

ON THE STORAGE OF HIGH-ENERGY PROTONS IN THE SOLAR CORONA: THE CYCLOTRON INSTABILITY

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Abstract. We consider the problem of long-time storage of high-energy protons, accelerated in the process of a flare, in coronal magnetic traps. From the viewpoint of the storage, one of the most important plasma instabilities is the kinetic cyclotron instability of the Alfvén waves. We carry out a detailed theoretical analysis of the instability for typical conditions of the solar corona. It is the refraction of the Alfvén waves in combination with a drastic decrease of the instability growth rate with an increase of the angle between the directions of the wave vector and the stationary magnetic field that leads to the possibility of the long-term storage of the flare protons. Sufficient conditions of the storage are determined.

1. Introduction

High-energy protons, accelerated in the process of solar flare and injected into coronal regions of closed magnetic fields, can be trapped by these magnetic configurations (Figure 1). Subsequent evolution of the trapped protons (quick loss in the solar atmosphere, long-time storage in the trap, or escape into interplanetary space?) is of great interest.

Simple estimates show that Coulomb collisions in the coronal plasma (the concentrations n_0 we are interested in are of the order of 10^7 – 10^8 cm^{-3}) limit the lifetime of

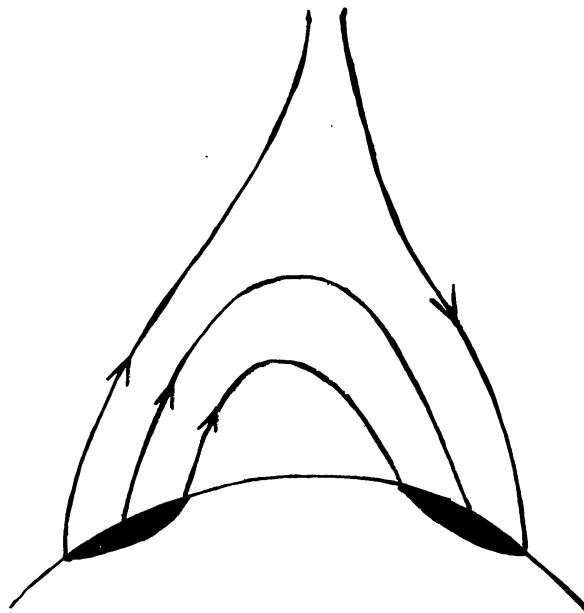


Fig. 1. The structure of the magnetic field of a coronal active region.

the protons with energies more than 10 MeV to the time of the order of 1 day. Protons with these energies also take a day to escape, due to the magnetic drift in a non-uniform and curved magnetic field (the intensity $B_0 \simeq 1-10$ G). However, there are processes which are able to reduce the life-time of the particles in such traps to some minutes. Meerson and Sasorov (1981) noted that if the energy density of flare-injected plasma (with allowance made for energetic particles) becomes comparable with, or greater than the pressure of the magnetic field, the magnetic configuration breaks down. It is the impossibility of combined 'magnetic field-plasma' equilibrium or MHD-instability of the equilibrium which cause the breakdown (Meerson and Sasorov, 1981). In this case, an appreciable amount of plasma together with energetic protons and frozen-in magnetic fields is ejected from the trap on a timescale of $\tau \simeq \mathcal{L}/c_A$, where \mathcal{L} is a characteristic size of the trap, $c_A = B_0(4\pi n_0 m_i)^{-1/2}$ is the Alfvén velocity, m_i is the ion mass. For typical values of $\mathcal{L} \simeq 10^{10}$ cm and $c_A \simeq 10^8$ cm s⁻¹ we have $\tau \simeq 10^2$ s.

If the pressure of both background plasma, p_0 , and high-energy protons, p_h , is much less than the pressure of the magnetic field, i.e., if

$$\beta_{0,h} \equiv 8\pi p_{0,h}/B_0^2 \ll 1, \quad (1.1)$$

the plasma in the trap can be stable with respect to fast large-scale MHD perturbations. In this case, however, various kinetic instabilities are essential. Some instabilities of this kind have already attracted attention. Meerson *et al.* (1978) investigated the bounce-resonant instability of the fast (magneto-acoustic) mode driven by high-energy protons with a peaked energy distribution. According to Meerson *et al.* (1978), this instability can support MHD-oscillations of a coronal condensation which give rise to a periodic modulation of type IV radio emission. Another example of kinetic instability driven by trapped high-energy protons was analyzed by Meerson and Sasorov (1981). They studied the gradient instability of the Alfvén waves due to the magnetic drift resonance with the protons.

However, it is the cyclotron instability (Sagdeev and Shafranov, 1960) that can be the most important in the situation considered. The instability arises due to anisotropy of longitudinal and transverse (with respect to the magnetic field) pressures of high-energy component of the plasma. In our case of a 'magnetic mirror' such anisotropy, namely, $p_{h\perp} > p_{h\parallel}$, is determined at least by a 'loss-cone' of the velocity distribution function of energetic protons. The cyclotron instability generates Alfvén waves by means of their resonant amplification by high-energy protons. If the plasma is homogeneous, the condition of cyclotron resonance takes the form

$$\omega - k_{\parallel} v_{\parallel} - n\omega_{Bi} = 0, \quad n = 0, \pm 1, \pm 2, \dots, \quad (1.2)$$

where ω is the wave frequency, ω_{Bi} is the proton gyrofrequency, v_{\parallel} is the component of proton's velocity along the direction of the stationary magnetic field \mathbf{B}_0 , $k_{\parallel} = k \cos \theta$, k is the absolute value of the wave vector \mathbf{k} , and θ is the angle between \mathbf{k} and \mathbf{B}_0 . The case of $n = 0$ in (1.2) corresponds to the Čerenkov resonance.

The scattering of high-energy protons by growing waves (Kennel and Petschek, 1966) can lead to a quick particle drift to the loss cone, i.e., to their precipitation into dense

chromospheric regions, where the particles lose their energy by collisions. The observations, however, indicate that there is a possibility of long-term storage of high-energy particles by coronal magnetic fields (Palmer *et al.*, 1975). How do we explain this?

Such a problem was first formulated by Wentzel (1976). However, his attempt to solve it had some shortcomings and that is why our results will differ considerably.

In this paper we carry out a consistent analysis of the cyclotron instability of the Alfvén waves with due regard for real conditions in the solar corona. On the basis of the analysis, we investigate the possibility of a long-term storage of high-energy protons in the coronal magnetic traps.

In Section 2 we consider the linear theory of the cyclotron instability of Alfvén waves. Here the expression for the instability growth rate is first obtained and investigated for general case of the wave propagation at arbitrary angle θ to the direction of the stationary magnetic field. The main result here is that the growth rate drastically falls with the increase of the angle θ . Possible damping processes of Alfvén waves in the coronal plasma are also discussed in Section 2.

In Section 3 the refraction of Alfvén waves propagating along the magnetic field lines is studied, the ‘geometrical optics’ equations being approximately solved.

Section 4 gives the estimates of amplification coefficients for Alfvénic wave packets. The refraction effects in combination with drastic fall of the growth rate with increase of θ leads to a possibility of a long-time storage of the particles. Sufficient conditions of the storage are found.

Finally, in Section 5 we present a brief discussion of the results and compare them with those of Wentzel (1976).

2. The Cyclotron Instability of the Alfvén Waves Driven by High-Energy Protons

Let us consider a magnetic trap filled with two-component plasma, namely, relatively cold ‘background’ and high-energy protons, the background plasma concentration, n_0 , being large compared to the high-energy proton concentration, n_1 . This model corresponds to a post-flare situation in the solar corona; it can be also applied to planetary magnetospheres. In this section we treat the plasma as homogeneous. Inhomogeneity effects will be analysed in Section 3.

The dispersion equation controlling frequencies of MHD waves and the character of their growth (damping) can be obtained by standard methods of linear theory of plasma (see, e.g., Ginzburg and Rukhadze, 1972, p. 402). This takes the form

$$\begin{vmatrix} \varepsilon_{11}^{(0)r} + \varepsilon_{11}^{(1)r} + \varepsilon_{11}^{(1)i} - N^2 \cos^2 \theta & \varepsilon_{12}^{(0)r} + \varepsilon_{12}^{(1)r} + \varepsilon_{12}^{(1)i} \\ -\varepsilon_{12}^{(0)r} - \varepsilon_{12}^{(1)r} - \varepsilon_{12}^{(1)i} & \varepsilon_{11}^{(0)r} + \varepsilon_{22}^{(0)i} + \varepsilon_{22}^{(1)r} + \varepsilon_{22}^{(1)i} - N^2 \end{vmatrix} = 0, \quad (2.1)$$

where $N = kc/\omega$ is the refractive index of the plasma, c is the velocity of light in vacuum, $\varepsilon_{\alpha\beta}^{(0)r}$ and $\varepsilon_{\alpha\beta}^{(1)r}$ are the real parts of the components of the dielectric constant tensors of the background plasma and of high-energy protons respectively, $\varepsilon_{\alpha\beta}^{(0)i}$ and $\varepsilon_{\alpha\beta}^{(1)i}$ are the

corresponding imaginary parts (see Appendix 1). Equation (2.1) was obtained under the following assumptions: (i) the components of the dielectric constant tensor, which define the imaginary parts of MHD wave frequencies (see below) are much less than the components defining the real parts of the frequencies; (ii) the contribution of high-energy protons to the real parts of MHD wave frequencies is much less than the corresponding contribution of the background plasma;* (iii) the Alfvén velocity is much less than the velocity of light in vacuum: $c_A \ll c$. It should also be remembered that we require $\beta_{0,h} \ll 1$ (see (1.1)).

From Equation (2.1) we have the following dispersion relation

$$2N^2 \cos^2 \theta = U \pm \sqrt{W}, \quad (2.2)$$

where

$$U = \varepsilon_{11}^{(0)r}(1 + \cos^2 \theta) + \varepsilon_+^r + \varepsilon_+^i + \varepsilon_{22}^{(0)i} \cos^2 \theta, \quad (2.3)$$

$$W = (\varepsilon_{11}^{(0)r} \delta(\theta, \alpha))^2 + 2\varepsilon_{11}^{(0)r}(\varepsilon_-^r \sin^2 \theta - 4\alpha i \varepsilon_{12}^{(1)r} \cos^2 \theta) + \\ + 2\varepsilon_{11}^{(0)r}(\varepsilon_-^i \sin^2 \theta - 4\alpha i \varepsilon_{12}^{(1)i} \cos^2 \theta) - 2\varepsilon_{22}^{(0)i} \cos^2 \theta (\varepsilon_-^r + \varepsilon_{11}^{(0)r} \sin^2 \theta), \quad (2.4)$$

$$\varepsilon_{\pm}^r = \varepsilon_{11}^{(1)r} \pm \varepsilon_{22}^{(1)r} \cos^2 \theta, \quad \varepsilon_{\pm}^i = \varepsilon_{11}^{(1)i} \pm \varepsilon_{22}^{(1)i} \cos^2 \theta,$$

$$\delta(\theta, \alpha) = (\sin^4 \theta + 4\alpha^2 \cos^2 \theta)^{1/2}, \quad \alpha = \omega/\omega_{Bi}.$$

The dispersion Equation (2.2) describes the modified MHD waves, the sign (+) in (2.2) corresponding to an ion-cyclotron (Alfvén) wave while the sign (−) corresponds to a fast (magneto-acoustic) wave. In formula (2.4) we omitted terms of a higher order with respect to small parameters n_1/n_0 and m_e/m_i ; m_e is the mass of electron.

Let us consider the physical meaning of different terms in expressions (2.3) and (2.4). The first two terms in (2.3) and (2.4) determine the frequencies of modified MHD waves. The third term, both in (2.3) and in (2.4), describes the resonant amplification (or damping) of these waves by high-energy protons. The last term, both in (2.3) and in (2.4), determines Landau damping of the modified MHD waves on background plasma electrons.

The velocity distribution function of the background plasma electrons may be assumed to be Maxwellian with the temperature T_b , the corresponding thermal velocity being $v_{Te} = (2T_b/m_e)^{1/2}$. As to the velocity distribution of the high-energy protons, the presence of a loss-cone in the magnetic trap causes a deficit of the particles with large longitudinal velocities. This fact enables us to assume the distribution function of high-energy protons ‘bi-Maxwellian’ with $T_{\perp} > T_{\parallel}$:

$$f(v) = n_1 \left(\frac{m_i}{2\pi T_{\perp}} \right) \left(\frac{m_i}{2\pi T_{\parallel}} \right)^{1/2} \exp \left(-\frac{m_i v_{\perp}^2}{2T_{\perp}} - \frac{m_i v_{\parallel}^2}{2T_{\parallel}} \right), \quad (2.5)$$

* It can be shown from Equations (A1.1)–(A1.6) in Appendix 1 that condition (i) is equivalent to $n_1/n_0 \ll \alpha \equiv \omega/\omega_{Bi}$. When $\alpha \ll 1$ (see below), this inequality can be rewritten as $\beta_h \alpha \ll 1$. Condition (ii) is fulfilled when $n_1/n_0 \ll \alpha^2$. In case of $\alpha \ll 1$, this is equivalent to $\beta_h \ll 1$.

where the indices \perp and \parallel stand for directions transverse and longitudinal to the stationary magnetic field, respectively, T is a characteristic energy of protons. The parameter $\eta = 1 - T_{\parallel}/T_{\perp}$ characterizes the degree of temperature anisotropy; $0 < \eta < 1$. As we know, such a character of anisotropy of the high-energy proton distribution leads to the possibility of a resonant excitation of ion-cyclotron (Alfvén) waves (Sagdeev and Shafranov, 1960). So we consider the case of sign (+) in dispersion Equation (2.2).

It is known, that the character of ion-cyclotron wave polarization is controlled by the competition between the parameters $\sin^2 \theta$ and $\alpha = \omega/\omega_{Bi}$ (see, e.g., Stix, 1962). When the angle θ is very small, $\sin^2 \theta \ll \alpha$, the wave polarization is close to circular, while in the opposite limit, $\sin^2 \theta \gg \alpha$,* the waves have almost linear polarization. We found this fact to significantly affect both the growth rate of the cyclotron instability and the wave damping rates on the background plasma electrons. So we first consider these limits prior to general case.

A. $\sin^2 \theta \ll \alpha$

In this case the expansion of the formula (2.4) in small parameter $\sin^2 \theta/\alpha$ yields the following equation for the real part of frequency of modified ion-cyclotron wave having almost circular polarization:

$$\omega = k_{\parallel} c_A (1 - \alpha)^{1/2}. \quad (2.6)$$

In (2.6) we have omitted small corrections taking into account the contribution of high-energy protons. These corrections can be easily found from general equations (2.2)–(2.4).

In the case considered, the wave growth rate has the form

$$\Gamma = \Gamma_0 \{1 - \theta^2 [a(\eta, \alpha, y) + (\theta/\alpha)^2 b(\eta, \alpha, y)]\}, \quad (2.7)$$

where

$$a(\eta, \alpha, y) = \left(2 + \frac{\alpha}{\eta - \alpha} \exp\left(\frac{1 - 2\alpha}{2y^2}\right) - \frac{2\eta - \alpha}{\eta - \alpha} \exp\left(\frac{2\alpha - 3}{2y^2}\right) \right) y^2 (1 - \eta)^{-1}, \quad (2.8)$$

$$b(\eta, \alpha, y) = (1/2)^4 (\eta - \alpha)^{-1} [1 + \exp(-2\alpha/y^2)] [\eta - \alpha \operatorname{th}(\alpha/y^2)], \quad (2.9)$$

$$\Gamma_0 = (\pi/2)^{1/2} \omega(n_1/n_0) \frac{(1 - \alpha)^2 (1 - \eta)^{-1}}{\alpha^2 y (2 - \alpha)} (\eta - \alpha) \exp\left(-\frac{(1 - \alpha)^2}{2y^2}\right), \quad (2.10)$$

$y = (k_{\parallel}/\omega_{Bi})(T_{\parallel}/m_i)^{1/2}$. Γ_0 is nothing but a well-known expression for the growth rate of unstable ion-cyclotron wave propagating along the magnetic field, $\theta = 0$ (Kennel and Petschek, 1966; Feigin and Yakimenko, 1969).

* To get this limit it is necessary to require $\alpha \ll 1$. In the case of $\beta_h \gg n_1/n_0$ the latter inequality is always true (see below).

In the case of $\eta \gtrsim \alpha$, $\alpha \ll 1$, and $y \simeq 1$, the order of magnitude estimated for Γ_0 is $\Gamma_0 \simeq \beta_h \eta \omega$. Note, that Equations (2.6)–(2.10) are valid for arbitrary $0 < \alpha < 1$. It follows from (2.10) that the instability criterion for $\theta = 0$ has a well-known form $\eta > \alpha$, i.e., $1 - T_{\parallel}/T_{\perp} > \omega/\omega_{Bi}$ (Sagdeev and Shafranov, 1960). When $\eta > \alpha$, the expressions $a(\eta, \alpha, y)$ and $b(\eta, \alpha, y)$ are both positive. Then Equation (2.7) means that the wave growth rate decreases monotonically with θ , i.e., the maximal growth rate is Γ_0 and it takes place at $\theta = 0$. The formula (2.7) describes small negative corrections to the growth rate Γ_0 , whose absolute values increase with θ .

From general expressions (2.2)–(2.4) we can derive the instability criterion for the case of $\sin^2 \theta \ll 1$:

$$\eta > \alpha \frac{\delta(\theta, \alpha) + 2\alpha \operatorname{th}(\alpha/y^2)}{2\alpha + \delta(\theta, \alpha) \operatorname{th}(\alpha/y^2)}. \quad (2.11)$$

When $\alpha \ll y^2$ the criterion is reduced to $\eta > \delta(\theta, \alpha)/2$; when $\theta = 0$ this converts to Sagdeev's–Shafranov's criterion $\eta > \alpha$. It follows from (2.11) that the instability threshold increases with θ . Given θ , the growth rate achieves maximum in the region of $y \simeq 1$ (for the case of $\theta = 0$ and $\alpha \ll 1$ the corresponding value of y is $2^{-1/2}$). It follows that unstable wave with a maximal growth rate has a frequency of the order of $\omega \simeq (c_A/v_0)\omega_{Bi}$, where $v_0 = (2T_{\parallel}/m_i)^{1/2}$ is a characteristic velocity of high-energy protons. Since for the typical coronal conditions we have $c_A/v_0 \simeq 10^{-2} - 10^{-1}$, the value of α is rather small, $\alpha = \omega/\omega_{Bi} \ll 1$. Therefore, in what follows we are interested in the case of $\alpha \ll 1$, when ion-cyclotron waves convert to 'purely' Alfvén waves.

Unstable Alfvén waves can be Landau-damped on thermal electrons. In the region of $\sin^2 \theta \ll \alpha$ the corresponding damping rate has the form (Stepanov, 1958)

$$\gamma = -\omega(\sqrt{\pi}/4)(m_e/m_i)(v_{Te}/c_A) \exp(-c_A^2/v_{Te}^2) \sin^2 \theta, \quad (2.12)$$

and it is usually small.

B. $\sin^2 \theta \gg \alpha$

In this case we may expand the expression (2.4) in small parameter $\alpha/\sin^2 \theta$. Then the wave growth rate is

$$\Gamma = (i\omega/2)(c_A/c)^2 (\varepsilon_{11}^{(1)i} - 2\alpha i \varepsilon_{12}^{(1)i} \operatorname{ctg}^2 \theta), \quad (2.13)$$

where now $\omega = k_{\parallel} c_A$ with an accuracy of small corrections of the order of β_h and α^2 .

The calculation of the components $\varepsilon_{11}^{(1)i}$ and $\varepsilon_{12}^{(1)i}$ for the distribution function (2.5) (see Appendix (1)) gives an expression for the wave growth rate Γ which can be tabulated. The final formula appears nearly as cumbersome, as in general case of arbitrary $\sin^2 \theta/\alpha$, so we do not present this here. It should be noted, however, that the expression (2.13) differs from corresponding results obtained by Dobeš (1968) and by Melrose and Wentzel (1970) by presence of the second term proportional to $\varepsilon_{12}^{(1)i}$. Since both the terms in (2.13) are comparable with each other, the account of the second term is indispensable.

It follows from (2.13) that the order of magnitude estimate of the wave growth (damping) rate Γ in this range of angles θ is $\Gamma \simeq \alpha \beta_h \omega$ (at $\eta \gtrsim \alpha$ and $y \simeq 1$). This suggests

that in the region of $\sin^2 \theta \gg \alpha$ a significant decrease of the growth rate occurs. For example, when $\eta \gg \alpha$, the growth rate decrease factor is of the order of η/α . It is essential that this decrease of the growth rate with θ takes place at very small values of θ , as long as $\alpha \ll 1$. This fact, unnoticed by Wentzel (1976), will be shown to play the most important role in the problem of the storage of high-energy protons (see Section 4).

It is to be recalled that in the case of $0 \ll \sin^2 \theta \ll 1$ the instability criterion (2.11) remains correct as well as its reduced form $\eta > \delta(\theta, \alpha)/2$ for $\alpha \ll y^2 \simeq 1$.

Now let us consider Alfvén wave damping in the region of $\sin^2 \theta \gg \alpha$. Since high-energy protons affects wave dispersion characteristics, the Alfvén wave gets some properties of fast (magneto-acoustic) mode, the fact leading to modified (additional) wave damping on thermal electrons of the coronal plasma. The corresponding damping rate can be easily obtained from (2.2)–(2.4) and this takes the form:

$$\gamma_m = -i(\omega/4)(c_A/c)^4 \varepsilon_{22}^{(0)i} \varepsilon_-^r \operatorname{ctg}^2 \theta. \quad (2.14)$$

It follows the order of magnitude estimate of the ratio between γ_m and Γ . When $\eta \gtrsim \alpha$, we have

$$\gamma_m/\Gamma \simeq (\beta_0 m_e/m_i)^{1/2} (\eta/\alpha). \quad (2.15)$$

It is clear that in certain conditions the modified damping rate can dominate over the instability growth rate in the region of $\sin^2 \theta \gg \alpha$. On the other hand, when $\eta \gtrsim \alpha$, the modified damping rate in the region of $\sin^2 \theta \gg \alpha$ is always much less than the instability growth rate in the region of $\sin^2 \theta \ll \alpha$, the ratio of γ_m/Γ being of the order of

$$\gamma_m/\Gamma \simeq (\beta_0 m_e/m_i)^{1/2} \ll 1. \quad (2.16)$$

Note, that the possibility of modified damping of the Alfvén wave on coronal electrons due to the ‘coupling’ with fast mode was discussed by Wentzel (1976), within the frame work of a phenomenological model. Wentzel assumed that the mode ‘coupling’ is caused by the resonant protons. It follows from Equation (2.14) that this is not the case. Indeed, we need only a contribution of high-energy protons to the wave dispersion. Equation (2.14) differs considerably from the heuristic result of Wentzel (see his formula (12)), the latter, in particular, does not include parameters of high-energy protons.

Apart from this damping mechanism, a ‘usual’ Landau damping of Alfvén waves on thermal electrons may also contribute to the wave damping; for corresponding damping rate see Appendix 2.

C. GENERAL CASE OF $\sin^2 \theta \simeq \alpha$

Carrying out the expansion of (2.4) in small parameters m_e/m_i and β_h (the parameter $\sin^2 \theta/\alpha$ has an arbitrary value), we obtain the wave growth rate in the form

$$\Gamma = (i\omega/4)(c_A/c)^2 (\varepsilon_+^i + (\sin^2 \theta/\delta(\theta, \alpha)) \varepsilon_-^i - 4\alpha \cos^2 \theta i \varepsilon_{12}^{(1)i} \delta_{(\theta, \alpha)}^{-1}), \quad (2.17)$$

where $\omega = k_{\parallel} c_A$ with an accuracy of corrections determined by small parameters β_h and α . The expressions for the components of the dielectric constant tensor for the distribution function (2.5) are given in Appendix 1.

We carried out numerical calculations of the dimensionless growth rate, $\tilde{\Gamma} = (n_0/n_1)\omega_{Bi}^{-1}\Gamma$, over a wide range of angles θ , fixing the value of $y = 2^{-1/2}$ which corresponds to the maximal growth rate of the 'straight' wave ($\theta = 0$). Figures 2 and 3 represent several results of these calculation for various cases typical for the solar corona. The numerical results confirm the analytical estimates presented in items (a) and (b) of this section. It is important that instability is followed by stability beginning with an angle $\theta = \theta_1$, unless the anisotropy parameter η is very large, $\eta < \eta_*$. In our case of $y = 2^{-1/2}$ we found η_* to be approximately equal to 0.41. The dependence of $\theta_1 = \theta_1(\eta)$

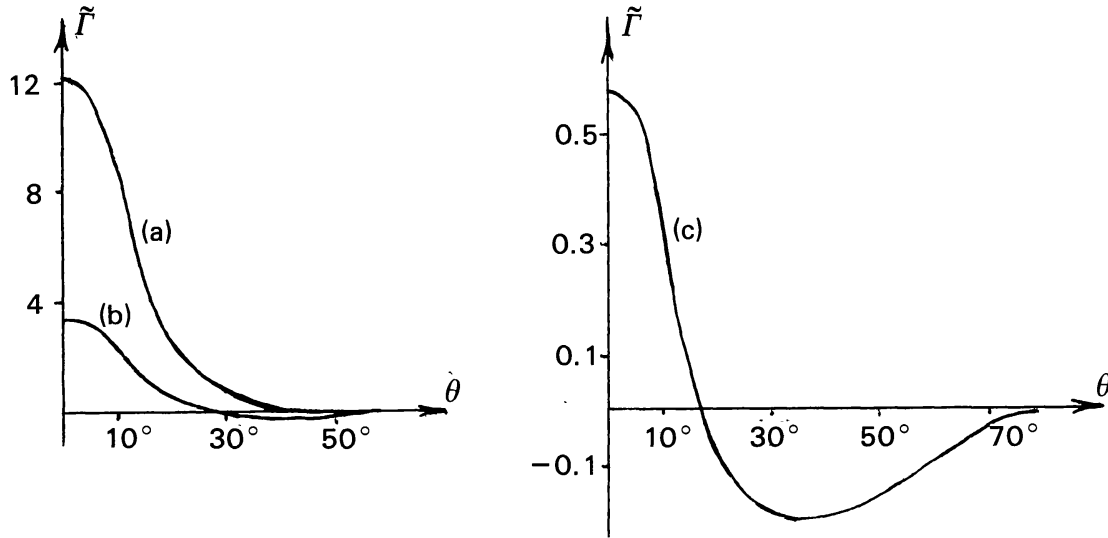


Fig. 2. The dimensionless growth rate, $\tilde{\Gamma} = (n_0/n_1)\omega_{Bi}^{-1}\Gamma$, is plotted against the angle θ ($y = 2^{-1/2}$, $\alpha = 7 \times 10^{-3}$) for various values of the anisotropy parameter η : (a) $\eta = 0.4$ (curve (a)); (b) $\eta = 0.17$ (curve (b)); (c) $\eta = 0.05$ (curve (c)).

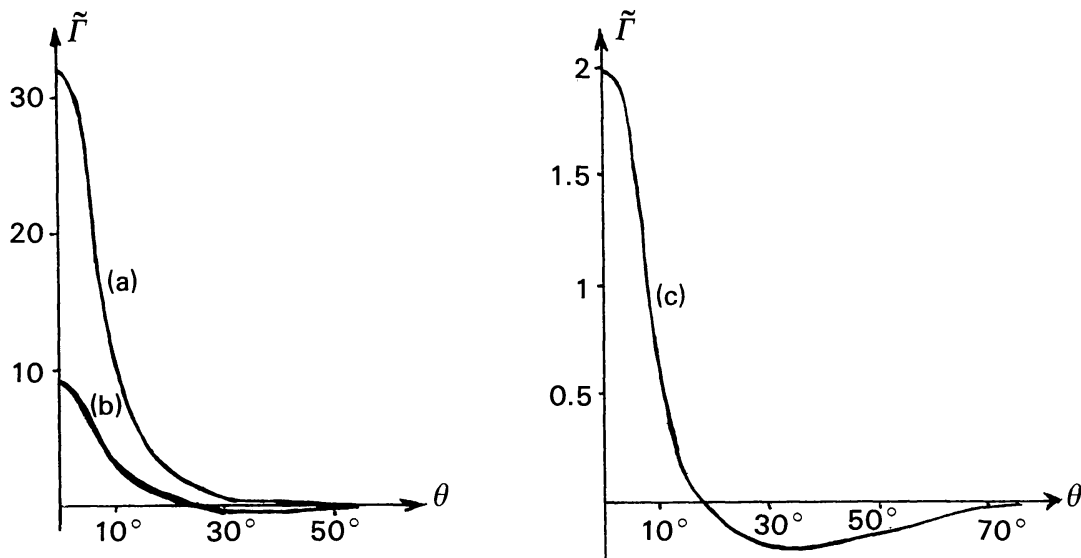


Fig. 3. The dimensionless growth rate, $\tilde{\Gamma} = (n_0/n_1)\omega_{Bi}^{-1}\Gamma$, is plotted against the angle θ ($y = 2^{-1/2}$, $\alpha = 1.8 \times 10^{-2}$) for various values of the anisotropy parameter η : (a) $\eta = 0.4$ (curve (a)); (b) $\eta = 0.17$ (curve (b)); (c) $\eta = 0.05$ (curve (c)).

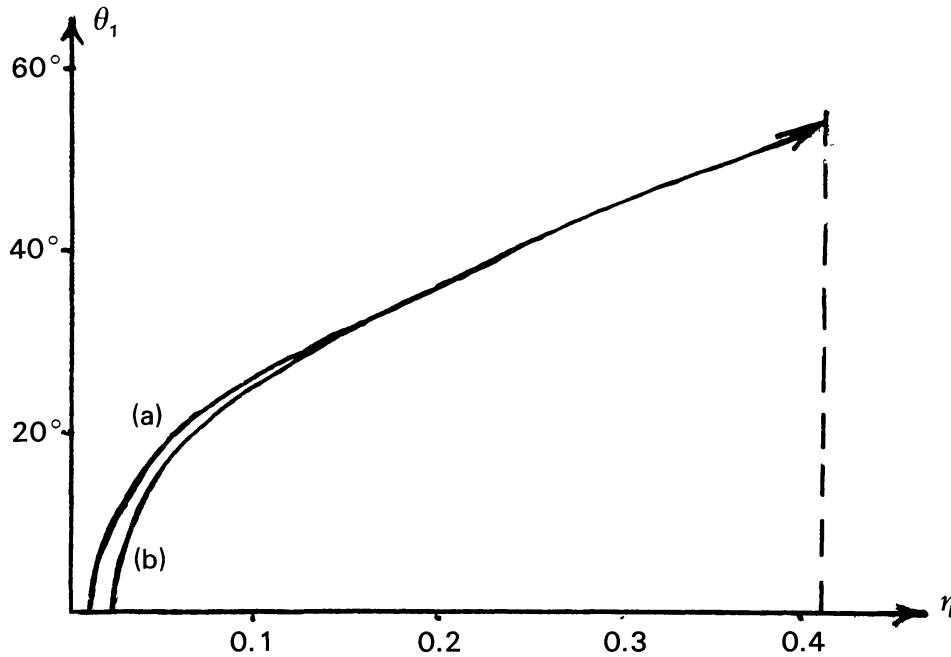


Fig. 4. The dependence of the angle θ_1 on the anisotropy parameter η for $y = 2^{-1/2}$ and various values of α : $\alpha = 7 \times 10^{-3}$ (curve (a)); $\alpha = 1.8 \times 10^{-2}$ (curve (b)).

is shown in Figure 4 for two typical values of α . The convergence of these two curves $\theta_1(\eta)$ in the region of $\sin^2 \theta \gg \alpha$ can be explained by independence of $\tilde{\Gamma}$ upon α for such angles (see Equations (2.13), (A1.4), and (A1.6)). When $\eta > \eta_*$, we have $\Gamma > 0$ for all angles, $0^\circ < \theta < 90^\circ$.

In the case of $\sin^2 \theta \simeq \alpha$, the Alfvén wave damping may be determined by high-energy protons themselves (when $\theta > \theta_1$) as well as by Landau damping on thermal electrons. The latter mechanism is characterized by the damping rate (Stepanov, 1958)

$$\gamma = (i\omega/4)(\epsilon_{22}^{(0)i}/\epsilon_{11}^{(0)r}) \cos^2 \theta (1 - \sin^2 \theta / \delta(\theta, \alpha)). \quad (2.18)$$

As to modified damping due to the mode coupling, it is always very small unless $\sin^2 \theta \gg \alpha$.

So far we have analysed the dependence of the wave growth rate on the angle θ for a fixed value of y . In contrast, the y -dependence of the growth rate for fixed θ is shown in Figure 5a, b for the anisotropy parameter $\eta = 0.1$ and 0.4 , respectively. It is seen that the characteristic scale of the wave growth rate decrease with y is of the order of unity for all angles. On the contrary, as we have seen earlier, the characteristic scale of the wave growth rate decrease with the angle θ is equal to $\alpha^{1/2}$, i.e. much less than unity in the situation considered.

So, in this section we have investigated the cyclotron instability of Alfvén waves and their possible damping mechanisms over a wide range of parameters θ , y , and η . This investigation is necessary for the determination of the role the cyclotron instability can play in real situations, such as solar corona or planetary magnetospheres.

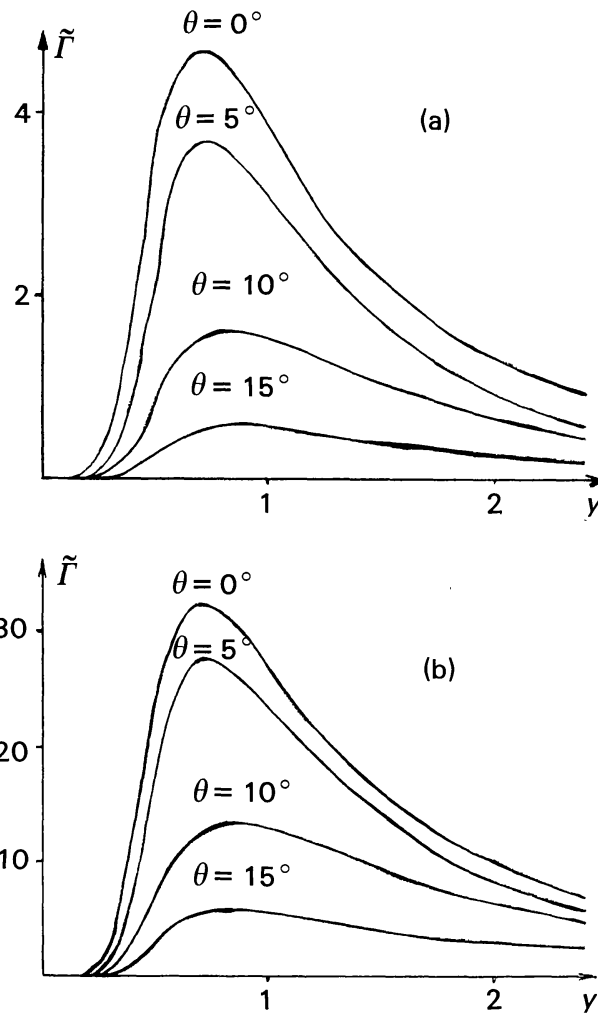


Fig. 5. The dimensionless growth rate, $\tilde{\Gamma} = (n_0/n_1)\omega_{Bi}^{-1}\Gamma$, is plotted against the parameter y ($c_A/v_0 = 7 \times 10^{-3}$) at $\theta = 0^\circ$; 5° ; 10° ; 15° for various values of the anisotropy parameter η : (a) $\eta = 0.1$; (b) $\eta = 0.4$.

3. Alfvén Wave Propagation and Amplification in Closed Magnetic Fields

In the solar corona above the active regions, the spatial distributions of plasma density and (especially) the magnetic field are strongly inhomogeneous if scales of the order of $0.5-1 R_\odot$ are meant. In this situation, Alfvén waves propagating along a magnetic field line of the active region are subjected to refraction.

To investigate this, let us consider the evolution of Alfvénic wave packet whose length is small if compared to the field line length \mathcal{L} , but large if compared to the wavelength, λ . In the frame of the WKB approximation, or the approximation of geometrical optics,* Weinberg (1962) obtained a system of evolution equations for the radius-vector \mathbf{r} of the center of the packet and for the characteristic wave-vector \mathbf{k} . These equations can be

* For waves of the frequency range considered, this approximation is valid with high precision; the parameter λ/\mathcal{L} being as small as 10^{-4} .

written in Hamiltonian form as follows:

$$d\mathbf{r}/dt = \partial\omega/\partial\mathbf{k} , \tag{3.1}$$

$$d\mathbf{k}/dt = -\partial\omega/\partial\mathbf{r} , \tag{3.2}$$

where $\omega = \mathbf{k}c_A(\mathbf{r})$, $c_A(\mathbf{r}) = \mathbf{B}_0(\mathbf{r})(4\pi m_i n_0(\mathbf{r}))^{-1/2}$ is the vector of Alfvén velocity.

Let us consider a coronal magnetic field line of the active region. For the sake of simplicity, we assume the field line to be plane. Introduce the local Cartesian coordinate system (x, y, z) , where the z axis coincides with the tangent to the chosen field line, while the x and y axes are opposite to the directions of the principal normal and binormal, respectively (Figure 6). Let us also introduce the arc length parameter l of the field line considered. Rewrite (3.1) in the following form:

$$(dl/dt)\mathbf{e}_z = \mathbf{c}_A , \tag{3.3}$$

where $\mathbf{e}_z \equiv d\mathbf{r}/dl$ is the unit vector directed along the z axis. Time derivative of the wave vector \mathbf{k} in the chosen local coordinate system takes the form

$$\frac{d\mathbf{k}}{dt} = \frac{dk_x}{dt} \mathbf{e}_x + \frac{dk_y}{dt} \mathbf{e}_y + \frac{dk_z}{dt} \mathbf{e}_z + k_x \frac{d\mathbf{e}_x}{dt} + k_z \frac{d\mathbf{e}_z}{dt} , \tag{3.4}$$

where $\mathbf{e}_x, \mathbf{e}_y$ are unit vectors directed along x and y axis, respectively. Since the field line is assumed to be plane, the direction of the vector \mathbf{e}_y is constant and $d\mathbf{e}_y/dt = 0$.

Now we make a transition from the derivatives with respect to t to the derivatives with respect to l in the right-hand side of (3.4). Then, using Frenet formulae which relate derivatives $d\mathbf{e}_i/dl$ to vectors \mathbf{e}_i themselves (see, e.g., Korn and Korn, 1968) we obtain the expression for $d\mathbf{k}/dt$ in the following form:

$$\frac{d\mathbf{k}}{dt} = \frac{dl}{dt} \left(\mathbf{e}_x \left(\frac{dk_x}{dl} - k_z R^{-1}(l) \right) + \mathbf{e}_y \frac{dk_y}{dl} + \mathbf{e}_z \left(\frac{dk_z}{dl} + k_x R^{-1}(l) \right) \right) , \tag{3.5}$$

where $R(l)$ is the local curvature radius of the chosen field line.

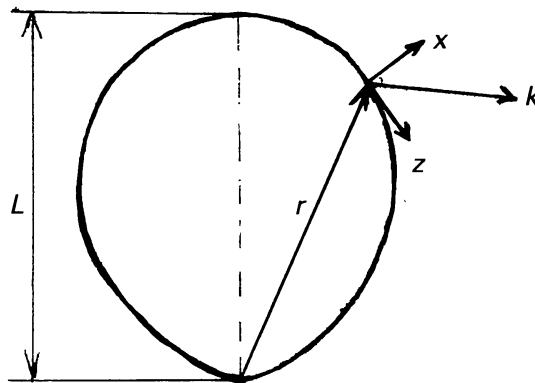


Fig. 6. The local Cartesian coordinate system (x, y, z) associated with the field line of the stationary magnetic field. The y -axis (not shown) is perpendicular to the plane of drawing and is directed to reader.

In the case of $\beta_{0,h} \ll 1$, the coronal magnetic field may be regarded as potential, $\text{curl } \mathbf{B}_0 = 0$. Besides, we may neglect the spatial inhomogeneity of the coronal plasma concentration. This can be justified by the fact that the concentration and magnetic field inhomogeneities in (3.2) appear in the function $c_A(\mathbf{r})$ only, the dependence of the magnetic field of the active region on \mathbf{r} being there much stronger than the corresponding dependence of $(n_0(\mathbf{r}))^{1/2}$. Then, the derivative $\partial\omega/\partial\mathbf{r}$ in (3.2) takes the form

$$\partial\omega/\partial\mathbf{r} = -(c_A/B_0)(\mathbf{k} \cdot \nabla)\mathbf{B}_0. \quad (3.6)$$

Combining (3.2)–(3.6) and using the equation $\text{div } \mathbf{B}_0 = 0$ we obtain equations describing the change of the wave vector's coordinates k_x , k_y , and k_z in the process of Alfvénic wave packet propagation along the stationary magnetic field:

$$\frac{dk_x}{dl} = k_x \frac{\partial B_{0z}}{B_{0z} \partial z} + k_z \left(R^{-1} - \frac{\partial B_{0z}}{B_{0z} \partial z} \right), \quad (3.7)$$

$$\frac{dk_y}{dl} = 0, \quad (3.8)$$

$$\frac{dk_z}{dl} = -k_x \left(R^{-1} + \frac{\partial B_{0z}}{B_{0z} \partial z} \right) - k_z \frac{\partial B_{0z}}{B_{0z} \partial z}. \quad (3.9)$$

It follows from (3.8) that the y component of wave-vector \mathbf{k} does not vary during propagation (it should be remembered that this is a consequence of the field line being a plane and of the plasma concentration inhomogeneity being weak). Thus, we may put $k_y = 0$: the account of non-zero k_y can be made by simply redenoting the final results.

For potential magnetic fields, the derivative $\partial B_0/\partial x$ is well-known to relate to the local curvature radius of the field line:

$$\partial B_0/B_0 \partial x = -R^{-1}(l). \quad (3.10)$$

Taking into account (3.10) and the equalities $B_{0z} = B_0$ and $\partial B_0/\partial z = \partial B_0/\partial l$, which take place for the chosen field line, we can rewrite the equation system (3.7) and (3.9) in the following form:

$$\frac{dk_x}{dl} = k_x \frac{\partial B_0}{B_0 \partial l} + k_z(2R^{-1}(l)), \quad (3.11)$$

$$\frac{dk_z}{dl} = -k_z \frac{\partial B_0}{B_0 \partial l}. \quad (3.12)$$

Integration of (3.12) yields the result: $k_z B_0 = \text{constant}$ (i.e. $\omega = \text{constant}$). Combining (3.11) and (3.12), we obtain two equations describing the evolution of the angle

$\theta = \text{arctg}(k_x/k_z)$ and the wave number $k = (k_x^2 + k_z^2)^{1/2}$:

$$\frac{dX}{dl} = 2 \frac{\partial B_0}{B_0 \partial l} X + 2R^{-1}(l), \quad (3.13)$$

$$k^{-1} \frac{dk}{dl} = - \frac{\partial B_0}{B_0 \partial l} \cos 2\theta + R^{-1}(l) \sin 2\theta, \quad (3.14)$$

where $X = \text{tg } \theta \equiv k_x/k_z$.

It follows that changes of both angle θ and wave number k during wave-packet propagation take place as a result of two factors: curvature of the field line and longitudinal non-uniformity of the magnetic field.

Equation (3.14) can be integrated in quadrature. Its solution, however, appears rather cumbersome and we do not write it down, since it will not be used further. As to Equation (3.13), its solution for an initial condition $X(l_0) = X$ takes the form

$$X(l) = \left(\frac{B_0(l)}{B_0(l_0)} \right)^2 \left(X_0 + 2 \int_{l_0}^l R^{-1}(l') \left(\frac{B_0(l_0)}{B_0(l')} \right)^2 dl' \right). \quad (3.15)$$

Expression (3.15) describes the evolution of the angle θ between wave vector k of the propagating packet and the direction of the stationary magnetic field.

Since the Alfvén wave growth rate was found to decrease rapidly, even for small angles θ (see Section 2), it is the limit of small X that we are primarily interested in. For the initial condition $\theta(l_0) \equiv \theta_0 = 0$ we have from (3.15) in the first order of $(l - l_0)/R \ll 1$:

$$\theta(l_0, \Delta l) \simeq 2R^{-1}(l_0) \Delta l, \quad (3.16)$$

where $\Delta l = l - l_0$.

It follows that Alfvén wave refraction is rather significant in typical conditions of the corona. Since the results of Section 2 indicate a strong dependence of the wave growth rates on the angle θ between \mathbf{k} and \mathbf{B}_0 , the consistent account of the refraction effects is indispensable in investigations of the cyclotron instability in the solar corona. It should be noted that a similar conclusion is also valid in the case of Alfvén-wave instabilities in the magnetospheres of the Earth and planets, where the refraction effects have not received sufficient attention, until recently.

As a result of the cyclotron instability, the test wave packet is amplified. Let us consider a packet which starts from a point l_0 and propagates along the field line. The wave amplification coefficient corresponding to the wave passage along a part of the field line with a length Δl takes the form

$$A(t) = \int_0^t \Gamma'(t') dt' = \int_{l_0}^{l_0 + \Delta l(t)} \Gamma'(l) dl / c_A(l), \quad (3.17)$$

where t is the time it takes for the wave packet to pass the distance Δl . If $l_0 = -\mathcal{L}/2$ and $t = \tau_w \equiv \int_{-\mathcal{L}/2}^{\mathcal{L}/2} dl/c_A(l)$, the amplification coefficient $A(t)$ converts to the integral amplification coefficient corresponding to one full passage of the packet along the total field line with the length \mathcal{L} .

The expression for net local growth rate $\Gamma' = \Gamma + \gamma$, which in fact depends on coordinates, can be obtained in the geometrical optics approximation. The results of these calculations, however, coincide with the corresponding formulae of Section 2, but all the parameters, except the universal constants and the wave frequency, become dependent on coordinates. The dependences of $\theta(l)$ and $k(l)$ which are necessary for calculation of $A(t)$, are determined by Equations (3.16) and (3.14), respectively.

In the next section we estimate the amplification coefficient $A(t)$ and use the estimates to determine the sufficient conditions of high-energy proton storage in the coronal magnetic trap. The last question we mention in this section is the bounce-motion of trapped protons, which is characterized by typical bounce-frequency Ω_b . These effects could be taken into account (see, e.g., Meerson *et al.* (1979), where the corresponding non-linear theory was developed). However, in the frequency range considered, $\omega \gg \Omega_b$, the account of the bounce-cyclotron effects in the linear theory would lead only to small oscillatory corrections to the wave growth rate (Meerson and Sasorov, 1979, unpublished).

4. Amplification Coefficients and the Time of Storage of High-Energy Protons

The excitation of the Alfvén wave turbulence due to the cyclotron instability can cause the scattering of the high-energy protons and lead to their precipitation into dense chromospheric regions. In this section we obtain some sufficient conditions of particle storage, i.e., the conditions of the life-time of protons in the trap being sufficiently large, despite the development of the cyclotron instability.

First of all, the validity of the linear theory of the Alfvén wave excitation, developed in Sections 2 and 3 requires the characteristic lifetime of protons in the trap to be much more than the wave-passage time τ_w .^{*} Since for high-energy protons in the coronal magnetic field, the loss-cone is rather narrow, the proton life-time is approximately equal to the characteristic time τ_D of quasi-linear diffusion (so called ‘weak-diffusion regime’, which is realized when $\tau_b \ll \tau_D$; $\tau_b = \Omega_b^{-1}$ is the typical bounce-period of high-energy proton). Based on this, let us require τ_D to be much greater than τ_w . What is more, the inequality of $\tau_D \gg \tau_w$ presents a reasonable condition for the storage.

It was shown in the Section 2 that noticeable wave growth occurs only in the limited angle interval of $0 \leq \theta \lesssim \alpha^{1/2} \ll 1$. That is why we may estimate the quasi-linear diffusion coefficient by its approximate value for the case of parallel propagation (Kennel and

* Since a typical wavelength of an unstable wave is much less than the scale of inhomogeneity of an Alfvén velocity profile at chromospheric altitudes, we may neglect the reflection of the wave from the ends of the magnetic trap.

Petschek, 1966):

$$D_0 \simeq (\omega_{Bi}^2/v_0) \frac{B_k^2(k \simeq \omega_{Bi}/v_0)/\Delta k}{B_0^2}, \quad (4.1)$$

where $\Delta k \simeq \omega_{Bi}/v_0$ is a characteristic bandwidth of wave spectrum and B_k is a Fourier component of the magnetic field of the Alfvén turbulence taken at the point of the cyclotron resonance.* The expression for B_k^2 as a function of time takes the form

$$B_k^2 = B_{fk}^2 \exp(2A(t)), \quad (4.2)$$

where B_{fk} is an initial level of Alfvén ‘noise’. In the range of wave-numbers considered, the initial noise level is probably determined by thermal fluctuations. Then, the estimate for B_{fk}^2 can be obtained by using well-known formulae for the spectral density of plasma fluctuations (see, e.g. Akhiezer *et al.*, 1974) with allowance made for the fact that the main source of fluctuations in our case is the high-energy protons rather than background plasma:

$$B_{fk}^2 \simeq (\omega/\omega_{Bi})(\omega_{Bi}/v_0)^3 \varepsilon_0. \quad (4.3)$$

Here we took $k \simeq \omega_{Bi}/v_0$, ε_0 is characteristic energy of accelerated protons and $\theta \lesssim \alpha^{1/2}$.

An analysis of the formulae for the instability growth rates obtained in Section 2 shows that there exist two parameters significantly affecting the wave amplification, namely, β_h and η . If we assume that the process of acceleration of protons in the flare does not lead to their appreciable anisotropy, the anisotropy parameter η will be determined by the size of the loss-cone. In this situation, the only parameter significantly controlling the wave growth rate is β_h .

Let us denote by β_* the critical value of β_h for which the time of proton diffusion, τ_D , becomes comparable with the wave-passage time τ_w . Using Equations (4.1)–(4.3) and taking into account the estimate $\tau_D \simeq D_0^{-1}$, we first get the critical value of A , for which $\tau_D \simeq \tau_w$:

$$A_* \simeq \frac{1}{2} \ln \left(\frac{B_0^2(v_0/\omega_{Bi})^3}{\varepsilon_0 \omega \tau_w} \right). \quad (4.4)$$

To obtain a sufficient condition of the storage, we calculate the amplification coefficient A (see (3.17)) for the waves, which have a maximal growth rate at an initial point l_0 . That is, we consider waves with initial conditions $\theta(l_0) \equiv \theta_0 = 0$, $k(l_0) \equiv k_0 = \omega_{Bi}/v_0$, and $k_y(l_0) = 0$. Besides, we estimate the particle diffusion due to the turbulence, whose final level is determined by amplification in the process of wave propagation up to the point where the amplification is followed by damping or becomes

* All the quantities in formula (4.1) and in a number of following expressions are, in fact, the functions of the coordinates. However, for order of magnitude estimates we are interested in, we may assume expression (4.1) to be somehow averaged with respect to coordinate l , the L -dependence being retained. Here L is the distance between the photosphere and the top of the field line.

very small (due to refraction). In other words, we calculate the amplification coefficient A taking into account the positive part of the curve $\Gamma'(\theta)$ only. Carrying out these calculations, we may neglect the evolution of the parameter $y = (k_{\parallel}/\omega_{Bi})(T_{\parallel}/m_i)^{1/2}$, in the process of the wave propagation, since the wave growth rate's dependence on θ is much stronger than its dependence on y in the range of angles $\theta \lesssim \alpha^{1/2}$ we are interested in. It should also be remembered that in our model of a plane field line and slowly varying plasma concentration, k_y remains equal to its initial value. Therefore, k_y remains zero for the wave with a maximal growth rate.

Numerical estimates show that the Alfvén wave damping by thermal electrons (including the 'modified' damping) can be neglected for typical values of coronal parameters. As to the amplification A , we can represent this in the form

$$A = \Gamma_0 \tau_0, \quad (4.5)$$

where Γ_0 is the growth rate (2.10) of the wave propagating along \mathbf{B}_0 , τ_0 is a parameter with the dimension of time. Since the function $\Gamma(\theta)$ rapidly decreases in the region of $\theta \sim \alpha^{1/2}$ we can take for τ_0 the value (see Equations (3.16) and (3.17))

$$\tau_0 \simeq (R/c_A) \alpha^{1/2}. \quad (4.6)$$

Making use of the estimate $\Gamma_0 \simeq \beta_h \eta \omega$ and substituting (4.6) for (4.5) we obtain an estimate for the amplification coefficient:

$$A \simeq \beta_h \eta \omega_{Bi} \alpha^{3/2} R/c_A. \quad (4.7)$$

Combining (4.4) with (4.7) we get an expression for the critical value of β_* :

$$\beta_* \simeq \frac{v_0^{3/2} R^{-1}}{2 \eta \omega_{Bi} c_A^{1/2}} \ln \left(\frac{B_0^2 (v_0/\omega_{Bi})^4}{\varepsilon_0 L} \right), \quad (4.8)$$

where we put $\tau_w \simeq L/c_A$.

For practical use, it is convenient to rewrite this in the form

$$\begin{aligned} \beta_* &= \beta_*(L, B_0(L), \varepsilon_0, \eta) \simeq \\ &\simeq 8 \times 10^{-6} \frac{\varepsilon_0^{3/4} (\text{MeV})}{B_0^{3/2} (\text{G}) L (R_{\odot}) \eta} \ln \left(5 \times 10^{15} \frac{\varepsilon_0 (\text{MeV})}{B_0^2 (\text{G}) L (R_{\odot})} \right). \end{aligned} \quad (4.9)$$

In formula (4.9) we took for n_0 the value of $n_0 \simeq 10^8 \text{ cm}^{-3}$ (the dependence on n_0 in (4.8) is very weak), and made use of the fact that $R \simeq L$. The parameter L is measured in solar radii, R_{\odot} . The formula (4.9) enables us to estimate the threshold value of β_* , such that the inequality of $\beta < \beta_*$ corresponds to a relatively long-term storage of the protons, $\tau_D \gg \tau_w$, in spite of the development of cyclotron instability. To carry out such an estimate it is necessary, apart from the typical energy of accelerated protons and the degree of their velocity distribution anisotropy, to know the typical value of height of the storage region as well as the magnetic field there.

We carried out illustrative calculations with formula (4.9) for two typical cases, characterized by the following parameters: (i) $L = 0.3 R_{\odot}$, $B_0 = 10 \text{ G}$ and (ii) $L = R_{\odot}$,

$B_0 = 1$ G. In both cases we considered accelerated protons with a typical energy of $\varepsilon_0 = 30$ MeV. The estimate of η can be made as follows. If the anisotropy parameter η is determined by the size of the loss-cone only, the expression for the effective value of η can be written as

$$\eta = 1 - \left(\int \frac{m_i v_{\parallel}^2}{2} f(v) dv \right) / \int \frac{m_i v_{\perp}^2}{4} f(v) dv. \quad (4.10)$$

Assuming the distribution function $f(v)$ is Maxwellian outside the loss-cone and zero inside it, we obtain, after calculations, $\eta = 3(2\mu + 1)^{-1}$, where $\mu = B_{\max}/B_{\min}$ is the ‘mirror ratio’ of the magnetic trap, B_{\min} is the magnetic field at the top of the field line, and B_{\max} is the magnetic field in a region of lower corona where Coulomb collisions become essential. For a typical coronal situation this corresponds to values of $\mu = 15$ and 150 in cases of (i) and (ii), respectively, the values of η being approximately equal to 0.1 and 0.01, respectively. Then formula (4.9) yields the following results for β_* : $\beta_* \simeq 4 \times 10^{-3}$ in the case of (i) and $\beta_* \simeq 0.4$ in the case of (ii). It should be noted that β_* increases rapidly with distance from the photosphere. Therefore, if the storage region of high-energy protons corresponds to moderate coronal heights, and the parameter β_h exceeds the value of β_* , the dynamics of the particles is essentially determined by cyclotron instability. The investigation of non-linear dynamics of the protons in this situation presents a difficult and interesting problem (cf. similar problems in magnetospheric physics; Bessipov and Trakhtengertz, 1980) which, however, falls outside the limits of this article. When $\beta_h < \beta_*$ long-term storage of the protons presents no problem. For a very high corona the threshold value of β_* may exceed the threshold of MHD instabilities (Meerson and Sasorov, 1981), so the main effect of $\beta_h > \beta_*$ in this case must be the ‘catastrophic’ escape of particles into interplanetary space.

5. Discussion

In this paper we considered the evolution of high-energy protons accelerated in the process of solar flare and trapped by coronal magnetic fields. One of the most important instabilities, governing storage time of the protons in such traps, is the cyclotron instability of the Alfvén waves, which develops due to ‘unavoidable’ anisotropy of perpendicular and parallel (with respect to the magnetic field) pressures of high-energy particles. The scattering of the particles by excited waves can lead to a quick loss in the dense chromospheric regions. However, there is evidence of the possibility of long-term storage of protons in coronal magnetic configurations. This presented a problem deserving attention.

It was Wentzel (1976) who first made an attempt to clarify the mechanism of ‘switching off’ the cyclotron instability. His idea was as follows. Since generated Alfvén waves undergo refraction, the angle θ changes in the process of the wave propagation. According to Wentzel, the wave stops growing as soon as it reaches a threshold value of θ , where a modified damping of the wave on coronal electrons becomes significant.

It was assumed that the damping is associated with coupling between Alfvén and magneto-acoustic modes by means of cyclotron-resonant protons.

However, the phenomenological approach used by Wentzel did not allow him to investigate the damping mechanism properly. Accurate calculations (see Section 2) show that modified damping results from the contribution of total distribution function of high-energy protons to the wave dispersion equation, rather than from the resonant protons only. And, what is still more important, this damping can be neglected for typical coronal conditions.* Hence, the problem in question has remained unsolved until recently.

In this paper we carried out detailed investigation of the cyclotron instability of the Alfvén waves and the possible mechanisms of wave damping in a broad region of angles, $0^\circ \leq \theta < 90^\circ$. It was established that an abrupt fall of the wave growth rate occurs when passing from the region of circular polarization of the wave, $\sin^2 \theta \ll \omega/\omega_{Bi}$, to the region of linear polarization, $\sin^2 \theta \gg \omega/\omega_{Bi}$. Since the ratio of ω/ω_{Bi} is usually much less than unity, this transition takes place at very small θ .

Further, the consistent treatment of the problem enabled us to investigate the wave refraction effects, taking into account both the curvature of the magnetic field line and the longitudinal nonuniformity of the magnetic field. We obtained simple relations describing change of the angle θ when the wave packet propagates along the field line. Therefore, we revealed a rapid decrease of the wave growth rate in the process of the wave propagation in the trap.

These effects lead to the possibility of long-term storage of accelerated protons by coronal magnetic fields. We determined some sufficient conditions for such storage. The most important parameter from the viewpoint of storage is the parameter β_h (it should be noted that it was the curvature radius of the magnetic field line that served as a key parameter in Wentzel's theory). If the value of β_h is less than a critical value β_* , determined by formulae (4.8) and (4.9) the storage time for the protons with energy of 10–30 MeV may amount to 1 day.

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Appendix 1

Real and imaginary components of the dielectric constant tensor for background plasma can be found, for example, in the review by Ginzburg and Rukhadze (1972), p. 466.

* As to the Alfvén wave refraction, Wentzel considered it only qualitatively.

As to the corresponding components of the dielectric constant tensor for high-energy protons, they can be obtained by calculation of the well-known general expressions for velocity distribution function (2.5). Then the real components of the tensor elements can be shown to have the following form:

$$\varepsilon_{11}^{(1)r} = \kappa(\eta, \alpha) (\exp(-s)/s) \sum_{n=1}^{\infty} n^2 I_n(s) \chi_n(\eta, \alpha, y), \quad (\text{A1.1})$$

$$\varepsilon_{22}^{(1)r} = \kappa(\eta, \alpha) (\exp(-s)/s) \left(\sum_{n=1}^{\infty} n^2 I_n(s) \chi_n(\eta, \alpha, y) \lambda_n(s) + 2\xi(s) v(\eta, \alpha, y) \right), \quad (\text{A1.2})$$

$$\begin{aligned} \varepsilon_{12}^{(1)r} = & i\kappa(\eta, \alpha) (\exp(-s)/y) \sum_{n=1}^{\infty} n^2 (I'_n(s) - I_n(s)) \tilde{\chi}_n(\eta, \alpha, y) + \\ & + i\varepsilon_{11}^{(0)r} (n_1/n_0) \alpha^{-1}, \end{aligned} \quad (\text{A1.3})$$

where

$$\kappa(\eta, \alpha) = (\pi/2)^{1/2} (c/c_A)^2 (n_1/n_0) \alpha^{-2} (1-\eta)^{-1},$$

$$\lambda_n(s) = (1-s/n)^2 + (s/n)^2 (1 - 2I_{n+1}(s)/I_n(s)),$$

$$\xi(s) = s^2 (I_0(s) - I_1(s)),$$

$$v(\eta, \alpha, y) = (2/\pi)^{1/2} \eta - (\alpha/y) u_0,$$

$$\chi_n(\eta, \alpha, y) = \eta((8/\pi) - (n/y)(u_{-n} - u_n)) - (\alpha/y)(u_{-n} + u_n),$$

$$\tilde{\chi}_n(\eta, \alpha, y) = \eta(u_{-n} + u_n) + (\alpha/n)(u_{-n} - u_n),$$

$s = (1-\eta)^{-1} y^2 \operatorname{tg}^2 \theta$; $I_n(w)$, and $I'_n(w)$ are the modified Bessel functions of the n th order and the derivative with respect to w , u_n is the imaginary part of the Kramp function, which has the following form:

$$u_n = (2/\sqrt{\pi}) \int_0^A \exp(z^2 - \Delta^2) dz, \quad \Delta = (\alpha - n)/y\sqrt{2}.$$

The second term in (A1.3) arises because of the quasi-neutrality of the total system 'plasma + high-energy protons'. Indeed, the component $\varepsilon_{12}^{(0)r}$ results from the summation of electron and ion contributions (see, e.g., Ginzburg and Rukhadze, 1972, p. 466), the contributions effectively compensating each other. If high-energy protons are absent, the condition of plasma quasi-neutrality takes the form of $n_{0i} = n_{0e}$. In this case $\varepsilon_{12}^{(0)r} = i\alpha\varepsilon_{11}^{(0)r}$. However, if there is a small group of high-energy protons in the system, the quasi-neutrality is restored through a 'drawing out' of additional electrons from the dense regions of background plasma. The corresponding quasi-neutrality condition now becomes $n_{0i} + n_1 - n_{0e} = 0$, where n_1 is the high-energy protons concentration. Then the summation of electron and ion contributions in $\varepsilon_{12}^{(0)r}$ yields an additional term,

proportional to n_1 . It is natural to attribute this term to the component $\varepsilon_{12}^{(1)r}$, just as it was done in (A1.3).

The imaginary components of the dielectric constant tensor elements contributed by high-energy protons are of the form

$$\varepsilon_{11}^{(1)i} = -i\tilde{\kappa}(\eta, \alpha, y) \exp(-s/s) \sum_{n=1}^{\infty} H_n(\alpha, y, s) \delta_n(\eta, \alpha, y), \quad (\text{A1.4})$$

$$\varepsilon_{22}^{(1)i} = -i\tilde{\kappa}(\eta, \alpha, y) \exp(-s/s) \left(\sum_{n=1}^{\infty} H_n(\alpha, y, s) \delta_n(\eta, \alpha, y) \lambda_n(s) - \xi(s) \exp(-\alpha^2/2y^2) \right), \quad (\text{A1.5})$$

$$\varepsilon_{12}^{(1)i} = \tilde{\kappa}(\eta, \alpha, y) \exp(-s) \sum_{n=1}^{\infty} H_n(\alpha, y, s) \tilde{\delta}_n(\eta, \alpha, y) (I'_n(s)/I_n(s) - 1), \quad (\text{A1.6})$$

where

$$\begin{aligned} \tilde{\kappa}(\eta, \alpha, y) &= \sqrt{2\pi} (c/c_A)^2 (n_1/n_0) (\alpha y)^{-1} (1-\eta)^{-1}, \\ H_n(\alpha, y, s) &= n^2 I_n(s) \exp(-(n^2 + \alpha^2)/2y^2), \\ \delta_n(\eta, \alpha, y) &= (n\eta/\alpha) \text{sh}(n\alpha/y^2) - \text{ch}(n\alpha/y^2), \\ \tilde{\delta}_n(\eta, \alpha, y) &= (\eta/\alpha) \text{ch}(n\alpha/y^2) - n^{-1} \text{sh}(n\alpha/y^2). \end{aligned}$$

Appendix 2

In the literature, one can find two formulae for the collisionless damping rate of the Alfvén waves in the region of $\sin^2 \theta \gg \alpha$, the formulae differing from each other. One of them was obtained by Stepanov (1958) (his formula (39)), the other was presented by Ginzburg and Rukhadze (1972), formula (25.46a). We carried out the consistent analysis which showed that the general expression for the damping rate has, in fact, the form

$$\begin{aligned} \gamma &= -\sqrt{\pi} \omega(m_e/m_i) \alpha^2 \exp[-(c_A/v_{Te})^2] \times \\ &\times [(v_{Te}/2c_A) \text{ctg}^2 \theta + (c_A/v_{Te})^3 \text{tg}^2 \theta + c_A/v_{Te}]. \end{aligned} \quad (\text{A2.1})$$

The term corresponding to the first summand in (A2.1) originates from the component $\varepsilon_{22}^{(0)i}$ of the dielectric constant tensor of the background plasma, and this coincides with corresponding result of Stepanov. The second summand is associated with the component $\varepsilon_{33}^{(0)i}$, and this term coincides with formula (25.46a) of Ginzburg and Rukhadze (1972). Besides, there exists the third term whose value may be comparable with those of the two mentioned above. This term owes to account of the component $\varepsilon_{23}^{(0)i}$ of the dielectric constant tensor of the background plasma.

The Alfvén wave damping is not exponentially small as long as $c_A/v_{Te} \ll 1$. In this case, the first term obtained by Stepanov (1958) dominates. However, presence of small factors such as α^2 and m_e/m_i makes the damping rate very small in almost all interesting situations.

Note that the ratio between damping rate γ and 'modified' damping rate γ_m of (2.14) has the form of $\gamma/\gamma_m \sim \alpha^2/\beta_h$ at $c_A/v_{Te} \lesssim 1$.

References

- Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G., and Stepanov, K. N.: 1974, *Plasma Electrodynamics* (in Russian), Nauka, Moscow, p. 528.
- Bespalov, P. A. and Trakhtengertz, V. Yu.: 1980, in A. B. Mikhailovskii (ed.), *The Problems of Plasma Theory* (in Russian), Atomizdat, Moscow, p. 88.
- Dobeš, K.: 1968, *Czech. J. Phys. Sec. B*, **18**, 1133.
- Feigin, F. Z. and Yakimenko, V. L.: 1969, *Geomagn. i Aeron.* **9**, 700.
- Ginzburg, V. L. and Rukhadze, A. A.: 1972, *Handbuch der Physik* **49/4**, 395.
- Kennel, C. F.: 1969, *Rev. Geophys.* **7**, 379.
- Kennel, C. F. and Petschek, H. E.: 1966, *J. Geophys. Res.* **71**, 1.
- Korn, G. A. and Korn, T. M.: 1968, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill, New York.
- Meerson, B. I. and Sasorov, P. V.: 1981, *Advances in Space Research* **1**, No. 3, 77.
- Meerson, B. I., Sasorov, P. V., and Stepanov, A. V.: 1978, *Solar Phys.* **58**, 165.
- Meerson, B. I., Sasorov, P. V., and Shklyar, D. R.: 1979, *Fizika Plazmi* **5**, 1107 (1979, *Soviet J. Plasma Phys.* **5**, 620).
- Melrose, D. B. and Wentzel, D. G.: 1970, *Astrophys. J.* **161**, 457.
- Mikhailovskii, A. B.: 1975, *The Theory of Plasma Instabilities* (in Russian), Vol. 1, Atomizdat, Moscow, p. 223.
- Palmer, I. D., Palmeira, R. A. R., and Allum, F. R.: 1975, *Solar Phys.* **40**, 449.
- Sagdeev, R. Z. and Shafranov, V. D.: 1960, *Zh. Experm. i Teor. Fiz.* **39**, 181 (1961, *Soviet Phys. JETP* **12**, 130).
- Stepanov, K. N.: 1958, *Zh. Experm. i Teor. Fiz.* **34**, 1292.
- Stix, T. H.: 1962, *The Theory of Plasma Waves*, McGraw-Hill, New York, p. 34.
- Weinberg, S.: 1962, *Phys. Rev.* **126**, 1899.
- Wentzel, D. G.: 1976, *Astrophys. J.* **208**, 595.