# Convergence of Iterative Voting

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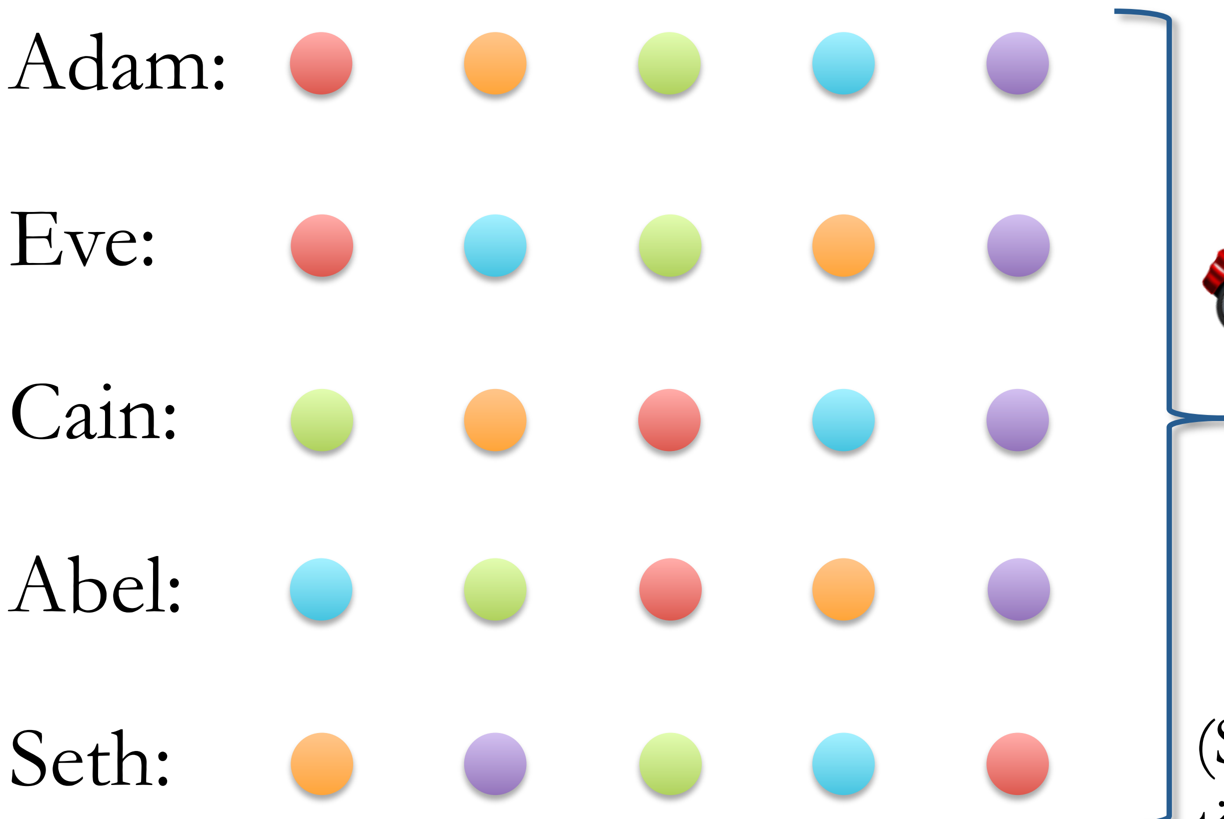
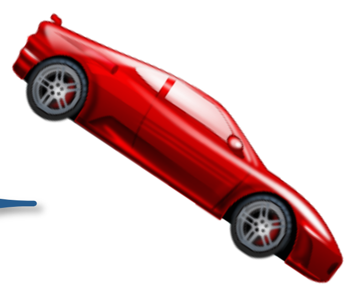
# What is Iterative Voting?

Color of the new car…



(Seth breaks ties)

# What is Iterative Voting?

Color of the new car…

Adam:

Eve:

Cain:

Abel:

Seth:

**Wait a minute!**

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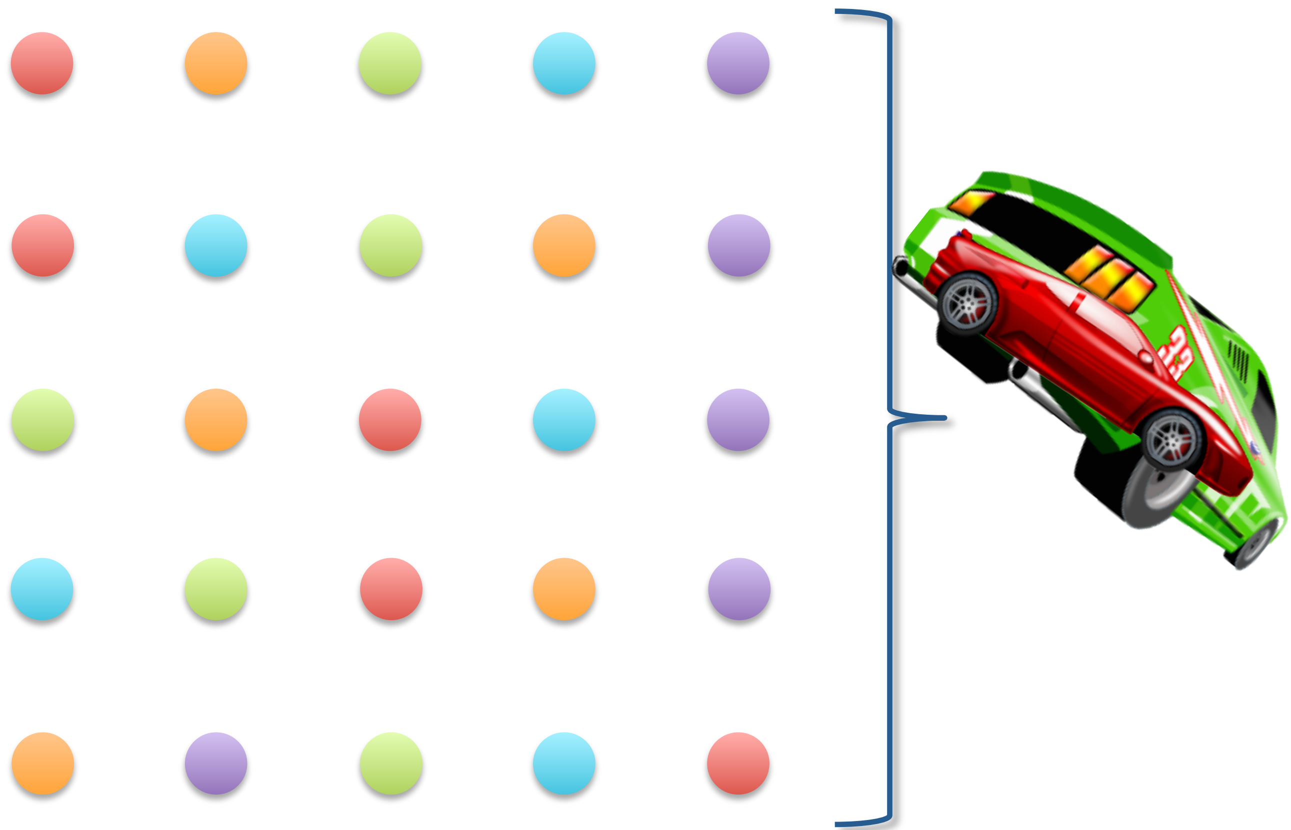

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Wait a minute!

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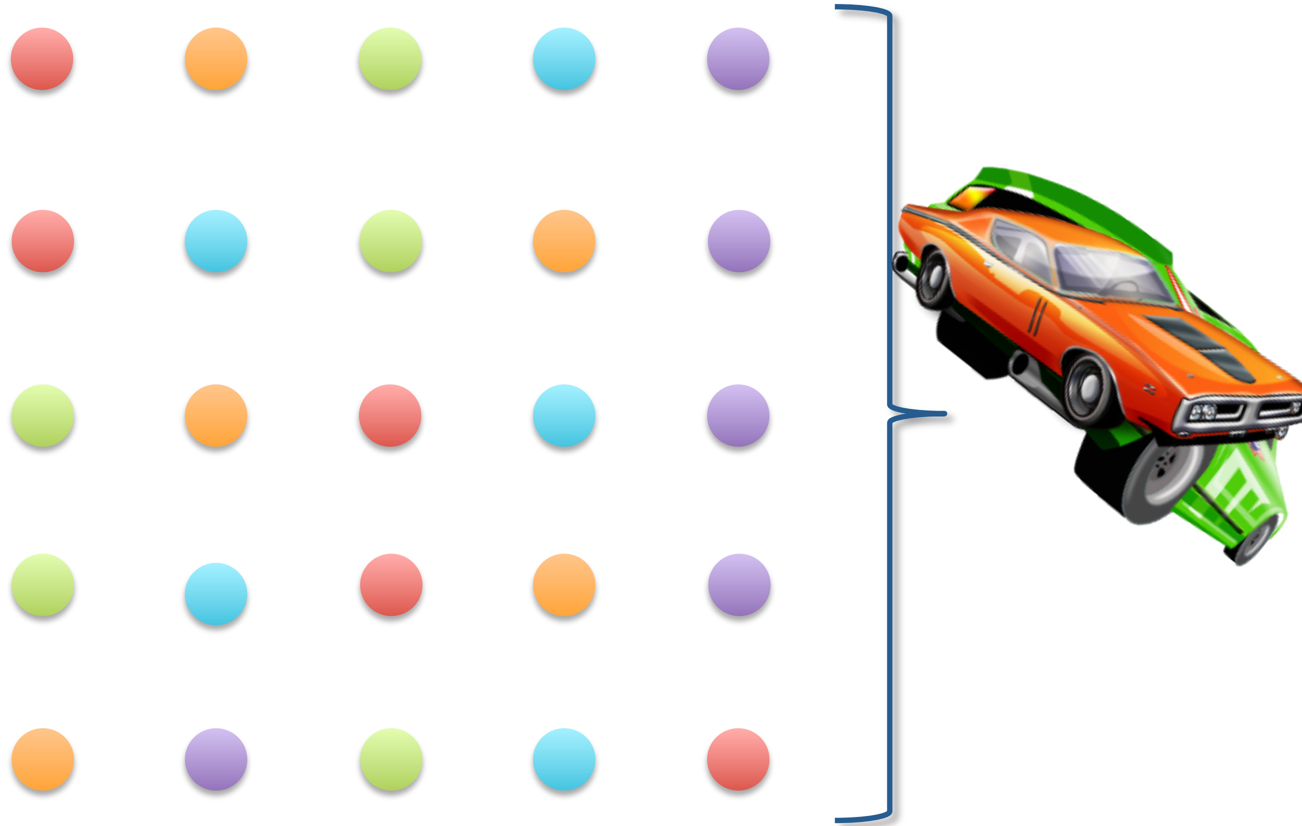


# What is Iterative Voting?

Color of the new car…

Adam:

Eve:

Can’t we all just get along?

Cain:

Abel:

Seth:

# What we know:
# (Meir et al. – AAAI 2010)

Assuming players play a myopic "best response" – reacting to the current state:

## Iterative Plurality converges

2 cases:

- Randomized tie breaking rule: from truthful state

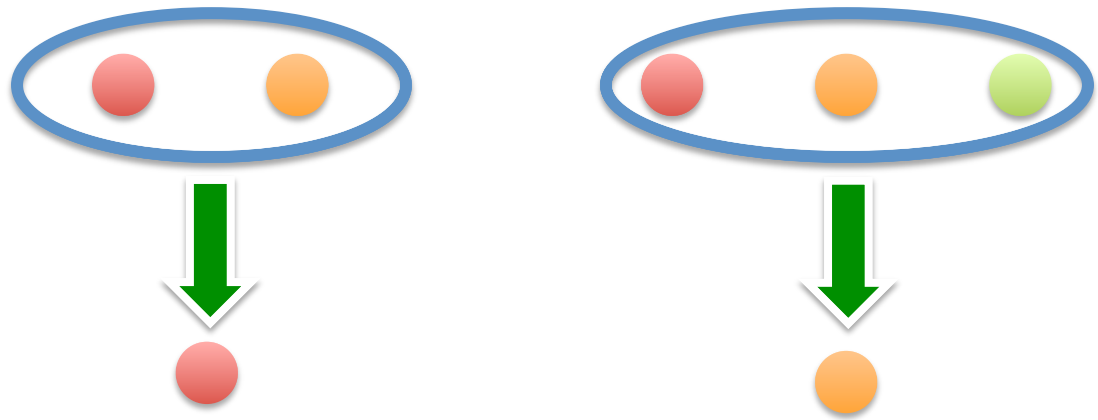
  And linear ordered – i.e., there is a fixed order between candidates, according to which ties are resolved

- Deterministic tie breaking rules: from any state (including non-truthful)

# Tie-breaking rules

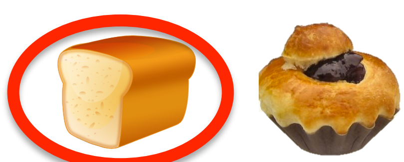
**Linear:**

**Non-linear:**



There is no set order between red and orange

**Pastry example:**
(thanks to
Ilan Nehama)



# Short aside:
# What are scoring rules

Scoring rules for *m* candidates define a scoring vector:

$$(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m)$$

under the condition

$$\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \ldots \geq \alpha_m = 0$$

A voter gives $\alpha_1$ points to his most preferred candidate, $\alpha_2$ points to his 2nd preference, etc.

**The winner is the candidate with most points**

# Short aside:
# Examples of scoring rules

**Plurality:** $(1,0,\ldots,0,0)$

**Veto:** $(1,1,\ldots,1,0)$

**Borda:** $(m-1,m-2,\ldots,1,0)$

***k*-approval:** $(\overbrace{1,1,\ldots,1}^{k\text{ candidates}},0,0,\ldots,0)$

***k*-veto:** $(1,1,\ldots,1,\overbrace{0,0,\ldots,0}^{k\text{ candidates}})$

# Theorem I:
# Tie-breaking rules matter

When using any arbitrary tie-breaking rule (i.e., not necessarily linear ones), every scoring rule & Maximin has tie-breaking rule for which it will not always converge

# Theorem 1: Proof sketch (scoring rules)

4 candidates, 2 voters, tie breaking rule makes *c* win if not tied with *b*. *b* wins if not tied with *d*. *d* wins if not tied with *a*.

a ≻…≻ b ≻ c ≻ d
c ≻…≻ d ≻ b ≻ a

→

b ≻…≻ a ≻ d ≻ c
c ≻…≻ d ≻ b ≻ a

↓

b ≻…≻ a ≻ d ≻ c
d ≻…≻ c ≻ a ≻ b

←

a ≻…≻ b ≻ c ≻ d
d ≻…≻ c ≻ a ≻ b

↑ (back to the first profile)

# Theorem II: Borda doesn't work

When using the Borda voting rule, regardless of tie-breaking rules, the iterative process may never converge

# Theorem II: Proof sketch

4 candidates, 2 voters (tie breaking doesn't matter):

a > b > c > d
c > d > b > a
d – 2; a, b – 3; c – 4

b > a > d > c
c > d > b > a
a – 2; c, d – 3; b – 4

a > b > c > d
d > c > a > b
b – 2; c, d – 3; a – 4

b > a > d > c
d > c > a > b
c – 2; a, b – 3; d – 4

# Theorem III:
# Iterative Veto converges

When using linear tie-breaking rules, iterative Veto will always converge – from truthful or non-truthful starting point

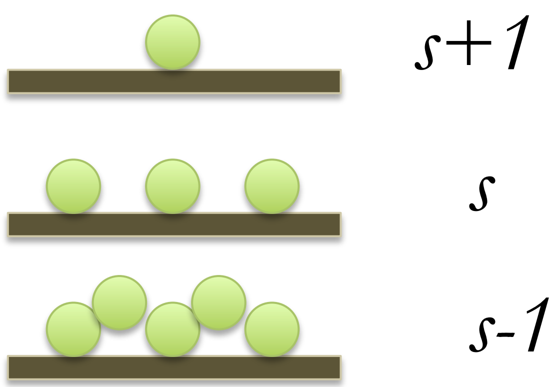
# Theorem III: Proof

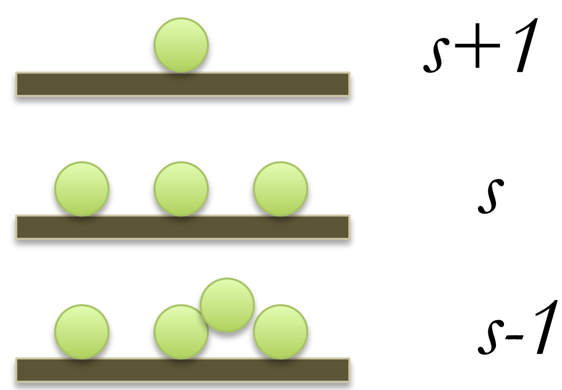
"Best response" straight-forwardly defined as **vetoing the current (unwanted) winner**.

*Lemma 1*: If there is a cycle, taking a stage in the cycle where there is more than one candidate with the maximal score, suppose winner score is $s$. Then **winning score at any other stage is $s$ or $s+1$**. Any stage with $s+1$ score has only one candidate with that score.

# Theorem III: Proof lemma I

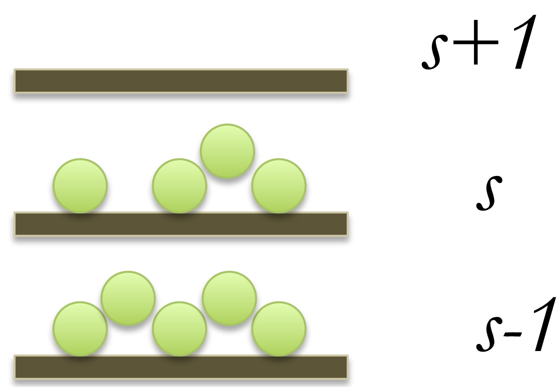
The futility of having a single winner – the score can't get higher, and you can't get multiple candidates to share the score:



# Theorem III: Proof

*Lemma 2*: If there is a cycle, **all stages with more than one candidate with the maximal score have the same number of candidates with maximal score and maximal-1 score**, and these are the same candidates in all the cycle.



# Theorem III: Proof

2 types of player moves:

A candidate with a score of *s* becomes winner with score of *s+1*

A candidate with a score of *s-1* gets point and becomes winner

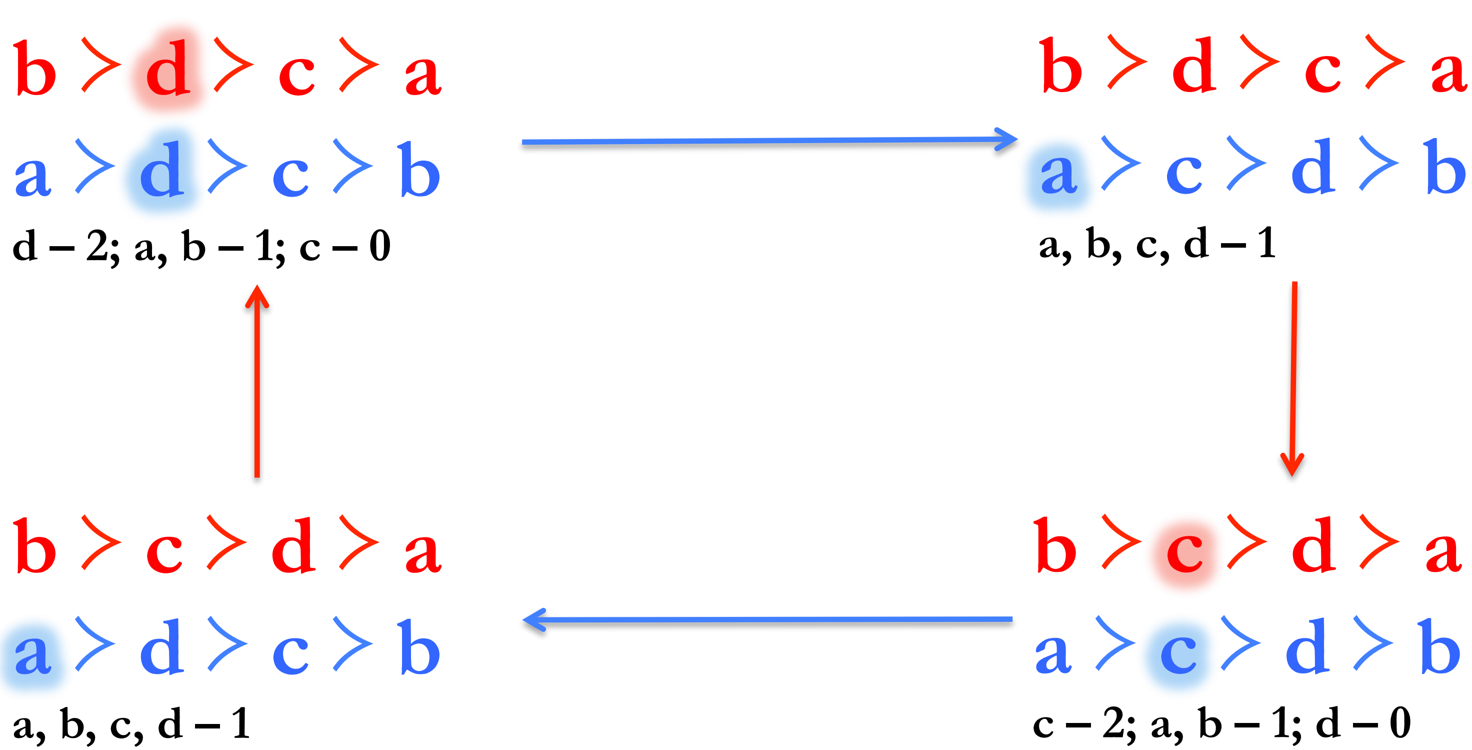
Previously vetoed candidates become winners (gaining a point), i.e., voters' situation progressively worse. This is a **finite process**

# Theorem IV:
# *k*-Approval doesn't work

**When using *k*-approval voting rule for *k*≥2, even with linear tie-breaking rule, the iterative process may never converge**

# Theorem IV: Proof sketch

4 candidates, 2 voters, and the tie breaking rule is alphabetical (a > b > c > d)



# Current problems: lazy-best Borda (with Maria Polukarov)

**Lazy-best** means we put the new winner in 1st place, and push everyone else back one spot.

**Does this converge with Borda?**

Using a simulator, it seems lazy-best Borda converges.

If we don't allow ties, it's easy to prove convergence. **Tie-breaking is key**.

Score increase may be high (up to m-1 points), but points are lowered one point at a time – so a cycle has many stages in which maximal score is either static or gets lowered.

# Current problems: Polynomial Veto (with Maria Polukarov)

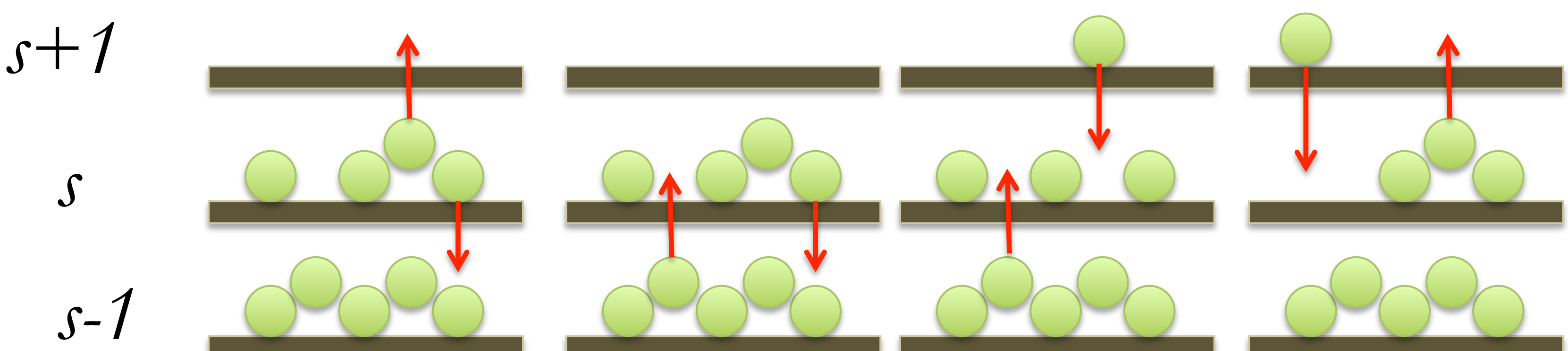
Plurality converges after a polynomial number of steps.

**Does Veto converge in polynomial time?**

Many characteristics found in convergence proof apply:

After initial moves, only candidates with top two scores are relevant

4 types of moves:



# Future work

Better understanding of what influences convergence (tie-breaking rules identified, what else?)

What is best-response for complex voting rules?

Moving beyond myopic best-response to more complex and varied responses

Computational complexity issues for best-response in complex voting rules

Weighted games

# *Fin*

(guess they decided to compromise on the car colors…)

**Thanks for listening!**