Temperature fluctuations and anomalous scaling in low-Mach-number compressible turbulent flow

Tov Elperin and Nathan Kleeorin
The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, P.O. Box 653, Israel

Igor Rogachevskii*
Racah Institute of Physics, The Hebrew University of Jerusalem, 91904 Jerusalem, Israel

(Received 25 October 1996; revised manuscript received 20 February 1997)

Temperature fluctuations in a low-Mach-number compressible turbulent fluid flow are studied. It is demonstrated that, due to compressibility and external pressure fluctuations, the anomalous scaling may occur in the second moment of the temperature field. The cause of the anomalous behavior is a compressibility-induced depletion of the turbulent diffusion of the second moment of the temperature. It is shown that temperature fluctuations in compressible fluid flow (without thermal instability) can be excited only by external pressure fluctuations. Experiments are suggested for the observation of the excitation of the temperature fluctuations. [S1063-651X(97)14206-4]

PACS number(s): 47.27.−i

I. INTRODUCTION

Problems of intermittency and anomalous scalings for scalar and vector fields passively advected by a three-dimensional isotropic turbulent fluid flow have been the subject of numerous investigations in the last years (see, e.g., Refs. [1–11]). The anomalous scaling means the deviation of the scaling exponents of the correlation function of a passive scalar (vector) field from their values obtained by the dimensional analysis. For incompressible turbulent flow, the anomalous scalings for a scalar field can occur only for a fourth-order correlation function, while for the vector field the anomalous scalings appear in the second moment.

For compressible ($\boldsymbol{\nabla}\cdot\mathbf{v}\neq 0$) turbulent fluid flow with low Mach numbers, the situation is quite different. In the present study it is shown that the compressibility of a turbulent fluid flow and external pressure fluctuations may result in the appearance of anomalous scaling in the second moment of the temperature field.

Note that in incompressible turbulent fluid flow, equations for temperature field and the number density of noninertial particles (or gaseous admixtures) coincide. On the other hand, in compressible ($\boldsymbol{\nabla}\cdot\mathbf{v}\neq 0$) turbulent fluid flows with low Mach numbers, these equations are different. Indeed, the equation for the number density has the form of a conservation law of the total number of particles. On the other hand, the equation for the temperature field does not have the form of a conservation law.

This results in different behaviors of particle number density and temperature advected by a compressible turbulent fluid flow. For example, fluctuations of particle number density can be excited even without an external source due to the compressibility of a turbulent fluid flow [10]. In the present study we show that temperature fluctuations are excited in the compressible turbulent fluid flow only if there are external fluctuations of pressure.

Different behaviors can also be observed in the dynamics of the mean fields. In particular, compressibility results in the formation of inhomogeneous spatial distributions of mean particle number density due to the effects of turbulent barodiffusion and turbulent thermal diffusion [12,13]. Inhomogeneities of the mean temperature in compressible turbulent fluid flow can be formed only in the presence of external fluctuations of pressure (we now discuss the case when thermal instability [14] is not excited). Excitation of temperature fluctuations and the formation of inhomogeneities of the mean temperature in a compressible turbulent fluid flow are caused by the work performed by external pressure. On the other hand, excitation of fluctuations and formation of inhomogeneities of the mean number density of particles advected by a compressible turbulent flow do not change the thermal energy of the system.

II. GOVERNING EQUATIONS

Evolution of the temperature field $T_f(t,\mathbf{r})$ in a compressible turbulent fluid flow is determined by the equation

$$\frac{\partial T_f}{\partial t}+(\mathbf{v}\cdot\boldsymbol{\nabla})T_f+(\gamma-1)T_f(\boldsymbol{\nabla}\cdot\mathbf{v})=\eta\Delta T_f+Q,\qquad(1)$$

where η is the molecular thermal conductivity, γ is the specific heat ratio, and Q is an external heat source. The density ρ_f and the velocity $\mathbf{v}$ of the fluid satisfy the continuity equation

$$\frac{\partial \rho_f}{\partial t}+\boldsymbol{\nabla}\cdot(\rho_f\mathbf{v})=0.\qquad(2)$$

The velocity $\mathbf{v}$ is determined by Navier-Stokes equation

*Present address: Department of Mechanical Engineering, Ben-Gurion University of the Negev, P. O. Box 653, Beer-Sheva 84105, Israel.

$$\rho_f \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} = -\boldsymbol{\nabla} P_f + \rho_f \mathbf{F}_\nu + \mathbf{F}, \tag{3}$$

where $\rho \mathbf{F}_\nu$ is the viscous force, and $\mathbf{F}$ is the stirring force. Fluid pressure P_f, temperature T_f, and density ρ_f (with characteristic values P_0, T_0, and ρ_0) satisfy the equation of state $P_f = \rho_f T_f / m_\mu$, (m_μ is the mass of molecules of the fluid). Consider turbulent flow with small Mach numbers. A solution of Eqs. (1)–(3) can be sought in the form of a power series of Mach number

$$\phi = \sum_{k=0}^{k=\infty} M^{2k} \phi_{k+1} \tag{4}$$

(see, e.g., Ref. [14]), where nondimensional functions $\phi = (\rho_f/\rho_0,\ T_f/T_0,\ P_f/P_0$, and $\mathbf{v}/v_0)$, the characteristic value of the velocity is $v_0 = (F_0 l_0/\rho_0)^{1/2}$, the characteristic value of the stirring force is F_0, the energy containing scale of turbulent motions l_0, the Mach number $M = v_0\sqrt{\gamma}/c_s$, and the sound speed $c_s = (\gamma T_0/m_\mu)^{1/2}$. Substitution of expansion (4) into Eqs. (1)–(3) and comparison the terms of the same order in M^{2k} yields a set of equations

$$\boldsymbol{\nabla} P_1 = \mathbf{0}, \tag{5}$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1\cdot\boldsymbol{\nabla})\mathbf{v}_1 = -\frac{1}{\rho_1}\boldsymbol{\nabla} P_2 + \mathrm{Re}^{-1}[\Delta \mathbf{v}_1 + \zeta \boldsymbol{\nabla}(\mathrm{div}\, \mathbf{v}_1)] + \frac{1}{\rho_1}\mathbf{F}_1, \tag{6}$$

$$\frac{\partial \rho_1}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_1 \mathbf{v}_1) = 0, \tag{7}$$

$$\frac{\partial T_1}{\partial t} + (\mathbf{v}_1\cdot\boldsymbol{\nabla})T_1 = -(\gamma-1)T_1(\boldsymbol{\nabla}\cdot\mathbf{v}_1) + \mathrm{Pe}^{-1}\Delta T_1 + Q_1, \tag{8}$$

where $Q_1 = Q l_0/(T_0 v_0)$, $\mathbf{F}_1 = \mathbf{F}/F_0$, and $\zeta = 1/3 + \zeta_b/\nu$, ζ_b is a bulk viscosity, $\mathrm{Re} = v_0 l_0/\nu$ is the Reynolds number, and ν is the kinematic viscosity. Note that Eq. (5) appears in the order of M^{-2}, whereas Eqs. (6)–(8) appear in the order of M^0. Equations (5)–(8) yield

$$\boldsymbol{\nabla}\cdot\mathbf{v}_1 = \frac{1}{\gamma T_1}\left(\mathrm{Pe}^{-1}\Delta T_1 + Q_1 - T_1 \frac{d}{dt}\ln P_1\right), \tag{9}$$

where $\mathrm{Pe} = v_0 l_0/\eta$. Equations (8) and (9) yield the equation for the fluid temperature,

$$\frac{\partial T_1}{\partial t} + (\mathbf{v}_1\cdot\boldsymbol{\nabla})T_1 = \mathrm{Pe}^{-1}\Delta T_1 + D(t)T_1 + Q_1, \tag{10}$$

where $D(t) = (\gamma^{-1}-1)(d/dt)\ln P_1$, and we changed notations $\gamma^{-1}\mathrm{Pe}^{-1} \to \mathrm{Pe}^{-1}$ and $\gamma^{-1}Q_1 \to Q_1$.

In order to study the temperature fluctuations, we derive an equation for the structure function $\langle \Theta(t,\mathbf{x})\Theta(t,\mathbf{y})\rangle$, where $T_1 = T + \Theta$, $T = \langle T_1\rangle$ is the mean temperature field, Θ is the fluctuating temperature field, and the angular brackets denote statistical averaging over the ensemble of turbulent fluid velocity. To this purpose we use the stochastic calculus (Feynman-Kac formula), which was applied in magnetohydrodynamics [15,16] and passive scalar transport in incompressible [15] and compressible [12,13] turbulent flows.

For simplicity the turbulent velocity field is assumed to be δ correlated in a time random process. However, the results also remain valid for the velocity field with a finite correlation time if the second moment Φ varies slowly in comparison with the correlation time of the turbulent fluid flow [15].

The use of the technique described in Ref. [13] allows us to derive the equation for the structure function $\Phi = \langle \Theta(t,\mathbf{x})\Theta(t,\mathbf{y})\rangle$:

$$\frac{\partial \Phi}{\partial t} = \hat{L}\Phi + 2\tau_D\langle D^2\rangle\Phi + I, \tag{11}$$

where

$$\hat{L} = -(\mathbf{V}_{\mathrm{eff}}\cdot\boldsymbol{\nabla})_{\mathbf{x}} - (\mathbf{V}_{\mathrm{eff}}\cdot\boldsymbol{\nabla})_{\mathbf{y}} + [\boldsymbol{\nabla}(\hat{\eta}\cdot\boldsymbol{\nabla})]_{\mathbf{x}} + [\boldsymbol{\nabla}(\hat{\eta}\cdot\boldsymbol{\nabla})]_{\mathbf{y}} + 2\langle \tau u_m(\mathbf{x})u_n(\mathbf{y})\rangle \frac{\partial^2}{\partial x_m \partial y_n},$$

$$\hat{\eta} = \eta_{pm} = \mathrm{Pe}^{-1}\delta_{pm} + \langle \tau u_p u_m\rangle, \quad \mathbf{V}_{\mathrm{eff}} = \mathbf{V} + \langle \tau \mathbf{u}(\boldsymbol{\nabla}\cdot\mathbf{u})\rangle,$$

$$I = 2\tau_Q\langle Q_1(\mathbf{x})Q_1(\mathbf{y})\rangle.$$

$\mathbf{v} = \mathbf{V} + \mathbf{u}$, where $\mathbf{V} = \langle \mathbf{v}\rangle$ is the mean velocity and $\mathbf{u}$ is the turbulent component of the velocity, τ is the momentum relaxation time of the random velocity field $\mathbf{u}$, which depends on the scale of turbulent motion, and τ_Q and τ_D are the momentum relaxation time of the external heat source and of the external flluctuations of pressure P_1, respectively, and $\langle D^2\rangle$ denotes averaging over external pressure fluctuations. We consider the case of $\boldsymbol{\nabla} T = \mathbf{0}$, where T is the mean temperature field. Equation (11) for $(\boldsymbol{\nabla}\cdot\mathbf{u}) = 0$ and $\langle D^2\rangle = 0$ was first derived by Kraichnan [17].

III. TEMPERATURE FLUCTUATIONS

Consider temperature fluctuations in a homogeneous and isotropic compressible turbulent fluid flow. In this case the correlation function $\langle \tau u_m u_n\rangle$ is given by

$$\langle \tau u_m(\mathbf{x})u_n(\mathbf{x}+\mathbf{r})\rangle = \eta_T\left[[F(r)+F_c(r)]\delta_{mn} + \frac{rF'}{2}\left(\delta_{mn} - \frac{r_m r_n}{r^2}\right) + rF_c'\frac{r_m r_n}{r^2}\right] \tag{12}$$

(for details see [13]), where $F' = dF/dr$, $\eta_T = u_0^2\tau_0/3$ is the turbulent thermal conductivity, u_0 is the characteristic velocity in the energy containing scale l_0 of turbulent motions, $\tau_0 = l_0/u_0$, and $F(0) = 1 - F_c(0)$. The function $F_c(r)$ describes the compressible (potential) component whereas $F(r)$ corresponds to the vortical part of the turbulence. Equation (11), by means of relation (12), reduces to

$$\frac{\partial \Phi}{\partial t} = \frac{1}{m(r)}\left[\Phi'' + 2\left(\frac{1}{r} + \chi\right)\Phi'\right] + 2\tau_D\langle D^2\rangle\Phi + I, \tag{13}$$

where

$$\frac{1}{m(r)}=\frac{2}{\mathrm{Pe}}+\frac{2}{3}[1-F-(rF_c)'],$$

$$\chi(r)=\frac{m(r)}{3}(2F_c'-F'),$$

and $\mathrm{Pe}=\gamma l_0 u_0/\eta \gg 1$ is the thermal Peclet number. We seek a solution of Eq. (13) with $I=0$ in the form

$$\Phi(t,r)=\frac{\psi(t,r)}{r}\exp\left[-\int_0^r \chi(x)dx\right], \tag{14}$$

where $\psi(t,r)=\Psi(r)\exp(2\Gamma t)$ and the unknown function $\Psi(r)$ in a nondimensional form is determined by the equation

$$\frac{1}{m(r)}\Psi''-[2\tilde{\Gamma}+U_0(r)]\Psi=0, \tag{15}$$

where

$$U_0(r)=\frac{1}{m(r)}\left(\frac{2\chi}{r}+\chi^2+\chi'\right),$$

$\tilde{\Gamma}=\Gamma-\tau_0^2\langle D^2\rangle$, distance r is measured in units of l_0, and time t is measured in units of τ_0.

We choose the following model of turbulence. Incompressible $F(r)$ and compressible $F_c(r)$ components are given by $F(r)=(1-\varepsilon)(1-r^{q-1})$, and $F_c(r)=\varepsilon(1-r^{q-1})$, where $r_d<r\ll 1$, q is the exponent in the spectrum of the function $\langle \tau u_m u_n\rangle$, and $r_d=\mathrm{Re}^{-1/(3-p)}$. The exponent p in the spectrum of the kinetic turbulent energy differs from that of the function $\langle \tau u_m u_n\rangle$ due to the scale dependence of the momentum relaxation time τ of turbulent velocity field **u**, and $q=2p-1$ (see Refs. [10,11]). We consider the case of the Prandtl number $\mathrm{Pr}=\nu/\eta\ll 1$, which is typical for gases.

Solution of Eq. (15) can be obtained using an asymptotic analysis (see, e.g., Refs. [10,13,15,16]). This analysis is based on the separation of scales. In particular, the solution of the Schrödinger equation (15) with a variable mass has different regions where the form of the potential $U_0(r)$, mass $m(r)$ and, therefore, eigenfunctions $\Psi(r)$ are different. Solutions in these different regions can be matched at their boundaries. Note that the most important part of the solution is localized in small scales (i.e., $r\ll 1$). The results obtained by this asymptotic analysis are presented below. Distributions of mass $m(r)$ are given by $1/m(r)=2[1+\beta_m \mathrm{Pe} r^{q-1}]/\mathrm{Pe}$, where $\beta_m=(1-\varepsilon)(1+q\sigma)/3$, and the parameter of compressibility $\sigma=\varepsilon/(1-\varepsilon)$. The solution of Eq. (15) has several characteristic regions. In region I, i.e., for $r_d\leqslant r\ll \mathrm{Pe}^{-1/(q-1)}$, the mass $m(r)$, the potential $U_0(r)$, and the functions $\Psi(r)$ and $\Phi(r)$ are given by

$$\frac{1}{m(r)}\sim\frac{2}{\mathrm{Pe}},\quad U_0(r)\sim\frac{2\beta_0}{r^{3-q}},$$

$$\Psi(r)=A_1 r^{1/2}J_\lambda(2\lambda\sqrt{|\beta_0|\mathrm{Pe}}\,r^{1/2\lambda}),$$

$$\Phi(r)=\frac{\Psi(r)}{r}[1+\beta_m \mathrm{Pe} r^{q-1}]^{-\beta_0/q(q-1)\beta_m},$$

where $\beta_0=q(q-1)(1-\varepsilon)(1-2\sigma)/6$, the parameter $\lambda=1/(q-1)$, J_λ is the Bessel function of the first kind, $\beta_0<0$, and $\sigma>\frac{1}{2}$. For $\sigma<\frac{1}{2}$ the derivative $(\partial\Phi/\partial r)_{r\to 0}>0$, and this solution cannot be a correlation function. In the region $0<r\leqslant r_d$ the exponent $q=3$ and the solution for the correlation function $\Phi(r)$ can be expressed in terms of the Legendre functions (see Ref. [13]). In region II ($\mathrm{Pe}^{-1/(q-1)}\ll r\ll 1$),

$$\frac{1}{m(r)}\sim 2\beta_m r^{q-1},\quad U_0(r)\sim-\frac{1-4b^2}{4mr^2},$$

$$\Psi(r)=A_2 r^{1/2+b}+A_3 r^{1/2-b},$$

$$\Phi(r)=A_2+A_3 r^{-2b},$$

where

$$b=\frac{q-\sigma(q-2)}{2(1+\sigma q)},$$

and $q>\sigma(q-2)$. In region III ($r\gg 1$),

$$\frac{1}{m(r)}\sim\frac{2}{3},\quad U_0(r)\sim 0,$$

$$\Psi(r)=A_4\cos[\sqrt{3|\tilde{\Gamma}|}(r-r_*)+\varphi],\ \ \Phi(r)=\Psi/r.$$

Matching functions $\Phi(r)$ and $\Phi'(r)$ at the boundaries of these regions yields the constants A_k and the damping (or growth) rate Γ of the temperature fluctuations. The latter is given by

$$\Gamma=\tau_0^2\langle D^2\rangle-\frac{g^2}{3},$$

where $g\propto\cot\varphi$ is the parameter of the continuous spectrum of the Schrödinger equation (15). From the latter formula it follows that, when $g\to 0$ and $D\to 0$, the damping rate of the temperature fluctuations $\Gamma\to 0$. Therefore the characteristic time of the turbulent thermal conductivity can be much longer than the turnover time of the turbulent eddies. The latter implies that the compressibility of the turbulent fluid flow results in a fairly strong reduction of the turbulent thermal conductivity for $D=0$.

The physics of this effect is as follows. The equation for the internal energy $U=c_V T_f$ is given by

$$\frac{dU}{dt}=-T_f\nabla\cdot\mathbf{u}+\nabla\cdot(\eta\nabla T_f). \tag{16}$$

It can be seen from Eq. (16) that the internal energy U can increase (or decrease) when $\nabla\cdot\mathbf{u}<0$ (when $\nabla\cdot\mathbf{u}>0$). This can result in a reduction of the rate of temperature leveling. Such an effect can be interpreted as a reduction of the turbulent thermal conductivity. The reason for the low turbulent thermal conductivity is that the increase and decrease of the internal energy in a small volume are separated in time and are not balanced in a compressible flow. Molecular thermal conductivity breaks the symmetry between the accumulation and outflow of thermal energy, i.e., it breaks a reversibility in time and does not allow a leveling of the thermal energy over the consecutive time intervals. It can be seen from Eq. (16) that in incompressible turbulent flow, when the molecular thermal conductivity $\eta=0$, the internal energy is conserved, whereas in compressible turbulent fluid flow the internal energy is not conserved due to the work performed by a pressure force. When $D\neq 0$ the self-excitation (exponential growth) of temperature fluctuations occurs. In this case spatial distribution of the temperature fluctuations is intermittent since the higher moments of the temperature field grow faster than flow moments (see Ref. [10]).

IV. ANOMALOUS SCALING OF TEMPERATURE FLUCTUATIONS

Consider temperature fluctuations in the presence of an external thermal source $I(r)$ in Eq. (13). Substituting Eq. (14) into Eq. (11) yields an equation for the unknown function $\psi(t,r)$,

$$\frac{\partial \psi}{\partial t}=\frac{1}{m}\,\frac{\partial^2 \psi}{\partial r^2}-U(r)\psi+f(r), \tag{17}$$

where $f(r)=rI(r)\exp[\int_0^r \chi(x)dx]$. Temperature fluctuations can only be damped without an external source (see Sec. III). Stationary temperature fluctuations require the presence of the nonzero external source. Determine the stationary solution of Eq. (17) in scales $\mathrm{Pe}^{-1/(q-1)}\ll r\ll 1$ (i.e., in region II, see Sec. III). The external source in these scales is chosen as follows: $I(r)=I_0(1-r^s)$, where $s>0$, and for $r>1$, $I(r)=0$. The general solution of Eq. (17) reads

$$\psi(r)=A_2\Psi_1+A_3\Psi_2+\int_0^\infty G(r,\xi)f(\xi)d\xi, \tag{18}$$

where $\Psi_1=r^{1/2+b}$ and $\Psi_2=r^{1/2-b}$ are solutions of Eq. (17) with $I=0$, and Green function $G(r,\xi)$ is given by

$$G(r,\xi)=m(\xi)H(r-\xi)\frac{\Psi_1(r)\Psi_2(\xi)-\Psi_2(r)\Psi_1(\xi)}{\Psi_1(\xi)\Psi_2'(\xi)-\Psi_2(\xi)\Psi_1'(\xi)},$$

and $H(y)$ is a Heaviside function. Equation (18) yields the formula for the second moment $\Phi(r)$,

$$\Phi(r)=A_2+A_3r^{-2b}-\frac{I_0}{2\beta_m(3-q)(3-q+2b)}r^{3-q}, \tag{19}$$

where we neglect small terms $\propto r^{3-q+s}$, because $r\ll 1$. Note that the term $\propto r^{3-q}$ in Eq. (19) corresponds to a normal scaling for the second moment of the temperature fluctuations, whereas the term $\propto r^{-2b}$ corresponds to the anomalous scaling.

Note that in incompressible turbulent flow only the fourth-order correlation function of the passive scalar fluctuations can have anomalous scaling (see Refs. [3]–[5]). On the other hand, we have found that in the compressible turbulent flow with external pressure fluctuations even the second moment can exhibit an anomalous scaling behavior. The reason for the anomalous scaling in the second moment is the compressibility, which results in a reduction of turbulent thermal conductivity (i.e., existence of modes with damping rate $\Gamma\ll 1$). Note that in view of the quantum mechanics analogy the appearance of the anomalous scaling in the second moment of the temperature fluctuations is related to the existence of the region with negative potential for $r\ll 1$. This yields a condition $\sigma>\frac{1}{2}$ for the anomalous scaling. On the other hand, the solution for the correlation function Φ exists in scales $\mathrm{Pe}^{-1/(q-1)}\ll r\ll 1$ when $q>\sigma(q-2)$. This yields conditions for the anomalous scalings of the temperature field in compressible turbulent fluid flow: when $1<q\leq 2$ the parameter of compressibility $\frac{1}{2}<\sigma<\infty$, and when $2<q<3$ the conditions for σ are given by $\frac{1}{2}<\sigma<q/(q-2)$.

V. CONCLUSIONS

It is demonstrated here that in a low-Mach-number compressible turbulent fluid flow with external pressure fluctuations the second-order correlation function of the temperature field can have anomalous scaling. The mechanism for the occurrence of anomalous scaling in the second moment is associated with the compressibility of the fluid flow. Mathematically the latter phenomenon is associated with the occurrence of the zero mode of the equation for the correlation function of the temperature field. For the two-point correlation function of the temperature field, this anomalous scaling is caused by a strong depletion of the relaxation of the second moment of the temperature field due to compressibility. In view of the quantum mechanics analogy, the equation for the second-order correlation function of the temperature field can be considered a Schrödinger equation, and the necessary condition for anomalous scaling is the appearance of the negative potential in the Schrödinger equation at small scales ($r\ll l_0$). This yields the condition $\sigma>\frac{1}{2}$. When the exponent in the spectrum of the function $\langle \tau u_m u_n\rangle$ is within the range $1<q\leq 2$ this condition is sufficient. On the other hand, when $2<q<3$, the conditions for the anomalous scaling is given by $\frac{1}{2}<\sigma<q/(q-2)$.

The turbulent velocity field is assumed to be described by a δ correlated in time random process. The derived equation for the second-order correlation function is valid as long as the momentum relaxation time of the velocity field is small in comparison with the the characteristic time of variations of the temperature fluctuations.

It is shown that in compressible turbulent flow the temperature fluctuations can be excited only by the external fluctuations of pressure. This mechanism of excitation of temperature fluctuations can be validated experimentally. Thus, e.g., it is conceivable to suggest installing a sound generator with a white-noise power spectrum in the wind tunnel used for the investigation of temperature fluctuations (see, e.g., Refs. [18,19]). Then the observed temperature correlation function will change strongly in comparison to a case without external pressure fluctuations.

ACKNOWLEDGMENTS

We are indebted to E. Balkovsky, G. Falkovich, and V. Lebedev for stimulating discussions. This work was partially supported by The German-Israeli Foundation for Scientific Research (Grant No. I-008-303.06/95).

[1] R. Kraichnan, Phys. Rev. Lett. **72**, 1016 (1994).
[2] R. Kraichnan, V. Yakhot, and S. Chen, Phys. Rev. Lett. **75**, 240 (1995).
[3] K. Gawedzki and A. Kupiainen, Phys. Rev. Lett. **75**, 3834(1995); D. Bernard, K. Gawedzki, and A. Kupiainen, Phys. Rev. E **54**, 2564 (1996).
[4] B. I. Shraiman and E. D. Siggia, C. R. Acad. Sci. **321**, 279 (1995).
[5] M. Chertkov, G. Falkovich, I. Kolokolov, and V. Lebedev, Phys. Rev. E **52**, 4924 (1995); M. Chertkov and G. Falkovich, Phys. Rev. Lett. **76**, 2706 (1996); M. Chertkov, G. Falkovich, and V. Lebedev, *ibid.* **76**, 3707 (1996).
[6] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1995).
[7] A. L. Fairhall, O. Gat, V. S. L'vov, and I. Procaccia, Phys. Rev. E **53**, 3518 (1996).
[8] M. Vergassola, Phys. Rev. E **53**, 3021 (1996).
[9] V. Borue and V. Yakhot, Phys. Rev. E **53**, 5576 (1996).
[10] T. Elperin, N. Kleeorin, and I. Rogachevskii, Phys. Rev. Lett. **77**, 5373 (1996).
[11] I. Rogachevskii and N. Kleeorin, Phys. Rev. E (to be published).
[12] T. Elperin, N. Kleeorin, and I. Rogachevskii, Phys. Rev. Lett. **76**, 224 (1996).
[13] T. Elperin, N. Kleeorin, and I. Rogachevskii, Phys. Rev. E **52**, 2617 (1995); **55**, 2713 (1997).
[14] B. Meerson, Rev. Mod. Phys. **68**, 215 (1996).
[15] Ya. B. Zeldovich, S. A. Molchanov, A. A. Ruzmaikin, and D. D. Sokoloff, Sov. Sci. Rev. C. Math Phys. **7**, 1 (1988); Ya. B. Zeldovich, A. A. Ruzmaikin, and D. D. Sokoloff, *The Almighty Chance* (World Scientific, London, 1990).
[16] N. Kleeorin and I. Rogachevskii, Phys. Rev. E **50**, 493 (1994).
[17] R. Kraichnan, Phys. Fluids **11**, 945 (1968).
[18] Jayesh and Z. Warhaft, Phys. Fluids A **4**, 2292 (1992).
[19] Jayesh, C. Tong, and Z. Warhaft, Phys. Fluids A **6**, 306 (1994).