Self-Excitation of Fluctuations of Inertial Particle Concentration in Turbulent Fluid Flow

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A mechanism for intermittency in spatial distribution of small inertial particles advected by a turbulent incompressible fluid flow is discussed. The mechanism is related to self-excitation (i.e., exponential growth without an external source) of fluctuations of concentration of small particles in a turbulent fluid flow. The effect is caused by the inertia of particles which results in a divergent velocity field of particles. An equation for the high-order correlation functions of concentration of small inertial particles is derived. It is shown that the growth rates of the higher moments of particle concentration are higher than those of the lower moments, i.e., particle spatial distribution is intermittent. Similar phenomena occur for noninertial admixtures advected by divergent turbulent velocity field. [S0031-9007(96)02006-6]

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Study of passive scalar fluctuations in a turbulent fluid flow is important in view of numerous applications [1]. The nature of the intermittency of scalar field passively convected by a fluid in the presence of an external source of passive scalar fluctuations was elucidated recently (see, e.g., [2–4]).

Without an external source the second moment of the passive scalar distribution in incompressible turbulent fluid flow can only decay due to turbulent diffusion [5]. Recently, it was found [6] that compressibility (i.e., $\mathrm{div}\,\mathbf{v} \propto \partial\rho/\partial t \neq 0$) of a turbulent fluid flow results in a strong inhibition of the diffusion of the second moment of mass concentration $C = m_p n_p/\rho$ for large Peclet numbers. Here $\mathbf{v}$ and ρ are the velocity and density of the fluid, m_p and n_p are the mass and number density of particles. However, a feasibility of self-excitation (i.e., exponential growth) of passive scalar fluctuations without an external source remained unexplored.

The main purpose of this Letter is to discuss an effect that causes a self-excitation (i.e., exponential growth) of fluctuations of inertial particle concentration in a turbulent fluid flow without an external source. This effect is responsible for the intermittency in particle spatial distribution. In particular, we have shown that the growth rates of the higher moments of particle concentration are higher than those of the lower moments.

Evolution of the number density $n_p(t,\mathbf{r})$ of small particles in a turbulent flow is determined by the equation,

$$\frac{\partial n_p}{\partial t} + \nabla\cdot(n_p\mathbf{v}_p) = D\Delta n_p\,, \tag{1}$$

where $\mathbf{v}_p$ is a random velocity field of the particles which they acquire in a turbulent fluid velocity field and D is the coefficient of molecular diffusion. We consider the case of large Reynolds and Peclet numbers. The velocity of particles $\mathbf{v}_p$ depends on the velocity of the fluid, and it can be determined from the equation of motion for a particle: $d\mathbf{v}_p/dt = [\mathbf{v} - \mathbf{v}_p]/\tau_p$, where τ_p is the Stokes time. In this study we consider incompressible turbulent flow $\nabla\cdot\mathbf{v} = 0$. However, the velocity field of particles is compressible, i.e., $\nabla\cdot\mathbf{v}_p \neq 0$. Indeed, a solution of the equation of motion for particles can be written in the form $\mathbf{v}_p = \mathbf{v} + \tau_p\mathbf{f}(\mathbf{v},\tau_p)$. The second term in this solution describes the difference between the local fluid velocity and particle velocity arising due to the small but finite inertia of the particle. We calculate the divergence of the equation of motion for particles, and, after simple manipulation, we obtain

$$\nabla\cdot\mathbf{v}_p = -\tau_p\nabla\cdot[(\mathbf{v}\cdot\nabla)\mathbf{v}] - \tau_p^2\nabla\cdot\left[\frac{\partial\mathbf{f}}{\partial v_k}\,\frac{dv_k}{dt}\right]. \tag{2}$$

When τ_p is very small Eq. (2) coincides with the results obtained in [7]. The Navier-Stokes equation for the fluid yields $\nabla\cdot[(\mathbf{v}\cdot\nabla)\mathbf{v}] = -\Delta P/\rho$, where P is the pressure of a fluid. From the latter equation and Eq. (2) it is seen that $\nabla\cdot\mathbf{v}_p \neq 0$ although for the fluid $\nabla\cdot\mathbf{v} = 0$.

Divergent velocity field of inertial particles is the main reason for a new effect, i.e., self-excitation (exponential growth) of fluctuations of concentration of small particles in a turbulent incompressible fluid flow. Indeed, multiplication of Eq. (1) by n_p and simple manipulations yield

$$\frac{\partial n_p^2}{\partial t} + (\nabla\cdot\mathbf{S}) = -n_p^2(\nabla\cdot\mathbf{v}_p) - 2D(\nabla n_p)^2, \tag{3}$$

where $\mathbf{S} = n_p^2\mathbf{v}_p - D\nabla n_p^2$. The latter equation implies that, if $\nabla\cdot\mathbf{v}_p < 0$, a perturbation of the equilibrium homogeneous distribution of inertial particles can grow in time, i.e., $(\partial/\partial t)\int n_p^2\, d^3r > 0$. However, the total number of particles is conserved. Averaging Eq. (3) over a volume V_*, we obtain

$$\frac{\partial\langle n_p^2\rangle}{\partial t} \sim -\langle n_p^2(\nabla\cdot\mathbf{v}_p)\rangle - 2D\langle(\nabla n_p)^2\rangle. \qquad (4)$$

Here we used $\int(\nabla\cdot\mathbf{S})dV_* = \int\mathbf{S}\cdot d\mathbf{A} \ll \int n_p^2(\nabla\cdot\mathbf{v}_p)dV_*$, and $\mathbf{A}$ is a closed surface. Equation (1) implies that variation of particle concentration during the time interval $\tau_0 = l_0/u_0$ around the value $n_p^{(0)}$ is of the order of $\delta n_p \sim -n_p^{(0)}\tau_0(\nabla\cdot\mathbf{v}_p)$, where u_0 is the characteristic velocity in the energy containing scale l_0. Substitution $n_p = n_p^{(0)} + \delta n_p$ into Eq. (4) yields $\partial\langle n_p^2\rangle/\partial t \sim 2\tau_0\langle n_p^2(\nabla\cdot\mathbf{v}_p)^2\rangle$. Therefore, the growth rate of fluctuations of particle concentration $\gamma \sim 2\tau_0\langle(\nabla\cdot\mathbf{v}_p)^2\rangle$. This estimate is in good agreement with the analytical results obtained below [see Eq. (6) for $r = 0$].

Now we discuss the physics of self-excitation (exponential growth) of fluctuations of particle concentration. The inertia causes particles inside the turbulent eddy to drift out to the boundary regions between eddies (the regions with decreased velocity of the turbulent fluid flow and maximum pressure of the fluid). Indeed, Eq. (2) shows that particle inertia results in $\nabla\cdot\mathbf{v}_p \propto \tau_p\Delta P/\rho$. On the other hand, for large Peclet numbers $\nabla\cdot\mathbf{v}_p \propto -dn_p/dt$ [see Eq. (1)]. Therefore, $dn_p/dt \propto -\tau_p\Delta P/\rho$. Thus there is accumulation of inertial particles (i.e., $dn_p/dt > 0$) in regions with the maximum pressure of a turbulent fluid (i.e., where $\Delta P < 0$). Similarly, there is an outflow of inertial particles from the regions with the minimum pressure of fluid.

This mechanism acts in a wide range of scales of a turbulent fluid flow. Turbulent diffusion results in the relaxation of fluctuations of particle concentration in large scales. However, in small scales where turbulent diffusion is of the order of molecular diffusion, the relaxation of fluctuations of particle concentration is very weak. Therefore the fluctuations of particle concentration are localized in the small scales.

This phenomenon is considered for the case when the density of fluid is much less than the material density of particles ($\rho \ll \rho_p$). However, the results of this study can be easily generalized to include the case $\rho \ge \rho_p$ using the equation of motion of particles in fluid flow presented in [1]. This equation of motion takes into account contributions due to the pressure gradient in the fluid surrounding the particle (caused by acceleration of the fluid) and the virtual ("added") mass of the particles relative to the ambient fluid. The results for $\rho \ge \rho_p$ coincide with those obtained for the case ($\rho \ll \rho_p$) except for the transformation $\tau_p \longrightarrow \beta\tau_p$, where

$$\beta = \left(1 + \frac{\rho}{\rho_p}\right)\left(1 - \frac{3\rho}{2\rho_p + \rho}\right).$$

For $\rho \ge \rho_p$ the value $dn_p/dt \propto -\beta\tau_p\Delta P/\rho$. Thus there is an accumulation of inertial particles (i.e., $dn_p/dt > 0$) in regions with the minimum pressure of a turbulent fluid since $\beta < 0$.

To study the fluctuations of inertial particle concentration we derive an equation for the high-order correlation function of particle concentration. For this purpose we use a method of path integrals (Feynman-Kac formula) [6,8–10]. The use of the technique described in [6,8] allows one to derive an equation for the high-order correlation function $\Phi_s = \langle\prod_{j=1}^s[n_p(\mathbf{x}^{(j)}) - N(\mathbf{x}^{(j)})]\rangle$:

$$\frac{\partial\Phi_s}{\partial t} + \sum_{j=1}^{s}\hat{L}^{(j)}\Phi_s = \sum_{j=1}^{s}\sum_{i=1}^{s-1}\hat{M}^{(ij)}\Phi_s + I, \qquad (5)$$

where $i \neq j$, $\hat{L}^{(j)}\Phi_s = \nabla^{(j)}\cdot\{[\mathbf{U}(\mathbf{x}^{(j)}) - \hat{D}(\mathbf{x}^{(j)})\times\nabla^{(j)}]\Phi_s\}$,

$$M^{(ij)}/2 = \langle\tau u_m(\mathbf{x})u_n(\mathbf{y})\rangle\frac{\partial^2}{\partial x_m\partial y_n} + \langle\tau b(\mathbf{x})b(\mathbf{y})\rangle + \langle\tau u_m(\mathbf{x})b(\mathbf{y})\rangle\frac{\partial}{\partial x_m} + \langle\tau u_m(\mathbf{y})b(\mathbf{x})\rangle\frac{\partial}{\partial y_m},$$

also $\mathbf{U}(\mathbf{x}^{(j)}) = \mathbf{V} - \langle\tau b\mathbf{u}\rangle$, $\hat{D}(\mathbf{x}^{(j)}) \equiv D_{mn} = D\delta_{mn} + \langle\tau u_p u_m\rangle$, $\mathbf{x} = \mathbf{x}^{(i)}, \mathbf{y} = \mathbf{x}^{(j)}$, $N = \langle n_p\rangle$ is the mean particle concentration, I is a source which depends on N and the structure functions of the lower orders, $\mathbf{v}_p = \mathbf{V} + \mathbf{u}, \mathbf{V} = \langle\mathbf{v}_p\rangle$ is the mean velocity, $\mathbf{u}$ is the random component of the velocity of particles, $b = \nabla\cdot\mathbf{u}$, and τ is the momentum relaxation time of the turbulent velocity $\mathbf{u}$, which depends on the scale of turbulent motions. We use here for simplicity the δ-correlated in time random process to describe a turbulent velocity field. However, the results also remain valid for the velocity field with a finite correlation time, if the high-order correlation functions Φ_s vary slowly in comparison with the correlation time of the turbulent flow (see, e.g., [8]). We seek a solution to Eq. (5) without the source I in the form $\Phi_s = \prod_{i,j}^s\Phi_2(\mathbf{x}^{(i)} - \mathbf{x}^{(j)})\exp(\gamma_s t)$, where $i \neq j$. Substitution of this solution into Eq. (5) yields $\gamma_s = s(s-1)\gamma_2/2$. This equation implies that, if the second moment of the particle concentration grows ($\gamma_2 > 0$), then all high-order correlation functions grow. It is shown below that under certain conditions $\gamma_2 > 0$. Note that the higher moments grow faster than the lower moments of particle concentration (i.e., $\gamma_s > s\gamma_2/2$). Therefore spatial distribution of fluctuations of particle concentration is intermittent. Since $\gamma_s > \gamma_{s-k}$ (where $0 < k < s$), a contribution of the source I into the obtained general solution is not essential.

Now we study the evolution of the second-order correlation function of particle concentration $\Phi \equiv \Phi_2$. Equation (5) implies an equation for the second moment Φ

$$\frac{\partial\Phi}{\partial t} = -2T_{mn}\frac{\partial^2\Phi}{\partial x_m\partial y_n} + 2\langle\tau b(\mathbf{x})b(\mathbf{y})\rangle\Phi - 4\langle\tau u_m(\mathbf{x})b(\mathbf{y})\rangle\frac{\partial\Phi}{\partial x_m} + I_2. \qquad (6)$$

where $T_{mn} = D_{mn}(0) - D_{mn}(\mathbf{r}), \mathbf{r} = \mathbf{y} - \mathbf{x}$, and $I_2 = 2\langle \tau b(\mathbf{x}) b(\mathbf{y}) \rangle N^2$.

Equation (6) for $b = 0$ was first derived by Kraichnan (see [5]). In this particular case, $b = 0$, this equation describes a relaxation of the second moment of particle concentration. On the other hand, when $b \neq 0$, i.e., when the velocity of particle is divergent, Eq. (6) implies a new effect of self-excitation (exponential growth) of fluctuations of particle concentration caused by the second term in (6). In particular, when $r \to 0$, the second moment can grow (i.e., $\partial\Phi/\partial t > 0$) due to the second term in Eq. (6) which is proportional to $\langle (\nabla \cdot \mathbf{v}_p)^2 \rangle$.

We consider a homogeneous and isotropic incompressible turbulent velocity field of fluid. In this case, the particle velocity field is also homogeneous and isotropic; however, it is compressible, i.e., $\nabla \cdot \mathbf{v}_p \neq 0$. The correlation function of a compressible homogeneous and isotropic random velocity field was derived in [6]. The second moment for the particle velocity can be chosen in the same form (see below):

$$\langle \tau u_m(\mathbf{x}) u_n(\mathbf{x} + \mathbf{r}) \rangle = D_T \bigg[[F(r) + F_c(r)] \delta_{mn} + \frac{rF'}{2} \bigg(\delta_{mn} - \frac{r_m r_n}{r^2} \bigg) + rF_c' \frac{r_m r_n}{r^2} \bigg] \qquad (7)$$

(for details see [6]), where $F' = dF/dr, F(0) = 1 - F_c(0)$, and $D_T = u_0 l_0/3$. The function $F_c(r)$ describes the potential component, whereas $F(r)$ corresponds to the vortical part of the turbulent velocity of particles.

We seek a solution to the equation for Φ without the source I_2 in the form,

$$\Phi(t, r) = \frac{\Psi(r)}{r} \exp\bigg[-\int_0^r \chi(x) dx \bigg] \exp(\gamma t), \qquad (8)$$

where $\gamma \equiv \gamma_2$. The substitution of (8) into Eq. (6) yields an equation for the unknown function $\Psi(r)$:

$$\frac{1}{m(r)} \frac{d^2\Psi}{dr^2} - [\gamma + U(r)]\Psi = 0, \qquad (9)$$

where

$$U(r) = \frac{1}{m(r)} \bigg(\frac{2\chi}{r} + \chi^2 + \chi' \bigg) - \kappa(r),$$

$$\frac{1}{m(r)} = \frac{2}{\mathrm{Pe}} + \frac{2}{3}[1 - F - (rF_c)'],$$

$$\chi(r) = \frac{m(r)}{3} \bigg(4F_c' - F' + \frac{1}{2} rF_c'' \bigg),$$

$$\kappa(r) = -\frac{1}{3} \bigg(8 \frac{F_c'}{r} + 7F_c'' + rF_c''' \bigg),$$

and distance r is measured in units of l_0, time t is measured in units of τ_0, and $\mathrm{Pe} = l_0 u_0/D \gg 1$ is the Peclet number.

Now we discuss the above model of a random velocity field of inertial particles. Consider a case when $\tau_\nu \ll \tau_p \ll \tau_0$, and the particle radius $a_* \ll l_\nu$, where τ_ν is the correlation time in the viscous dissipation scale l_ν of a fluid flow. The viscous scale is $l_\nu \sim \mathrm{Re}^{-1/(3-p)}$, where $\mathrm{Re} = l_0 u_0/\nu \gg 1$ is the Reynolds number, ν is the kinematic viscosity of the fluid and p is the exponent in the spectrum of the turbulent kinetic energy of fluid. This model is valid when the material density ρ_p of particles is much larger than the density ρ of fluid. We introduce a scale r_a in which $\tau_p = \tau(r = r_a)$, where $l_\nu \ll r_a \ll 1$, and $\tau(r)$ is the correlation time of the turbulent fluid velocity field in the scale r. In the range $r_a \ll r < 1$ the effect of inertia of particles is very small, and particle velocity is close to the fluid velocity. In this case, $F = 1 - r^{q-1} + O(\tau_p^2/\tau_0^2)$ and $F_c = O(\tau_p^2/\tau_0^2)$, where $q = 2p - 1$. The exponent p in the spectrum of kinetic turbulent energy is different from that of the function $\langle \tau u_m u_n \rangle$ due to the scale dependence of the momentum relaxation time τ of the turbulent velocity of fluid [11]. Thus, in the scales $r_a \ll r < 1$, the effects of compressibility of the particle velocity field is negligible.

On the other hand, in scales $l_\nu \ll r < r_a$ the effect of inertia is important so that $\nabla \cdot \mathbf{v}_p \neq 0$. In these scales incompressible $F(r)$ and compressible $F_c(r)$ components of the turbulent velocity field of particles can be chosen as $F(r) = (1 - \varepsilon)(1 - r^{q-1})$, and $F_c(r) = \varepsilon(1 - r^{q-1})$. We take into account that in the equation of motion for particles $d\mathbf{v}_p/dt = (\mathbf{v} - \mathbf{v}_p)/\tau_p$ the last term $|\mathbf{v}_p/\tau_p| \ll |d\mathbf{v}_p/dt|$ in the scales $l_\nu \ll r < r_a$. In this case the equation of motion for particles coincides with Navier-Stokes equation for fluid in the inertial range (where the viscous term is dropped out) except for the term $\propto \nabla P$. In the latter equation for particles the term $\mathbf{v}/\tau_p$ can be interpreted as a stirring force. Thus in this case it is plausible to suggest that the exponent in the spectrum of the second moment for particle velocity coincides with that of the turbulent fluid velocity. However, $|\nabla \cdot \mathbf{v}_p| \propto |\nabla \cdot [(\mathbf{v}_p \cdot \nabla)\mathbf{v}_p]| \neq 0$.

We consider the case of large Schmidt numbers, $\mathrm{Sc} = \nu/D \gg 1$. This condition is always satisfied for Brownian particles. The solution of Eq. (9) can be obtained using an asymptotic analysis (see, e.g., [6]). This analysis is based on the separation of scales. In particular, the solution of the Schrödinger equation (9) with a variable mass has different regions where the potential $U(r)$, mass $m(r)$, and eigenfunctions $\Psi(r)$ are different. The solutions in these different regions can be matched at their boundaries. Note that the most important part of the solution is localized in small scales (i.e., $r \ll 1$). The results obtained by this asymptotic analysis are presented below.

The solution of Eq. (9) for $l_\nu \le r \ll r_a$ yields the function $\Phi(t, r) = \tilde{\Phi}(r) \exp(\gamma t)$, where $\tilde{\Phi}(r) = A_1 r^{-a} \cos(c \ln r + \varphi_1)$, and $a = (2q - \sigma q_1)/4(1 + q\sigma)$, $\sigma = \varepsilon/(1 - \varepsilon)$, and $c = [(q - 3a)(q + a)/3]^{1/2}, q_1 = q^2 + 3q - 6$. Note that $q > 3a$ and $a > 0$. This yields the range of values of σ: $2q/q_2 < \sigma < 2q/q_1$, where $q_2 = 7q^2 + 9q - 18$

and $q_1 > 0$. For example, when $p = 5/3$ this inequality yields $0.11 < \sigma < 0.73$. Parameters A_1 and φ_1 are determined by conditions $\tilde{\Phi}(r = l_\nu) = 1$ and $d\tilde{\Phi}/dr(r = l_\nu) = 0$. In the range $r_a \ll r \ll 1$ the function $\tilde{\Phi}(r) = A_2 + A_3 r^{-q}$. In large scales, i.e., $r \gg 1$, functions $F(r)$ and $F_c(r)$ tend to zero and therefore $1/m(r) \to 2/3$ and $U \to 0$. The solution of Eqs. (8) and (9) in this range is given by $\tilde{\Phi}(r) = A_4 r^{-1} \exp(-r\sqrt{3\gamma/2})$, where we consider a case $\gamma \geq 0$. Parameters A_2, A_3, A_4, and the growth rate of fluctuations γ are determined by matching functions $\tilde{\Phi}(r)$ and $\tilde{\Phi}'$ at $r \approx r_a$ and $r \approx 1$. In particular, the growth rate of fluctuations of particle concentration is given by

$$\gamma = \frac{[c^2 + (q - a)^2]^2}{6(3 - q)^2 r_a^{2q}} \ln^2\left(\frac{\mathrm{Re}}{\mathrm{Re}^{(\mathrm{cr})}}\right), \qquad (10)$$

where $r_a = (\tau_p/\tau_0)^{1/(p-1)}$, $\mathrm{Re} > \mathrm{Re}^{(\mathrm{cr})}$ and the critical Reynolds number $\mathrm{Re}^{(\mathrm{cr})}$

$$\mathrm{Re}^{(\mathrm{cr})} \simeq r_a^{p-3} \exp\left[\frac{3 - p}{c}\left(\pi k + \arctan\frac{a}{c} + \arctan\frac{9 - a}{c}\right)\right], \qquad (11)$$

where $k = 1, 2, 3, \ldots$. When $\tau_p \geq \tau_0$ in Eqs. (10) and (11) r_a is set equal to 1. This analysis shows that the characteristic scales of localization of the fluctuations are of order $l_f \sim r_a \exp(\pi n/c)$, where $n \leq k$. Thus we have shown that the fluctuations of particle concentration can be excited without an external source.

Remarkably, a condition for the exponential growth of the mean concentration of inertial particles requires gradients of the external fields (temperature or pressure) and inhomogeneous turbulent fluid flow [10]. Thus inertia of particles in a turbulent fluid flow results in formation of large-scale (mean field) inhomogeneities in the spatial distribution of particle concentration in the vicinity of the minimum (or maximum) of the mean temperature of the fluid, depending on the ratio of material particle density to that of the fluid [10].

The analyzed effect of self-excitation (exponential growth) of fluctuations of particle concentration is important in turbulent fluid flows of a different nature with inertial particles or droplets (e.g., in atmospheric turbulence, combustion, and in a laboratory turbulence). In particular, this effect causes the formation of inhomogeneities in the spatial distribution of fuel droplets in internal combustion engines. Indeed, characteristic parameters of turbulence in a cylinder of an internal combustion engine are maximum scale of turbulent flow $l_0 \sim 0.5$–1 cm; velocity in the scale l_0, $u_0 \sim 100$ cm/s; Reynolds number $\mathrm{Re} \sim (0.7$–$7) \times 10^3$. When the exponent of the spectrum of turbulent kinetic energy of fluid $p = 5/3$ and the degree of compressibility of particle velocity field $\sigma = 0.3$, we obtain from Eq. (11) that the critical Reynolds number $\mathrm{Re}^{(\mathrm{cr})} \sim 200$ for $a_* = 30\ \mu$m. Since $\mathrm{Re} > \mathrm{Re}^{(\mathrm{cr})}$, the fluctuations of droplet concentration can be easily excited in a cylinder of an internal combustion engine. The excited fluctuations are localized in scales $\sim 0.6 \times 10^{-2}$ cm.

The self-excitation of fluctuations of particle concentration is observed in atmospheric turbulence. Using the parameters of the atmospheric turbulent boundary layer, $u_0 \sim 30$–100 cm/s, $l_0 \sim 10^3$–10^4 cm, we find that the excited fluctuations of particle concentration are localized in scales ~ 0.3–1 cm. This effect causes formation of small-scale inhomogeneities in droplet clouds ("inch clouds") which were discovered recently [12].

In summary, it is demonstrated that inertia of particles in a homogeneous and isotropic incompressible turbulent fluid flow causes a self-excitation of fluctuations of particle concentration. The growth rates of the higher moments of particle concentration are higher than those of the lower moments, i.e., particle spatial distribution is intermittent. This process can be damped by the nonlinear effects (e.g., two-way coupling between fluctuations of particle concentration and turbulent fluid flow). When the particle velocity field is divergence free, i.e., $\nabla \cdot \mathbf{v}_p = 0$, all the moments of the concentration field do not grow, and there is no intermittency without an external source of fluctuations of particle concentration. When the inertia effect is negligible (e.g., for a small size of particle or gaseous admixture) but the fluid velocity field is divergent, i.e., $\nabla \cdot \mathbf{v} \neq 0$, the moments of the concentration field grow, and there is intermittency without an external source of fluctuations of particle concentration. In this case, Eqs. (10) and (11) with $r_a = 1$ determine the growth rate of fluctuations of particle concentration and the critical Reynolds number, respectively.

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