to bribe individual A and still be better off since, in the neighborhood of $(X_M, y_M)$, $\phi^A - \frac{1}{3} < \frac{1}{3} - \phi^B$ (for $\phi^A > \frac{1}{3}$, $\phi^B < \frac{1}{3}$). Depending on the curvature of the utility functions, individual B can have larger excess of private good $y$ to be distributed in compensation. The case of $\sum_i \phi^i < 1$ implies that the mean of marginal rates of substitution between environmental amenities and consumption goods is smaller than the median's marginal substitution rate. If we were dealing with renewable resources, this case will be as frequent as the case where $\sum_i \phi^i > 1$. However, with exhaustible resources, amenity-inclined individuals internalize, in their vote, the future benefits generated by the stock of resources and, thus, they increase the evaluation of their marginal rate of substitution. Therefore, with exhaustible resources, assuming that $\sum_i \phi^i \geqq 1$ is perfectly plausible.

## 3. CONCLUSION

The MMD outcome, with equal partitioning of private goods among society members, does not always achieve Pareto-optimality since it does not respond to intensity of preferences. Because of the asymmetry of individual trade between public and private goods, bribes can be given only by individuals who are more amenity inclined if the sum of marginal rates of substitution between public and private goods exceeds one. This, of course, raises the question of the true revelation of preferences since conservation-inclined individuals will be induced to declare larger marginal rates of substitution than their true ones, and consumption-minded individuals will announce lower $\phi^i$'s. However, this problem will only complicate the vote-trading process and is beyond the scope of this paper.

One may argue about the ethical implications of the bribes procedure proposed here to reveal intensity of preferences. It is, however, quite similar to complex real-life situations which involve either changing preferences or technological changes. Changing preferences are usually achieved by educational processes. Conservation-inclined individuals may devote their excess of private goods to advertising and educational campaigns in order to influence the vote in a subsequent ballot. In a dynamic framework, assuming short memory effect (i.e., educational promotion has to be continuous), only conservationists are able to dedicate a share of their income to educational purposes. On the other hand, individuals whose conservation motive is weak (i.e., whose consumption motive is strong) may invest some present income in research and development in order ultimately to find more efficient ways to transform the extracted good into the consumption commodity. This does not imply that environmental amenities are diminished. On the contrary, since consumption is increased only by means of technological changes, these amenities are preserved. In a dynamic analysis, the vote consists of determining the rate of extraction and the time horizon, which will be infinite in a continuous society. The resulting outcome depends on the size of the subjective discount rate of society's members. Thus, a complete dynamic analysis needs exploring the democratic solution of a conflict of interests when the resources available to each individual are subject to his preferences over time and his preferences for the environment.

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