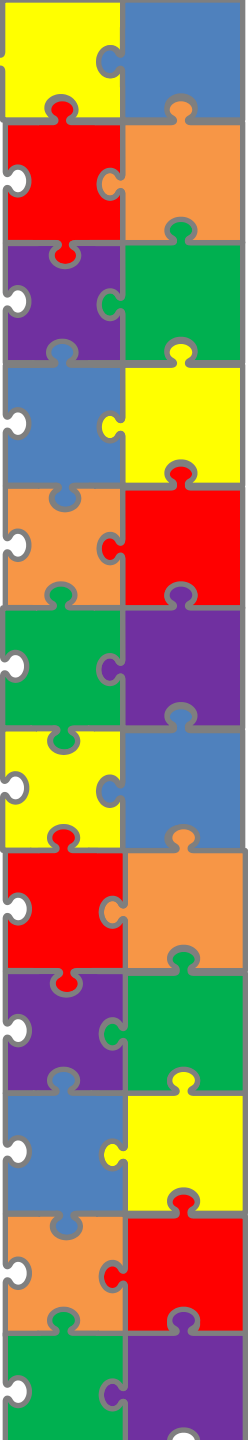


# Misrepresentation in District Voting

*Yoram Bachrach, Omer Lev, Yoad Lewenberg & Yair Zick*

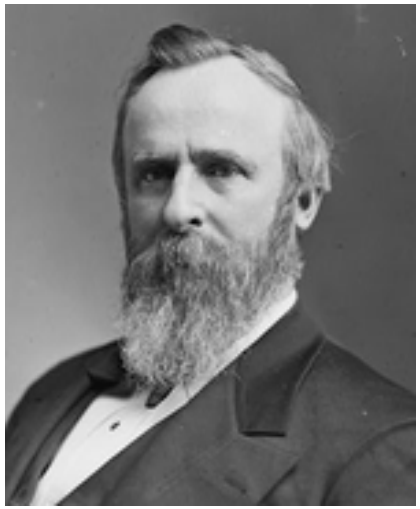
# The problem UK

1951 UK elections:

| | | |
|---|---|---|
| Popular vote: | 48% | **48.8%** |
| Parliament seats: | **321 (51.6%)** | 295 (47.2%) |

# The problem US

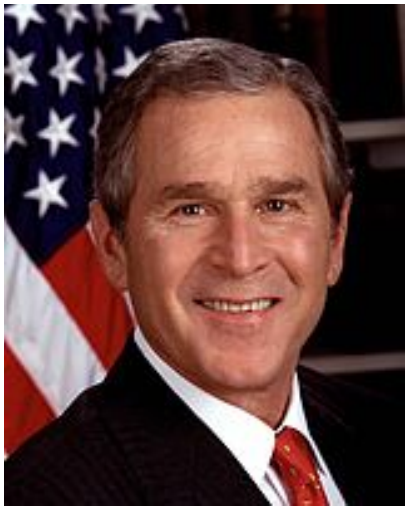
1876 US elections:



| | | |
|---|---|---|
| Popular vote: | 47.9% | **50.9%** |
| Electoral votes: | **185** | 184 |

# The problem US

2000 US elections:



| | | |
|---|---|---|
| Popular vote: | 47.9% | **48.4%** |
| Electoral votes: | **271** | 266 |

# District voting setup

Set C of $m$ candidates.

Set V of voters divided into a partition $D_1, \ldots, D_z$ of equal size, so that each district has $n$ voters.

# District voting setup

Each district uses voting rule *f* to determine winner.

The candidate that wins over the plurality of the districts is the winner of the overall election.

# Score-monotone voting rules

A voting rule $f$ is score-monotone if it assigns some type of score to a candidate, and selects the candidate maximizing/minimizing this score.

E.g.:
Scoring rules
Copeland
Maximin
…

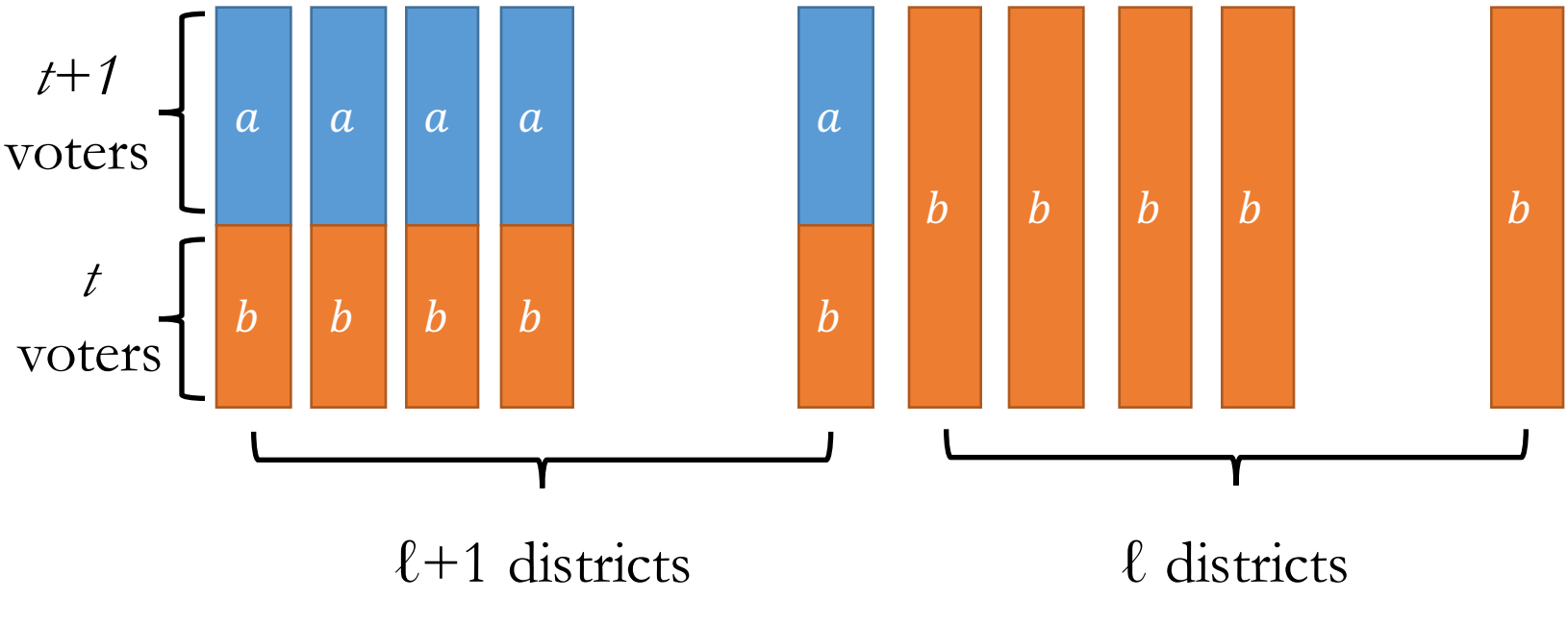
# Price of districting

How much are voters being misrepresented?
(for score-based voting rules *f*)

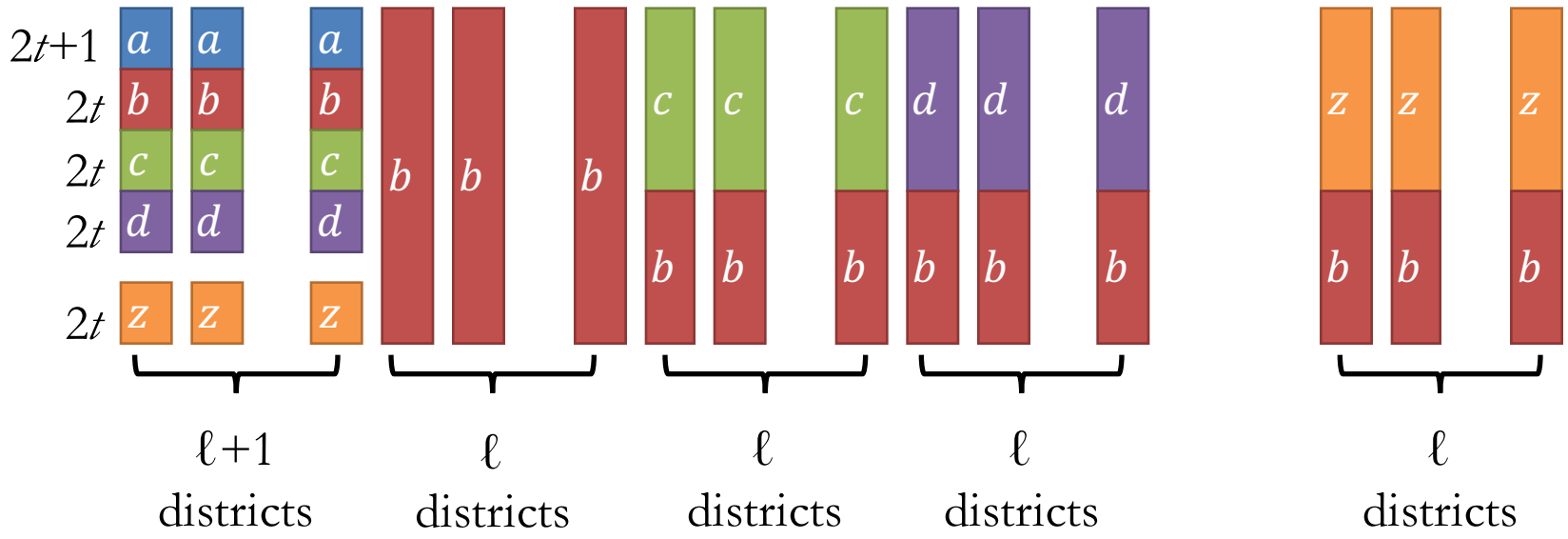
$$\max_{i \in C} \frac{\text{score of candidate } i \text{ in } f(V)}{\text{score of winning candidate in } f(V)}$$

# Plurality 2 candidates

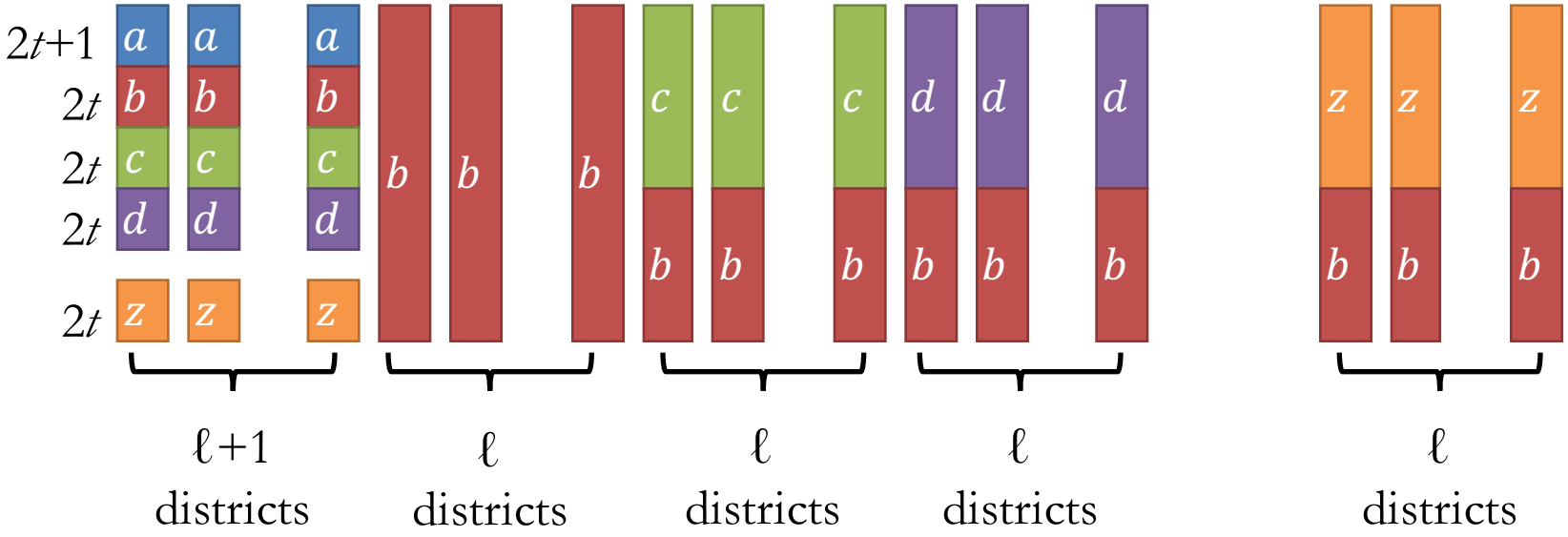
2 $\ell$+1 districts, each with 2$t$+1 voters



# Plurality *m* candidates



# Plurality *m* candidates




$$1 + \frac{n - 2\lceil \frac{n}{2} \rceil + 1}{q+2} + \frac{(z+1)(\lceil \frac{n}{2} \rceil - 1) - n}{(\ell+2)(q+2)} \approx \Theta(m^2)$$

$$q = \lfloor \frac{n}{m} \rfloor$$

# Plurality majority twist
# *m* candidates

$$\frac{q+1}{q+2} + \frac{n(\lceil \frac{n}{2} \rceil - 1)}{(q+2)(\lfloor \frac{n}{2} \rfloor + 1)} \quad \approx \quad \Theta(m)$$

$q = \lfloor \frac{n}{m} \rfloor$

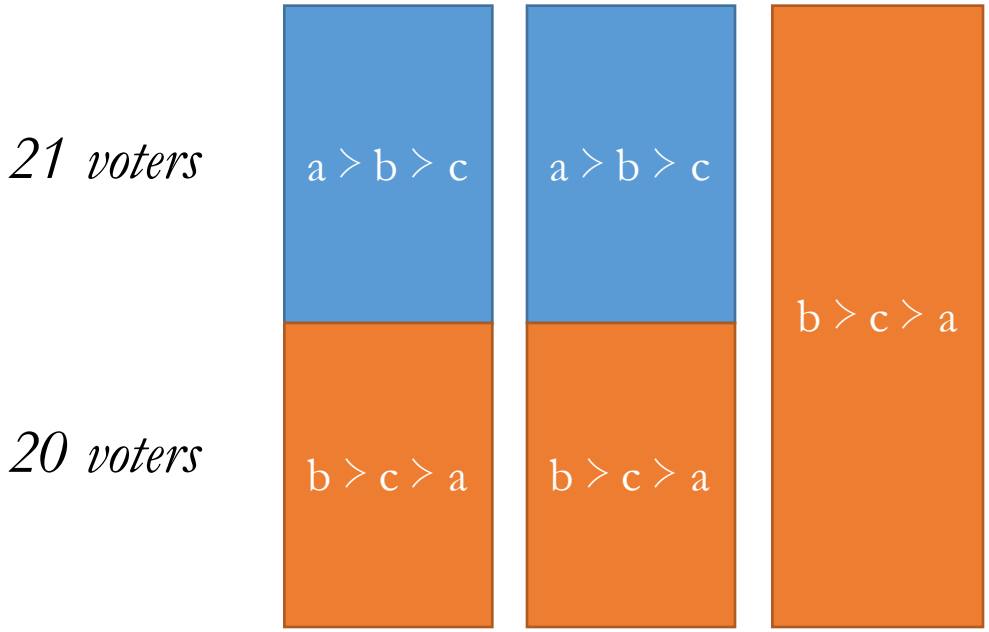
# Other scoring rules

$k$-approval: $\Theta(m^2/k)$

Veto: $\Theta(m)$

Borda: $\Theta(m^2)$

# Copeland




District winner is *a*.

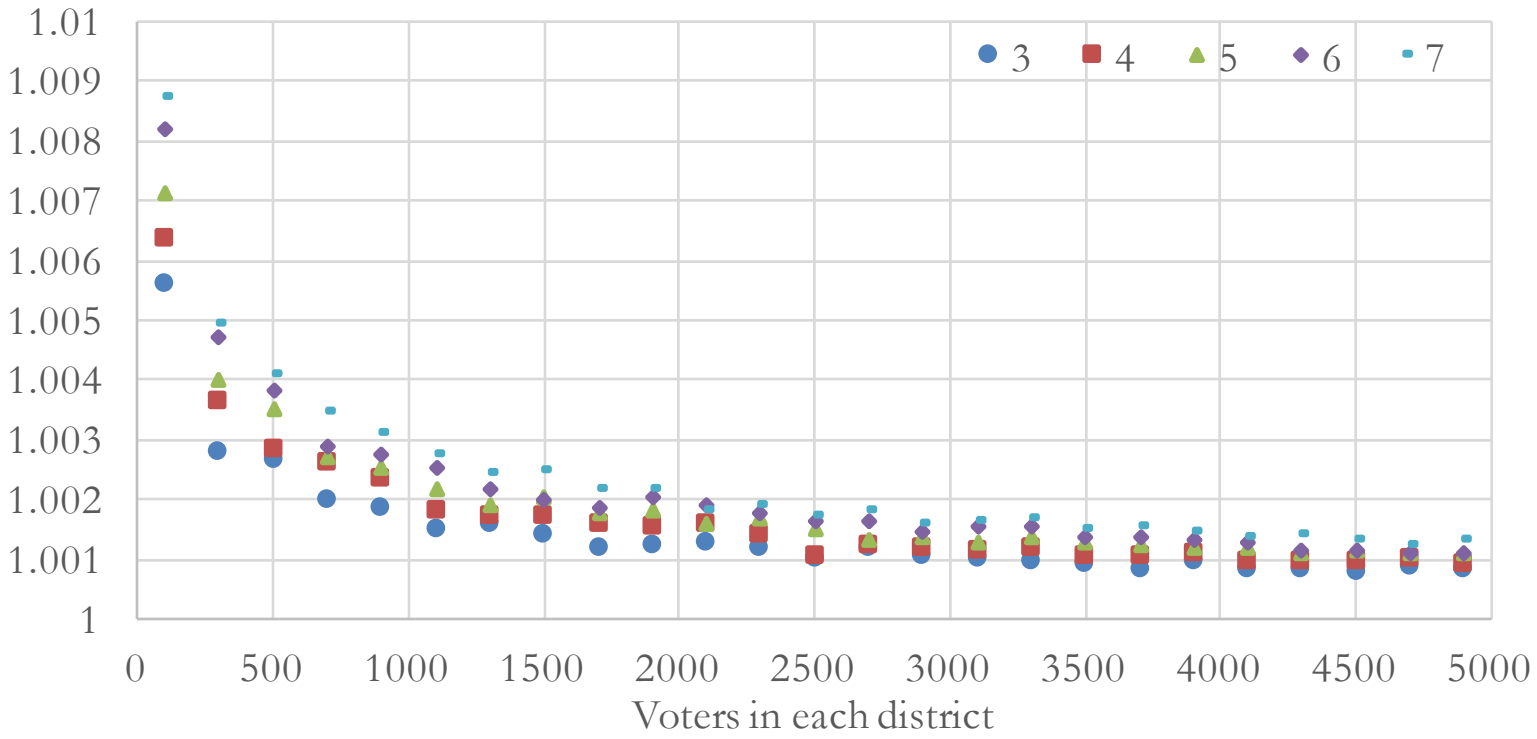
Copeland winner is *b* with score 2, *a* with score 0.

# Copeland Price of districting

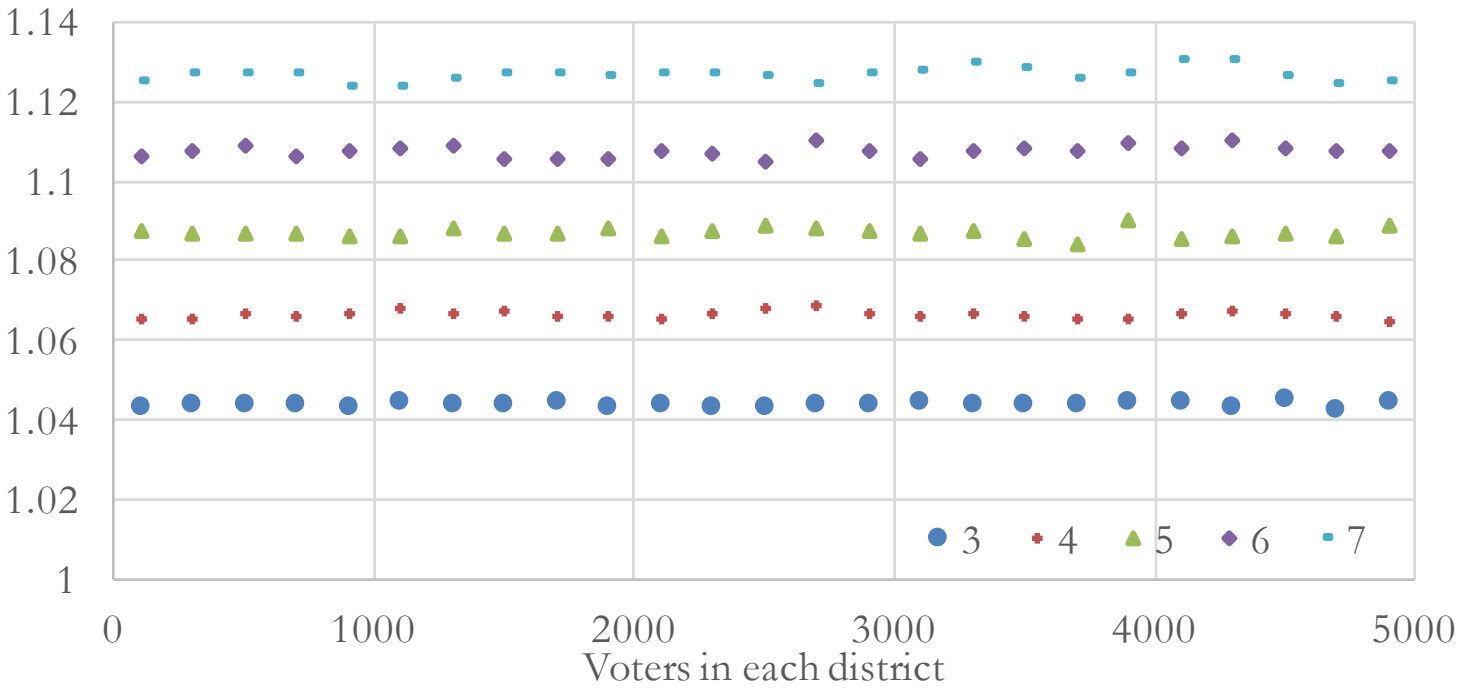
$$\max_{i \in C} \frac{\text{score of candidate } i \text{ in } f(V)+m}{\text{score of winning candidate in } f(V)+m}$$

District winner may have worst possible score, while Copeland winner has best possible score.

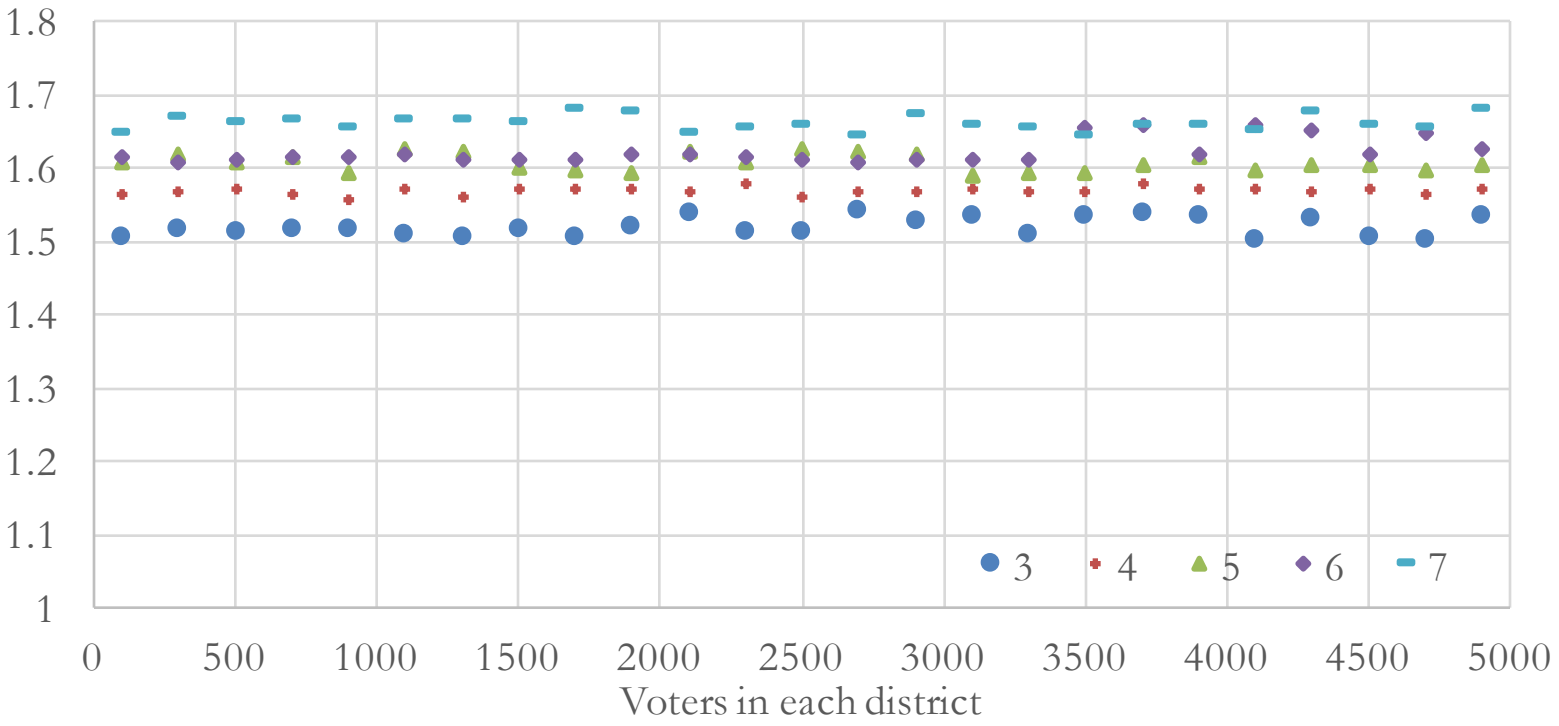
# Simulations: Borda uniform



# Simulations: plurality Mallows



# Simulations: Copeland Mallows



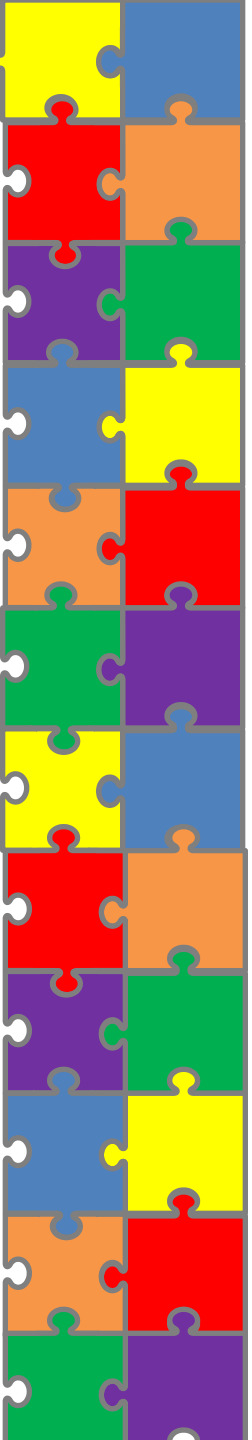
# What's next?

Another paper with Yoad…
(complexity, geography, real world data)

More voting methods

Is Homogeneity/heterogeneity of districts good or bad?

More effects of districts on outcomes and their representability.



# *Fin*

*Thanks for listening!*