False-name-proof and Strategy-proof Voting Rules under Separable Preferences

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Abstract

We consider the problem of a society that uses a voting rule to choose a subset from a given set of objects (candidates, binary issues, or alike). We assume that voters' preferences over subsets of objects are separable: Adding an object to a set leads to a better set if and only if the object is good (as a singleton set, the object is better than the empty set). A voting rule is strategy-proof if no voter benefits by not revealing their preferences truthfully and it is false-name-proof if no voter gains by submitting several votes under other identities. We characterize all voting rules that satisfy false-name-proofness, strategy-proofness, and ontoness as the class of voting rules in which an object is chosen if it has either at least one vote in every society or a unanimous vote in every society. To do this, we first prove that if a voting rule is false-name-proof, strategy-proof, and onto, then the identities of the voters are not important.

1 Introduction

Societies take decisions by means of voting rules, mapping profiles of voters' preferences into social alternatives. But since individual preferences are private information, voters may behave strategically by not submitting their preferences truthfully. Voting rules that are immune to this kind of manipulation are called strategy-proof. Another way of behaving strategically, especially when the identities of voters cannot or are not easily verified, is by voting several times under other identities. This is in fact a real and growing phenomenon in anonymous online voting. Many social decisions processes are held online around the world by voting, particularly during the Covid-19 lockdowns. Selection processes for Massive Open Online (MOO) Courses or MOO Schools, rating systems for goods and services, Internet auctions (a popular part of Electronic Commerce), or Facebook allowing users to vote on its future terms of use (see Zuckerberg [2009]), are all examples where a voter could benefit from voting multiple times. As many institutions or websites may not have enough resources to correctly identify each voter in these kinds of voting situations, repeated voting can be a highly relevant issue. An online anonymous poll on a specific political issue, run by a popular news website, although not binding, may lead to overwhelming pressure on the government. A voting rule where voters cannot benefit by voting several times is usually known in the literature as false-name-proof (see Yokoo et al. [15]).[1]

When voters' preferences do not have a specific structure, results on voting rules satisfying some form of false-name-proofness are rather negative, even for random voting rules. Bu [3] shows that, under unrestricted domains of voters' preferences, if a voting rule is strategy-proof, anonymous, and population monotonic, then it is strongly false-name-proof; moreover, under strict preferences, the converse also holds.[2] Conitzer [5] characterizes all

[1] Here, we shall say that a voting rule is *false-name-proof* if no voter can benefit by repeating the same vote several times, while a voting rule is *strongly false-name-proof* if no voter can benefit by casting several votes (not necessarily the same). The proof of our main result (Theorem 1) shows that, in our setting with separable preferences, the class of false-name-proof and the class of strongly false-name-proof voting rules do coincide on the family of voting rules satisfying strategy-proofness and ontoness.

[2] A voting rule is anonymous if the names of the voters are not important. Population monotonicity is a strong requirement: when new voters arrive and vote, each voter initially present should not be strictly

anonymous-proof and neutral randomized voting rules under strict preferences over a finite set of alternatives.[3] Each element in the class identified by Conitzer[5] is described by a probability $p \in [0, 1]$ with which an alternative is chosen with uniform probability and with probability $1 - p$ a pair of alternatives is chosen with uniform probability and if all voters unanimously prefer one alternative over the other, this preferred alternative is chosen, and otherwise a fair coin is used to decide between the two. These voting rules are perceived as being very unresponsive to voters' preferences.

Nevertheless, in many applications, the particular structure of the set of alternatives suggests that not all voters' preferences are conceivable. A large literature on social choice presumes that a natural restriction on the domain of voters' preferences holds, one that is meaningful with respect to that structure. A prominent example of this kind of domain restriction is when the set of alternatives has a linear order structure relative to which single-peaked preferences can be naturally defined. Then, the median voter rule (that selects the median of the profile of voters' best alternatives relative to the linear order) is strategy-proof (see Moulin [8]). However, the median voter rule is not false-name-proof since a voter with the lowest best alternative can manipulate the voting rule by casting several votes for her own best alternative. The latter was shown by Todo et al. [11], who also characterize the class of all strongly false-name-proof, efficient, and anonymous voting rules as those that, for each set of voters N (with cardinality n), the voting rule selects the median of the n reported best alternatives together with $n-1$ fixed ballots for a given *a priory* alternative.[4] Some form of false-name-proofness has also been studied in other settings, often under the name of duplication as in Congar and Merlin [4] and García-Lapresta and Martínez-Panero [6]. For instance, in social networks (see Brill et al. [2]), in matching problems (see Todo and Conitzer [10]), in sybil attacks (see Shahaf et al. [9] and Meir et al. [7]), or together with other properties (see Wagman and Conitzer [14] and Waggoner et al. [13]). What most of these papers have in common, is that they assume that voting rules are anonymous, so anonymity does not appear explicitly in their characterizations. Since we are considering problems where voters' identities are not easily verifiable, anonymity is a very natural requirement. However, we show that in our setting with separable preferences, anonymity follows from false-name-proofness, strategy-proofness, and ontoness (see Proposition 3).

In this paper, we consider social choice problems where societies have to choose a subset from a given set of objects (candidates, binary issues, or alike), and voters have separable preferences over subsets of objects. A voter's preference is *separable* over the family of all subsets of objects if the ranking of subsets is guided by the partition separating the set of objects into the set of good objects (as singleton sets, objects that are better than the empty set) and bad objects (as singleton sets, objects that are worse than the empty set). Adding a good object to any set leads to a better set, while adding a bad object leads to a worse set. Note that the best subset of objects of a separable preference is the union of all good objects and that all additively representable preferences are separable. This is the setting considered by Barbera et al. [1], where they characterize the family of all strategy-proof, anonymous, onto, and neutral voting rules as the class of all voting by quota.[5] A (neutral) voting by quota for N specifies (and it can be identified with) an integer q_N between 1 and

better off than she was before.

[3]A randomized voting rule is anonymous-proof if it is anonymous and satisfies strong false-name-proofness and participation (namely, all voters prefer to vote than to abstain), which all together imply strategy-proofness (see Proposition 6 in Subsection 3.4). A randomized voting rule is neutral if it does not depend on the names of the alternatives.

[4]Todo et al.[11] and Todo et al. [12] extend the analysis to the case where the set of alternatives has a tree structure.

[5]They characterize the larger family of all strategy-proof and onto voting rules as the class of voting by committees.

n, where $n = |N|$. Then the choice of the subset of objects made by the voting by quota q_N at a profile of separable preferences is done object-by-object as follows: object x belongs to the chosen set if and only if the set of voters for which x is a good object has cardinality larger or equal to q_N. Hence, voting by quota can be seen as a family of qualified majority voting where the two alternatives at stake (in each voting) are whether or not x belongs to the collectively chosen subset of objects.

We want to identify here, for this setting under separable preferences, simple voting rules that are simultaneously immune to two types of manipulations: not voting according to the true preferences (strategy-proofness) and voting many times (false-name-proofness). Observe that our notion of false-name-proofness is weaker than most of the notions that can be found in the literature (for example in Conitzer [5], Todo et al. [11], and Bu[3]). We consider situations where each agent must submit one vote under its identity, but it can also repeat this vote using other identities, while the most common notion allows each agent to additionally vote several times submitting any kind of vote, not necessarily equal to the vote cast by the agent under its identity. Our notion of false-name-proofness protects against an extremely easy way to manipulate an election by a (not necessarily human) voter: clicking several times to submit the same vote seems easier than voting several times, changing the vote each time.

On the way to our characterization result, we prove some auxiliary results. The first one is important by itself since it states that any false-name-proof, strategy-proof, and onto voting rule is indeed anonymous. Hence, anonymity does not have to be explicitly imposed (as most of the literature does). This implication is also very intuitive: if no voter has the incentive to use other identities to repeat her vote, then voters' identities should not matter at all. Additionally, we prove two lemmas that identify implications that false-name-proofness has in this setting: an object should be chosen if either it has at least one vote in every possible set of voters or if it has a unanimous vote in every possible set of voters. These results help us to build the proof for our main result (Theorem 1) that characterizes all voting rules that satisfy false-name-proofness, strategy-proofness, and ontoness as the class of voting by quota where to be chosen, each object needs either at least one vote or a unanimous vote. This means that each voter can either impose the object (by voting for it) or veto the object (by not voting for it). Our proof of Theorem 1 shows that false-name-proofness in this characterization can be replaced by strong false-name-proofness (Corollary 1). Moreover, we show in Proposition 4 that if a voting rule is strategy-proof, false-name-proof, and onto, then it satisfies participation. In Proposition 6 we establish that, in any setting and any domain of preferences, if a voting rule is strongly false-name-proof and satisfies anonymity and participation, then it is strategy-proof as well. Finally, Example 1 indicates that the statement of Proposition 6 does not hold if strong false-name-proofness is replaced by false-name-proofness, even under the domain of separable preferences.

The rest of the paper is organized as follows. Section 2 contains preliminaries, the definition of separable preferences, properties of voting rules, and the definition of voting by quota. Section 3 contains the results. In Subsection 3.1 we state Theorem 1, the main result of the paper. Subsection 3.2 contains preliminary results that are used in the proof of Theorem 1. In Subsection 3.3 we prove Theorem 1 and state two of its Corollaries. Subsection 3.4 contains additional results. Finally, in Subsection 3.5 we show that the axioms used in Theorem 1 are independent.

2 Preliminaries and definitions

2.1 Voters, alternatives, separable preferences, and voting rules

We are interested in studying social choice procedures under which societies choose a subset from a given set of objects, as in Barbera et al. [1]. Our aim is to identify those that are simultaneously immune to voters' manipulations by revealing non-truthful preferences and by providing additional preferences under other identities. While the first property assumes that the society is fixed, the second one requires considering different societies. For this reason, we consider societies with a variable set of voters. Let $\mathcal{N}$ be the family of all finite and no-empty subsets of the set of positive integers $\mathbb{Z}_+$. An element $N \in \mathcal{N}$ is interpreted as a society. We denote the cardinality of N by n and refer to an element $i \in N$ as a *voter*. Each set of voters $N \in \mathcal{N}$ has to collectively choose a subset from a given finite set $\mathcal{M} = \{1, \ldots, M\}$ of objects. Then the set of *alternatives* from which any society has to choose from is the family of all subsets of objects $2^{\mathcal{M}}$.

For each voter i, let P_i be voter i's preference over $2^{\mathcal{M}}$. We assume that P_i is a strict linear order and denote by $\mathcal{D}$ a generic set of strict preferences over $2^{\mathcal{M}}$, which we will refer to as a *domain*. We denote the weak counterpart of P_i by R_i. A *profile* (for $N \in \mathcal{N}$) is an ordered list of preferences $P = (P_i)_{i\in N} \in \mathcal{D}^N$, one for each voter in N. By convention, we set $\mathcal{D}^{\emptyset} = \emptyset$. For $N, N' \in \mathcal{N}$ (with $N \cap N' = \emptyset$) and two profiles of preferences $P = (P_i)_{i\in N} \in \mathcal{D}^N$ and $P' = (P'_i)_{i\in N'} \in \mathcal{D}^{N'}$, we denote the profile $((P_i)_{i\in N}, (P'_i)_{i\in N'}) \in \mathcal{D}^{N\cup N'}$ by (P, P'). Let $P = (P_i)_{i\in N} \in \mathcal{D}^N$ be a profile, let $i \in N$ be a voter, and let $S \subset N$ be a subset of N, we denote by $P_{-i} \in \mathcal{D}^{N\setminus\{i\}}$ the profile obtained from P after deleting P_i and by $P_S \in \mathcal{D}^S$ and by $P_{-S} \in \mathcal{D}^{N\setminus S}$ the profiles obtained from P restricted to S and to $N \setminus S$ respectively.

Let P_i be a preference over $2^{\mathcal{M}}$. An object is good (respectively, bad) according to P_i if as a singleton set is strictly preferred (respectively, less preferred) to the empty set. A preference P_i is separable if the division between good and bad objects guides the ordering between some pairs of subsets: adding a good object to any set leads to a better set, while adding a bad object to any set leads to a worse set. Formally, a preference $P_i \in \mathcal{D}$ is *separable* if for all $T \in 2^{\mathcal{M}}$ and $x \notin T$,

$$T \cup \{x\}\ P_i\ T \text{ if and only if } \{x\}\ P_i\ \emptyset.$$

Let $\mathcal{S}$ be the set of all separable preferences over $2^{\mathcal{M}}$.

A preference $P_i \in \mathcal{D}$ is *additive* if there exists a function $u : \mathcal{M} \cup \{\emptyset\} \to \mathbb{R}$ such that $u(\emptyset) = 0$ and for all $T, T' \in 2^{\mathcal{M}}$,

$$T\ P_i\ T' \text{ if and only if } \sum_{x\in T} u(x) > \sum_{x\in T'} u(x),$$

where by convention we set $\sum_{x\in\widehat{T}} u(x) = 0$ for $\widehat{T} = \emptyset$. In this case, we say that u (additively) represents P_i. Of course, all additive preferences are separable. Let $\mathcal{A}$ be the set of all additive preferences over $2^{\mathcal{M}}$.

Given $P_i \in \mathcal{D}$, let $b(P_i)$ and $w(P_i)$ be respectively the best and the worst subsets of $2^{\mathcal{M}}$ according to P_i; namely, $b(P_i)P_iT$ for all $T \in 2^{\mathcal{M}}\setminus b(P_i)$ and $TP_iw(P_i)$ for all $T \in 2^{\mathcal{M}}\setminus w(P_i)$. It is easy to see that if $P_i \in \mathcal{S}$, then $b(P_i) = \{x \in \mathcal{M} \mid \{x\}P_i\emptyset\}$ and $w(P_i) = \{x \in \mathcal{M} \mid \emptyset P_i\{x\}\}$.

A voting rule for $N \in \mathcal{N}$ on a domain $\mathcal{D}$ selects a subset of $\mathcal{M}$ for each profile $P \in \mathcal{D}^N$. Namely, given a domain $\mathcal{D}$, a *voting rule for* N is a mapping $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$. A *voting rule* $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ *(on* $\mathcal{D}$*)* is a family of voting rules, one for each $N \in \mathcal{N}$.

2.2 Properties of voting rules

We define desirable properties for voting rules.

The first property states that all possible subsets of objects should be feasible to be selected; that is, for each $N \in \mathcal{N}$, f_N is onto. Barbera et al. [1] refer to ontoness as voters' sovereignty.

Definition 1 *A voting rule $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ for N is* onto *if, for all $T \in 2^{\mathcal{M}}$, there exists a $P \in \mathcal{D}^N$ such that $f_N(P) = T$. A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is* onto *if f_N is onto for each $N \in \mathcal{N}$.*

The following two properties are very natural in online environments, and state that no voter nor object should receive a differential treatment.

Definition 2 *A voting rule $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ for N is* anonymous *if, for each bijection $\sigma : N \to N$ and each $P \in \mathcal{D}^N$, $f_N(P) = f_N(\sigma(P))$, where $\sigma(P) = (P_{\sigma(i)})_{i \in N}$. A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is* anonymous *if f_N is anonymous for each $N \in \mathcal{N}$.*

Definition 3 *A voting rule $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ for N is* neutral *if, for each bijection $\mu : \mathcal{M} \to \mathcal{M}$ and each $P \in \mathcal{D}^N$, $\mu(f_N(P)) = f_N(\mu(P))$, where $\mu(T)$ and $\mu(P) = (P'_i)_{i \in N}$ are the subset of objects and the preference profile obtained respectively from $T \in 2^{\mathcal{M}}$ and $P \in \mathcal{D}^N$ by permuting the objects according to μ; namely, $\mu(T) = \{x \in \mathcal{M} \mid x = \mu(y) \text{ for } y \in T\}$ and, for each $i \in N$ and pair $S, T \in 2^{\mathcal{M}}$, $\mu(S)\ P'_i\ \mu(T)$ if and only if $S\ P_i\ T$. A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is* neutral *if f_N is neutral for each $N \in \mathcal{N}$.*

The first manipulation property that we are interested in is the one that requires that the voting rule should give voters incentives to be truthful.

Definition 4 *A voting rule $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ for N is* strategy-proof *if for all $P \in \mathcal{D}^N$, $i \in N$ and $P'_i \in \mathcal{D}$,*

$$f_N(P_i, P_{-i})\ R_i\ f_N(P'_i, P_{-i}).$$

A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is strategy-proof *if f_N is strategy-proof for each $N \in \mathcal{N}$.*

Another important requirement, especially in online voting, is that a voter should never have incentives to cast repeated votes.

Definition 5 *A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is* false-name-proof *if for all $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, all $i \in N$ and all $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ such that $N \cap N' = \emptyset$ and $P'_j = P_i$ for all $j \in N'$,*

$$f_N(P)\ R_i\ f_{N \cup N'}(P, P').$$

Conitzer [5] refers to false-name-proof (randomized) voting rules as those satisfying a stronger version of our notion for non-randomized voting rules. Conitzer's [5] condition imposes stronger restrictions on the voting rule by not requiring that the additional preferences submitted by voter $i \in N$ coincide with agent i's original preference P_i.[6] For this reason, we refer here to Conitzer's [5] original notion as strong false-name-proofness.

Definition 6 *A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is* strongly false-name-proof *if for all $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, all $i \in N$ and all $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ such that $N \cap N' = \emptyset$,*

$$f_N(P) R_i f_{N \cup N'}(P, P').$$

[6]Even more, in Conitzer [5] it is not required that the voter in N submits its true preference at all.

Moreover, Conitzer [5] also requires that (randomized) voting rules induce voters to vote. We shall show that, in our context, this participation property follows from false-name-proofness, strategy-proofness, and ontoness. For non-randomized voting rules, participation is as follows.

Definition 7 *A voting rule* $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ *satisfies* participation *if, for all* $(N, P) \in \mathcal{N} \times \mathcal{D}^N$ *such that* $n \geq 2$ *and all* $i \in N$*,*

$$f_N(P) \; R_i \; f_{N \setminus \{i\}}(P_{-i}).$$

[5] refers to a voting rule as anonymous-proof if it satisfies strong false-name-proofness, anonymity, and participation. We shall show in Proposition 6 below that, in any domain of preferences, strategy-proofness follows from these three properties.

2.3 Voting by committees and voting by quota

To define a class of anonymous voting rules required to state our main result, we need the concept of a committee.

Definition 8 *A committee* $\mathcal{W}_N$ *is a non-empty family of non-empty coalitions of* N*, which satisfies coalition monotonicity; that is, if* $I \in \mathcal{W}_N$ *and* $I \subseteq J$*, then* $J \in \mathcal{W}_N$*. Such coalitions are called winning. A minimal winning coalition is an* $I \in \mathcal{W}_N$ *such that for every* $J \subsetneq I$*,* $J \notin \mathcal{W}_N$*. We denote by* $\mathcal{W}_N^m$ *the family of minimal winning coalitions of* $\mathcal{W}_N$*.*

Following Barbera et al. [1], we introduce *voting by committees.*

Definition 9 *A voting rule* $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ *is voting by committees for* N *if, for each* $x \in \mathcal{M}$*, there exists a committee* $\mathcal{W}_{N,x}$ *such that, for every* $P \in \mathcal{D}^N$*,*

$$x \in f_N(P) \text{ if and only if } \{i \in N \mid x \in b(P_i)\} \in \mathcal{W}_{N,x}.$$

A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ *is* voting by committees *if for each* $N \in \mathcal{N}$*,* f_N *is voting by committees for* N*.*

Let $N \in \mathcal{N}$ be a given set of voters with cardinality n, and let $x \in \mathcal{M}$ be an object. A *quota* for N and x is an integer $q_{N,x} \in \{1, \ldots, n\}$. Set $q_N = (q_{N,x})_{x \in \mathcal{M}}$.

Definition 10 *A voting rule* $f_N : \mathcal{D}^N \to 2^{\mathcal{M}}$ *for* N *is* voting by quota $q_N = (q_{N,x})_{x \in \mathcal{M}}$ *if, for all* $P \in \mathcal{D}^N$*,*

$$x \in f_N(P) \text{ if and only if } |\{i \in N \mid x \in b(P_i)\}| \geq q_{N,x}.$$

A voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ *is* voting by quota *if, for each* $N \in \mathcal{N}$*,* f_N *is voting by quota* q_N*.*

Note that by definition, a voting by quota rule is very simple. It can be seen as a family of extended majority voting, one for each object x, where voters have to decide whether x is chosen or not; specifically, it is anonymous, as the voting rule only takes into account the number of votes that each object receives, not the voters' identities.

For a fixed society, Barbera et al. [1] characterize the class of strategy-proof and onto voting rules when the preferences are separable or additive representable. The following proposition follows from their results.

Proposition 1 *Let* $\mathcal{D}$ *be a domain such that* $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$*. Then, a voting rule* $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ *is strategy-proof and onto if and only if* f *is voting by committees.*

Adding anonymity to the requirements gives us, also from Barbera et al. [1], the following result.

Proposition 2 *Let* $\mathcal{D}$ *be a domain such that* $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$*. Then, a voting rule* $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ *is strategy-proof, anonymous, and onto if and only if* f *is voting by quota.*

3 Results

3.1 Main result

We now state our main result characterizing the family of all false-name-proof, strategy-proof, and onto voting rules, on any separable domain that contains all additively representable preferences.

Theorem 1 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is false-name-proof, strategy-proof, and onto if and only f is a voting by quota such that, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$.*

For the proof of the main axiom, we need some preliminary results, which are introduced in the following Subsection.

3.2 Preliminary results for Theorem 1

In this Subsection, we introduce additional notation and obtain some preliminary results that will be useful to prove Theorem 1.

Given $x \in \mathcal{M}$, we denote by $u^x : \mathcal{M} \cup \{\emptyset\} \to \mathbb{R}$ any function such that $u^x(x) > |u^x(y)| > u^x(\emptyset) = 0$ for all $y \in \mathcal{M} \setminus \{x\}$, and if $x \in T$ and $x \notin T'$ then $\sum_{y \in T} u^x(y) > \sum_{y \in T'} u^x(y)$ and, accordingly, if u^x represents P_i, then $x \in T$ and $x \notin T'$ imply $T \; P_i \; T'$. Similarly, given $y \in \mathcal{M}$, we denote by $u_y : \mathcal{M} \cup \{\emptyset\} \to \mathbb{R}$ any function such that $|u_y(y)| > |u_y(x)| > u_y(\emptyset) = 0 > u_y(y)$ for all $x \in \mathcal{M} \setminus \{y\}$, and if $y \in T$ and $y \notin T'$ then $\sum_{x \in T} u_y(x) < \sum_{x \in T'} u_y(x)$ and, accordingly, if u_y represents P_i, then $y \in T$ and $y \notin T'$ imply $T' \; P_i \; T$. Let $u^{\emptyset} : \mathcal{M} \cup \{\emptyset\} \to \mathbb{R}$ be any function such that $u^{\emptyset}(x) < 0$ for all $x \in \mathcal{M}$ and, accordingly, if $u^{\emptyset}$ represents P_i, $\{\emptyset\} \; P_i \; T$ for all $T \in 2^{\mathcal{M}} \setminus \{\emptyset\}$. Finally, for each $S \subset N$, we denote by $(P^x_S, P^{\emptyset}_{N \setminus S}) \in \mathcal{A}^N$ an arbitrary additive profile for which $b(P^x_i) = \{x\}$ for all $i \in S$ and $b(P^{\emptyset}_i) = \{\emptyset\}$ for all $i \in N \setminus S$.

We first show that in our context, false-name-proofness, strategy-proofness, and ontoness imply anonymity.

Proposition 3 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be a false-name-proof, strategy-proof, and onto voting rule. Then, f is anonymous.*

Proof. Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be a false-name-proof, strategy-proof, and onto voting rule. Then, by Proposition 1, for each $N \in \mathcal{N}$, f_N is voting by committees for N. Let $(\mathcal{W}_{N,x})_{x \in \mathcal{M}}$ be the family of committees associated to f_N.

Claim *Let $S, N, N' \in \mathcal{N}$ be such that $N \cap N' = \emptyset$ and $S \subset N$, and let $x \in \mathcal{M}$. Then, $S \in \mathcal{W}_{N \cup N',x}$ if and only if $S \in \mathcal{W}_{N,x}$.*

Proof of the Claim. ($\Rightarrow$) We prove it by the contrapositive. Assume $S \notin \mathcal{W}_{N,x}$ holds. Then, for any profile of the form $(P^x_S, P^{\emptyset}_{N \setminus S}) \in \mathcal{A}^N$, $f_N(P^x_S, P^{\emptyset}_{N \setminus S}) = \{\emptyset\}$. Since f is false-name-proof, $f_{N \cup N'}(P^x_S, P^{\emptyset}_{N \setminus S}, P^x_{N'}) = \{\emptyset\}$; otherwise, any $i \in S$ would have the incentive to replicate its vote under the identities of agents in N'. But this means that $S \cup N' \notin \mathcal{W}_{N \cup N',x}$. Since $S \subset N$ and the committee satisfies coalition monotonicity, $S \notin \mathcal{W}_{N \cup N',x}$ holds as well, which is what we wanted to prove.
($\Leftarrow$) We prove it by the contrapositive. Assume $S \notin \mathcal{W}_{N \cup N',x}$ holds. Then, for any profile of the form $(P^x_S, P^{\emptyset}_{N \setminus S}, P^{\emptyset}_{N'}) \in \mathcal{A}^{N \cup N'}$, $f_{N \cup N'}(P^x_S, P^{\emptyset}_{N \setminus S}, P^{\emptyset}_{N'}) = \{\emptyset\}$. Since f is false-name-proof, $f_N(P^x_S, P^{\emptyset}_{N \setminus S}) = \{\emptyset\}$; otherwise, any $i \in N \setminus S$ would have the incentive to replicate

its vote under the identities of agents in N'. But this means that $S \notin \mathcal{W}_{N,x}$ holds as well, which is what we wanted to prove. □

We now prove that f_N is anonymous by induction on the number of voters $|N|$.

For $|N| = 1$, f_N is trivially anonymous.

Induction hypothesis: Assume f_N is anonymous for all $N \in \mathcal{N}$ such that $|N| = n$. By Proposition 2, f_N is voting by quota $q_N = (q_{N,x})_{x \in \mathcal{M}}$.

Consider any $N' \in \mathcal{N}$ such that $|N'| = |N| + 1$. Let $t \notin N$ be the agent for which $N' = N \cup \{t\}$. To obtain a contradiction, assume $f_{N'}$ is not anonymous. Then, there exist $x \in \mathcal{M}$ and two different sets of voters $S', T' \in 2^{N'}$ such that $|T'| = |S'| \leq |N|$ and $S' \in \mathcal{W}_{N',x}$ but $T' \notin \mathcal{W}_{N',x}$. Fix $i \in S' \setminus T'$ and $j \in T' \setminus S'$. We proceed by distinguishing among several cases and subcases.

Case A: $t \notin S'$.

Case A.1: $|S'| < |N|$.

Case A.1.1: $t \notin T'$. We have that $S' \subset N$. Then, by the Claim, $S' \in \mathcal{W}_{N,x}$. By the anonymity of f_N, and since $|S'| = |T'|$, we have that $T' \in \mathcal{W}_{N,x}$. Then, again by the Claim, $T' \in \mathcal{W}_{N',x}$. This is a contradiction with one of the initial hypotheses.

Case A.1.2: $t \in T'$. Define $N_1 = N \backslash \{i\} \cup \{t\}$ and $S'_1 = S' \cap N_1$. Thus, we have that $T' \subset N_1$ and $|S'_1| \leq |T'|$. Assume that $f_{N_1}(P^x_{S'_1}, P^{\emptyset}_{N_1 \backslash S'_1}) = \{x\}$ for a profile of the form $(P^x_{S'_1}, P^{\emptyset}_{N_1 \backslash S'_1}) \in \mathcal{A}^{N_1}$. This would mean that $S'_1 \in \mathcal{W}_{N_1,x}$. As $|N_1| = |N|$ and f_N is anonymous, $T' \in \mathcal{W}_{N_1,x}$. Then, by the Claim and the fact that $N_1 \cup \{i\} = N'$, we have that $T' \in \mathcal{W}_{N',x}$, a contradiction with one of the initial hypothesis.

Case A.2: $|S'| = |N|$. Since $j \in T' \setminus S'$, $t = j \in T'$ holds. Consider the following sets of agents: $N' = N \cup \{t\} = N \cup \{j\}$, $N_1 = N \backslash \{i\} \cup \{j, z\}$, with $z \notin N \cup \{j\}$, and $N_2 = N \cup \{j, z\}$. Assume $T' \notin \mathcal{W}_{N_2,x}$. Then, as $T' \subset N_1$, by the Claim, $T' \notin \mathcal{W}_{N_1,x}$. Accordingly, for any profile of the form $(P^x_{T'}, P^{\emptyset}_z) \in \mathcal{A}^{N_1}$, $f_{N_1}(P^x_{T'}, P^{\emptyset}_z) = \{\emptyset\}$. But then, an agent in T' can vote with i's identity and obtain $f_{N_2}(P^x_{T'}, P^{\emptyset}_z, P^x_i) = \{x\}$. To see that this last equality holds, observe that $S' \subset T' \cup \{i\} = N'$. Then, $S' \in \mathcal{W}_{N',x}$ implies that, by coalition monotonicity of the committee, $T' \cup \{i\} \in \mathcal{W}_{N',x}$. Therefore, by the Claim, $T' \cup \{i\} \in \mathcal{W}_{N_2,x}$, and so the equality holds, which constitutes a violation of false-name-proofness. Hence, our assumption that $T' \notin \mathcal{W}_{N_2,x}$ was not correct. Thus, $T' \in \mathcal{W}_{N_2,x}$. Then, by the Claim, we obtain that $T' \in \mathcal{W}_{N',x}$, a contradiction with one of the initial hypothesis.

Case B: $t \in S'$. Define the set of n agents $N^* = N' \backslash \{j\}$. Then, considering that $N' = N^* \cup \{j\}$, we apply the same reasoning as the one used in Cases A.1.2 and A.2 to reach a contradiction. Thus, $f_{N'}$ is anonymous as well. ■

The second preliminary result shows the effect that false-name-proofness has on the quotas of a voting rule.

Lemma 1 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be a false-name-proof, strategy-proof, and onto voting rule. Then, f is voting by quota $q = (q_{N,x})_{N \in \mathcal{N}, x \in \mathcal{M}}$ where, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$.*

Proof. Assume $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is a false-name-proof, strategy-proof, and onto voting rule. By Propositions 1, 2, and 3, for each $N \in \mathcal{N}$, f_N is a voting by quota $q_N = (q_{N,x})_{x \in \mathcal{M}}$. For an object $x \in \mathcal{M}$, a set of voters N, and a profile $P \in \mathcal{D}^N$, define $x_N(P) = \{i \in N \mid x \in b(P_i)\}$.

Claim *Let $N, N' \in \mathcal{N}$ be such that $N \cap N' = \emptyset$. Then, $q_{N,x} \leq q_{N \cup N',x}$ for all $x \in \mathcal{M}$.*

Proof of the claim. To obtain a contradiction, suppose that $q_{N,x} > q_{N \cup N',x}$ for an object $x \in \mathcal{M}$. Hence, $q_{N,x} > 1$. Consider $i \in N$ and $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u^x and $x_N(P) = q_{N,x} - 1$. Observe that $x \in b(P_i)$ by definition of u^x, and $x \notin f_N(P)$ by definition of voting by quota q_N. Consider now a profile $P' \in \mathcal{D}^{N'}$ such that $P'_j = P_i$ for all $j \in N'$. Since $N \cap N' = \emptyset$, $x_{N\cup N'}(P, P') = x_N(P) + x_{N'}(P') = q_{N,x} - 1 + n' > q_{N\cup N',x}$, by our contradiction hypothesis and $n' \geq 1$. Since $f_{N\cup N'}$ is voting by quota $q_{N\cup N'}$, $x \in f_{N\cup N'}(P, P')$ and, by definition of u^x, $f_{N\cup N'}(P, P')\ P_i\ f_N(P)$, a contradiction with false-name-proofness. □

To proceed with the proof of Lemma 1, first observe that by the definition of voting by quota, anonymity, and the Claim, for a given x, $q_{1,x} = 1 \leq q_{2,x} \leq 2$ holds, where $q_{k,x} = q_{S,x}$ for any $S \in \mathcal{N}$ with $|S| = k$. Fix $x \in \mathcal{M}$. We distinguish between the two possible cases and, for each of them, we show that the statement of Lemma 1 holds by induction on n (the number of voters).

Case 1: $q_{1,x} = q_{2,x} = 1$. *Induction hypothesis*: Assume $q_{1,x} = \cdots = q_{n,x} = 1$.
Consider the case $n + 1$, where $n \geq 2$. We want to show that $q_{n+1,x} = 1$. To obtain a contradiction, suppose $q_{n+1,x} > q_{n,x} = 1$. Consider $i \in N$ and a profile $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u_x and $x_N(P) = 1$. Observe that $x \notin b(P_i)$ by definition of u_x, and $x \in f_N(P)$ by definition of voting by quota $q_{n,x}$. Let $j \notin N$ and $P_j = P_i$. Then, $x_{N\cup\{j\}}(P, P_j) = 1 < q_{x,n+1}$ and, accordingly, $x \notin f_{N\cup\{j\}}(P, P_j)$, which is a contradiction with false-name-proofness because, by definition of u_x, $f_{N\cup\{j\}}(P, P_j)\ P_i\ f_N(P)$.

Case 2: $q_{1,x} = 1 < 2 = q_{2,x}$. *Induction hypothesis*: Assume $q_{x,t} = t = |T|$ for every $T \in \mathcal{N}$ such that $1 \leq t \leq n$.
Consider the case $n + 1$, where $n \geq 2$. We want to show that $q_{n+1,x} = n + 1$. By the definition of quota, the Claim, and the induction hypothesis we have that $n \leq q_{n+1,x} \leq n + 1$. Suppose that $q_{n+1,x} = n$. Consider $i \in N$, $x \in \mathcal{M}$ and a profile $P \in \mathcal{D}^N$ such that $P_i \in \mathcal{A}$ is additively represented by u^x and $x_N(P) = q_{n,x} - 1 = n - 1$. Observe that $x \in b(P_i)$ by definition of u^x, and $x \notin f_N(P)$ by definition of voting by quota q_n. Let $j \notin N$ and $P_j = P_i$. Then, $x_{N\cup\{j\}}(P, P_j) = n - 1 + 1 = q_{n+1,x}$ and, accordingly, $x \in f_{N\cup\{j\}}(P, P_j)$, which is a contradiction with false-name-proofness because, by definition of u^x, $f_{N\cup\{j\}}(P, P_j)\ P_i\ f_N(P)$.

Thus, f is voting by quota $q = (q_{N,x})_{N\in\backslash,x\in\mathcal{M}}$ where, for every $x \in \mathcal{M}$, either $q_{N,x} = 1$ for every $N \in \mathcal{N}$ or $q_{N,x} = n$ for every $N \in \mathcal{N}$. ■

The third preliminary result states that voting by quota in this subclass satisfies false-name-proofness.

Lemma 2 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ be a voting by quota $q = (q_{N,x})_{N\in\mathcal{N},x\in\mathcal{M}}$ such that, for every $x \in \mathcal{M}$, either $q_{N,x} = 1$ for every $N \in \mathcal{N}$ or $q_{N,x} = n$ for every $N \in \mathcal{N}$. Then f is false-name-proof.*

Proof. Let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ be a voting by quota $q = (q_{N,x})_{N\in\mathcal{N},x\in\mathcal{M}}$.

Assume first that, for a given $x \in \mathcal{M}$, $q_{N,x} = n$ for all $N \in \mathcal{N}$. We show that f is strongly false-name-proof, which would imply that f is false-name-proof. Let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, $i \in N$, and $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ be arbitrary, and assume $N \cap N' = \emptyset$. As f_N and $f_{N\cup N'}$ are quota n and $n + n'$ respectively, $f_N(P) = \cap_{j\in N} b(P_j) \supseteq \cap_{j\in N\cup N'} b(P_j) = f_{N\cup N'}(P, P')$. Therefore, $f_{N\cup N'}(P, P') = f_N(P) \setminus B$ for some $B \subseteq b(P_i)$. By iteratively applying separability to each object in B and transitivity of P_i, we obtain that $f_N(P)\ R_i\ f_{N\cup N'}(P, P')$ and so f is strongly false-name-proof. Therefore f is also false-name-proof.

Assume now that, for a given $x \in \mathcal{M}$, $q_{N,x} = 1$ for all $N \in \mathcal{N}$. We show that f is strongly false-name-proof, which would imply that f is false-name-proof. Let $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, $i \in N$, and $(N', P') \in \mathcal{N} \times \mathcal{D}^{N'}$ be arbitrary, and assume $N \cap N' = \emptyset$. As f_N and $f_{N\cup N'}$ are both quota one, $b(P_i) \subseteq f_N(P) = \cup_{j\in N} b(P_j) \subseteq \cup_{j\in N\cup N'} b(P_j) = f_{N\cup N'}(P, P')$. Therefore, $f_{N\cup N'}(P, P') = f_N(P) \sqcup W$ for some $W \subseteq w(P_i)$, where $\sqcup$ stands for the disjoint union. By iteratively applying separability to each object in W and transitivity of P_i, we obtain that $f_N(P)\ R_i\ f_{N\cup N'}(P,P')$ and so f is strongly false-name-proof. Therefore f is also false-name-proof. ■

3.3 Proof of Theorem 1 and two corollaries

Proof. $\Longleftarrow$) Let f a voting by quota such that, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$. By Proposition 2, f is strategy-proof and onto and, by Lemma 2, f is false-name-proof.
$\Longrightarrow$) Let f be a false-name-proof, strategy-proof and onto voting rule. Then, by Lemma 1, f is voting by quota $q = (q_{N,x})_{N\in\mathcal{N},x\in\mathcal{M}}$ where, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$. ■

Since strong false-name-proofness implies false-name proofness, we obtain, as a consequence of the proof of Lemma 1, that the statement in Theorem 1 still holds under this stronger notion.

Corollary 1 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ is strongly false-name-proof, strategy-proof, and onto if and only f is a voting by quota such that, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$.*

Finally, asking for neutrality means that all objects nedd to have the same quota.

Definition 11 *Let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ be a voting by quota $q = (q_{N,x})_{N\in\mathcal{N},x\in\mathcal{M}}$. We say that f is* voting by quota one *if, for each $x \in \mathcal{M}$, $q_{N,x} = 1$ for every $N \in \mathcal{N}$. We say that f is* voting by full quota *if, for each $x \in \mathcal{M}$, $q_{N,x} = n$ for every $N \in \mathcal{N}$.*

These two voting by quota are very extreme, and each of them can be seen as the dual of the other. Voting by quota one gives to each voter i the power of imposing x (*i.e.*, i is decisive for x), while voting by full quota gives to each voter i the power to veto x, or imposing that x is not selected (*i.e.*, i is decisive for not x).

Corollary 2 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ is strategy-proof, false-name-proof, onto and neutral if and only if f is either voting by quota one or voting by full quota.*

3.4 Additional results

In this subsection, we present additional results. We first show that if objects only need one vote or a unanimous vote to be chosen, then the rule must verify participation. Hence, participation follows from false-name-proofness, strategy-proofness, and ontoness.

Proposition 4 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and assume the voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ is false-name-proof, strategy-proof, and onto. Then, f satisfies participation.*

Proof. Let f be a voting rule that satisfies false-name-proofness, strategy-proofness, and ontoness. Then, by Theorem 1, f is a voting by quota such that, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$. Let Q_1 be the subset of objects that have quota 1 and Q_n be the subset of objects that have quota n. Let $(N,P) \in \mathcal{N} \times \mathcal{D}^N$ be such that $n \geq 2$ and let $i \in N$ be arbitrary. Since each object has either quota 1 or quota n, we have that $f_N(P) = [\bigcup_{j\in N}(b(P_j) \cap Q_1)] \cup [\bigcap_{j\in N}(b(P_j) \cap Q_n)]$. Accordingly, we also have $f_{N\setminus\{i\}}(P_{-i}) = [\bigcup_{j\in N\setminus\{i\}}(b(P_j) \cap Q_1)] \cup [\bigcap_{j\in N\setminus\{i\}}(b(P_j) \cap Q_n)]$. Therefore, $f_N(P) = [(f_{N\setminus\{i\}}(P_{-i}) \cup (b(P_i)) \cap Q_1] \cup [(f_{N\setminus\{i\}}(P_{-i}) \cap b(P_i)) \cap Q_n]$, which means that

$f_{N\setminus\{i\}}(P_{-i}) = (f_N(P) \setminus B) \sqcup W$ for some $B \subseteq b(P_i)$ and for some $W \subseteq w(P_i)$. By iteratively applying separability to each object in B and W, and transitivity of P_i, we obtain that $f_N(P)\ R_i\ f_{N\setminus\{i\}}(P_{-i})$ and so f satisfies participation. ■

For the case of a unique object, which corresponds to the general setting where there are only two alternatives (identified as $\{x\}$ and $\{\emptyset\}$), participation implies strategy-proofness.[7]

Proposition 5 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N\in\mathcal{N}}$ be a voting rule and assume $\mathcal{M} = \{x\}$. If f satisfies participation, then f is strategy-proof.*

Proof. Assume f satisfies participation and, to obtain a contradiction, suppose f is not strategy-proof. Let $N \in \mathcal{N}$, $P \in \mathcal{D}^N$, $i \in N$ and $P'_i \in \mathcal{D}$ be such that $P'_i \neq P_i$ and

$$f_N(P'_i, P_{-i})\ P_i\ f_N(P_i, P_{-i}). \tag{1}$$

Since $\mathcal{M} = \{x\}$, $\mathcal{D}$ contains only two preferences and so either $\{x\}\ P_i\ \{\emptyset\}$ or $\{\emptyset\}\ P_i\ \{x\}$. Suppose that the first hold. Then,

$$\{\emptyset\}\ P'_i\ \{x\}. \tag{2}$$

Observe that with only one object, the chosen alternative is either $f_N(P) = \{x\}$ or $f_N(P) = \{\emptyset\}$. In the former case, it is clear that $f_N(P_i, P_{-i})\ R_i\ f_N(P'_i, P_{-i})$, which contradicts (1). Therefore, we must have that $f_N(P) = \{\emptyset\}$ and, according to (1) and $\{x\}\ P_i\ \{\emptyset\}$,

$$f_N(P'_i, P_{-i}) = \{x\}\ P_i\ \{\emptyset\} = f_N(P_i, P_{-i}) \tag{3}$$

holds. By participation, $f_N(P)\ R_i\ f_{N\setminus\{i\}}(P_{-i})$, and so by (3), $f_{N\setminus\{i\}}(P_{-i}) = \{\emptyset\}$. Applying now participation to agent i with preference P'_i, $f_N(P'_i, P_{-i})\ R'_i\ f_{N\setminus\{i\}}(P_{-i})$ holds as well. But then, by (3), we have $\{x\}\ R'_i\ \{\emptyset\}$, a contradiction with (2). The proof is analogous for the case when $\{\emptyset\}\ P_i\ \{x\}$. ■

It is easy to see that if there are two or more objects, participation does not imply strategy-proofness: the Borda count, combined with a tie-breaking that selects one among the potentially many subsets of winners, is an example of a voting rule that satisfies participation and it is not strategy-proof.

We now show that in *any* setting (and, in particular, in *any* domain of preferences) strong false-name-proofness, anonymity, and participation imply strategy-proofness.[8] Let A be any set of social alternatives, let $\mathcal{U}$ be any set of complete and transitive preferences over A, and adapt the properties of a rule $f_N : \mathcal{U}^N \to A$ for N as previously defined for our setting. Denote by R_i a generic (and possibly weak) preference of voter i over A in $\mathcal{U}$ and let $R = (R_i)_{i\in N} \in \mathcal{U}^N$ be a profile.

Proposition 6 *Let $\mathcal{U}$ be any domain of preferences over A and let $f = \{f_N : \mathcal{U}^N \to A\}_{N\in\mathcal{N}}$ be a strongly false-name-proof voting rule that satisfies anonymity and participation. Then, f is strategy-proof.*

Proof. Fix $N \in \mathcal{N}$, $R \in \mathcal{U}^N$, $i \in N$, and let $R'_i \in \mathcal{U}$ be arbitrary. Consider any $j \notin N$ and set $R_j = R'_i$. Then,

$$\begin{array}{llll}
f_N(R_i, R_{-i}) & R_i & f_{N\cup\{j\}}(R, R_j) & \text{by strong false-name-proofness} \\
 & = & f_{N\cup\{j\}}((R_{-i}, R_j), R_i) & \text{by anonymity} \\
 & R_i & f_{(N\cup\{j\})\setminus\{i\}}(R_{-i}, R_j) & \text{by participation} \\
 & = & f_{(N\setminus\{i\})\cup\{j\}}(R_{-i}, R_j) & \text{by anonymity} \\
 & = & f_{(N\setminus\{i\})\cup\{i\}}(R_{-i}, R_j) & \text{by anonymity} \\
 & = & f_N(R_{-i}, R'_i) & \text{since } R_j = R'_i.
\end{array}$$

[7]This result can be seen as an indirect consequence of Lemma 1 in Wagman and Conitzer [14] when submitting an extra vote does not convey any cost. For completeness, we state and prove directly this result as Proposition 5.

[8]Conitzer [5] already observes that this holds in his setting as a consequence of his characterization.

Hence, f is strategy-proof. ■

Therefore, Corollary 3 below follows from Corollary 1 and Propositions 3, 4 and 6.

Corollary 3 *Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$. Then, a voting rule $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ is strong false-name proof, onto, anonymous and verifies participation if and only f is voting by quota such that, for each $x \in \mathcal{M}$, either $q_{N,x} = 1$ for all $N \in \mathcal{N}$ or $q_{N,x} = n$ for all $N \in \mathcal{N}$.*[9]

Example 1 below shows that the statement of Proposition 6 does not hold if strong false-name-proofness is replaced by false-name-proofness, even if the domain of the voting rule is restricted to be the set of separable preferences.

Example 1 *For $P_i \in \mathcal{S}$ denote by $s(P_i)$ the most-preferred singleton set from the set of good objects or the empty set if there is none. Let $f^s = \{f^s_N : \mathcal{S}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be the voting rule where, for each $N \in \mathcal{N}$, $f^s_N : \mathcal{S}^N \to 2^{\mathcal{M}}$ is defined by setting, for each $P \in \mathcal{S}^N$, $f^s_N(P) = \cup_{i \in N} s(P_i)$. It is easy to check that f^s is false-name-proof and verifies participation. The following example for $N = \{1, 2\}$ and $\mathcal{M} = \{x, y\}$ shows that f^s is neither strategy-proof nor strongly false-name-proof. Consider the separable profile $(P_1, P_2, P'_1) \in \mathcal{S}^{N \cup \{3\}}$, where*
$\{x, y\}\ P_1\ \{x\}\ P_1\ \{y\}\ P_1\ \{\emptyset\}$, $\{x\}\ P_2\ \{\emptyset\}\ P_2\ \{x, y\}\ P_2\ \{y\}$ and $\{x, y\}\ P'_1\ \{y\}\ P'_1\ \{x\}\ P'_1\ \{\emptyset\}$. Then, by definition of f^s_N, $f^s_N(P'_1, P_2) = \{x, y\}\ P_1\ \{x\} = f^s_N(P_1, P_2)$, which means that f^s_N is not strategy-proof, so neither is f^s. Moreover, $f^s_{\{1,2,3\}}(P_1, P_2, P'_1) = \{x, y\}\ P_1\ \{x\} = f^s_{\{1,2\}}(P_1, P_2)$, which means that f^s is not strongly false-name-proof.

3.5 Independence of the axioms of the main result

We conclude by showing that the axioms in Theorem 1 are independent. For each of the axioms in the statement of Theorem 1, we exhibit an example of a voting rule, different from the voting by quota characterized in Theorem 1, that satisfies all axioms but one.

Let $\mathcal{D}$ be a domain such that $\mathcal{A} \subseteq \mathcal{D} \subseteq \mathcal{S}$ and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be a voting rule.

- All but Strategy-proofness: The voting rule f^s defined in Example 1.
- All but false-name-proofness: Fix $m \in \mathbb{Z}_+$ with $m \geq 2$. For each $N \in \mathcal{N}$, let f_N be voting by quota n if $n \leq m$ and quota $n-1$ if $n > m$, and let $f = \{f_N : \mathcal{D}^N \to 2^{\mathcal{M}}\}_{N \in \mathcal{N}}$ be the voting rule defined accordingly. By Proposition 2, f is strategy-proof and onto. To see that f is not false-name-proof, consider the case where $m = 3$, $N = \{1, 2, 3\}$, and the profile $P = (P_1, P_2, P_3, P_4) \in \mathcal{D}^{N \cup \{4\}}$ with $b(P_i) = \{x\}$ for $i = 1, 2, 4$, $P_1 = P_4$, and $b(P_3) = \{\emptyset\}$. Then, since $f_{N \cup \{4\}}(P_1, P_2, P_3, P_4) = \{x\}\ P_1\ f_N(P_1, P_2, P_3) = \{\emptyset\}$, f is not false-name-proof.
- All but Ontoness: For each $(N, P) \in \mathcal{N} \times \mathcal{D}^N$, define $f_N(P) = \mathcal{M}$. Then, f is trivially false-name-proof and strategy-proof but it is not onto.

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[9] It is easy to check that every voting by quota verifies participation.

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