Monotonicity in Condorcet Jury Theorem

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Abstract. Consider a committee of experts dealing with dichotomous choice problem, where the correctness probabilities are all greater than $\frac{1}{2}$. We prove that, if a random subcommittee of odd size m is selected randomly, and entrusted to make a decision by majority vote, its probability of deciding correctly increases with m. This includes a result of Ben-Yashar and Paroush (2000), who proved that a random subcommittee of size $m \geq 3$ is preferable to a random single expert.

1. Introduction and model

There is a variety of situations where a group of decision makers is required to select one of two alternatives, of which exactly one is correct. This gives rise to the *dichotomous choice model*, which goes back more than two centuries, as far as Condorcet (1785).

Each expert i has a correctness probability p_i, indicating his ability to identify the correct alternative. We assume that each $p_i \geq \frac{1}{2}$ (cf. Nitzan and Paroush 1985). The selections of the experts are assumed to be independent.

A *decision rule* is a rule for translating the individual opinions into a group decision. One of the most popular decision rules is the simple majority rule.

There are several directions in the study of the dichotomous choice model. One of them is concerned with Condorcet's Jury Theorem in various setups. Condorcet's original statement may be phrased as composed of two parts (see Ben-Yashar and Paroush 2000):

(i) A group decision, utilizing the simple majority rule, is more likely to be correct than that of any of the members.

(ii) The probability of the group to make a correct choice using the simple majority rule tends to 1 as the number of members tends to infinity.

Part (i) is referred to as the nonasymptotic part of Condorcet's statement, and (ii) as the asymptotic part (see Ben-Yashar and Paroush 2000). This statement provides the theoretical justification of democratic voting in public affairs and in social choice. A Condorcet's Jury Theorem (CJT) is a formulation of conditions substantiating Condorcet's belief. The classical conditions of this theorem are the independence of the decision makers and the same value $p > 1/2$ of the individual correctness probabilities. Note that the first proof of the classical CJT is due to Laplace in 1812.

Attempts to generalize the theorem were made in several directions. In particular, several studies considered the case of heterogeneous correctness probabilities (cf. Grofman, Owen and Feld 1983; Miller 1986; Young 1989; Paroush 1998; Berend and Paroush 1998). Others compared decision groups of different sizes (cf. Feld and Grofman 1984; Paroush and Karotkin 1989; Maranon 2000; Karotkin and Paroush 2003). Another aspect was concerned with identifying the optimal decision rule under partial information on correctness probabilities (cf. Sapir 1998; Berend and Sapir 2002).

Most of the previous studies were focused on the asymptotic part of Condorcet's statement. For example, Berend and Paroush (1998) found necessary and sufficient conditions for the asymptotic part of Condorcet's statement for independent voters in heterogeneous teams.

The nonasymptotic part of Condorcet's statement is also not always valid (Nitzan and Paroush 1982; Ben-Yashar and Paroush 2000). For instance, Ben-Yashar and Paroush (2000) calculated that the nonasymptotic part of Condorcet's statement is valid if the competence vector is $(0.8, 0.7, 0.7)$, in which case the simple majority rule yields a probability 0.826 of correct choice. However, it is invalid for competence vectors $(0.8, 0.7, 0.6)$ or $(0.9, 0.6, 0.6)$, where the probabilities of a simple majority to choose correctly are 0.788 and 0.792, respectively.

The motivation for this paper derives from the results of Ben-Yashar and Paroush (2000). They considered a slight adjustment of Condorcet's statement, which is valid regardless of the specific competence structure of the group. Their main result is that the probability of a group of odd size n with competence structure $(p_1, p_2, \dots p_n)$, where $p_i > \frac{1}{2}$ for each i, to reach the correct decision when utilizing the simple majority rule is larger than the probability $\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i$ of a random group member to do so.

To illustrate it, consider again the examples given above with competence structures $(0.8, 0.7, 0.6)$ or $(0.9, 0.6, 0.6)$, in which the nonasymptotic part of Condorcet's statement is invalid. In each of these cases $\bar{p} = 0.7$, while the probabilities of the simple majority to choose correctly are $0.788 > 0.7$ and $0.792 > 0.7$.

Moreover, Ben-Yashar and Paroush (2000) concluded that the same holds for a subset of an odd number of experts chosen at random from the original group. Namely, the probability of a randomly selected subcommittee of odd size $m \geq 3$ to decide correctly when utilizing the simple majority rule is larger than the probability $\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i$ of a random single member to do so.

The result of Ben-Yashar and Paroush (2000) raises the question as to how the probability of making a correct decision varies when we use the simple majority rule for randomly selected subcommittees of various sizes. Their result implies that it is preferable to let a random set of an odd size $m \geq 3$ decide rather than selecting a random single member and letting him decide by himself. Is it, more generally, better to select a random large subcommittee rather than a smaller one? The purpose of this paper is to answer this question.

Obviously, in these results it is immaterial whether the group size n is odd or even, as only the subcommittees are important. Moreover, it is both interesting and technically convenient to consider subcommittees of even size as well. In this case, if a vote results in a draw, the decision is made randomly, with a 50 percent chance of accepting each of the two opinions (see Berg 1993). It will turn out that increasing the committee size from m to $m+1$ always increases the probability of making a correct choice if m is even. However, if m is odd, then increasing it by 1 leaves the probability exactly the same.

In Section 2 we present the main results and in Section 3 their proofs. Section 4 discusses the results.

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2. Main Results

We start with several notations. The committee of n members will be denoted by $\bar{n} = \{1, 2, \ldots, n\}$. As mentioned previously, p_i is the probability of the i-th expert to choose the correct alternative, and denote $q_i = 1 - p_i$ his probability to make a mistake. $E = \{e_1, e_2, \ldots, e_m\}$ is a typical set of m members, where $1 \leq e_1 < e_2 < \ldots < e_m \leq n$.

$M_{m,j}(E) = M_{m,j}(e_1, e_2, \ldots, e_m)$ for $0 \le j \le m$ is the probability that exactly $m - j$ of the members of E will choose the correct alternative, while all the others will choose the incorrect one:

$$M_{m,j}(E) = \begin{cases} 1, & m = 0, \\ \displaystyle\sum_{\substack{E_1 \subseteq E \\ |E_1| = m-j}} \prod_{i \in E_1} p_i \cdot \prod_{i \in E \setminus E_1} q_i, & m \ge 1. \end{cases}$$

For example,

$$M_{3,1}(1,2,3) = p_1 p_2 q_3 + p_1 q_2 p_3 + q_1 p_2 p_3.$$

$S_{n,m,j}$ is the sum of all $M_{m,j}(E)$'s as E ranges over the $\binom{n}{m}$ subsets of $\bar{n}$ of size m. Namely, denoting by $\bar{n}_m$ this collection of subsets:

$$S_{n,m,j} = \sum_{E \in \bar{n}_m} M_{m,j}(E).$$

For example,

$$S_{4,3,1} = M_{3,1}(1,2,3) + M_{3,1}(1,2,4) + M_{3,1}(1,3,4) + M_{3,1}(2,3,4).$$

For E of odd size $m \ge 1$, let $M(E) = M(e_1, e_2, \ldots, e_m)$ be the probability of E to make the correct choice when utilizing the simple majority rule:

$$M(E) = \sum_{j=0}^{(m-1)/2} M_{m,j}(E).$$

For example, if $E = \{1,2,3\}$ then $M(E) = M_{3,0}(E) + M_{3,1}(E)$.

Strictly speaking, the simple majority rule is not well-defined for even-sized committees. It is natural, though, to extend the rule to this case by tossing a coin if the votes are split evenly within the committee. Namely, we define the probability of E of even size $m \ge 0$ to make the correct choice when utilizing the simple majority rule by:

$$M(E) = \sum_{j=0}^{m/2-1} M_{m,j}(E) + \frac{1}{2} M_{m,m/2}(E).$$

Denote by $\bar{M}_m$ the probability of a randomly selected subcommittee of m members to decide correctly when utilizing the simple majority rule. That is, for odd $m \ge 1$

$$\bar{M}_m = \frac{1}{\binom{n}{m}} \sum_{E \in \bar{n}_m} M(E) = \frac{1}{\binom{n}{m}} \sum_{j=0}^{(m-1)/2} S_{n,m,j},$$

and for even $m \geq 0$:

$$\bar{M}_m = \frac{1}{\binom{n}{m}} \sum_{E \in \bar{n}_m} M(E) = \frac{1}{\binom{n}{m}} \left(\sum_{j=0}^{m/2-1} S_{n,m,j} + \frac{1}{2} S_{n,m,m/2} \right).$$

For example, if $n = 4$ and $m = 3$ then

$$\bar{M}_3 = \frac{1}{4}(M(1,2,3) + M(1,2,4) + M(1,3,4) + M(2,3,4)),$$

where

$$\begin{aligned} M(1,2,3) &= M_{3,0}(1,2,3) + M_{3,1}(1,2,3), \\ M(1,2,4) &= M_{3,0}(1,2,4) + M_{3,1}(1,2,4), \\ M(1,3,4) &= M_{3,0}(1,3,4) + M_{3,1}(1,3,4), \\ M(2,3,4) &= M_{3,0}(2,3,4) + M_{3,1}(2,3,4), \end{aligned}$$

or, equivalently,

$$\bar{M}_3 = \frac{1}{4}(S_{4,3,0} + S_{4,3,1}),$$

where

$$\begin{aligned} S_{4,3,0} &= M_{3,0}(1,2,3) + M_{3,0}(1,2,4) + M_{3,0}(1,3,4) + M_{3,0}(2,3,4), \\ S_{4,3,1} &= M_{3,1}(1,2,3) + M_{3,1}(1,2,4) + M_{3,1}(1,3,4) + M_{3,1}(2,3,4). \end{aligned}$$

In particular, $\bar{M}_0 = \frac{1}{2}$ and $\bar{M}_1 = \bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i$. If n is odd, then $\bar{M}_n$ is the probability that a committee of size n utilizing a simple majority rule will make the correct choice.

With these notations, we may rephrase the basic result of Ben-Yashar and Paroush (2000) mentioned above as the assertion that, for a group of odd size $n \geq 3$ with competence structure $(p_1, p_2, \ldots p_n)$, where $p_i > \frac{1}{2}$ for each i, we have:

$$\bar{M}_1 < \bar{M}_n.$$

Their generalized result is that, for a group of (odd or even) size $n \geq 3$ with competence structure $(p_1, p_2, \ldots p_n)$, where $p_i > \frac{1}{2}$ for each i, we have:

$$\bar{M}_1 < \bar{M}_m, \qquad 3 \leq m \leq n, \ (m \text{ odd}).$$

Our main result is

THEOREM 1. *For a group of size n with any competence structure $(p_1, p_2, \ldots p_n)$, where $p_i \geq \frac{1}{2}$ for each i, we have:*

$$\bar{M}_m(E) \begin{cases} \leq \bar{M}_{m+1}, & 0 \leq m \leq n-1, \ m \text{ even}, \\ = \bar{M}_{m+1}, & 1 \leq m \leq n-1, \ m \text{ odd}. \end{cases}$$

Moreover, for any even m we have $\bar{M}_m = \bar{M}_{m+1}$ if and only if $p_i = \frac{1}{2}$ for $1 \leq i \leq n$.

In particular, Theorem 1 implies both of the above theorems of Ben-Yashar and Paroush (2000). In addition, it provides the monotonicity of the probability of a correct choice as a function of the size of the selected subcommittee. Namely

$$\bar{M}_0 \leq \bar{M}_1 = \bar{M}_2 \leq \bar{M}_3 = \bar{M}_4 \leq \ldots = \bar{M}_{n-1} \leq \bar{M}_n$$

if n is odd, and

$$\bar{M}_0 \leq \bar{M}_1 = \bar{M}_2 \leq \bar{M}_3 = \bar{M}_4 \leq \ldots \leq \bar{M}_{n-1} = \bar{M}_n$$

if n is even.

3. Proofs

To prove Theorem 1 we will need the following two lemmas. Denote by $\bar{n}_{m,r} = \{(E, E') : \ E \in \bar{n}_m, \ E' \in \bar{n}_r, \ E \bigcap E' = \emptyset\}$.

LEMMA 1. *For a group of size n and nonnegative integers m and r, $m + r \leq n$ and $0 \leq j \leq m$ and $0 \leq k \leq r$ we have:*

$$\sum_{(E,E')\in\bar{n}_{m,r}} M_{m,j}(E) M_{r,k}(E') = \binom{j+k}{k} \binom{m+r-j-k}{r-k} S_{n,m+r,j+k}.$$

Proof of Lemma 1. Each term in the sum defining $M_{m,j}(E)$ is of the form $\prod_{i\in E_1} p_i \cdot \prod_{i\in E\setminus E_1} q_i$, where $E_1 \subseteq E$ is of size $m-j$. Similarly, $M_{r,k}(E')$ consists of terms of the form $\prod_{i\in E_1'} p_i \cdot \prod_{i\in E'\setminus E_1'} q_i$, with $|E_1'| = r-k$. Hence, for $E \bigcap E' = \emptyset$, the product $M_{m,j}(E) M_{r,k}(E')$ is made of terms of the form $\prod_{i\in F_1} p_i \cdot \prod_{i\in(E\bigcup E')\setminus F_1} q_i$, where $F_1 \subseteq E \bigcup E'$ and $|F_1| = m+r-j-k$. Let F and $F_1 \subseteq F$ be arbitrary fixed sets of sizes $m+r$ and $m+r-j-k$, respectively. Letting E and E' vary over all pairs of disjoint sets of sizes m and r, respectively, the product $M_{m,j}(E) M_{r,k}(E')$ contains the term $\prod_{i\in F_1} p_i \cdot \prod_{i\in F\setminus F_1} q_i$ if and only if

$$E \subseteq F \qquad \text{and} \qquad |E \bigcap F_1| = m - j, \qquad \text{and} \qquad E' = F \setminus E.$$

Choosing the $m-j$ elements of $E \bigcap F_1$ out of the $m+r-j-k$ elements of F_1 may be done in $\binom{m+r-j-k}{m-j}$ ways, choosing the remaining j elements of E (i.e., those of $E \bigcap (F \setminus F_1)$) may be done in $\binom{j+k}{j}$ ways, and the set E' is then uniquely determined. Hence the term $\prod_{i \in F_1} p_i \cdot \prod_{i \in F \setminus F_1} q_i$ appears in $\sum_{(E,E') \in \bar{n}_{m,r}} M_{m,j}(E) M_{r,k}(E')$ exactly $\binom{m+r-j-k}{m-j}\binom{j+k}{j}$ times. Obviously, $S_{n,m+r,j+k}$ is the sum of the same terms, each taken only once. This proves the lemma.

LEMMA 2. *For a group of size n we have*

$$S_{n,2k+1,k} \geq S_{n,2k+1,k+1}, \qquad 1 \leq k \leq (n-1)/2,$$

with equality if and only if $p_i = \frac{1}{2}$, $i = 1, 2, \ldots, n$.

Proof of Lemma 2. Since

$$S_{n,2k+1,k} = \sum_{E \,\in\, \bar{n}_{2k+1}} M_{m,k}(E)$$

and

$$S_{n,2k+1,k+1} = \sum_{E \,\in\, \bar{n}_{2k+1}} M_{m,k+1}(E),$$

it suffices to show that

$$M_{2k+1,k}(E) \geq M_{2k+1,k+1}(E), \qquad E \in \bar{n}_{2k+1}. \tag{1}$$

Take an arbitrary set E of size $2k+1$. To prove (1) for E, we recall that its left hand side consists of the sum of the probabilities of all voting results in which exactly $k+1$ of the members of E vote correctly and the other k incorrectly:

$$M_{2k+1,k}(E) = \sum_{\substack{E_1 \subseteq E \\ |E_1| = k+1}} \prod_{i \in E_1} p_i \cdot \prod_{i \in E \setminus E_1} q_i.$$

Since all $p_i's$ are at least $\frac{1}{2}$, we have:

$$\prod_{i \in E_1} p_i \cdot \prod_{i \in E \setminus E_1} q_i \geq \prod_{i \in E_1 \setminus \{i_0\}} p_i \cdot \prod_{i \in (E \setminus E_1) \bigcup \{i_0\}} q_i, \qquad E_1 \subseteq E,\ i_0 \in E_1. \tag{2}$$

Summing (2) over all $i_0 \in E_1$, we obtain:

$$(k+1) \cdot \prod_{i \in E_1} p_i \cdot \prod_{i \in E \setminus E_1} q_i \geq \prod_{i \in E \setminus E_1} q_i \cdot M_{k+1,1}(E_1). \qquad (3)$$

Summing (3) over all sets $E_1 \subseteq E$ with $|E_1| = k+1$ we arrive at:

$$(k+1) M_{2k+1,k}(E) \geq (k+1) M_{2k+1,k+1}(E).$$

Clearly, equality takes place if and only if $p_i = \frac{1}{2}$ for all $i \in E$. This proves (1), and thereby the lemma.

Proof of Theorem 1. First we will show that:

$$\bar{M}_{2k+1} = \bar{M}_{2k+2}, \qquad 0 \leq k \leq n/2 - 1. \qquad (4)$$

To this end, recall that the probability of a randomly selected subcommittee of size $2k+1$ to decide correctly when utilizing the simple majority rule is

$$\bar{M}_{2k+1} = \frac{1}{\binom{n}{2k+1}} \sum_{E \in \bar{n}_{2k+1}} M(E). \qquad (5)$$

Now:

$$\begin{aligned}
(n-2k-1) \sum_{E \in \bar{n}_{2k+1}} M(E) &= \sum_{E \in \bar{n}_{2k+1}} M(E) \sum_{i \in \bar{n} \setminus E} 1 \\
&= \sum_{E \in \bar{n}_{2k+1}} M(E) \sum_{i \in \bar{n} \setminus E} (p_i + q_i) \\
&= \sum_{(E,E') \in \bar{n}_{2k+1,1}} M(E)(M_{1,0}(E') + M_{1,1}(E')) \\
&= \sum_{j=0}^{k} \sum_{(E,E') \in \bar{n}_{2k+1,1}} M_{2k+1,j}(E) M_{1,0}(E') \\
&\quad + \sum_{j=0}^{k} \sum_{(E,E') \in \bar{n}_{2k+1,1}} M_{2k+1,j}(E) M_{1,1}(E').
\end{aligned}$$

Using Lemma 1 twice with $m = 2k+1$ and $r = 1$, the first time with $k = 0$ and the second with $k = 1$, we obtain:

$$
\begin{aligned}
(n-2k-1)\sum_{E\in\bar{n}_{2k+1}} M(E) &= \sum_{j=0}^{k}(2k+2-j)S_{n,2k+2,j} + \sum_{j=1}^{k+1} jS_{n,2k+2,j} \\
&= (2k+2)\left(\sum_{j=0}^{k} S_{n,2k+2,j} + \frac{1}{2}S_{n,2k+2,k+1}\right).
\end{aligned}
\tag{6}
$$

Substituting (6) into (5) we find that

$$
\begin{aligned}
\bar{M}_{2k+1} &= \frac{2k+2}{\binom{n}{2k+1}\binom{n-2k-1}{1}}\left(\sum_{j=0}^{k} S_{n,2k+2,j} + \frac{1}{2}S_{n,2k+2,k+1}\right) \\
&= \frac{2k+2}{\binom{n}{2k+2}\binom{2k+2}{1}}\left(\sum_{j=0}^{k} S_{n,2k+2,j} + \frac{1}{2}S_{n,2k+2,k+1}\right) \\
&= \frac{1}{\binom{n}{2k+2}}\left(\sum_{j=0}^{k} S_{n,2k+2,j} + \frac{1}{2}S_{n,2k+2,k+1}\right) = \bar{M}_{2k+2},
\end{aligned}
$$

which proves (4).
It remains to prove that:

$$
\bar{M}_{2k} \le \bar{M}_{2k+1}, \qquad 0 \le k \le (n-1)/2.
$$

Similarly to the first part of the proof

$$
\bar{M}_{2k} = \frac{1}{\binom{n}{2k}} \sum_{E\in\bar{n}_{2k}} M(E) \tag{7}
$$

and

$$
\begin{aligned}
(n-2k)\sum_{E\in\bar{n}_{2k}} M(E) &= (n-2k)\left(\sum_{j=0}^{k-1}\sum_{E\in\bar{n}_{2k}} M_{2k,j}(E) + \frac{1}{2}\sum_{E\in\bar{n}_{2k}} M_{2k,k}(E)\right) \\
&= \sum_{j=0}^{k-1}\sum_{E\in\bar{n}_{2k}} M_{2k,j}(E)\sum_{i\in\bar{n}\setminus E}(p_i+q_i) \\
&\quad +\frac{1}{2}\sum_{E\in\bar{n}_{2k}} M_{2k,k}(E)\sum_{i\in\bar{n}\setminus E}(p_i+q_i) \\
&= \sum_{j=0}^{k-1}\sum_{(E,E')\in\bar{n}_{2k,1}}\left(M_{2k,j}(E)M_{1,0}(E') + M_{2k,j}(E)M_{1,1}(E')\right) \\
&\quad +\frac{1}{2}\sum_{(E,E')\in\bar{n}_{2k,1}}\left(M_{2k,k}(E)M_{1,0}(E') + M_{2k,k}(E)M_{1,1}(E')\right).
\end{aligned}
$$

By Lemma 1 this yields

$$
\begin{aligned}
(n-2k)\sum_{E\in\bar{n}_{2k}} M(E) &= \sum_{j=0}^{k-1}(2k+1-j)S_{n,2k+1,j} + \sum_{j=0}^{k-1}(j+1)S_{n,2k+1,j+1} \\
&\quad +\frac{1}{2}(k+1)S_{n,2k+1,k} + \frac{1}{2}(k+1)S_{n,2k+1,k+1} \\
&= (2k+1)\sum_{j=0}^{k-1} S_{n,2k+1,j} + \frac{3k+1}{2}S_{n,2k+1,k} + \frac{k+1}{2}S_{n,2k+1,k+1} \\
&= (2k+1)\sum_{j=0}^{k} S_{n,2k+1,j} + \frac{k+1}{2}\left(S_{n,2k+1,k+1} - S_{n,2k+1,k}\right).
\end{aligned}
$$

In a view of (7), this gives:

$$\begin{aligned}\bar{M}_{2k} &= \frac{1}{\binom{n}{2k}\binom{n-2k}{1}}\left((2k+1)\sum_{j=0}^{k} S_{n,2k+1,j} + \frac{k+1}{2}\left(S_{n,2k+1,k+1} - S_{n,2k+1,k}\right)\right) \\ &= \frac{(2k+1)}{\binom{n}{2k+1}\binom{2k+1}{1}}\left(\sum_{j=0}^{k} S_{n,2k+1,j} + \frac{k+1}{2(2k+1)}\left(S_{n,2k+1,k+1} - S_{n,2k+1,k}\right)\right) \qquad (8) \\ &= \frac{1}{\binom{n}{2k+1}}\left(\sum_{j=0}^{k} S_{n,2k+1,j} + \frac{k+1}{2(2k+1)}\left(S_{n,2k+1,k+1} - S_{n,2k+1,k}\right)\right).\end{aligned}$$

Now

$$\bar{M}_{2k+1} = \frac{1}{\binom{n}{2k+1}}\sum_{j=0}^{k} S_{n,2k+1,j},$$

so that (8) we obtain

$$\bar{M}_{2k+1} - \bar{M}_{2k} = \frac{1}{\binom{n}{2k+1}} \cdot \frac{k+1}{2(2k+1)}(S_{n,2k+1,k} - S_{n,2k+1,k+1}),$$

which by Lemma 2 completes the proof.

4. Discussion

Condorcet's statement is sometimes split into three parts (see Karotkin and Paroush 2003). Added to it is the assertion that the advantage of a group decision over that of a sole expert increases with the size of the deciding body. This may be interpreted in various ways, and only few authors have elaborated on it. Karotkin and Paroush (2003) analyze the question of what happens to the probability of the group to decide correctly if a committee of size $2k+1$ is enlarged by adding 2 experts. They find sufficient conditions for the new probability to be higher than the old one. Similarly, they compare the probability of a homogeneous group of $2k+1$ experts, each with correctness probability p, and that of a group of $2k+3$ experts with lower probabilities $q < p$.

Our approach represents another interpretation of this part of Condorcet's statement. Namely, we have a fixed committee, but have to select between various decision strategies. The extreme possibilities are taking the whole body of experts or picking just one of them at random. The results of Ben-Yashar and Paroush (2000) mean that the first strategy is preferable, at least as far as the probability of making a correct decision is concerned. Of course, it may be very costly, if not utterly infeasible, to convene the whole group to make a decision, and one may consider middle-path strategies − pick up several members at random and let them decide. In this paper we have shown that the success probability when using this strategy increases consistently with the number of selected experts.

Let us emphasize that, augmenting the deciding body is not always advantageous. For example, a committee with competence structure $(0.9, 0.9, 0.9)$ has a probability of 0.972 of making the correct choice. Adding 2 experts of competence 0.6 each reduces the probability to 0.943. In this paper we have offered an alternative interpretation, under which the added part in Condorcet's statement is always valid (for experts with competence $p_i \geq \frac{1}{2}$). For example, for a committee with competence structure $(0.9, 0.9, 0.9, 0.6, 0.6)$ we have:

$$\bar{M}_0 = 0.5 < \bar{M}_1 = \bar{M}_2 = 0.78 < \bar{M}_3 = \bar{M}_4 = 0.886 < \bar{M}_5 = 0.943.$$

The situation is similar to that encountered for the statement that a large body performs better than each of its members separately. As we have mention, this may fail if the one member selected is one of the top experts in the group. However, if this single member is a random one, the statement becomes unconditionally correct (Ben-Yashar and Paroush 2000).

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