Proton electromagnetic instabilities in the expanding solar wind

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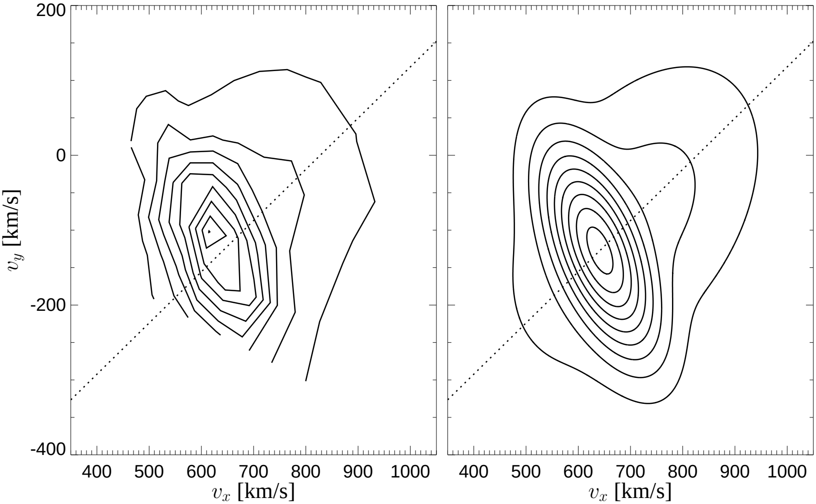
Protons in the fast solar wind

- collisionless
- temperature anisotropies $T_\parallel \neq T_\perp$
- secondary/beam populations
- signatures of wave-particle interactions
- signatures of kinetic instabilities

Helios 2 proton velocity distribution function

In situ data at 0.3 AU in the fast solar wind vs fitted data



Linear dispersion $\omega = \omega(\boldsymbol{k})$

For $f_s = f_s(v_\parallel, v_\perp)$ at oblique propagation the linear dispersion reads

$$\left| \left(\frac{k^2 c^2 + \omega_p^2}{\omega^2} - 1 \right) \mathbf{1} - \boldsymbol{k}\boldsymbol{k}\frac{c^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} \int \frac{k_\parallel \frac{\partial f_s}{\partial v_\parallel} - n \frac{\omega_{cs}}{v_\perp} \frac{\partial f_s}{\partial v_\perp}}{\omega - k_\parallel v_\parallel + n\omega_{cs}} \mathbf{T}_{ns} \, \mathrm{d}^3 v \right| = 0$$

where $\mathbf{T}_{ns} = \begin{pmatrix} \frac{n^2 \omega_{cs}^2}{k_\perp^2} J_n^2 & \frac{in\omega_{cs}}{k_\perp} v_\perp J_n J_n' & -\frac{n\omega_{cs}}{k_\perp} v_\parallel J_n^2 \\ -\frac{in\omega_{cs}}{k_\perp} v_\perp J_n J_n' & v_\perp^2 J_n'^2 & i v_\parallel v_\perp J_n J_n' \\ -\frac{n\omega_{cs}}{k_\perp} v_\parallel J_n^2 & -i v_\parallel v_\perp J_n J_n' & v_\parallel^2 J_n^2 \end{pmatrix}$ and $J_n = J_n\left(\frac{k_\perp v_\perp}{\omega_{cs}}\right)$

At parallel propagation:

$$\frac{k_\parallel^2 c^2 + \omega_p^2}{\omega^2} - \sum_s \frac{\omega_{ps}^2}{\omega^2} \int \frac{v_\perp^2}{2} \frac{k_\parallel \frac{\partial f_s}{\partial v_\parallel} \mp \frac{\omega_{cs}}{v_\perp} \frac{\partial f_s}{\partial v_\perp}}{\omega - k_\parallel v_\parallel \pm \omega_{cs}} \, \mathrm{d}^3 v = 1$$

Electromagnetic proton instabilities

Instabilities for bi-Maxwellian beam and core distribution functions:
Core proton temperature anisotropy driven

- proton cyclotron instability
- mirror instability
- parallel fire hose
- oblique fire hose
- …

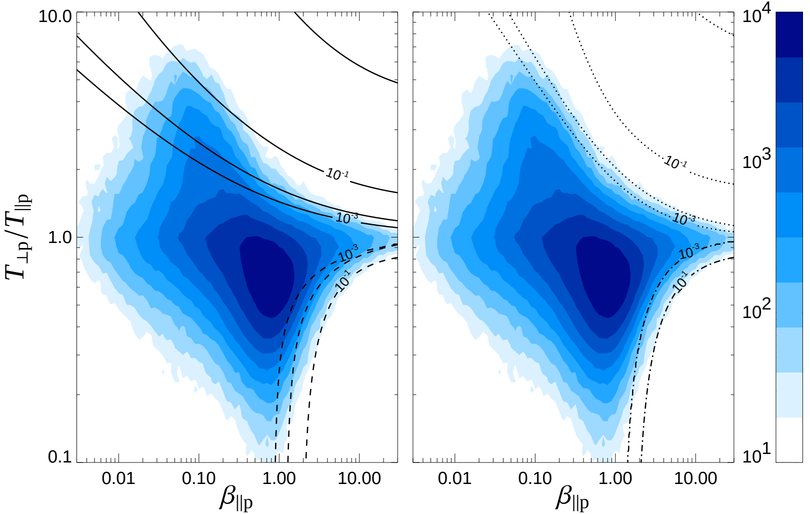
Beam driven (*Daughton & Gary*, 1998)

- parallel magnetosonic instability ($\sim$ parallel fire hose)
- oblique Alfvén instability
- …

These instabilities are typically resonant.

WIND/SWE observations vs linear theory

Fitted Faraday cup results (*Kasper et al.*, 2002; *Hellinger et al.*, 2006)



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Quasilinear diffusion

Linear superpositon of waves $\boldsymbol{E} = \sum_{\boldsymbol{k}} \delta\boldsymbol{E}(\boldsymbol{k},\omega)\, \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega t}, \ldots$ (*Kennel & Engelmann*, 1966) ⇒ second order diffusion

$$\frac{\partial f_\mathrm{s}}{\partial t} = \frac{\partial}{\partial v_\|}\left(D_{\|\|\mathrm{s}}\frac{\partial f_\mathrm{s}}{\partial v_\|} + D_{\|\perp\mathrm{s}}\frac{\partial f_\mathrm{s}}{\partial v_\perp}\right) + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \left(D_{\perp\|\mathrm{s}}\frac{\partial f_\mathrm{s}}{\partial v_\|} + D_{\perp\perp\mathrm{s}}\frac{\partial f_\mathrm{s}}{\partial v_\perp}\right)$$

where

$$\begin{aligned}
D_{\|\|\mathrm{s}} = & \sum_{\boldsymbol{k}} \sum_{n=-\infty}^{\infty} \frac{q_\mathrm{s}^2}{m_\mathrm{s}^2} \frac{1}{k_\perp^2 |\omega|^2} \frac{\gamma}{|\omega - k_\| v_\| - n\omega_{c\mathrm{s}}|^2} \times \\
& \Re\Big[|\delta E_x|^2 n^2 J_n^2 k_\|^2 \omega_{c\mathrm{s}}^2 + |\delta E_y|^2 (J_n')^2 k_\|^2 k_\perp^2 v_\perp^2 + |\delta E_z|^2 J_n^2 k_\perp^2 \, |\omega - n\omega_{c\mathrm{s}}|^2 \\
& + 2i\delta E_x \overline{\delta E}_y n J_n J_n' k_\|^2 k_\perp \omega_{c\mathrm{s}} v_\perp - 2i(\overline{\omega} - n\omega_{c\mathrm{s}})\delta E_y \overline{\delta E}_z J_n J_n' k_\| k_\perp^2 v_\perp \\
& + 2(\overline{\omega} - n\omega_{c\mathrm{s}})\delta E_x \overline{\delta E}_z n J_n^2 k_\| k_\perp \omega_{c\mathrm{s}} \Big] \\
D_{\|\perp\mathrm{s}} = & \ldots
\end{aligned}$$

for general oblique propagation.

Quasilinear diffusion

Linear superpositon of waves $\boldsymbol{E} = \sum_{\boldsymbol{k}} \delta \boldsymbol{E}(\boldsymbol{k},\omega)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega t}, \ldots$ (*Kennel & Engelmann*, 1966) ⇒ second order diffusion

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and for parallel propagation

$$D_{\|\|\mathrm{s}} = v_\perp^2 \sum_k \omega_{c\mathrm{s}}^2 \frac{|\delta B_\pm|^2}{2B_0^2}\frac{\gamma}{|\omega - kv_\| \pm \omega_{c\mathrm{s}}|^2}$$

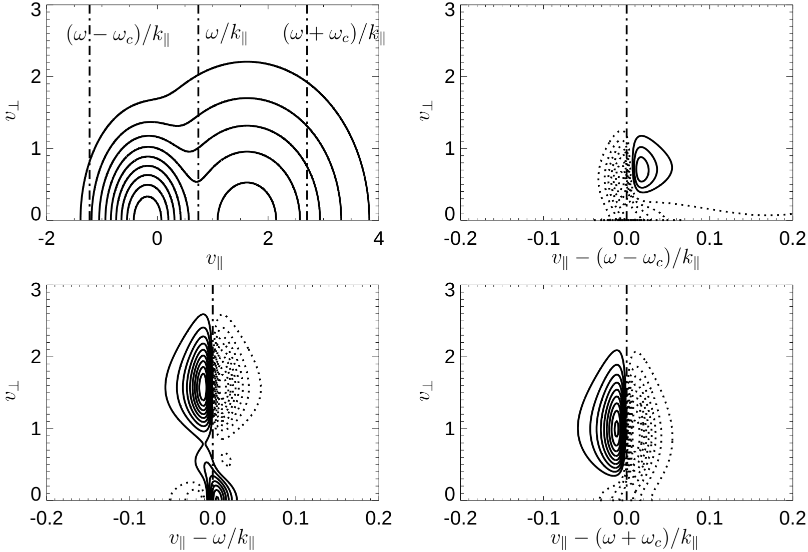
$$D_{\|\perp\mathrm{s}} = \mp v_\perp \sum_k \frac{\omega_{c\mathrm{s}}^3}{k}\frac{|\delta B_\pm|^2}{2B_0^2}\frac{\gamma}{|\omega - kv_\| \pm \omega_{c\mathrm{s}}|^2}$$

$$D_{\perp\|\mathrm{s}} = v_\perp \sum_k \frac{\omega_{c\mathrm{s}}^2}{k}\frac{|\delta B_\pm|^2}{2B_0^2}\frac{\gamma(2\omega_r - 2kv_\| \pm \omega_{c\mathrm{s}})}{|\omega - kv_\| \pm \omega_{c\mathrm{s}}|^2}$$

$$D_{\perp\perp\mathrm{s}} = \sum_k \frac{\omega_{c\mathrm{s}}^2}{k^2}\frac{|\delta B_\pm|^2}{2B_0^2}\frac{\gamma|\omega - kv_\||^2}{|\omega - kv_\| \pm \omega_{c\mathrm{s}}|^2}$$

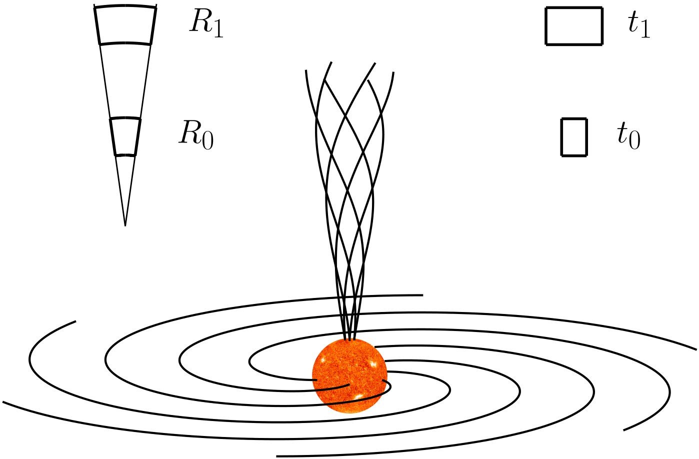
Quasilinear prediction – oblique Alfvén instability

Landau, anomalous and standard cyclotron resonances



Expanding box model

Spherical expansion | Expanding box



Hybrid expanding box model

Liewer et al. (2001), *Hellinger et al.* (2003)

- assuming a constant solar wind velocity v_{sw}
- radial distance evolves as $R = R_0 + v_{sw}t = R_0(1 + t/t_e)$ where $t_e = R_0/v_{sw}$ is the characteristic expansion time
- transverse scale expands $\propto 1 + t/t_e$
- hybrid approximation, kinetic ions & fluid electrons

Slow expansion

- double adiabatic evolution when no wave activity is present
- self-consistent competition between the expansion and instabilities.

2-D hybrid expanding box simulation

Parameters

- radial $\boldsymbol{B}$
- $t_e = R_0/v_{\mathrm{sw}0} = 10^4\omega_{cp0}^{-1}$
- $\Delta x = \Delta y = v_A/\omega_{cp0}$
- $N_x \times = N_y = 512 \times 512$
- core 2048 p/c, beam 1024 p/c
- $\beta_{\mathrm{p}\|0} = 0.2$
- $\beta_{\mathrm{b}\|0} = 0.1$
- $T_{\mathrm{p}\perp 0}/T_{\mathrm{p}\|0} = 1.8$
- $T_{\mathrm{b}\perp 0}/T_{\mathrm{b}\|0} = 1$
- $n_{\mathrm{p}} = 0.9 n_{\mathrm{e}}$
- $n_{\mathrm{b}} = 0.1 n_{\mathrm{e}}$,
- $v_{\mathrm{bp}0} = 1.3 v_{A0}$

Theoretical expectation

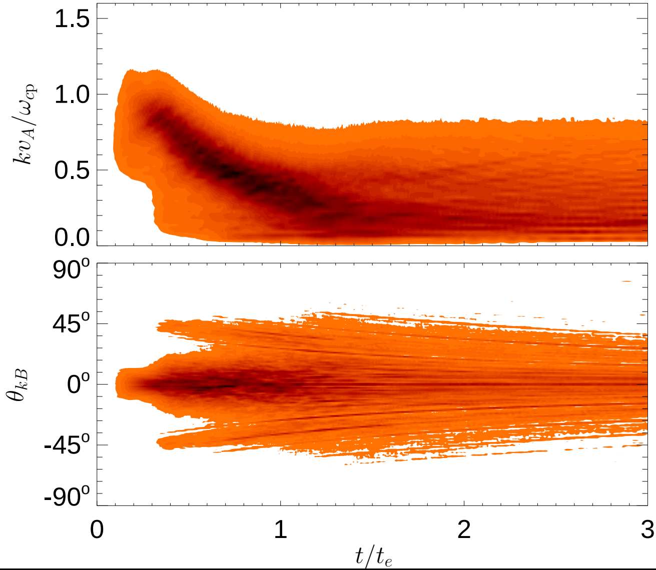
- $B \propto (1 + t/t_e)^{-2}$
- $n \propto (1 + t/t_e)^{-2}$
- $T_\perp \propto (1 + t/t_e)^{-2}$
- $T_\| = \mathrm{const.}$
- $v_{\mathrm{bp}} = \mathrm{const.}$
- $v_A \propto (1 + t/t_e)^{-1}$

$T_\perp/T_\|$ decreases

v_{bp}/v_A increases

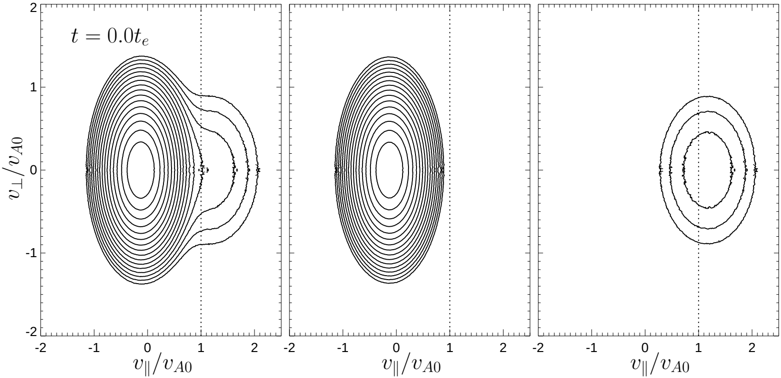
Wave activity

δB^2 as a function of time and wave vector/angle



Proton velocity distribution function

Evolution of VDF



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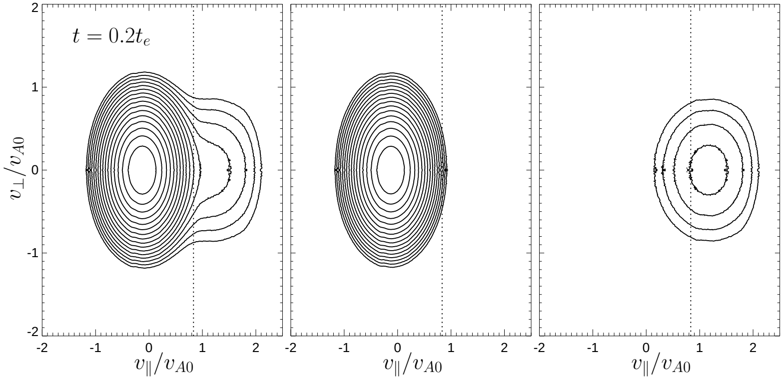

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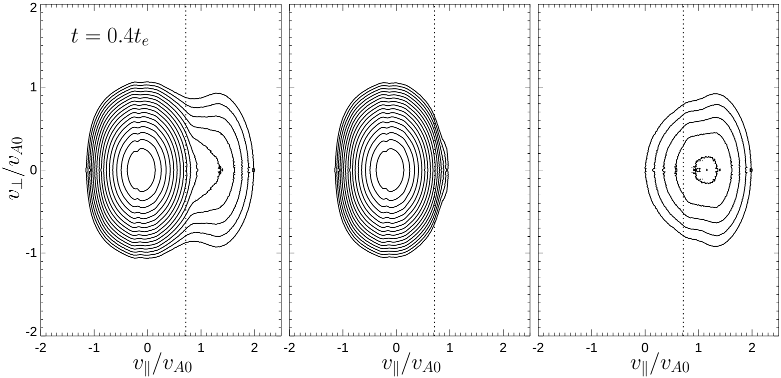

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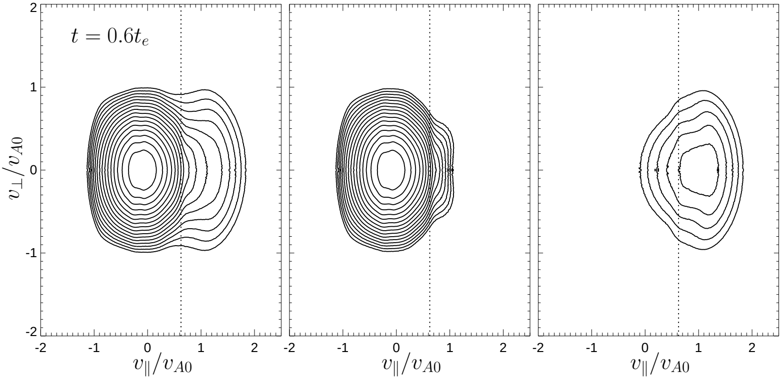

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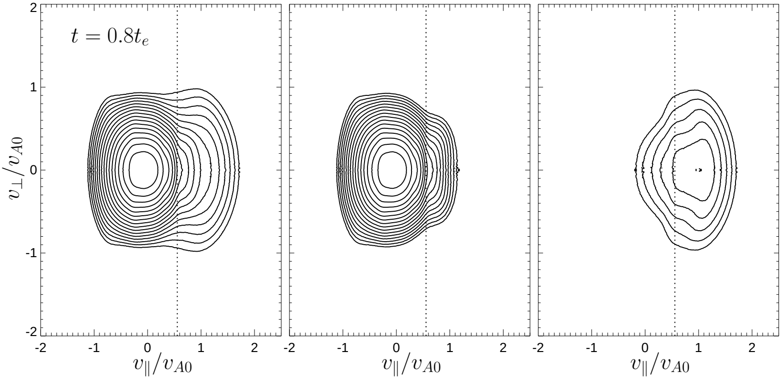

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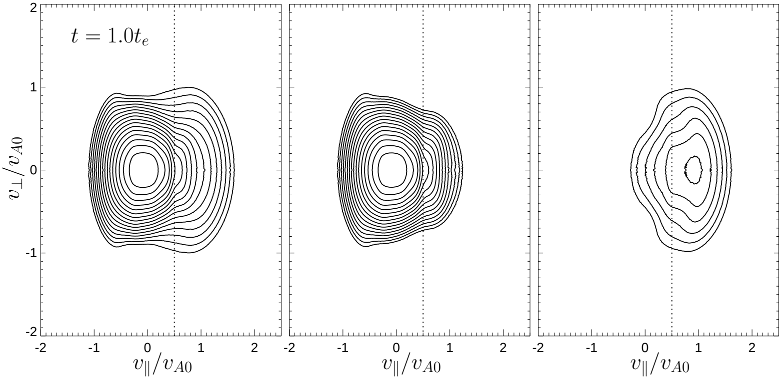

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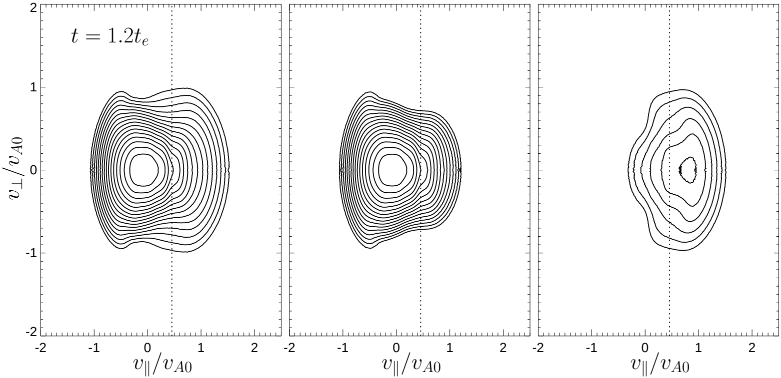

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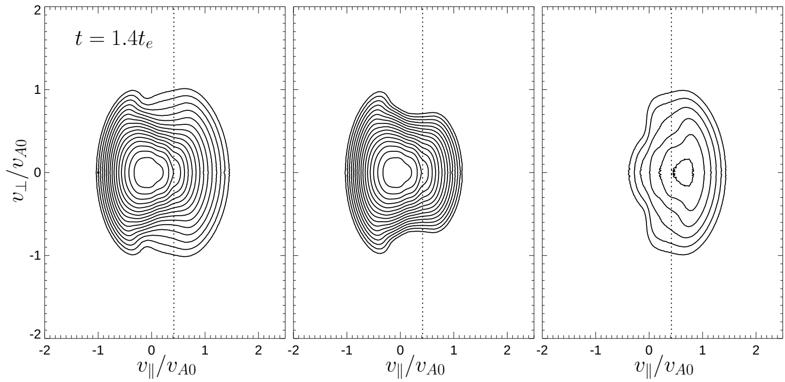

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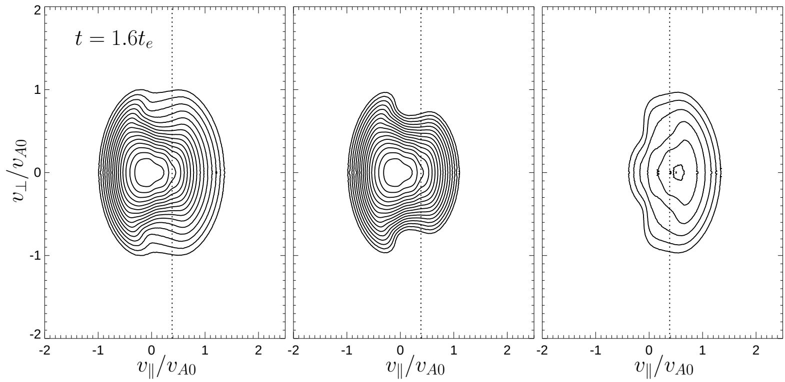

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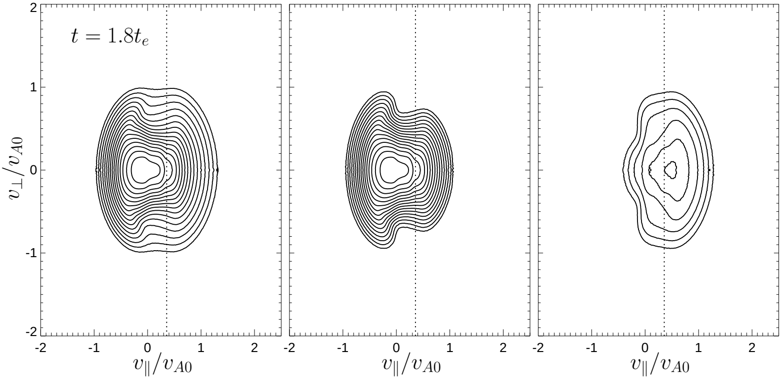

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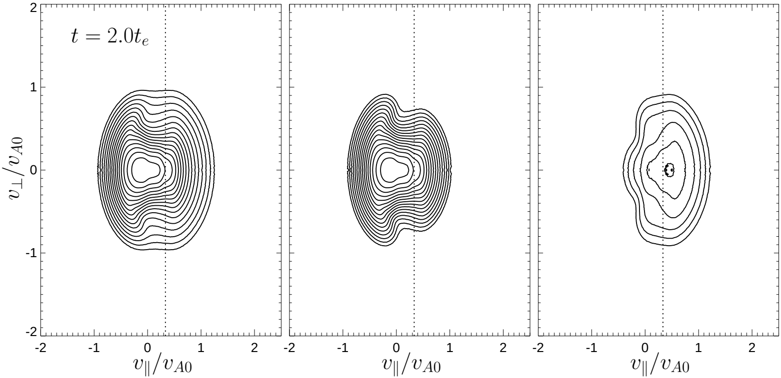

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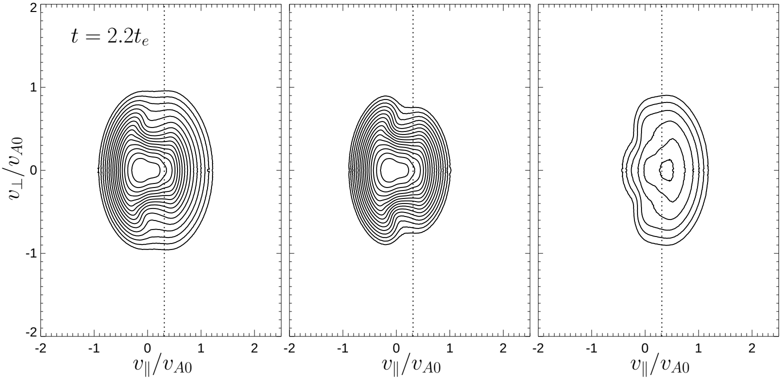

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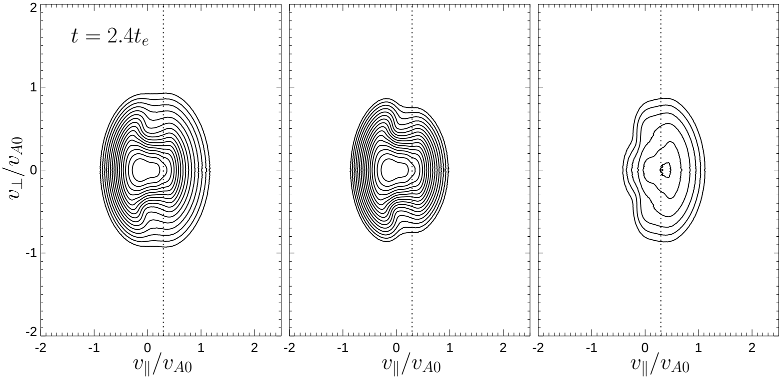

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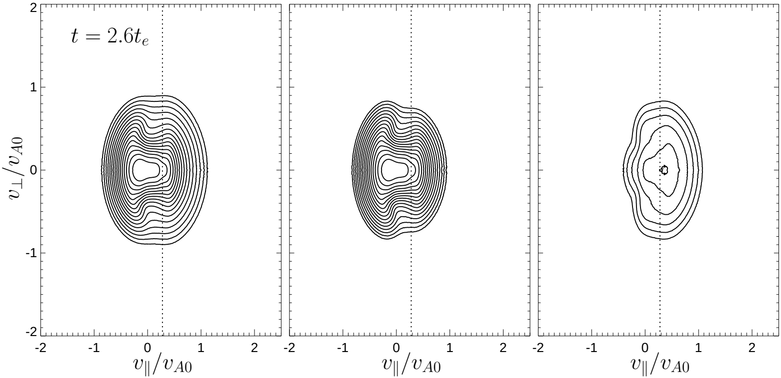

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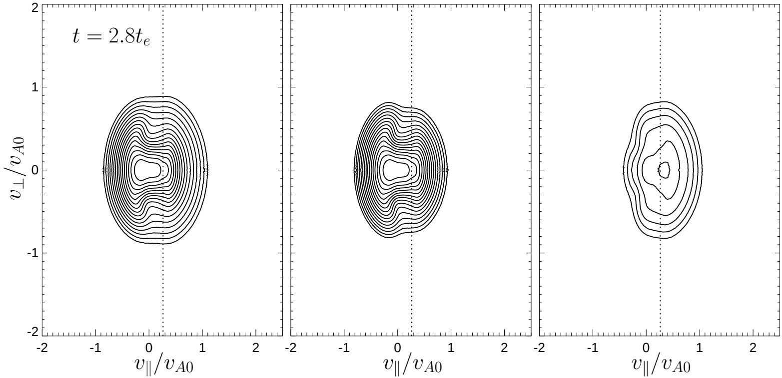

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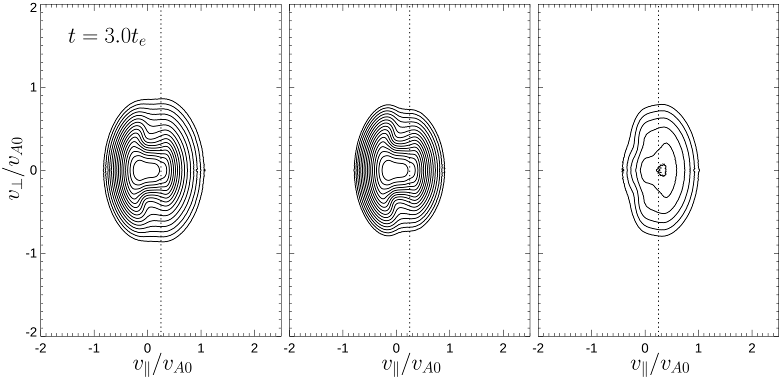
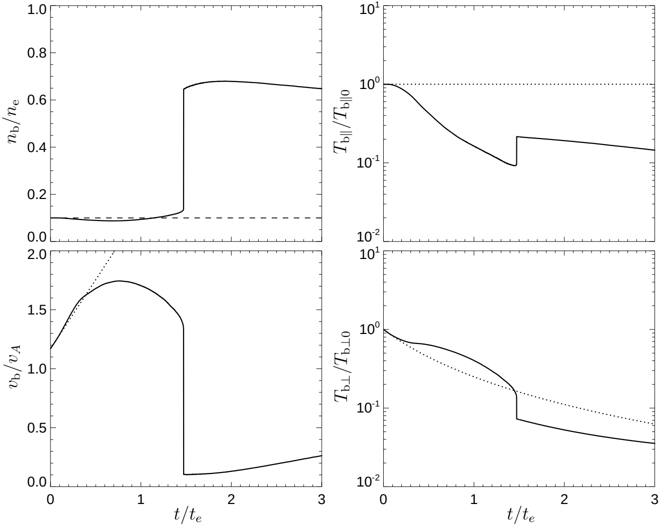


Separation of beam and core

Beam moments – fitted results (2 drifting bi-Maxwellian VDF)



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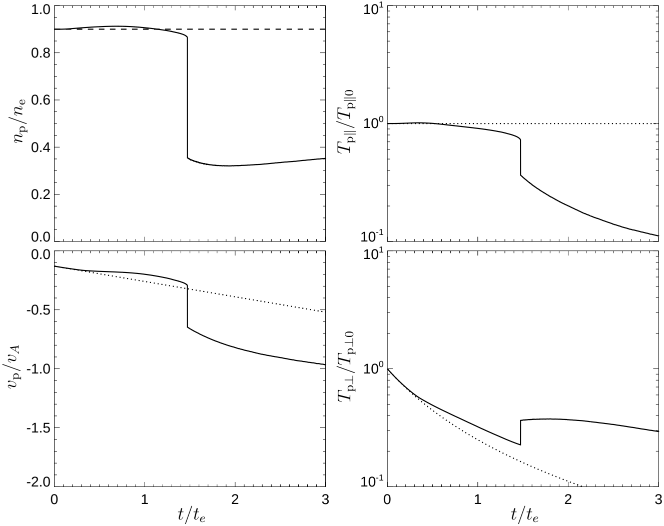

Linear dispersion $\omega = \omega(\boldsymbol{k})$

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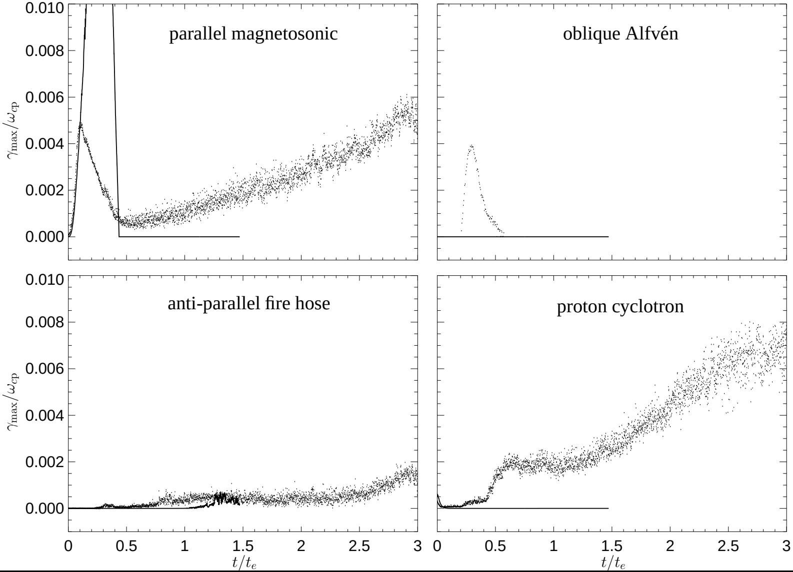
where $\mathbf{T}_{n\mathrm{s}} = \begin{pmatrix} \frac{n^2\omega_{c\mathrm{s}}^2}{k_\perp^2} J_n^2 & \frac{in\omega_{c\mathrm{s}}}{k_\perp} v_\perp J_n J_n' & -\frac{n\omega_{c\mathrm{s}}}{k_\perp} v_\| J_n^2 \\ -\frac{in\omega_{c\mathrm{s}}}{k_\perp} v_\perp J_n J_n' & v_\perp^2 J_n'^2 & i v_\| v_\perp J_n J_n' \\ -\frac{n\omega_{c\mathrm{s}}}{k_\perp} v_\| J_n^2 & -i v_\| v_\perp J_n J_n' & v_\|^2 J_n^2 \end{pmatrix}$ and $J_n = J_n\left(\frac{k_\perp v_\perp}{\omega_{c\mathrm{s}}} \right)$

At parallel propagation:

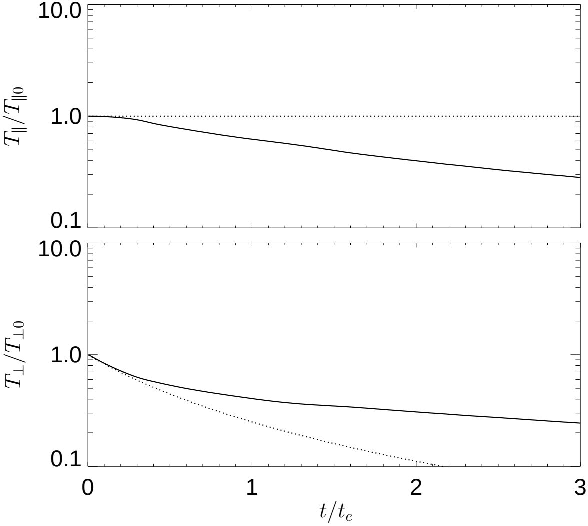
$$\frac{k_\|^2 c^2 + \omega_p^2}{\omega^2} - \sum_\mathrm{s} \frac{\omega_{p\mathrm{s}}^2}{\omega^2} \int \frac{v_\perp^2}{2} \frac{k_\| \frac{\partial f_\mathrm{s}}{\partial v_\|} \mp \frac{\omega_{c\mathrm{s}}}{v_\perp} \frac{\partial f_\mathrm{s}}{\partial v_\perp}}{\omega - k_\| v_\| \pm \omega_{c\mathrm{s}}} \, \mathrm{d}^3 v = 1$$

$\gamma_{\max}$ from VDF vs bi-Maxwellian fitting



Total (effective) temperatures

Temperatures as function of time



Summary

Expansion for the beam-core system

- complex evolution destabilizing
  - parallel magnetosonic/fire hose instability
  - oblique Alfvén instability
  - proton cyclotron instability
  - anti-parallel fire hose
- marginal stability most of the time
- linear analysis based on real (detailed) VDF necessary
- instabilities compete/drive each other
- mostly resonant interaction $\Rightarrow$ complicated VDF
- effective parallel cooling and perpendicular heating (isotropization)

Qualitative agreement with Helios data.