Viewpoint

Geometrical Analysis of Radiolaria and Fullerene Structures: Who Gets the Credit?

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The Viewpoint *column offers readers of* The Mathematical Intelligencer *the opportunity to write about any issue of interest to the international mathematical community. Disagreement and controversy are welcome. The views and opinions expressed here, however, are exclusively those of the author, and the publisher and editor-in-chief do not endorse them or accept responsibility for them.* Viewpoint *should be submitted to the editor-in-chief, Marjorie Senechal.*

The 1985 discovery of the C_{60} molecule, with carbon atoms at the 60 vertices of a truncated icosahedron (Fig. 1), by Harold W. Kroto, Richard E. Smalley, Robert F. Curl, and coauthors [1] was an important event in the nanotechnology revolution. The discoverers named it *buckminsterfullerene*, after the American architect Buckminster Fuller. The now-famous family of *fullerenes*—molecules of pure carbon in the shape of convex polyhedra with degree-3 vertices and pentagonal and hexagonal faces—soon followed [2].

For any convex polyhedron with F faces, E edges, and V vertices, we have the Euler relation $V - E + F = 2$. It is easy to show that the faces cannot all be hexagons. For fullerenes, where f_6 and f_5 are the numbers of hexagonal and pentagonal faces, respectively, it is almost as easy to show that $f_5 = 12$ and $V = 2(10 + f_6)$. Thus the number of pentagonal faces is always 12. The value of f_6 can be any number but 1 [3]. Accordingly, the smallest fullerene, C_{20}, has a shape of the dodecahedron, formed only by pentagons. The next fullerenes are C_{24}, C_{26}, C_{28}, …, C_{60}, C_{70}, $C_{2(10+h)}$ …

But these polyhedra were studied much earlier. The distinguished Scottish biologist and classics scholar D'Arcy Thompson (1860–1948) mentioned the Euler formula in connection with radiolaria in the first, 1917, edition of his book *On Growth and Form* [4]. Radiolaria are planktonic microorganisms whose sizes range from 0.04 mm to 1 mm. These fascinating geometrical creatures (Fig. 2) produce their skeletons from mineral compounds absorbed from seawater. Radiolarian skeletons are light, strong, and stable, the very requirements that led Fuller to his concept of geodesic domes.

In later editions, Thompson analyzed the now-called fullerenes in detail, and Kroto, *et al.*, cited his work. But was Thompson the first to carry out this analysis? In 2006, while working on my book *Fullerenes, Carbon Nanotubes and Nanoclusters: Genealogy of Forms and Ideas* [5], I found a reference to a book called *Geometry of Radiolaria* by Dmitry Morduhai-Boltovskoi, published in Russian in 1936 by

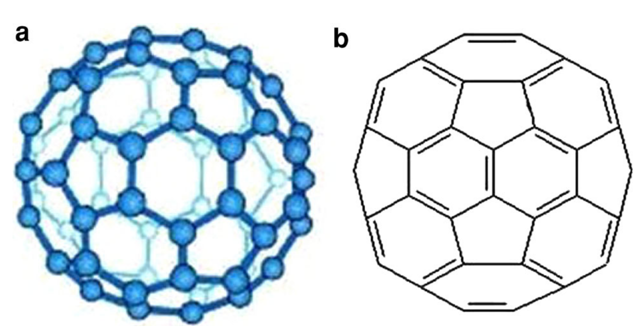

Figure 1. Molecule of *buckminsterfullerene*, C_{60}. Carbon atoms and carbon-carbon bonds are shown in a and b, respectively.

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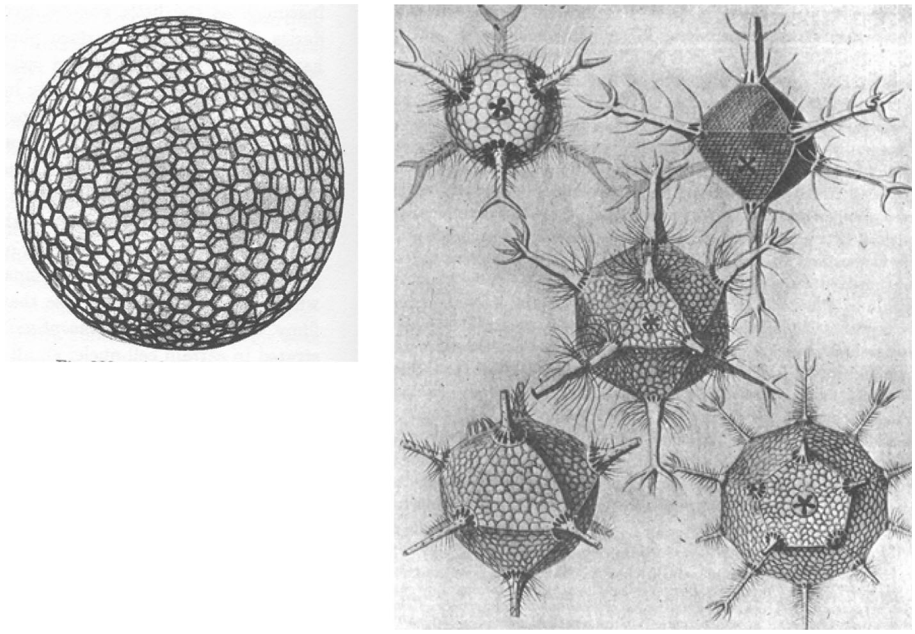

Figure 2. Radiolaria drawings by Ernst Haeckel (1834–1919).

Rostov-on-Don University Press [6]. His name did not mean anything to me at that time and it seemed absolutely impossible to find his book.

A long internet search eventually brought me to the website of V. Pyrkov from Rostov-on-Don, the archivist of works by Morduhai-Boltovskoi [7]. Thanks to the materials Pyrkov sent me, I finally read *The Geometry of Radiolaria* and much more by this remarkable scholar and thinker.

Dmitry Morduhai-Boltovskoi (1876–1952) (Fig. 3) was a prominent Russian mathematician, the author of an annotated Russian translation of Euclid's *Elements*. His mathematical interests included analysis, differential Galois theory, number theory, hyperbolic geometry, topology, and mathematical biology. Many of his students founded their own scientific schools. The famous Russian writer A. Solzhenitsyn was his student in Rostov University; Morduhai-Boltovskoi was a

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Figure 3. D. D. Morduhai-Boltovskoi, 1906 (photo provided by V. Pyrkov).

model for the character Professor Dmitri Dmitrievich Goryainov-Shakhovskoy in Solzhenitsyn's novel *The First Circle*.

Geometry of Radiolaria displays the power of the author's mathematical arsenal. To analyze radiolarian forms he applied the theory of polyhedra, calculus of variations, and differential equations. "Regular forms in living organisms can be explained by the economy of materials," he says. The chapter on "Discrete radiolarian geometry" includes a section on "Euler's theorem and its direct consequences" for polyhedra with n-sided faces and vertices of arbitrary degrees.

He concludes that, to model a radiolarian,

(a) A polyhedron should have either triangular faces or degree-3 vertices;
(b) A polyhedron without triangular and tetragonal faces should have at least 12 pentagonal faces; a polyhedron without degree-3 and degree-4 vertices should have at least 12 degree-5 vertices;
(c) A polyhedron with only degree-3 vertices and an arbitrary number of hexagonal faces should have 4 triangular or/and 6 tetragonal or/and 12 pentagonal faces.

Evidently Morduhai-Boltovskoi did not know of Thompson's 1917 work, nor would he have learned much from it had he read it. On the other hand, it's possible that Thompson read *Geometry of Radiolaria* while preparing the second, 1942 edition of *On Growth and Form*. For, in the *Autobiography of Prof. D. D. Morduhai-Boltovskoi, February 7, 1946* (sent to me by V. Pyrkov), the scientist wrote: "I have managed to print only some of my works on mathematical biology, including *Geometry of Radiolaria*. The latter became known to some foreign biologists through translations. D'Arcy Thompson in Scotland read it in Russian. I've got a letter from him with the very positive review."

In 2012, *Geometry of Radiolaria* was reprinted in Russian. I hope an English translation is on the way. This pioneering work deserves a place in the international history of science.

I should add, however, that in 1935, a year before Morduhai-Boltovskoi published his book, the American mathematician Michael Goldberg discovered the family of polyhedra, now called Goldberg polyhedra, that includes the fullerenes [8]. For a recent detailed account of his work, see [9].

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