$$q = -\frac{225}{8}\nu M_1 H^2 \{2(\mu\Omega - \omega) + e^2(27\mu\Omega - 46\omega)\}, \quad (18)$$

$$N_{xy} = \frac{225}{8}\nu M_1 H^2 \sqrt{1-\mu^2}\{\mu\Omega - 2\omega + 9e^2(\mu\Omega - 3\omega)\}. \quad (19)$$

Equations (17)-(19) have been obtained after averaging the fast-oscillating terms in the expressions for the energy dissipation rate and the projections of the tidal torque on the OXYZ coordinate axes. The inertial axis OZ has been taken along the binary system's total angular momentum vector, which is equivalent to the condition

$$K\dot{\omega}^{-1/3}\ \dot{\zeta}\sin J = I\Omega\sin(\beta - J). \quad (20)$$

In deriving Eqs. (13)-(16) we have made use of the relation [1] $\Phi + U = \pi$. The angle Λ remains practically unchanged in the present approximations.

One can solve the system of evolutionay equations (13)-(16) numerically. The results we have obtained for two of the most typical cases are here shown in graphical form.

Figure 3 plots the parameters Ω, ω, and $\mu = \cos\beta$ against elapsed time t for a binary system comprising white dwarfs with parameters $\nu_1 \sim 10^{12}$ cm ^{2}/sec, $M_1 = M_2 \approx 1\ M_\odot$ and for initial values $\beta_0 = 60°$, $\Omega_0 \approx 0.35\cdot10^{-2}$ sec^{-1}, $\omega_0 = \Omega_0/2$, $e_0 = 0.01$. Figure 4 illustrates the evolution of the eccentricity e for the same case.

In Fig. 5 we track the evolution of Ω, ω, μ for a pair of white dwarfs with $\nu_1 \sim 10^9$ cm ^{2}/sec and white dwarfs with $\nu_1 \sim 10^9$ cm ^{2}/sec and $M_1 = M_2 \approx 0.33\ M_\odot$ in which the initial angular velocity of orbital motion oxceeds each stars's axial angular rotation speed: $\omega_0 = 2\Omega_0 \approx 0.04$ sec^{-1}. The inclination $\beta_0 = 45°$. The two viscosities we have considered, $\nu \sim 10^{12}$, 10^9 cm ^{2}/sec, are the limiting values found by Durisen. [6]

It is evident from these diagrams that if the axial rotation has a higher angular velocity than the orbital motion ($\Omega > \omega$, Figs. 3 and 4), then synchronization ($\Omega = \omega$) and alignment of the rotation axis ($\beta = 0°$) will be achieved almost simultaneously; a far longer time will be required for the orbital eccentricity to disappear. On the other hand, if $\Omega < \omega$ (Fig. 5), synchronization will set in well before the axis becomes aligned.

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Negative magnetic pressure as a trigger of large-scale magnetic instability in the solar convective zone

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If small-scale magnetic fluctuations are generated in a medium subject to MHD turbulence, the total (hydrodynamic + magnetic) turbulent pressure will drop; and if the turbulence has become thoroughly established, this effect may reverse the sign of the large-scale magnetic force in the medium, rendering the large-scale field unstable.

1. INTRODUCTION

The magnetic fields of solar active regions have a highly nonuniform structure: a configuration of flux ropes develops, by some mechanism not yet fully understood. It has been suggested (see the monographs of Parker [1] and Priest,[2] and the references therein) that the magnetic flux ropes might originate from the prevailing large-scale field in the sun's convective zone when magnetic buoyancy triggers instabilities there. However, in order for such an instability to set in, the initial magnetic field would have to be strongly nonuniform in the direction of gravity; to the point where the characteristic scale for changes in the field is smaller than the density scale-height. Magnetic fields as inhomogeneous as we observe could be excited only by powerful, localized generators — sources which do not seem typical of the convective zone.[3,4]

In this letter we propose a net type of instability which might, if it were to develop in the turbulent solar convective zone, give rise to irregularities in the large-scale magnetic field taking the form of ropes or sheets. This instability could be triggered if a negative effective magnetic pressure were to emerge in a medium having well-developed small-scale hydrodynamic turbulence. Fine-scale turbulent pulsations would act as the energy source driving the instability.

Although several authors have investigated instabilities that would result from reversals in the sign of various forces,[5-7] the effects stemming from a sign reversal of magnetic forces within an ambient large-scale field, engendering unstable MHD perturbations in a medium subject to hydrodynamic turbulence, have not previously been considered.

2. EFFECTS OF NEGATIVE MAGNETIC PRESSURE

Suppose that a medium is characterized by magnetohydrodynamic turbulence having a basic scale ℓ_0 of hydrodynamic motion and a scale ℓ_m of magnetic pulsation. One example is the sun's turbulent convective zone: granules of scale $\ell_0 \sim (5\text{-}10)\cdot 10^7$ cm stochastically form and decay, while the energy of random hydrodynamic pulsations generates turbulent magnetic fields. We have recently shown[8,9] that the minimum scale, the thickness of the threads in the pulsational magnetic field, should be roughly $\ell_m \sim \ell_0 R_m^{-1/2}$, where R_m is the magnetic Reynolds number.

When turbulent magnetic fields are generated, the total (hydrodynamic + magnetic) turbulent pressure P_t will diminish. This phenomenon is quite general in character and might serve as a mechanism for producing flux tubes (ropes) in the large-scale solar magnetic field.

The principle of this effect qualitatively becomes apparent if we take isotropic turbulence as an illustration. In this case, one will recall, $\langle u_i(\mathbf{r}) \cdot u_j(\mathbf{r})\rangle = \langle u^2\rangle\delta_{ij}/3$ and $\langle h_i(\mathbf{r})\, h_j(\mathbf{r})\rangle = \langle h^2\rangle\delta_{ij}/3$, where the vectors $\mathbf{h}$ and $\mathbf{u}$ represent random pulsations of the hydrodynamic and magnetic fields, δ_{ij} is the Kronecker delta, and the angle brackets signify averages over some scale longer than ℓ_0. For isotropic turbulence the combined turbulent pressure may be written as

$$P_t = W_m/3 + 2W_k/3, \tag{1}$$

where $W_m = \langle h^2\rangle/8\pi$ is the energy density of the magnetic pulsations and $W_k = \langle\rho u^2\rangle/2$ is that of the turbulent hydrodynamic motions[10,11] (ρ denotes the plasma density).

Now suppose that the turbulence is maintained by some "inexhaustible" energy reservoir. This is in fact the typical situation in the convective zones of the sun and stars, where changes of state occur only over long evolutionary time spans. Accordingly when turbulent magnetic fields are generated, the total energy of of turbulence will be conserved (the dissipation of energy will be offset by replenishment).

In light of this circumstance ($W_k + W_m \equiv$ const) we can express the change of the turbulent pressure in an unbounded, statistically homogeneous medium in terms of the change ΔW_m in the magnetic energy desnity:

$$P_t = P_t^{(0)} - \Delta W_m/3, \tag{2}$$

where $P_t^{(0)}$ denotes the initial (excluding the generated field) turbulent pressure. When turbulent magnetic fields are generated, ΔW_m will be positive, so the turbulent pressure will diminish.

Next let us superimpose on the fine-scale isotropic MHD turbulence a large-scale ($L_B \gg \ell_0$) magnetic field. In particular, in the solar convection zone such a field will be induced by the effects of differential rotation and helicity.[12,4] On becoming "entangled" by the hydrodynamic turbulence, this large-scale field will give rise to additional small-scale magnetic pulsations.[13,3] One can show that the energy density W_m of the turbulent magnetic pulsations will then depend primarily on two quantities, W_k and the energy density $W_B = B^2/8\pi$ of the large-scale magnetic field B.

For the weak-field case ($W_B \ll W_k$), we may expand W_m in powers of W_B, obtaining

$$W_m = W_m^{(0)} + a_t(W_k)\, B^2/8\pi + \ldots,$$

where $W_m^{(0)}$ is the magnetic energy density that the pulsations would have if the large-scale field were absent. Combining this expression with Eq. (2), we may write the turbulent pressure P_t in the form

$$P_t = P_t^{(0)} - a_t B^2/24\pi. \tag{3}$$

For large-scale processes the total pressure P_{tot} will play an important role:

$$P_{tot} = P_k + P_t + P_B,$$

where P_k is the ordinary gas-dynamical pressure of the plasma and $P_B = B^2/8\pi$ is the magnetic pressure of the large-scale field. With Eq. (3), the total pressure becomes.

$$P_{tot} = P_k + P_t^{(0)} + (1 - q_P)\, B^2/8\pi, \tag{4}$$

where $q_P = a_t/3$. Thus in the presence of well-developed MHD turbulence the sign of the effective magnetic pressure $P_m = (1 - q_P)B^2/8\pi$ can change if $q_P > 1$.

When a large-scale field $\mathbf{B}$ is superimposed on turbulence that is initially isotropic, the isotropy will break down. Nevertheless Eq. (4) will remain valid; only the ratio between q_P and a_t will change. Our calculations indicate[1] that in the case of Kolmogorov turbulence with a superposed large-scale magnetic field the effective magnetic force may be expressed in the form

$$\mathbf{F}_m = -\nabla(1 - q_P)\, B^2/8\pi + (\mathbf{B}\cdot\nabla)(1 - q_S)\, \mathbf{B}/4\pi. \tag{5}$$

In deriving Eq. (5) we have presupposed[14] that the statistical moments have a characteristic relaxation time.

In two limiting cases the functions $q_P(R_m, \varepsilon_0)$, $q_S(R_m, \varepsilon_0)$ are given by simple expressions:

$$\text{a) } \varepsilon_0 \ll 1,\ R_m^{-1/4}:\ q_P \approx 4\ln R_m/15,\quad q_S \approx 2q_P/3; \tag{6a}$$

$$\text{b) } R_m^{-1/4} \ll \varepsilon_0 \ll 1:\ q_P \approx 8\,(1 - 5\ln\varepsilon_0/2)/25,\quad q_S \approx 8\,(1 - 15\ln\varepsilon_0)/15^2, \tag{6b}$$

where $\varepsilon_0 = 4v_A^2/u_0^2$, $v_A = B/\sqrt{4\pi\rho}$ is the Alfven velocity, and $u_0 = \sqrt{\langle u^2\rangle}$ is the turbulence velocity.

We point out that along with the possibility that the effective magnetic pressure may be negative (when $q_P > 1$), Eq. (5) also provides for a possible reversal in the sign of the magnetic stress force (when $q_S > 1$). For given ε_0 one can estimate from the expressions (6) the values of the magnetic Reynolds number for which the sign of the magnetic forces will reverse.

It further is worth noting that calculations support our assumption that the total energy of MHD turbulence is conserved if the energy reservoir is inexhaustible. Indeed, the superposition of a quasihomogeneous large-scale field upon well-developed MHD turbulence will not affect the total turbulence energy, but will merely redistribute it between the hydrodynamic and the magnetic pulsations.

The negative magnetic-pressure effect should not be confused with the lowering of magnetic pressure by turbulent diamagnetism.[15] Although diamagnetism can reduce the magnetic pressure, it cannot change the sign ($P_1 \approx P_2/R_m$, where P_1, P_2 denote the magnetic pressures inside and outside the turbulent region). Moreover, turbulent diamagnetism tends to "expel" the magnetic field from the zone occupied by turbulent plasma, whereas the negative magnetic-pressure effect may strengthen the field in that region.

We see, then, that once fine-scale turbulence has become established, the elasticity of the large-scale magnetic field will largely be suppressed. This circumstance might be capable of triggering large-scale MHD perturbations.

3. LARGE-SCALE MHD INSTABILITY EXCITED BY NEGATIVE VALUE OF EFFECTIVE MAGNETIC PRESSURE

Let us examine the large-scale phenomena that will develop on scales L in the range $\ell_0 \lesssim L \ll L_B$. The influence of the small-scale turbulence on these processes will be described in terms of the parameters q_P, q_S, and the turbulent viscosity coefficient $\nu_t \sim \ell_0 u_0/6$. Even though q_P, q_S, ν_t may all be determined by one and the same turbulence, the three parameters characterize different effects. The first two measure the nonlinear (with respect to the large-scale field) negative contribution of the turbulence to the effective magnetic force, while the parameter ν_t specifies the turbulent diffusion of the large-scale magnetic field (an effect linear in the field strength).

We first consider the properties of magnetic buoyancy in the presence of small-scale turbulence. Let the x axis of a Cartesian coordinate system be directed opposite the free-fall vector g, and let the z axis lie along the large-scale magnetic field $\mathbf{B}(x)$. To simplify matters we regard the field as horizontal.

We single out a magnetic flux tube located along the x axis at level 1, say, where the density is ρ_1 and the field strength is B_1. Now we gradually move the flux tube as a whole upward from level 1 to level 2, at which the ambient medium has corresponding parameters ρ_2, B_2. If the density ρ_2^* within the tube (in position 2), once the total pressures inside and outside the tube have equalized turns out to be lower than the density ρ_2 of the surrounding plasma, the flux tube will continue to float upward due to Archimedean forces.

Provided dissipative processes are absent and the thermal conductivity is high enough, the density excess $\Delta\rho = \rho_2^* - \rho_2$ can be determined, as usual, from the laws of conservations of mass and magnetic flux within the tube. We need merely recognize that in the presence of fine-scale turbulence the criterion for balance between the total pressures inside and outside the flux tube (at level 2) will take the form

$$c_s^2\rho_2 + \frac{K_P B_2^2}{8\pi} = c_s^2\rho_2^* + \frac{K_P B_2^{*2}}{8\pi},$$

where $K_P = 1 - q_P$, C_S is the speed of sound, and B_2^* is the magnetic field strength within the tube at point x_2. Assuming the displacement $\xi = x_2 - x_1$ to be small, we shall write the density ρ_2 and field strength B_2 as

$$\rho_2 = \rho_1(1 - \xi/\Lambda_\rho), \quad B_2 = B_1(1 - \xi/\Lambda_B),$$

where $\Lambda_\rho = -\rho\cdot dx/d\rho$, $\Lambda_B = -B\cdot dx/dB$ are the scale heights of the density and the magnetic field. As a result we obtain a density differential

$$\Delta\rho = \frac{B_1^2 K_P(\Lambda_B - \Lambda_\rho)}{4\pi c_s^2 \Lambda_B \Lambda_\rho}\xi. \tag{7}$$

The flux tube can become buoyant, that is, instability can set in, if

$$K_P(\Lambda_B - \Lambda_\rho)/\Lambda_B\Lambda_\rho < 0. \tag{8}$$

In mildly turbulent plasma having a comparatively small magnetic Reynolds number (with the quantity $K_P \approx 1$), the small-scale turbulence will not affect large-scale processes. Then in view of the condition (8), the criterion for instability due to magnetic buoyancy will take the form $\Lambda_B < \Lambda_\rho$. In other words, instability will develop only if the scale for change in the initial magnetic field is less than the density scale-height.[3,1]

The situation will be radically different, however, in a medium whose fine-scale hydrodynamic turbulence has become well established. Thus, when $q_P > 1$ the effective magnetic pressure of the plasma will become negative ($K_P < 0$), and conventional magnetic buoyancy in a highly nonuniform magnetic field will no longer exist [see the inequality (8)]. On the other hand, when $\Lambda_B > \Lambda_\rho$ and $K_P < 0$ instability will be excited in the large-scale magnetic field. It is evident from the condition (8) that instability will develop even in a large-scale field that is initially quasihomogeneous.

Let us estimate the growth rate of this instability. Neglecting dissipative processes for simplicity's sake, we shall retain only the Archimedean force in the equation of motion of the magnetic flux tube:

$$d^2\xi/dt^2 = -\left(\frac{v_A}{c_s}\right)^2 \frac{gK_P(\Lambda_B - \Lambda_\rho)}{\Lambda_B\Lambda_\rho}\xi, \tag{9}$$

where $v_A = B_1/\sqrt{4\pi\rho_1}$ is the Alfvén velocity. We seek a solution to Eq. (9) of the form $\xi \propto \exp(\gamma t)$; the instability growth rate will then be given by

$$\gamma = (v_A/\Lambda_\rho)\sqrt{-K_P(1 - \Lambda_\rho/\Lambda_B)}, \tag{10}$$

which makes use of the fact that $\Lambda_\rho = c_s^2/g$.

The small-scale turbulent pulsations are the energy source for this instability. This circumstance represents a fundamental distinction between the instability we are discussing and that considered by Parker.[1] The Parker instability is triggered by gravity forces in a highly nonuniform magnetic field ($\Lambda_B < \Lambda_\rho$), and in that sense is analogous to Rayleigh–Taylor instability.

As for the role of the turbulent viscosity, it will serve either to weaken the instability or to stabilize it completely.[2)] As a consequence our mode of instability will have a threshold character. It is evident from Eq. (10) that the instability growth rate is proportional to the large-scale field strength B. To obtain an estimate, we may regard the turbulent damping decrement γ_d of the MHD perturbations as being of order $\nu_t/\Lambda_\rho{}^2$. Then instability will set in only if the large-scale field is sufficiently strong: $B > B_{cr}{}^*\gamma_t, \rho)$. The threshold of instability for the large-scale field can be determined from the equation

$$\varepsilon_0\,(q_P\,(\varepsilon_0, R_m) - 1) = (l_0/3\Lambda_\rho)^2. \tag{11}$$

We therefore have here a new channel capable of transferring energy from small-scale turbulence to the large-scale magnetic field. The resultant instability could serve to generate irregularities in the large-scale field taking the form of sheets or ropes.

4. INSTABILITY IN TURBULENT SOLAR CONVECTIVE ZONE

It is important to decide whether the instability triggered by negative magnetic pressure might produce magnetic nonuniformities whose scale is comparable to that of sunspots. To be specific, let us take Spruit's model[16] for the convective zone. In this model the plasma at a depth of order 10^4 km (at least the size of a sunspot) would have parameters $R_m = u_0\ell_0/v_m = 3.1\cdot10^7$, a magnetic viscosity coefficient $\nu_m = 1.1\cdot10^5$ cm^2/sec, $\rho = 5\cdot10^{-4}$ g/cm^3, $\Lambda_\rho = 4.3\cdot10^8$ cm, $\nu_t = 5\cdot10^{11}$ cm^2/sec, $u_0 = 1.2\cdot10^4$ cm/sec, and $\ell_0 = 2.8\cdot10^8$ cm. The time scale for rotation of a turbulent convection cell will be roughly $\tau_0 \sim \ell_0/u_0 \approx 2.3\cdot10^4$ sec; we emphasize that our discussion is valid only for time spans longer than τ_0. In a field $B = 100$ gauss, the Alfvén velocity $v_A \approx 1.3\cdot10^3$ cm/sec, and $\varepsilon_0 \approx 4.7\cdot10^{-2}$.

By Eqs. (6b), the coefficient $K_P \approx -1.8$. The effective magnetic pressure will then be negative (see Secs. 2, 3), triggering an instability which will develop on a time scale $\tau_\nu \sim \Lambda_\rho/(v_A\sqrt{-K_P}) \approx 2.5\cdot10^5$ sec. For definiteness we assume that $\Lambda_\rho/\Lambda_B \ll 1$. At the levels in question the characteristic time for decay of MHD perturbations due to turbulent viscosity is roughly $\tau_d \sim \Lambda_\rho{}^2/\nu_t \approx 3.7\cdot10^5$ sec. According to Eq. (11), the threshold of instability for the large-scale field will be $B_{cr}(\nu_t\rho) \approx 65$ gauss. Since $\tau_0 < \tau_\gamma < \tau_d$, the instability when $B > B_{cr}$ will not be suppressed by the turbulent viscosity, and it would indeed be able to develop on scales comparable to the spot size.

Both theoretical estimates and the observations indicate that the toroidal magnetic field on the sun is nearly zero in the polar cap regions and on the equator; it reaches a maximum at low latitudes. For the most part, then, it is only at low latitudes that the large-scale field can become unstable. Presumably this is the circumstance that would account for the so-called royal zone of spot formation on the sun.

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1)These results will be discussed in full in a separate paper.
2)In the solar convective zone the turbulent viscosity significantly exceeds both the magnetic and the kinematic viscosity.

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