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APPLIED WELFARE ANALYSIS FOR CONSUMERS WITH COMMODITY INCOME

BY

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The concept of economic surplus has been applied to a wide variety of practical problems even though its place in economic theory has always been controversial.[1] For expository convenience, the distinction between surpluses which accrue to buyers and surpluses which accrue to sellers is firmly embedded in the literature.[2] In general equilibrium analysis, however, consumer's income depends on the value of his endowment of commodities which in turn depends on their prices. Thus, his losses as a consumer of certain goods with the rise in their price are not separable from his gains as the seller of these goods.

The purpose of this paper is to analyse the concept of economic surplus for the consumer who receives income in commodities. The main question is whether the usual measure of welfare change by the areas between the Marshallian demand and supply curves is still valid for this consumer.

The analysis applies to a wide range of practical problems in which the concept of economic surplus is used in cost-benefit analysis. Most notably, it applies to the agricultural sector in developing countries, where farmers typically consume a considerable share of their own products. In this case, the analyst cannot even estimate separately supply and demand functions because observable data are of *excess* demand and supply only.

1 THE MEASURE OF WELFARE CHANGES

In this section, we present the definitions and conceptual tools of measuring costs or benefits of price changes for an individual consumer with commodity

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1 For instance, see Burns [1], Mishan [3], Silberberg [4], to cite a few.

2 Currie, Martin and Schmitz [2] have reviewed the use of consumer's and producer's surplus in economic analysis.

income. We examine therefore the consumption behaviour of an individual whose income is given by y=**pw**, where **w**=(w_1 ,..., w_n) is the bundle of his initial endowments and **p**=(p_1 ,..., p_n) is the vector of prices. The consumer chooses his consumption bundle **x**=(x_1 ,..., x_n) so as to maximize an increasing strictly quasi-concave utility function u(x) subject to the budget constraint

$$\mathbf{p}(\mathbf{x} - \mathbf{w}) = 0 \tag{1}$$

Fig. 1 presents the simple case of a consumer receiving all of his income in a single commodity. This, for instance, would be the case of the monoculture farmers. The consumer is endowed with the quantity $\overline{OW}$ of x_1 and is faced with the market prices p^0, represented by the slope of the budget line $\overline{VW}$. At these prices, his position at A indicates that he chooses to consume the quantity $\overline{OM}$ of his own endowment and sell the quantity $\overline{MW}$. When the price of x_1 rises, the consumer's new budget line would be $\overline{UW}$, and the effect

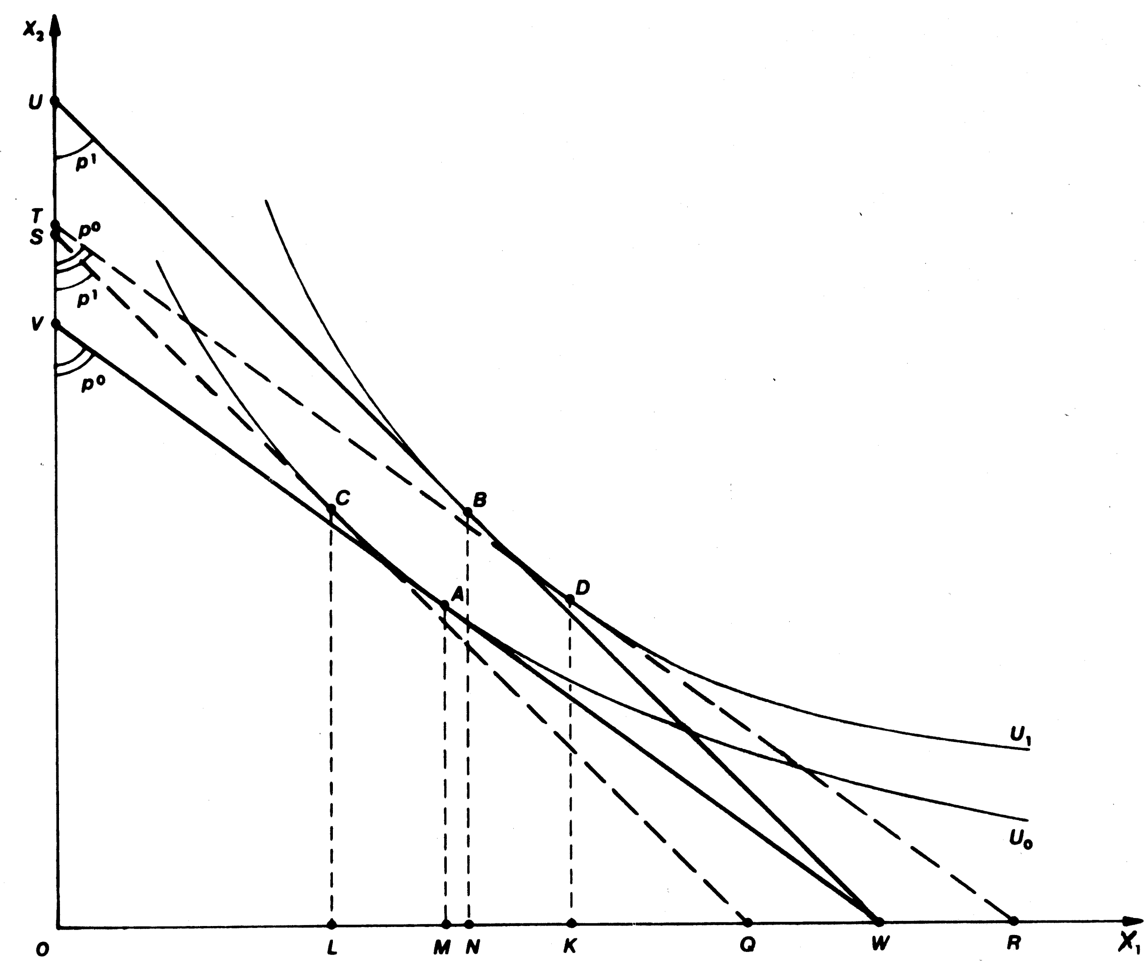

Figure 1

would be to *raise* his welfare (except for the trivial case in which he consumes all of his endowment). Thus, unlike the ordinary consumer whose money income is fixed, the consumer with fixed endowment of commodities will *gain* from a price rise of those commodities of which he has excess supply. The consumer's new position at B indicates that he will increase his consumption of x_1 to $\overline{ON}$ (assuming x_1 to be a normal good), but the higher prices p' will allow him to increase his consumption of x_2 and thus ultimately gain.

The standard procedure in evaluating the welfare change undergone by an individual in moving from one price situation to another is to obtain an index involving price-weighted quantity changes. Hicks's measures of the money equivalent of a welfare change define the *Compensating Variation* (CV), or the amount of money income needed following a price rise in order to leave the consumer at the original level of welfare; and the *Equivalent Variation* (EV), or the amount of money income needed in order to permit the individual the same level of welfare with the old prices as he now has with the new prices.[3] Corresponding to the money equivalent of a welfare change, we can define for this consumer the quantity equivalent which determines the amount needed to be added to (or subtracted from) his original endowment in order to permit this consumer the same level of welfare, following a price change. This would provide a quantifiable measure of the welfare change in terms of that commodity.

In Fig. 1, the quantity CV is given by $-\overline{QW}$, where the minus sign indicates that this quantity must be subtracted from his original endowment in order to leave him at his original level of welfare following a price rise of x_1. This, in turn, provides a measure of welfare *gain* from the price rise to that consumer. The EV expressed in terms of money (*i.e.* in terms of x_2 in Fig. 1) is given by $\overline{VT}$. In terms of commodity, the EV is given by $+\overline{WR}$, indicating that additional endowment would have to be given to that consumer to permit the welfare level u' if the price of x_1 were to fall back to its original level.

3 CV, thus, corresponds to a quantity index which uses the original prices as weights, whereas EV is a quantity index which uses the new prices as weights. If x_2 represents the bundle of all other commodities (or the money income), then, following a price rise of x_1, the amount of money representing the welfare change which uses the original prices would be given by $\overline{OS}/\overline{OV}$. Hicks's CV would be given by the interval $\overline{VS}$. Notice that the measure $\overline{VS}$ can be divided into $\overline{VV} - \overline{SV}$ where $\overline{VV}$ registers the effect of the increase in the price of x_1 on the money income of the consumer while $\overline{SV}$ is exactly the *equivalent* variation for the ordinary consumer.

We can now turn to the formal analysis. Maximizing the utility function $u(\mathbf{x})$ subject to the budget constraint (1), we obtain, under the usual conditions, the ordinary demand functions describing the optimal quantities in terms of prices and the initial *endowment* (rather than income as for the ordinary consumer). Denote these demand functions by $x_i(\mathbf{p},\mathbf{w})$. Substituting these demand equations for the quantities in the utility function, we thus obtain the indirect utility function[4]

$$u[x_1(\mathbf{p},\mathbf{w}),\ldots,x_n(\mathbf{p},\mathbf{w})] = V(\mathbf{p},\mathbf{w}) \tag{2}$$

In the case of a consumer with commodity income, there are two ways to define the compensations. One is in terms of commodities and for that we compare the minimal endowment necessary in one price situation with the minimal endowment necessary in another price situation when the consumer attains the same level of utility in the two situations. In these terms the above compensations can be expressed as follows:

Let the initial situation be characterized by prices $\mathbf{p}^0$ and endowments $\mathbf{w}^0$ and let the prices in the alternative situation be $\mathbf{p}'$. The compensating endowments are given by

$$V(\mathbf{p}^0, \mathbf{w}^0) = V(\mathbf{p}', \mathbf{w}^0 - \mathbf{c}) \tag{3}$$

where $\mathbf{c}$ is the vector denoting the amounts that have to be substracted from the initial endowment in order to leave the consumer at the original level of utility.

The compensating variation is the money value of the amounts that have to be subtracted, evaluated at the new price situation, *i.e.*

$$\mathrm{CV} = \sum_{j=1}^{n} p_j' \, c_j$$

Define now the *compensating endowment,* denoted by $w(\mathbf{p},u)$, to be the minimum bundle which is necessary to secure the utility level u at the price situation $\mathbf{p}$. Obviously, $w(\mathbf{p},u)$ is *any* point on the budget line which is tangent to the indifference curve at the utility level u. By definition,

4 It is important to note that the properties of this indirect utility are quite different than those of the indirect utility function for the ordinary consumer, the arguments of which are income and prices. Most importantly, it is no longer true that $V(\mathbf{p},w)$ is non-increasing.

$$\text{CV} = \sum_{j=1}^{n} p_j' \frac{\partial w_j(\mathbf{p}',u)}{\partial p_j} \tag{4}$$

However, only in the case of a consumer receiving all of his income in a single commodity will the bundle $w(\mathbf{p},u)$ be single valued and the sign of $\partial w_j(\mathbf{p},u)/\partial p_i$ be uniquely determined. In the more general case it is possible to overcome this difficulty by assuming the compensation to be given in only *one* commodity. In this case the vector $\mathbf{c}$ will have the form $(c_1, 0,\ldots,0)$ if the compensation is given in the first commodity and therefore $\partial w_1(\mathbf{p}', u)/\partial p_j < 0$ while $\partial w_i(\mathbf{p},u)/\partial p_j = 0$ for $i = 2,\ldots, n$.

Similarly the equivalent endowments are given by

$$V(\mathbf{p}^0, \mathbf{w}^0 + \mathbf{e}) = V(\mathbf{p}', \mathbf{w}^0),$$

where $\mathbf{e}$ is the vector denoting the amounts that have to be added to the initial endowment so as to have the same welfare impact on the consumer as does the change in prices from $\mathbf{p}^0$ to $\mathbf{p}'$.

The equivalent variation is given by

$$\text{EV} = \sum_{j=1}^{n} p_j^0 e_j$$

or, using the concept of the compensating endowment $\mathbf{w}(\mathbf{p},u)$, we get

$$\text{EV} = \sum_{j=1}^{n} p_j^0 \frac{\partial w(\mathbf{p}^0,u)}{\partial p_j} \tag{5}$$

Again it is possible to require the compensation is given in a single commodity in which case $\partial w_1(\mathbf{p}^0,u)/\partial p_j > 0$ while $\partial w_i(\mathbf{p}^0,u)/\partial p_j = 0$ for $i = 2,\ldots,n$.

The other way of defining the compensation is in monetary terms via the *expenditure function.* This function is the inverse of the indirect utility function and relates levels of utility to the minimum amount of income necessary to achieve that level at a given price situation. Let z_j denote the excess demand for the j^{th} product, *i.e.*, $z_j = x_j - w_j$. The net expenditure function is defined to be the value of the minimum in the following problem

$$\text{Min} \sum_{j=1}^{n} p_j z_j \text{ ; s.t. : } u(\mathbf{w}_0 + \mathbf{z}) = u \tag{6}$$

The $z_j(\mathbf{p}, w_j, u) = x_j(\mathbf{p},u) - w_j$ which solves this problem are the compensated excess demands and the $x_j(\mathbf{p},u)$'s are the usual compensated demands. Denote the net expenditure function by $M(\mathbf{p},\mathbf{w},u)$. By definition

$$M(\mathbf{p},\mathbf{w},u) = m(\mathbf{p},u) - \sum_{j=1}^{n} p_j w_j = \sum_{j=1}^{n} p_j \cdot x_j(\mathbf{p},u) - \sum_{j=1}^{n} p_j w_j, \tag{7}$$

where $m(\mathbf{p},u)$ is the gross expenditure function.

The Hicksian compensating variation is the difference between the expenditure function in price situation $\mathbf{p}'$ and in price situation $\mathbf{p}^0$, both at the original level of utility

$$\text{CV} = M(\mathbf{p}', \mathbf{w}^0, u^0) - M(\mathbf{p}^0, \mathbf{w}^0, u^0) \tag{8}$$

The boundary condition for utility maximization implies that

$$M(\mathbf{p}^0, \mathbf{w}^0, u^0) = \sum_{j=1}^{n} p_j^0 x_j(\mathbf{p}^0, u^0) - \sum_{j=1}^{n} p_j^0 w_j^0 = 0$$

and therefore,

$$\text{CV} = \sum_{j=1}^{n} p_j' w_j(\mathbf{p}', u^0) - \sum_{j=1}^{n} p_j' w_j^0 \tag{9}$$

In Fig. 1 we can identify the CV by the difference between the money value of the initial endowment at the new prices — which is denoted by the distance $\overline{\text{OU}}$ and the money value of the compensating consumption bundle — which is denoted by the distance $\overline{\text{OS}}$. Hence, the compensating variation is exactly the distance $-\overline{\text{SU}}$, where the minus sign indicates that this amount of money income has to be subtracted from this consumer's income in order to leave him at the original level of utility. In terms of the quantity endowed, the compensating variation is given by the distance $-\overline{\text{QW}}$, and one can easily verify that the two definitions of the compensating variation in (4) and in (9) are exactly identical.

In a similar way, the Hicksian equivalent variation is given by

$$\mathrm{EV} = M(\mathbf{p}', \mathbf{w}^0, u') - M(\mathbf{p}^0, \mathbf{w}^0, u') \tag{10}$$

and since the boundary condition implies that $M(\mathbf{p}',\mathbf{w}^0,u') = 0$, we get

$$\mathrm{EV} = \sum_{j=1}^{n} p_j^0 w_j^0 - \sum_{j=1}^{n} p_j x_j (\mathbf{p}^0, u') \tag{11}$$

In Fig. 1 the EV is given either directly by the distance + $\overline{\mathrm{VT}}$ on the "money" axes or indirectly by the money value of the quantity change given by the distance + $\overline{\mathrm{WR}}$ on the quantity axes. We can thus straightforwardly verify the equivalence between the two definitions of the equivalent variation in (5) and (11).

By the well-known properties of the expenditure function we have

$$\frac{\partial M(\mathbf{p},\mathbf{w},u)}{\partial p_j} = z_j (\mathbf{p},\mathbf{w},u) = x_j (\mathbf{p},u) - w_j \tag{12}$$

Consider now the change in a single price, say p_1. Using rudimentary calculus together with (9) and (11) we can write (8) and (10) in terms of the areas under the compensated demand function as

$$\mathrm{CV} = \int_{p_1^0}^{p_1'} (x_1 (\mathbf{p},u^0) - \mathbf{w1})\, \mathrm{d}p_1 \tag{13}$$

$$\mathrm{EV} = \int_{p_1^0}^{p_1'} (x^1 (\mathbf{p},u') - w_1)\, \mathrm{d}p_1 \tag{14}$$

These formulae express the compensating and the equivalent variations in income as areas under the *Hicksian compensated excess demand curves,* between the old and the new price horizontals. The only distinction between CV and EV is the level of utility the compensation is designed to reach.

The areas under the Hicksian compensated excess demand curves can be quite different from those under the Marshallian excess demand curve, as we can see in Fig. 2. The Hicksian compensated demand curve $x(\mathbf{p},u)$, which registers the substitution effect, is always downward sloping. Given the initial endowment w_1^0, the CV is given in Fig. 2 by the area [a-e-g-d] between the compensated demand curve $x_1(\mathbf{p},u^0)$ and the endowment line w_1^0 and between the price horizontals p_1^0 and p'_1 while the EV is given by the area [f-g-d-b] between the curves $x_1(\mathbf{p},u')$ and w_1^0 and between the same price horizontals.

Obviously, neither one of the true measures of welfare changes is actually measurable as both depend on the abstract notion of compensated demand. The only measurable index of welfare change is the area [e-g-d-b], denoted by MV and defined by the observable Marshallian demand curve $x(\mathbf{p},\mathbf{w}^0)$ and the fixed endowment line w_1^0. This is exactly the consumer's surplus. Notice that for the consumer receiving his income in commodities, the Marshallian demand curve can have a positive slope even for a non-inferior good – as indeed is the case illustrated in Fig. 2 – since the income effect is not necessarily negative. To see this, we can derive the Slutsky equation for our case, noting that at the optimum

$$x_i(\mathbf{p},u) = x_i(\mathbf{p},y) = x_i(\mathbf{p}, M(\mathbf{p},\mathbf{w},u) + \mathbf{p}\cdot\mathbf{w}) \text{ for } i = 1,\dots, n, \qquad (15)$$

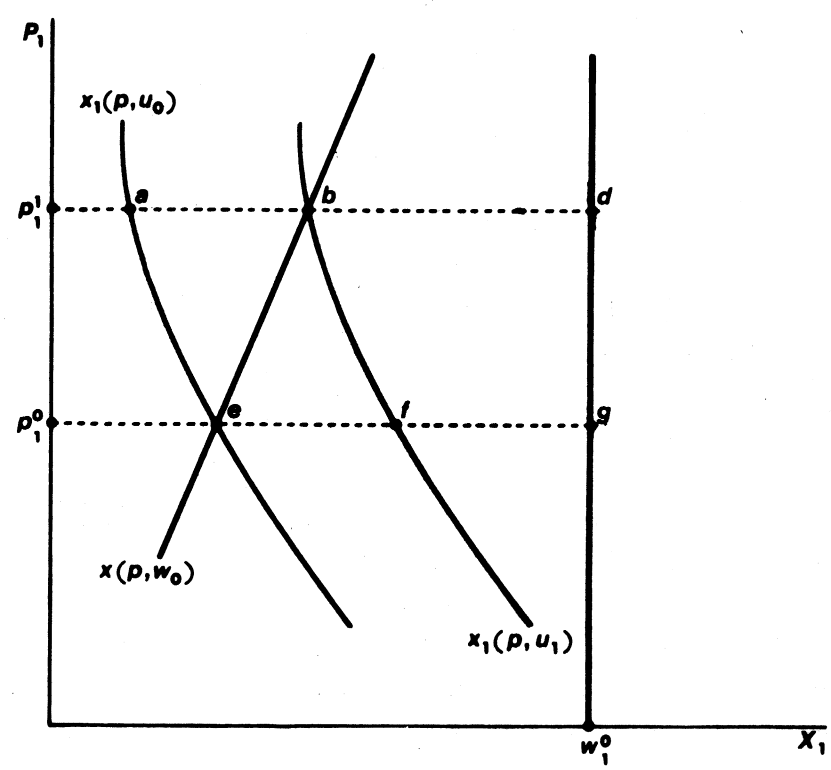

Figure 2

and

$$M(\mathbf{p},\mathbf{w},u) = 0 \tag{15a}$$

Thus, by differentiating x_i (**p**, **p**w) with respect to p_j, one obtains

$$\frac{\mathrm{d}x_i\,(\mathbf{p},\mathbf{pw})}{\mathrm{d}p_j} = \frac{\partial x_i\,(\mathbf{p},\mathbf{pw})}{\partial p_j} + w_j\,\frac{\partial x_i\,(\mathbf{p},\mathbf{pw})}{\partial y} \tag{16}$$

However by (15) and (12)

$$\frac{\partial x_i\,(\mathbf{p},u)}{\partial p_j} = \frac{\partial x_i\,(\mathbf{p},\mathbf{p}\cdot\mathbf{w})}{\partial p_j} + \frac{\partial x_i}{\partial y}\cdot[x_j\,(\mathbf{p},u)\cdot w_j + w_j]\quad i,j=1,\ldots,n \tag{17}$$

Thus

$$\frac{\mathrm{d}x_i\,(\mathbf{p},\mathbf{pw})}{\mathrm{d}p_j} = \left.\frac{\partial x_i\,(\mathbf{p},u)}{\partial p_j}\right|_{\bar{u}} + (w_j - x_j)\left.\frac{\partial x_i\,(\mathbf{p},y)}{\partial y}\right|_{\bar{\mathbf{p}}}\quad i,j=1,\ldots,n \tag{18}$$

The first element on the right hand side of (18) is the substitution effect, which is always negative for a change in its own price. The second element is the income effect. If x_i is a normal good *i.e.*, $\partial x_i/\partial y > 0$, the income effect is positive provided that $(w_j - x_j) > 0$, *i.e.*, the consumer has an excess supply of that commodity. In that case, the income effect outweights the substitution effect and the entire Marshallian demand curve is positively sloped.

Hence, while the compensating variation is given by the area [a-e-g-d], the consumer surplus is given by the area [b-e-g-d]. The difference between the two depends to a large extent on the *net* income effect which in turn depends on the excess demand. Whether or not the consumer surplus concept is still useful in a cost benefit analysis for this type of consumers depends on whether the observed index MV is a sufficiently close approximation of the "true" measures CV or EV. In Fig. 2, we can see that the difference can be quite substantial depending on the slopes of the various curves.

2 CONSUMER SURPLUS AND WELFARE GAINS

Suppose that prices change from $\mathbf{p}^0$ to $\mathbf{p}'$ while the consumer's endowment remains fixed at $\mathbf{w}^0$. Accordingly, his level of welfare changes from $u^0 = V(\mathbf{p}^0, \mathbf{w}^0)$ to $u' = V(\mathbf{p}', \mathbf{w}^0)$. As noted above, two sets of compensation measures can be defined – one set in commodity, the other in money. Henceforth we focus mainly on the latter. The monetary compensation required to offset the effect of the price is

$$\mathrm{CV} = M(\mathbf{p}^0, \mathbf{w}^0, u^0) - M(\mathbf{p}', \mathbf{w}^0, u^0) = [m(\mathbf{p}^0, u^0) - \sum_{j=1}^{n} w_j^0 p_j^0] + [m(\mathbf{p}', u^0) - \sum w_j^0 p_j'] = [m(\mathbf{p}^0, u^0) - m(\mathbf{p}', u^0)] + \sum_{j=1}^{n} w_j^0 (p_j' - p_j^0) \tag{19}$$

In a similar way we can define the equivalent variation.

In terms of the areas under the compensated demand function, this becomes

$$\mathrm{CV} = \int_{\mathbf{p}^1}^{\mathbf{p}^0} \sum_{j=1}^{n} z^i(\mathbf{p}, w_j^0, u^0)\, \mathrm{d}p_j = \int_{\mathbf{p}^1}^{\mathbf{p}^0} \sum_{j=1}^{n} x_j(\mathbf{p}, u^0)\, \mathrm{d}p_j + \sum_{j=1}^{n} w_j^0 (p_j' - p_j^0) \tag{20}$$

The gross compensated demand functions $x_j(\mathbf{p}, u^0)$ are identical for the ordinary consumer with money income and the consumer with commodity income. Thus, the first expression on the right-hand side of (20) is simply the area under the compensated demand curve between the two price boundaries. The second expression on the right-hand side of (20) is simply the change in income resulting from the price change. This result shows that total welfare effect of a price change on consumers with commodity income can be decomposed into a consumer effect - measured by the area under the consumer's compensated demand curve – and a producer effect – measured by the income change.

To relate this exact measure of the compensating variation to the approximation given by the area under the Marshallian demand curve we can draw on the rigorous analysis of Willig [5] who produced error bounds on the extent to which compensating and equivalent variations are approximated by the change in the consumer surplus of the ordinary consumer, following a price change.

The following proposition simplifies somewhat Willig's results, showing that his extension of the analysis for nonconstant income elasticity of demand and the derived upper and lower error bounds are quite unnecessary and a single parameter can be derived which characterizes entirely the error of approximation. As a consequence, the proposition suggests that the consumer surplus concept is a useful tool in applied welfare analysis over a much wider range of products and markets than that indicated by Willig. Furthermore, the approximation error is computed based on measurable parameters such as the share of the commodity in the consumer's total expenditure, income elasticity of demand, and the percentage change in price.

PROPOSITION: The error of approximation in using the consumer surplus change as a measure of the compensating or equivalent variation is proportional to the income elasticity of demand and the share of the budget spent on the commodity.

The proof of this proposition is given in the appendix.

For sufficiently small changes in price, one can approximate the upper-bound error of using MV instead of EV or CV by $\frac{1}{2}\frac{dp_1}{p_1}\eta^0\theta_1$. This error is calculated in table 1 for some selected values of η and θ_1, assuming that the rate of change in price, dp/p, is 5%.

The results of the proposition apply to the ordinary consumer as well as to a consumer with commodity income. In regard to the latter it should be noted that the income elasticity measures the percentage change in demand as

TABLE 1[a] – ERROR OF APPROXIMATION IN USING CONSUMER'S SURPLUS IF THE RATE OF PRICE CHANGE IS 5%

θ_1 / η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
0.3	.001	.002	.002	.003	.004	.005	.005	.006	.007	.007
0.5	.001	.003	.004	.005	.006	.008	.009	.010	.011	.012
0.7	.002	.004	.005	.007	.009	.011	.012	.014	.016	.017
1.0	.003	.005	.008	.010	.013	.015	.018	.020	.023	.024
1.2	.003	.006	.009	.012	.015	.018	.021	.024	.027	.029
1.5	.004	.008	.011	.015	.019	.023	.026	.030	.034	.036
2.0	.005	.010	.015	.020	.025	.030	.035	.040	.045	.048
3.0	.008	.015	.022	.030	.038	.045	.053	.060	.068	.071
5.0	.013	.025	.038	.050	.063	.075	.088	.100	.113	.119
10.0	.025	.050	.080	.100	.125	.150	.175	.200	.225	.238
20.0	.050	.100	.150	.200	.250	.300	.350	.400	.450	.475
30.0	.075	.150	.225	.300	.375	.450	.525	.600	.675	.713

a θ_1 is the share of budget, η is the income elasticity of demand.

an effect of a change in income *when prices are held constant.* If the consumer's income is given by $y = \mathbf{p}.\mathbf{w}$, then the income elasticity is given by

$$\eta_{ij} = \frac{\partial x_j}{\partial w_i} \cdot \frac{w_i}{x_j} \quad i, j = 1, \ldots, \mathrm{n}$$

We can thus term this elasticity the *endowment elasticity.* Under the assumption of free competitive markets we can assume $\eta_{ij} = \eta_{kj} = \eta_j$ for all $i,k = 1, \ldots, n$. Writing the Slutsky equation (18) in elasticity terms we get

$$\epsilon_1 = \epsilon_1^c + (s_1 - \theta_1)\eta \tag{21}$$

where ϵ_1^c is the price elasticity of compensated demand and s_1 is the share of the first commodity in consumer's total income. Hence, the percentage error of approximation is given, via equations (A.4), and (A.5) in the appendix and equation (21), by

$$\frac{\mathrm{CV} - \mathrm{MV}}{\mathrm{MV}} \lesssim \frac{1}{2} \frac{\mathrm{d}P_1}{P_1} \cdot \frac{\theta_1}{s_1 - \theta_1} \quad (\epsilon_1 - \epsilon_1^c) \tag{22}$$

In terms of the quantity changes this error can be written as

$$\frac{\mathrm{CV} - \mathrm{MV}}{\mathrm{MV}} \lesssim \frac{1}{2} \frac{\theta_1}{s_1 - \theta_1} \cdot \frac{(\Delta Q - \Delta Q^c)}{Q} \tag{23}$$

where ΔQ is the change in quantity as an effect of the change in price along the ordinary demand curve and ΔQ^c is the change in quantity along the compensated demand curve.

For the ordinary consumer with money income this percentage error is given, in price elasticity terms, by

$$\frac{\mathrm{CV} - \mathrm{MV}}{\mathrm{MV}} \leqslant \frac{1}{2} \frac{\mathrm{d}P_1}{P_1} \cdot (\epsilon_1 - \epsilon_1^c)$$

and, in terms of quantity changes, by

$$\frac{\mathrm{CV} - \mathrm{MV}}{\mathrm{MV}} \leqslant \frac{1}{2} \cdot \frac{(\Delta Q - \Delta Q^c)}{Q}$$

For all consumers ΔQ_c and ϵ_1^c are negative. For the ordinary consumer however ϵ_1 and ΔQ also are negative if the good is normal whereas for the consumer with commodity income ϵ_1 and ΔQ can be positive even if the good is normal. Hence the upper bound on the error of approximation for this consumer is likely to be larger than for the ordinary consumer.

Empirical studies that apply the consumer surplus concept for consumers with commodity income must be aware of the possibility that the error of approximation involved in this measure may be unacceptably high. This error is proportional to the *endowment* elasticity and the analyst must establish an estimate for this elasticity in order to determine an upper bound on the error.

APPENDIX

PROOF OF THE PROPOSITION

The differential equation $\eta(y) = (\partial x_1/\partial y)\cdot(y/x_1)$ which defines the income elasticity can be integrated over the interval (y^0, y') to yield

$$\int_{y^0}^{y'} \frac{dx_1\,(\mathbf{p},y)}{dx_1} = \int_{y^0}^{y'} \eta\,\frac{dy}{y} \tag{A.1}$$

Let η(y) be a monotonic decreasing and continuous function of y. By the mean value theorem for integrable functions, equation (21) becomes

$$x_1\,(\mathbf{p},y) = x_1\,(\mathbf{p},y^0)\left(\frac{y}{y^0}\right)^{\eta(y'')}$$

where y'' is some mean value in the interval of integration (y^0, y'), Thus, by the fundamental equation of the expenditure function theory,

$$\frac{\partial m(\mathbf{p},u)}{\partial p_1} = x_1\,(\mathbf{p},y^0)\left(\frac{m(\mathbf{p},u)}{y^0}\right)^{\eta''}$$

where $\eta'' \equiv \eta(y'')$.

Consider a price increase of commodity x_1 from $\mathbf{p}^0$ **to** $\mathbf{p}'$. By definition, $m(\mathbf{p}',u) \geqslant y^0$ for $\mathbf{p}' > p^0$ and $m(\mathbf{p}^0,u) = y^0$.
Therefore,

$$\frac{\partial m(\mathbf{p},u)}{\partial p_1} \leqslant x_1\,(\mathbf{p},y^0)\quad \left(\frac{m(\mathbf{p},u)}{y^0}\right)^{\eta^0} \tag{A.2}$$

where $\eta^0 \equiv \eta(y^0)$ is constant for constant y^0.

By integrating (20) over the price range $[p_1^0, p_2']$, one obtains

$$\frac{m(\mathbf{p}'u)^{1-\eta^0} - (y^0)^{1-\eta^0}}{1-\eta^0} \leqslant (y^0)^{-\eta^0} \int_{p_1^0}^{p_1'} x_1\,(\mathbf{p},y^0)\,\mathrm{d}p_1$$

Hence

$$m(\mathbf{p}',u) \leqslant y\quad \left[1 + \frac{(1-\eta^0)\mathrm{MV}}{y^0}\right]^{\frac{1}{1-\eta^0}} \tag{A.3}$$

where $\mathrm{MV} = \int_{p_1^0}^{p_1'} x_1\,(\mathbf{p},y^0)\,\mathrm{d}p_1$ is the consumer's surplus.

The concluding step is based on Willig's approximation analysis. We loosely apply the Taylor approximation (see Willig, p. 593) to (A.3) and by the definition of compensating variation obtain the upper bound for the percentage error of approximation CV with MV.

$$\mathrm{CV} = m(\mathbf{p}',u) - y^0 \lesssim \mathrm{MV} = \frac{\eta^0(\mathrm{MV})^2}{2y^0}$$

Similarly $\mathrm{EV} = m(\mathbf{p}',u') - m(\mathbf{p}^0,u') \lesssim \mathrm{MV} - \dfrac{\eta^0(\mathrm{MV})^2}{2y^0}$

Thus, the percentage error of approximation cannot exceed

$$\frac{\mathrm{CV}\text{-}\mathrm{MV}}{\mathrm{MV}} \lesssim \frac{\mathrm{MV}}{2y^0} \cdot \eta^0 \tag{A.4}$$

To provide some insight to this upper bound, one can develop this expression as dependent on the share spent on the commodity, the income elasticity and the rate of price change, as follows:

$$\eta^0 \cdot \frac{MV}{2y^0} \simeq \frac{1}{2} \frac{dp_1}{p_1} \eta^0 \theta_1 \; [\, 1 + \frac{1}{2} \frac{dp_1}{p_1} \; \epsilon_1 \,] \tag{A.5}$$

where θ_1 = share of budget spend on commodity 1 and ϵ_1 = price elasticity of demand for commodity 1. *Q.E.D.*

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Summary

APPLIED WELFARE ANALYSIS FOR CONSUMERS WITH COMMODITY INCOME

The paper analyzes the relevance and validity of the economic surplus concept for consumers receiving their income in commodities. For these consumers, such as farmers in developing countries, it is shown that the measure of welfare gains or losses via the area under the Marshallian demand and supply curves may lead to a considerable error. The paper provides boundaries for the error of approximation as a function of the share of the product in the consumer's budget and the income elasticity of demand.