Dynamics of Particles Advected by Fast Rotating Turbulent Fluid Flow: Fluctuations and Large-Scale Structures

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Dynamics of particles advected by fast rotating incompressible turbulent fluid flow is studied. Fast rotation and particle inertia imply the divergent particle velocity field and result in both intermittency in spatial distribution of particles and formation of the large-scale inhomogeneous structures. A nonzero mean helicity of fluid flow causes an additional mean nondiffusive turbulent flux of inertial particles. Intermittency in the systems with and without external pumping is studied. Fast rotation causes anomalous scaling already in the second moment of inertial particle number density and may result in excitation of a small-scale instability of inertial particle distribution, which leads to the formation of small-scale particle clusters. We discuss the relevance of our results for atmospheric, astrophysical, and industrial turbulent rotating flows. [S0031-9007(98)07241-X]

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Dynamics of particles advected by turbulent rotating flows is of fundamental importance in view of a great number of applications in naturally occurring and laboratory flows [e.g., dynamics of particles in the atmospheric tornado and dust storms, formation of planetesimals (progenitors of planets) in protoplanetary disks, industrial cyclone separators, etc.]. Interesting phenomena in turbulent transport of particles and gases were discovered recently, e.g., turbulent thermal diffusion, turbulent barodiffusion [1], and self-excitation, i.e., exponential growth of fluctuations of the number density of inertial particles [2]. The turbulent thermal diffusion and turbulent barodiffusion cause formation of large-scale inhomogeneous structures of particle concentration. The self-excitation of fluctuations of the number density of particles results in the intermittency in spatial distribution of inertial particles. Small-scale structures in particle distribution were observed experimentally [3]. All these effects are caused by inertia of particles which results in a divergent velocity field of particles [4].

In the present Letter we show that the fast rotation results in very interesting dynamics of particles advected by incompressible turbulent fluid flow. In particular, fast rotation causes both intermittency in the spatial distribution of particles and formation of the large-scale inhomogeneous structures.

Governing equations.—Consider particles advected by fast rotating turbulent incompressible fluid flow. The equation of motion for particles is given by

$$d\mathbf{u}/dt = (\mathbf{v} - \mathbf{u})/\tau_p + 2\mathbf{u} \times \mathbf{\Omega} + \mathbf{f}_1/m_p \,, \quad (1)$$

where the fluid velocity $\mathbf{v}$ is determined by equation

$$d\mathbf{v}/dt = -\nabla P/\rho + \nu \Delta \mathbf{v} + 2\mathbf{v} \times \mathbf{\Omega} + \mathbf{f}_2/\rho \,, \quad (2)$$

$\tau_p = m_p/(6\pi\rho\nu a_*)$ is the Stokes time, $\mathbf{u}$ is the velocity of particle of a mass m_p and the size a_*, $\mathbf{\Omega}$ is the angular velocity, $\mathbf{v}$ is the velocity of fluid of the density ρ and the pressure P, ν is the kinematic velocity, and $\mathbf{f}_1$ and $\mathbf{f}_2$ are the sums of external and centrifugal forces. We consider an incompressible fluid flow (i.e., $\nabla \cdot \mathbf{v} = 0$). By means of Eqs. (1) and (2) we derive equations for $b = \nabla \cdot \mathbf{u}$ and $\mathbf{W} = \nabla \times \mathbf{U}$,

$$\partial b/\partial t = -b/\tau_p + 2\mathbf{\Omega} \cdot (\mathbf{W} + \nabla \times \mathbf{v}) \,, \quad (3)$$

$$\partial \mathbf{W}/\partial t = -\mathbf{W}/\tau_p - 2b\mathbf{\Omega} \quad (4)$$

(see [5]), where $\mathbf{U} = \mathbf{u} - \mathbf{v}$. For derivation of Eqs. (3) and (4) we consider the case of fast rotation; i.e., we assume that for fluctuations $\nabla P/\rho \simeq 2\mathbf{v} \times \mathbf{\Omega}$, $\Omega \gg |\mathbf{W}|$, and $\Omega \gg |\nabla \times \mathbf{v}|$. We also assume that $|b| \gg |\partial \mathbf{U}/\partial Z|$, where the axis Z is directed along $\mathbf{\Omega}$. We study the dynamics of particles for the times which are much larger than τ_p. Therefore we may neglect the time derivatives in Eqs. (3) and (4) and obtain the steady state solution of Eqs. (3) and (4): $b = \boldsymbol{\omega} \cdot (\nabla \times \mathbf{v})/(1 + \omega^2)$, $\mathbf{W} = -b\boldsymbol{\omega}$, where $\boldsymbol{\omega} = 2\mathbf{\Omega}\tau_p$. The velocity $\mathbf{U}$ can be presented as a sum of the vortical and the potential components, i.e., $\mathbf{U} = \nabla \times \mathbf{A} + \nabla\phi$. In $\mathbf{k}$ space $b(\mathbf{k}) = -k^2\phi(\mathbf{k})$ and $\mathbf{W}(\mathbf{k}) = k^2\mathbf{A}(\mathbf{k})$, where we use the condition $\nabla \cdot \mathbf{A} = 0$. This yields $\mathbf{U}(\mathbf{k}) = i(\mathbf{k} + \mathbf{k} \times \boldsymbol{\omega})\phi(\mathbf{k})$, where $\phi(\mathbf{k}) = i(\mathbf{k} \times \boldsymbol{\omega}) \cdot \mathbf{v}(\mathbf{k})/k^2(1 + \omega^2)$. Fast rotation renders fluid flow to become two-dimensional in the plane normal to the rotation axes $\mathbf{\Omega}$. In the two-dimensional case we use the following identities: $(\boldsymbol{\omega} \times \mathbf{k})_i = \omega\epsilon_{ij}k_j$, $k_i(\boldsymbol{\omega} \times \mathbf{k})_j = k^2\omega\epsilon_{in}P_{nj}(k)$, and $(\boldsymbol{\omega} \times \mathbf{k})_i(\boldsymbol{\omega} \times \mathbf{k})_j = k^2\omega^2P_{ij}(k)$, where $P_{ij}(a) = \delta_{ij} - a_ia_j/a^2$, $\epsilon_{11} = \epsilon_{22} = 0$, $\epsilon_{21} = -\epsilon_{12} = 1$, and δ_{ij} is the Kronecker tensor. Using the above solutions and the identities we derive the tensor for the second moment of the velocity field of particles advected by two-dimensional, homogeneous, isotropic, and reflectionary invariant turbulent incompressible fluid flow:

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$$\langle \tau u_m(\mathbf{k}) u_n(-\mathbf{k}) \rangle = D_\perp [k^2 P_{mn}(k) + \omega^2 k_m k_n - \omega (k_m \epsilon_{ns} + k_n \epsilon_{ms}) k_s] \times f_\omega(k)/(1 + \omega^2), \quad (5)$$

where $D_\perp = u_0^2 \tau_0 / 2(1 + \omega^2)$, u_0 is the characteristic turbulent velocity in the maximum scale l_0 of turbulent motions, $\tau_0 = l_0/u_0$, and k is measured in the units l_0^{-1}. In $\mathbf{r}$ space this tensor is given by

$$\langle \tau u_m(\mathbf{x}) u_n(\mathbf{x} + \mathbf{r}) \rangle = D_\perp \{ (F + F_c) \delta_{mn} + r F' P_{mn}(r) + (F_c'/r) [r_m r_n - (r_m \epsilon_{ns} + r_n \epsilon_{ms}) r_s / \omega] \}, \quad (6)$$

where $F = -f_\omega'/r(1 + \omega^2)$, $F_c = \omega^2 F$, $F' = dF/dr$, and r is measured in the units l_0. The function $F_c(r)$ describes the potential component, whereas $F(r)$ corresponds to the vortical part of the turbulent velocity of particles. The degree of compressibility is $\sigma = F_c/F = \omega^2$. Therefore the particle velocity field is compressible due to both fast rotation and particle inertia, while the turbulent fluid flow is incompressible. Note that in the limit of very small particle inertia, i.e., $\tau_p \to 0$, the degree of compressibility $\sigma \to 0$ and the particles velocity coincides with the fluid velocity. On the other hand, for very large particle inertia, i.e., $\tau_p \gg \Omega^{-1}$, the correlation function $\langle \tau u_m u_m \rangle \to 0$. The latter implies that very heavy particles are not affected by the turbulence. Note that the external body force does not explicitly appear in the final expression for the correlation function $\langle \tau u_m(\mathbf{x}) u_n(\mathbf{x} + \mathbf{r}) \rangle$ of the particle velocity field because the parameter $\sigma \gg \tau_p/\tau_0$. However, the body force in the Navier-Stokes equation for the fluid determines the spectrum of the fluid velocity field.

Number density $n_p(t, \mathbf{r})$ of small particles in a turbulent flow is determined by the equation

$$\partial n_p / \partial t + \nabla \cdot (n_p \mathbf{u}) = D \Delta n_p ,$$

where D is the coefficient of molecular diffusion. We study one-way coupling whereby particles are advected by a prescribed turbulent velocity field. This approximation is valid when $m_p n_p \ll \rho$. Particle-particle interactions become important only when $m_p n_p \sim \rho$.

Small-scale fluctuations.—We consider the case of large Reynolds and Peclet numbers. To study the fluctuations of particle number density we derive an equation for the second moment of particle concentration. For this purpose we use a method of path integrals and modified Feynman-Kac formula (for details see, e.g., [1,2,6]). The equation for the second-order correlation function $\Phi = \langle \Theta(\mathbf{x}) \Theta(\mathbf{y}) \rangle$ is given by

$$\frac{\partial \Phi}{\partial t} = -2[D_{mn}(0) - D_{mn}(\mathbf{r})] \frac{\partial^2 \Phi}{\partial x_m \partial y_n} + 2 \langle \tau b(\mathbf{x}) b(\mathbf{y}) \rangle \Phi - 4 \langle \tau u_m(\mathbf{x}) b(\mathbf{y}) \rangle \frac{\partial \Phi}{\partial x_m} + I, \quad (7)$$

where $\Theta = n_p - N$, $\mathbf{r} = \mathbf{y} - \mathbf{x}$, $I = 2 \langle \tau b(\mathbf{x}) b(\mathbf{y}) \rangle N^2$, $D_{pm} = D \delta_{pm} + \langle \tau u_p u_m \rangle$, $N = \langle n_p \rangle$ is the mean number density of particles, and τ is the momentum relaxation time of random velocity field $\mathbf{u}$, which depends on the scale of turbulent motion. We use here for simplicity the δ correlated in the time random process to describe a turbulent velocity field. However, the results remain valid also for the velocity field with a finite correlation time, if all moments of the number density of the particles vary slowly in comparison with the correlation time of the turbulent velocity field (see, e.g., [6]).

Equation (7) for $b = 0$ was first derived by Kraichnan (see [7]). In this particular case, $b = 0$, this equation describes a relaxation of the second moment of particle number density. On the other hand, when $b \neq 0$, i.e., when the velocity of particle is divergent, Eq. (7) implies both an effect of self-excitation of fluctuations of particle number density caused by the second term in (7) and anomalous scalings for the fluctuations.

We seek a solution to the equation for the function Φ in the form

$$\Phi(t, r) = \Psi(r) r^{(1-d)/2} \exp\left[-\int_0^r \chi(x)\, dx \right] \exp(\gamma t) ,$$

where d is the dimensionality of space, and the function $\Psi(r)$ is determined by

$$\Psi''/m(r) - [\gamma + U_0(r)] \Psi = 0 , \quad (8)$$

where $U_0(r) = [(d - 1)\chi/r + \chi^2 + \chi' + (d - 1) \times (d - 3)/(4r^2)]/m(r) - \kappa(r)$, $1/m(r) = 2/\mathrm{Pe} + 2[1 - F - (rF_c)']/d$, $\chi(r) = m(r)[(3d + 1)F_c' - F' + 2rF_c'']/d$, $\kappa(r) = -2[(d^2 - 1)F_c'/r + (2d + 1)F_c'' + rF_c''']/d$, distance r is measured in units of l_0, time t is measured in units of τ_0, $\mathrm{Pe} = l_0 u_0 / D \gg 1$ is the Peclet number, and $\mathrm{Re} = l_0 u_0/\nu \gg 1$ is the Reynolds number for the turbulent flow.

We consider the case of large Schmidt numbers, $\mathrm{Sc} = \nu/D \gg 1$. The solution of Eq. (8) can be obtained using an asymptotic analysis (see, e.g., [2,6]). This analysis is based on the separation of scales. In particular, the solution of the Schrödinger equation (8) with a variable mass has different regions where the form of the potential $U_0(r)$, mass $m(r)$, and, therefore, eigenfunctions $\Psi(r)$ are different. Solutions for Φ and Φ' in these different regions can be matched at their boundaries. Consider two-dimensional turbulent fluid flow ($d = 2$). The asymptotic analysis yields the growth rate of fluctuations of particle concentration

$$\gamma = \frac{[b_2^2 + (1 - a)^2]^2}{(3 - p)^2} \ln^2 \left(\frac{\mathrm{Re}}{\mathrm{Re}^{(\mathrm{cr})}} \right), \quad (9)$$

where $\mathrm{Re} > \mathrm{Re}^{(\mathrm{cr})}$ and the critical Reynolds number $\mathrm{Re}^{(\mathrm{cr})}$ is given by $\mathrm{Re}^{(\mathrm{cr})} \simeq \exp\{(3 - p)[\pi k + \arctan((1 - a)/b_2) - \arctan((\xi - a)/b_2)]/b_2\}$. Here $k = 1, 2, 3, \ldots$, $b_2^2 = (q - 1)^2 M / 4(1 + q\sigma)^2$, $M = -1 + 2\sigma(5 + 4q) - \sigma^2(9 + 8q)$, $q = 2p - 1$, p is the exponent in the energy spectrum of the turbulent fluid flow, $a = (q - 1)[1 - \sigma(3 + 2q)/2(1 + q\sigma)]$, and $\xi = (1 - 9\sigma)/(1 + \sigma)$. Therefore, the fluctuations of particle number density can be excited without an external source. The mechanism of excitation of these fluctuations is associated with the fact that the Coriolis force concentrates the particles either at boundaries of turbulent eddies or at their centers; i.e., it causes formation of small-scale inhomogeneities in particle distribution. This effect acts in a wide range of scales of turbulent motions. Scale-dependent turbulent diffusion causes relaxation of clusters of particles. Since in small scales the turbulent diffusion is weak, the fluctuations of particle number density are localized in small scales. The scale of localization of fluctuations is given by $l_f \sim l_\nu \exp(\pi k/b_2)$, where $l_\nu \sim \mathrm{Re}^{-1/(3-p)}$ is the viscous dissipation scale of a fluid flow. It was found in [2] that the growth rate γ_s of the s-order correlation function of particle number density is given by $\gamma_s \propto s^2\gamma_2$ for $s \gg 1$ (where $\gamma_2 = \gamma$ is the growth rate of the second-order correlation function). This implies that when $\gamma > 0$ (i.e., fluctuations of particle number density are excited), higher moments grow faster than lower moments, i.e., $\gamma_s > \gamma_{s-1}$ and $\gamma_s > s\gamma_2/2$. This results in intermittency, i.e., the appearance of sharp peaks in which the main part of the field intensity is concentrated.

Anomalous scalings for a passive scalar advected by a turbulent fluid flow are a subject of active research in recent years. For an incompressible turbulent velocity field the anomalous scalings for scalar field can occur only beginning with a fourth-order correlation function (see, e.g., [8]). Here we have shown that the anomalous scalings appear already in the second moment of the number density of inertial particles due to the effects of rotation and inertia. Now we consider the case when there is no self-excitation of the fluctuations of the number density of particles, i.e., when $\mathrm{Re} < \mathrm{Re}^{(\mathrm{cr})}$, and fluctuations are sustained by a source $I(r)$. We study a zero mode for Eq. (7), i.e., the mode with $\gamma = 0$. The external source is chosen as follows: $I(r) = I_0(1 - r^s)$, where $0 \le r \le 1$ and $s > 0$, and for $r > 1, I(r) = 0$. The second moment $\Phi(r)$ in scales $l_\nu \le r < 1$ is given by $\Phi = r^{-a}(A_3 r^{|b_2|} + A_4 r^{-|b_2|}) - I_0 r^{3-q}/c_-$ [for $1/33 < \sigma < \min(\sigma_1, 1/9)$], and $\Phi = A_3 r^{-a}\cos(b_2 \ln r + \varphi_2) - I_0 r^{3-q}/c_+$ [for $\max(1/33, \sigma_1) < \sigma < \min(\sigma_2, 1/9)$], where $c_\pm = 2\beta_m[(3 - q + a)^2 \pm b_2^2]$, $\sigma_{1,2}$ are the roots of the equation $M = 0$, and $\beta_m = (1 + q\sigma)/3(1 + \sigma)$. The term $\propto r^{3-q}$ in these equations corresponds to a normal scaling for the second moment of particle concentration, whereas the term $\propto r^{-|b_2|}$ corresponds to the anomalous scaling. When $\max(1/33, \sigma_1) < \sigma < \min(\sigma_2, 1/9)$ the anomalous scaling in the range $l_\nu \le r < 1$ is complex ($\propto r^{-a \pm i|b_2|}$).

Note that Eqs. (7)–(9) in the case $d = 2$ describe also formation of particle clusters on the surface of water (e.g., scum on the sea surface). In this case the horizontal motions of the surface of water are compressible, i.e., $\nabla \cdot \mathbf{v}_\perp = -\partial \mathbf{v}_Z/\partial Z$, where $\mathbf{v}_Z$ is the vertical component of water velocity in the vicinity of the surface. The degree of compressibility is determined by the surface waves.

Large-scale effects.—Now we consider the large-scale dynamics of particles in a fast rotating turbulent fluid flow. The equation for the mean field $N = \langle n_p \rangle$ reads

$$\partial N/\partial t + \nabla \cdot [N\mathbf{V}_{\mathrm{eff}} - \hat{D}\nabla_m N] = 0$$

(see [1]), where $\hat{D} \equiv D_{pm}$, $\mathbf{V}_{\mathrm{eff}} = \mathbf{V} - \langle \tau b \mathbf{u} \rangle$, and $\mathbf{V} = \langle \mathbf{u} \rangle$ is the mean velocity. Using the expression for b we calculate the effective velocity $(\mathbf{V}_{\mathrm{eff}})_j = V_j + \omega_i \alpha_{ij}/(1 + \omega^2)$, where $\alpha_{ij} = \langle \tau u_i (\nabla \times \mathbf{v})_j \rangle$. Consider the following model of three-dimensional turbulent fluid velocity field:

$$\langle \tau v_m(\mathbf{x}) v_n(\mathbf{y}) \rangle = D_\perp[F_a(\rho, \mathbf{R})(\delta_{mn} - \lambda_m\lambda_n) + \rho F_a'(\delta_{mn} - \lambda_m\lambda_n - \rho_m\rho_n/\rho^2) + \mu(\rho, \mathbf{R})\varepsilon_{mnk}\rho_k] + D_\parallel \eta(\rho, \mathbf{R})\lambda_m\lambda_n \,, \tag{10}$$

where ε_{mnk} is the Levi-Civita tensor, $\mathbf{r} = \mathbf{y} - \mathbf{x} = \boldsymbol{\rho} + (r_m\lambda_m)\boldsymbol{\lambda}$, $D_\parallel = v_Z^2\tau_0$, $\boldsymbol{\lambda}$ is the unit vector in the direction of $\boldsymbol{\Omega}$, $\mathbf{R} = (\mathbf{y} + \mathbf{x})/2$, and $F_a' = \partial F_a/\partial\rho$. When $D_\parallel = 0$ and $\mu = 0$, Eq. (10) recovers a well-known equation for two-dimensional incompressible turbulent flow. When $D_\parallel \neq 0$ and $\mu \neq 0$, Eq. (10) describes nearly two-dimensional homogeneous and isotropic helical turbulence with slow vertical (along $\boldsymbol{\Omega}$) motions of fluid ($v_\perp \gg v_Z$). Now we calculate $\alpha_{ij} = \alpha_{ij}^{(0)} + \alpha_{ij}^{(1)}$, where $\alpha_{ij}^{(0)} = \langle \tau v_i (\nabla \times \mathbf{v})_j \rangle = \alpha_0 \delta_{ij}$, and $\alpha_{ij}^{(1)} = \langle \tau U_i (\nabla \times \mathbf{v})_j \rangle = \alpha_0[\omega^2 P_{ij}(\omega) - \varepsilon_{ijp}\omega_p]/2(1 + \omega^2)$, where $\alpha_0 = -\langle \tau \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle/3$ is the α effect (see, e.g., [9]). Here we use Eq. (10) and the equation for $\mathbf{U}(\mathbf{k})$ which is rewritten in $\mathbf{r}$ space. The derived formula for α_{ij} yields the effective velocity of particles $\mathbf{V}_{\mathrm{eff}} = \mathbf{V} + \alpha_0\boldsymbol{\omega}/(1 + \omega^2)$. This formula implies that the large-scale turbulent flux of particles has a component in the direction of the rotation axis. The latter can result in the large-scale instability and formation of large-scale inhomogeneities in particle spatial distribution.

The mechanism for the appearance of an additional mean turbulent flux of particles along the rotation axis is as follows. The Coriolis force concentrates particles at the centers of eddies when they rotate in the direction which is opposite to the vector of main rotation $\boldsymbol{\Omega}$, i.e., for those eddies where the vorticity $\nabla \times \mathbf{v}$ and $\boldsymbol{\Omega}$ are antiparallel. On the other hand, the eddies which rotate with $\boldsymbol{\Omega}$ do not contain particles because the Coriolis force carries out particles to the boundaries of the eddies. The particles containing eddies which move in the direction opposite to $\boldsymbol{\Omega}$ (i.e., along the vorticity $\nabla \times \mathbf{v}$) have the positive helicity, while the eddies which move along $\boldsymbol{\Omega}$

have the negative helicity. When the mean helicity is zero, the numbers of eddies with positive and negative helicities are the same, and the additional mean turbulent flux of particles vanishes. When the mean helicity is negative (i.e., the α effect is positive) the number of eddies with negative helicity (which move in the direction of $\mathbf{\Omega}$) is larger than that with positive helicity. This results in the appearance of an additional mean turbulent flux of particles along the rotation axis. On the other hand, when the mean helicity is positive (i.e., the α effect is negative) the number of eddies with positive helicity (which move in the direction opposite to $\mathbf{\Omega}$) is larger than that with negative helicity. This causes an additional mean turbulent flux of particles in the direction which is opposite to the rotation axis $\mathbf{\Omega}$. Thus, the nonzero mean helicity breaks a symmetry between particle containing eddies which move in the direction of $\mathbf{\Omega}$ and in the opposite direction.

Applications.—The analyzed effects of formation of small-scale and large-scale inhomogeneities of particles distribution in fast rotating turbulent fluid flow are important in atmospheric turbulence (e.g., dynamics of particles and hydrometeors in the atmospheric tornadoes and dust storms), in astrophysical turbulence (e.g., formation of planetesimals in accretion protoplanetary disks), and in industrial turbulent rotating flows (e.g., cyclone dust collectors and vortex chambers). Indeed, characteristic parameters in the atmospheric tornado are $l_0 \sim (3-10) \times 10^3$ cm, $u_0 \sim 30-100$ cm/s, and $\Omega \sim 1$ s^{-1} (see, e.g., [10]). The condition $(2\Omega\sqrt{33})^{-1} < \tau_p < (6\Omega)^{-1}$ implies that the particles of the size $a_* \sim 90-170$ μm can be accumulated in the small-scale clusters. The typical size of the clusters is about 5–10 cm. The critical Reynolds numbers which are necessary for formation of the small-scale particle clusters are about 130 (for particles of the size $a_* \sim 100$ μm). The typical Reynolds number in the atmospheric turbulent fluid flow Re $\sim (1-10) \times 10^6$. Therefore, fluctuations of particle number density can be easily excited and the small-scale particle clusters are formed. Now we estimate the large-scale effective velocity $\mathbf{V}_{\rm eff}$ of particles in the atmospheric tornado. The α effect is estimated as $\alpha_0 \sim u_0 \sim 30-100$ cm/s. Therefore the large-scale effective velocity of particles of the size $a_* \sim 100-200$ μm is of the order of $\mathbf{V}_{\rm eff} \sim 25-50$ cm/s. This causes the mean turbulent flux of particles and formation of large-scale inhomogeneities in particle spatial distribution. The characteristic time of particle inhomogeneities formation is more than 10^3 sec.

Now we consider particles in astrophysical turbulence, i.e., formation of planetesimals (progenitors of planets) in accretion protoplanetary disks (see, e.g., [5]). Planetesimals are formed from grains and dust in the gaseous protostellar disks or the solar nebula due to coagulation. Inertia of particles advected by turbulent rotating fluid flow causes formation of small-scale and large-scale inhomogeneities of particle distribution. The typical parameters of the protosolar nebula are $l_0 \sim 10^{11}$ cm, $u_0 \sim 4 \times 10^5$ cm/s, and $\Omega \sim 4 \times 10^{-7}$ s^{-1}, the Reynolds number Re $\sim 10^{10}$. A value $\omega = 2\Omega\tau_p = 0.2$ corresponds to $a_* \sim 10$ cm, the Stokes time in accretion protoplanetary disks $\tau_p \propto a_*$ because the mean free path of molecules the protosolar nebula is much larger than the size of particles. Particles of the size $a_* \sim 10-20$ cm can be accumulated in planetesimals. The characteristic size of planetesimals for the first mode ($k = 1$) is $l_1 \sim 10^4$ cm and for the second mode ($k = 2$) is $l_2 \sim 10^6$ cm. There is also a possibility for the change of a settling velocity due to the large-scale effective velocity. For the protosolar nebula the effective velocity is $\mathbf{V}_{\rm eff} \sim 2 \times 10^5$ cm/s.

An additional application of these effects is the industrial rotating turbulent flows, e.g., the cyclone dust collectors (see, e.g., [11]). The typical flow parameters in the cyclone dust collectors are $l_0 \sim 10$ cm and $\Omega \sim 50-75$ s^{-1}, the Reynolds number Re $\sim 3 \times 10^3$. Therefore particles with the size $a_* \sim 10-25$ μm can be accumulated in small-scale clusters of about 2 cm during a time interval of order of 10 sec. The latter can explain some peculiarities observed in the cyclone dust collectors (see, e.g., [11]).

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