Effective Ampère force in developed magnetohydrodynamic turbulence

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The interaction between a large-scale uniform weak magnetic field **B** and a developed small-scale magnetohydrodynamic (MHD) turbulence is studied. It is found that the effective mean Ampère force in the turbulence is given by $\mathbf{F}_m = -\nabla(Q_p\mathbf{B}^2/8\pi)+(\mathbf{B}\cdot\nabla)Q_s\mathbf{B}/4\pi$. The turbulent magnetic coefficients Q_p and Q_s are drastically decreased at large magnetic Reynolds numbers, whereas in the absence of turbulence $Q_p=Q_s=1$. This phenomenon arises due to a negative contribution of the MHD turbulence to the mean magnetic force. This is caused by the generation of magnetic fluctuations at the expense of fluctuations of the velocity field. This effect is nonlinear in terms of the large-scale magnetic field. It is shown here that in turbulence with a mean large-scale magnetic field, a universal k^{-1} spectrum of magnetic fluctuations exists; this spectrum is independent of the exponent of the spectrum of the turbulent velocity field. A variant of the renormalization group (RNG) method allows the derivation of the scaling and amplitude of the turbulent transport coefficients: turbulent viscosity, turbulent magnetic diffusion, and turbulent magnetic coefficients. A small parameter in the RNG method is the ratio $\varepsilon=B^2/(8\pi W_k)$ of the large-scale magnetic energy density to the energy density $W_k=(1/2)\rho\langle u^2\rangle$ of the turbulent velocity field.

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I. INTRODUCTION

Investigations of developed magnetohydrodynamic (MHD) turbulence are important from the point of view of various cosmic and laboratory applications. The main property of the MHD turbulence is that random motion of a conducting fluid can generate both a large-scale mean magnetic field (see, e.g., [1–4]) and magnetic fluctuations [5,6].

MHD turbulence at large magnetic Reynolds numbers, R_m, was studied mainly in an approximation that is linear in terms of the mean magnetic field. This treatment leads to well-known effects: the α effect, the eddy viscosity, the turbulent magnetic diffusion, and the turbulent diamagnetism (see, e.g., [1–4]). These effects vanish in the case of homogeneous and isotropic turbulence with a uniform mean magnetic field.

In this paper we investigate an effect which is nonlinear in terms of the mean magnetic field. It is the effect of variations of the mean magnetic force by developed MHD turbulence. This means that the mean Ampère force is given by

$$\mathbf{F}_m = -\nabla\left[\frac{Q_p}{8\pi}\mathbf{B}^2\right]+\frac{1}{4\pi}(\mathbf{B}\cdot\nabla)Q_s\mathbf{B} ,$$

where **B** is the mean magnetic field. The magnetic coefficients Q_p and Q_s determine the effect of the MHD turbulence on the mean magnetic force. In the absence of turbulence $Q_p=Q_s=1$, while in developed MHD turbulence the magnetic coefficients drastically decrease. This phenomenon arises due to a negative contribution of the MHD turbulence to the mean magnetic force. For $R_m \gg 1$, Q_p can be negative and the "effective" magnetic pressure, $p_m=Q_p\mathbf{B}^2/8\pi$, changes sign. The case $\beta \gg 1$ is considered, where β is the ratio of the gas pressure to that of the mean magnetic field. Therefore the total (magnetic plus gas) pressure is always positive. In contrast to both the α effect and turbulent magnetic diffusion this phenomenon can arise in uniform magnetic field.

This effect can excite a large-scale magnetic instability (see [7–10]). It leads to the formation of inhomogeneities of the regular magnetic field at the expense of the energy transferred from small-scale turbulent pulsations. This phenomenon may serve as a mechanism of magnetic flux rope formation in the turbulent convective zone of the sun, stars, and spiral galaxies [7–10]. This effect causes an observed anomalous increase (decrease) of the solar radius at the minimum (maximum) of the solar activity [11,12]. In addition, this phenomenon may determine the fine structure of the observed solar torsional oscillations and solar meridional motions [12].

The renormalization group (RNG) procedure is employed for the investigation of the MHD turbulence at large magnetic Reynolds numbers. The RNG method comprises a replacement of real turbulence with a medium characterized by effective turbulent transport coefficients. This procedure enables us to derive equations for the transport coefficients: turbulent viscosity, turbulent magnetic diffusion, and turbulent magnetic coefficients. The RNG procedure used here (see also [13]) is slightly different from the "classic" RNG method (see, e.g., [14–20]). Indeed, the differences between these approaches are as follows.

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(1) We study here the interaction between the large-scale magnetic field and background MHD turbulence. In contrast to the "classic" RNG method, spectrum and statistical properties of the background turbulence (a medium with zero-mean fields) are assumed to be given here. Furthermore, the background turbulence can be arbitrary. In the "classic" RNG method, on the other hand, an external random stirring force with Gaussian statistics is introduced (see, e.g., [19]).

(2) A small parameter in the RNG procedure applied here is $\varepsilon = W_B/W_k$, where W_B is the energy density of the mean magnetic field and W_k is the energy density of the hydrodynamic motions of the background turbulence. On the other hand, the parameter of the nonlinear interaction is assumed to be small in the "classic" RNG method.

(3) The RNG procedure employed here performs a direct renormalization of the MHD equations, whereas in the "classic" RNG method the Green's function is renormalized; the "classic" RNG method has mainly beer used in the study of the Navier-Stokes equation.

A limitation of the RNG procedure used here is associated with the assumption that the background turbulence is considered to be known. For the RNG procedure, an equation invariant under the renormalization of the turbulent transport coefficients must be determined. For this purpose, the recent results [8] for a simple model with the high-order closure procedure are used.

II. "EFFECTIVE" MAGNETIC PRESSURE

Let us consider fully developed MHD turbulence with $\mathrm{Re} \gg 1$ and $R_m \gg 1$, where $\mathrm{Re} = u_0 l_0/\nu_0$ is the Reynolds number, $R_m = u_0 l_0/\eta_m$ is the magnetic Reynolds number, l_0 is the maximal scale of turbulent motions, u_0 is the characteristic turbulent velocity, ν_0 is the kinematic viscosity, $\eta_m = c^2/4\pi\sigma$ is the magnetic diffusion, c is the speed of light, and σ is the electrical conductivity of the fluid. The dissipation due to the molecular viscosities ν_0 and η_m is intrinsic only in the region $l \le l_d$ and $l_d \ll l_0$.

Now we discuss the meaning of the "effective" magnetic pressure (see also [7,8]). For isotropic turbulence the equation of state is given by

$$P_T = \tfrac{1}{3} W_m + \tfrac{2}{3} W_k \tag{1}$$

(see, e.g., [21,22]). Here P_T is the total (hydrodynamic plus magnetic) turbulent pressure, $W_m = \langle \mathbf{h}^2 \rangle / 8\pi$ is the energy density of the magnetic fluctuations, and $W_k = \langle \rho \mathbf{u}^2 \rangle / 2$ is the energy density of the turbulent hydrodynamic motion where $\mathbf{u}$ and $\mathbf{h}$ are random pulsations of the hydrodynamic and magnetic fields and ρ is the density of the conducting fluid. The angle brackets denote averaging over the ensemble of turbulent pulsations.

The equation describing the evolution of the total energy density $W_T = W_k + W_m$ of the homogeneous turbulence with uniform mean magnetic field $\mathbf{B}$ is given by (see Appendix A)

$$\frac{\partial W_T}{\partial t} = I_T - \frac{W_T}{\tau} , \tag{2}$$

where τ is the correlation time of the turbulence in the scale l_0. The second term in (2), W_T/τ, determines the dissipation of the turbulent energy. For a given time-independent source of the turbulence I_T, the solution of Eq. (2) is given by

$$W_T = I_T \tau \left[1 - \exp\left\{ -\frac{t}{\tau} \right\} \right] + \tilde{W}_T \exp\left\{ -\frac{t}{\tau} \right\} ,$$

where $\tilde{W}_T = W_T(t=0)$. Note that a time-independent source of the turbulence exists, for example, in the sun. For $t \gg \tau$ the total energy density of the turbulence reaches a stationary value $W_T = \mathrm{const} = I_T \tau$. It is independent of the mean magnetic field $\mathbf{B}$. Therefore the total energy density W_T of the homogeneous turbulence with uniform mean magnetic field is *conserved* (the dissipation is compensated by the supply of energy), i.e.,

$$W_k + W_m = \mathrm{const.} \tag{3}$$

For a statistically homogeneous medium Eq. (3) is equivalent to the conservation of the total turbulent energy. Note that the uniform large-scale magnetic field performs no work on the turbulence. It can only redistribute the energy between hydrodynamic pulsations (i.e., fluctuations of the velocity) and magnetic fluctuations.

By combining Eqs. (1) and (3), one can express the change of turbulent pressure δP_T in terms of the change of the magnetic energy density δW_m: $\delta P_T = -\delta W_m/3$. It thus follows that the turbulent pressure is reduced when magnetic fluctuations are generated (i.e., $\delta W_m > 0$).

The total turbulent pressure is also decreased by the "tangling" of the large-scale regular magnetic field $\mathbf{B}$ with hydrodynamic pulsations (see, e.g., [1–4]). The regular magnetic field, "entangled" with the hydrodynamic pulsations, generates supplementary small-scale magnetic fluctuations. In this case the density of the magnetic energy W_m depends on W_k and W_B, where $W_B = B^2/8\pi$ is the energy density of the large-scale magnetic field $\mathbf{B}$. For weak mean magnetic fields ($W_B \ll W_k$), expanding the function W_m in a series in W_B, one obtains

$$W_m = W_m^{(0)} + a_p(W_k) \frac{B^2}{8\pi} + \cdots , \tag{4}$$

where $W_m^{(0)}$ is the energy density of the magnetic fluctuations in the absence of a large-scale magnetic field. This expression yields the variation of the magnetic energy δW_m. The sign of a_p is determined by the direction of energy transfer. It is positive when magnetic fluctuations are generated and negative when they are damped. In view of Eq. (4) the turbulent pressure takes the form

$$P_T = P_T^{(0)} - a_p B^2/24\pi ,$$

where $P_T^{(0)}$ is the turbulent pressure in the absence of the mean magnetic field.

The turbulent magnetic pressure P_h as well as the turbulent hydrodynamic pressure P_u are given by

$$P_h \equiv \frac{1}{3} \left[\frac{\langle h^2 \rangle}{8\pi} \right] = P_h^{(0)} + q_h \frac{B^2}{8\pi} ,$$

$$P_u \equiv \frac{2}{3} \left[\frac{\langle \rho u^2 \rangle}{2} \right] = P_u^{(0)} - q_u \frac{B^2}{8\pi} .$$

Here $P_\alpha^{(0)}$ is a component of pressure in a medium with zero mean field. Generation of magnetic fluctuations at the expense of the energy of the hydrodynamic pulsations corresponds to $q_h > 0$ and $q_u > 0$. The total turbulent pressure is $P_T = P_h + P_u = P_T^{(0)} - (q_u - q_h) B^2/8\pi$, where $a_p = 3(q_u - q_h)$. The total pressure is $P = P_k + P_T + P_B$, where P_k is the usual gas dynamic pressure of the plasma and $P_B = B^2/8\pi$ is the magnetic pressure of the mean field.

Let us extract a component which depends on the mean (regular) magnetic field **B**:

$$p_m(B) = P_B + (q_h - q_u)\frac{B^2}{8\pi} = (1 + q_h - q_u)\frac{B^2}{8\pi} \equiv Q_p \frac{B^2}{8\pi}$$

from the total pressure P. Then

$$P = p + p_m(B) = p + Q_p \frac{B^2}{8\pi} , \tag{5}$$

where $p = P_k + P_T^{(0)}$. The pressure $p_m(B)$ is called the *effective* magnetic pressure. It follows that in the presence of developed MHD turbulence it is possible to reverse the sign of the effective magnetic pressure $p_m(B) = Q_p B^2/8\pi$ if $Q_p < 0$ (i.e., $1 + q_h < q_u$). Note that both the hydrodynamic pulsations and the magnetic fluctuations contribute to the mean effective magnetic pressure. However, the gain in the turbulent magnetic pressure P_h is not as large as the reduction of the turbulent hydrodynamic pressure P_u by the mean magnetic field **B**, due to the different coefficients of W_m and W_k in the equation of state (1). Therefore a negative contribution of the MHD turbulence to the mean magnetic force can arise.

We consider the case when $p \gg B^2/8\pi$, so the total pressure P is always positive. Only the effective magnetic pressure $p_m(B)$ can be negative (when $Q_p < 0$) while the pressure P_B as well as the values p, P_u, P_h, and P_T are positive.

Note that $Q_p = 1 - a_p/3$. When mean magnetic field **B** is superimposed on the isotropic turbulence, the isotropy will break down. Nevertheless Eq. (5) will remain valid; only the relationship between Q_p and a_p will change. In the next section we shall obtain the expressions for Q_p and other turbulent coefficients. Note that we use the conservation law for the total turbulent energy only to demonstrate the principle of the effect, but we shall not employ this law to develop the theory of this effect (see Sec. III and Appendix B).

III. THE RNG METHOD AND TURBULENT TRANSPORT COEFFICIENTS

In this section we describe the properties of the developed MHD turbulence located within the range $l_d < l < l_0$. Here l_0 is the maximal scale of the turbulent motions. In the very small scales $l < l_d$ the molecular dissipation is important. The RNG method is employed here for investigation of the MHD turbulence. Numerous works on turbulence are confined to a study of the large-scale properties of flows by averaging the equations over the ensemble of the turbulent pulsations (see, e.g., 1,23,24]). This averaging is carried out over the pulsations of all scales of turbulence. Such approaches cannot allow one to investigate the turbulent transport.

On the other hand, the averaging in the RNG method is performed over the pulsations of scales from l_d to l_* within the inertial range of turbulence $l_d < l_* < l_0$. Therefore turbulent viscosity ν, turbulent magnetic diffusion η, and turbulent magnetic coefficients Q_p, Q_s depend on the scale of the averaging l_*. The next stage of the RNG method comprises a step-by-step increase of the scale of the averaging. This procedure allows the derivation of equations for the turbulent transport coefficients. For the RNG method an equation invariant under the renormalization of the turbulent transport coefficients must be determined.

Perform the first step of the RNG procedure, i.e., we average MHD equations over the pulsations of the scales from l_d to l_*. The averaged equations for velocity **v** and magnetic field **H** are given by

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla\left[p + Q_p\frac{H^2}{8\pi}\right] + \frac{Q_s}{4\pi}(\mathbf{H}\cdot\nabla)\mathbf{H} + \rho\nu\Delta\mathbf{v} + \rho\mathbf{f} , \tag{6}$$

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{H} - \eta\nabla\times\mathbf{H} + c\mathbf{E}) , \tag{7}$$

where $\nabla\cdot\mathbf{v} = 0$, $\rho\mathbf{f}$ is the external force, and **E** is the external electric field. The turbulent coefficients ν, η, Q_p, and Q_s depend on the scale of averaging l_*. If l_* tends to the dissipation scale l_d, the functions ν and η approach the molecular magnitudes ν_0, η_m, and the magnetic coefficients Q_p and Q_s tend to 1.

The form of Eq. (6) is chosen according to the recent results [8] obtained with the aid of a simple model for the high-order closure procedure. We will show that this equation is invariant under the renormalization of the turbulent transport coefficients. For simplicity nonhelical turbulence is considered. So the term $\nabla\times(\alpha\mathbf{H})$ is dropped in Eq. (7), where α is the helicity of turbulence (see, e.g., [1–4]).

After this average, the range $l > l_*$ corresponds to "mean" fields, whereas the turbulent pulsations are in the range $l < l_*$. Note that usually in a theory of turbulence the space-time scales of mean fields are larger than the main scales of the turbulence. However, in real conditions such separation of the scales is usually done by convention. For example, such requirements are violated in many experiments with turbulence. The influence of the fluctuations on the "mean" fields is described by the turbulent coefficients ν, η, Q_p, and Q_s.

Let us change the scale of the averaging on a small value $|\Delta\mathbf{k}| \ll k_*$, where the wave number $k_* = l_*^{-1}$. After that we carry reayeraged Eqs. (6) and (7) over the turbulent pulsations. Now in the region $k < k_* - |\Delta\mathbf{k}|$ the fields **V** and **B** are mean ones, while the region $k > k_* - |\Delta\mathbf{k}|$ corresponds to the turbulent fields (see Fig. 1). Here **k** is the wave vector. Since Eqs. (6) and (7) have been already averaged over the pulsations of the scales that are smaller than l_*, it is sufficient to average these equations over pulsations of the velocity **u** and the mag-

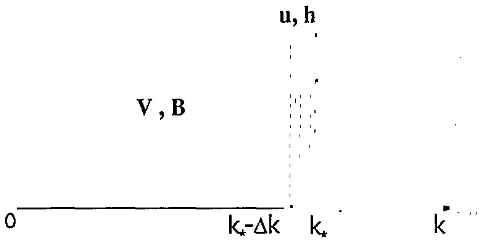

FIG. 1. The procedure of scales elimination and reaveraging.

netic field $\mathbf{h}$ located in the small region $k_* - |\Delta \mathbf{k}| < |\mathbf{k}| < k_*$. Here $\mathbf{v}=\mathbf{V}+\mathbf{u}$, $\mathbf{H}=\mathbf{B}+\mathbf{h}$, $\mathbf{V}=\langle \mathbf{v} \rangle$, $\mathbf{B}=\langle \mathbf{H} \rangle$, and the angle brackets denote averaging over the ensemble of turbulent pulsations in the domain $k_* - |\Delta \mathbf{k}| < |\mathbf{k}| < k_*$. Then the equations for the "mean" fields $\mathbf{V}$ and $\mathbf{B}$ are given by

$$\rho \left[\frac{\partial V_j}{\partial t} + (\mathbf{V}\cdot\nabla) V_j \right] + \frac{\partial}{\partial x_j} \left[p + Q_p \frac{B^2}{8\pi} \right] - \frac{Q_s}{4\pi} (\mathbf{B}\cdot\nabla) B_j - \rho \nu \Delta V_j - \rho f_j = \frac{\partial}{\partial x_j} \delta\sigma_{ij} , \quad (8)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{V}\times\mathbf{B} - \eta \nabla\times\mathbf{B} + c\mathbf{E}) = \nabla \times \langle \mathbf{u}\times\mathbf{h} \rangle , \quad (9)$$

where $\nabla\cdot\mathbf{V}=0$, the generalized Maxwell-stress tensor (including the Reynolds turbulent-stress tensor) is

$$\delta\sigma_{ij} = -Q_p \frac{\langle h^2 \rangle}{8\pi} \delta_{ij} + \frac{Q_s}{4\pi} \langle h_i h_j \rangle - \rho \langle u_i u_j \rangle , \quad (10)$$

δ_{ij} is the Kronecker delta. The equations for "mean" fields contain the second moments for the turbulent fields. To obtain a closed system of equations it is important to find the dependence of the second moments $\langle h_i h_j \rangle$, $\langle u_i u_j \rangle$, and $\langle h_i u_j \rangle$ on the "mean" fields. For this purpose we shall perform the following procedure.

(1) We shall derive equations for the turbulent fields $\mathbf{u}=\mathbf{v}-\mathbf{V}$ and $\mathbf{h}=\mathbf{H}-\mathbf{B}$. Here we shall change to a frame moving with a local velocity of the "mean" flows $\mathbf{V}$.

(2) We shall introduce a background MHD turbulence. It is the turbulence without the mean fields ($\mathbf{V}=0$ and $\mathbf{B}=0$). For simplicity the background turbulence is assumed to be homogeneous and isotropic. The solutions $\mathbf{u}^{(0)}$ and $\mathbf{h}^{(0)}$ correspond to the background MHD turbulence. The goal of the present paper is the investigation of a shift from the background turbulence level due to the presence of the mean fields.

(3) We shall derive equations for the fields $\mathbf{u}^{(1)}=\mathbf{u}-\mathbf{u}^{(0)}$ and $\mathbf{h}^{(1)}=\mathbf{h}-\mathbf{h}^{(0)}$. These equations describe the shift from the background turbulence level due to the presence of the mean fields.

(4) We shall solve the integro-differential equations for the fields $\mathbf{u}^{(1)}$ and $\mathbf{h}^{(1)}$ by means of a method of iterations. We consider here the effects that are quadratic in terms of the mean magnetic field $\mathbf{B}$ and are linear in the spatial derivatives of the mean fields $\mathbf{V}$ and $\mathbf{B}$. Note that the small parameter in the RNG procedure is $\varepsilon = W_B / W_k$, where $W_B = B^2/8\pi$ is the energy density of the mean magnetic field, W_k is the energy density of the hydrodynamic motions of the background turbulence.

(5) We shall calculate the second moments for the turbulent fields in order to find the generalized Maxwell-stress tensor and the effective electric field:

$$\delta\sigma_{ij} = \delta\sigma_{ij}^{(0)} - \frac{Q_p}{8\pi} (2\langle h_p^{(1)} h_p^{(0)} \rangle + \langle h_p^{(1)} h_p^{(1)} \rangle) \delta_{ij} + \frac{Q_s}{4\pi} (\langle h_i^{(1)} h_j^{(0)} \rangle + \langle h_i^{(0)} h_j^{(1)} \rangle + \langle h_i^{(1)} h_j^{(1)} \rangle) - \rho (\langle u_i^{(1)} u_j^{(0)} \rangle + \langle u_i^{(0)} u_j^{(1)} \rangle + \langle u_i^{(1)} u_j^{(1)} \rangle) , \quad (11)$$

$$\delta\mathbf{E} \equiv \langle \mathbf{u}\times\mathbf{h} \rangle = \langle \mathbf{u}^{(1)} \times \mathbf{h}^{(1)} \rangle + \langle \mathbf{u}^{(0)} \times \mathbf{h}^{(1)} \rangle + \langle \mathbf{u}^{(1)} \times \mathbf{h}^{(1)} \rangle , \quad (12)$$

where $\delta\sigma_{ij}^{(0)}$ is the Maxwell-stress tensor for the background turbulence. Note that for the nonhelical turbulence $\langle \mathbf{u}^{(0)} \times \mathbf{h}^{(0)} \rangle = \mathbf{0}$.

Substitution of (11) and (12) into Eqs. (8) and (9) yields the equations for the mean fields. The described procedure enables us to derive equations for the transport coefficients: turbulent viscosity ν, turbulent magnetic diffusion η, and turbulent magnetic coefficients Q_p, Q_s. The details of the calculations are presented in Appendix B. The result is given by

$$\frac{d\nu}{dk} = -\frac{7}{60\nu k^2} \left[W_0(k) + \frac{5Q_s}{14\pi\rho} P_m M_0(k) \right] , \quad (13)$$

$$\frac{d\eta}{dk} = -\frac{P_m}{3\nu k^2 (1+P_m)} W_0(k) , \quad (14)$$

$$\frac{dQ_s}{dk} = Q_s \frac{P_m^2 (3+P_m)}{15\nu^2 k^2 (1+P_m)^2} \times \left[W_0(k) - \frac{Q_s}{4\pi\rho} \left[\frac{1+3P_m}{3+P_m} \right] M_0(k) \right] , \quad (15)$$

$$\frac{dQ_p}{dk} = 4\frac{dQ_s}{dk} - Q_p \frac{P_m}{3\nu^2 k^2 (1+P_m)} \times \left[W_0(k) - \frac{Q_s}{2\pi\rho} \frac{M_0(k)}{(1+P_m)} \right] . \quad (16)$$

Here $P_m(k) = \nu(k)/\eta(k)$ is the effective magnetic Prandtl number. The functions W_0 and M_0 are the hydrodynamic and magnetic energy spectra of the background turbulence:

$$W_0(k) = (\beta_0 - 1) \left[\frac{u_0^2}{k_0} \right] \left[\frac{k}{k_0} \right]^{-\beta_0} ,$$

$$M_0(k) = (\beta_m - 1) \left[\frac{h_0^2}{k_m} \right] \left[\frac{k}{k_m} \right]^{-\beta_m} ,$$

where h_0 is the characteristic value of the turbulent magnetic field and $k_m = l_m^{-1} \sim \sqrt{R_m} k_0$ is determined by the main scale of the magnetic fluctuations l_m and $k_0 = l_0^{-1}$ (see, e.g., [6]). For example, for the Kolmogorov spectrum of hydrodynamic pulsations $\beta_0 = 5/3$, while the Kraichnan spectrum of the magnetic fluctuations corresponds to $\beta_m = 3/2$ (see, e.g., [1,4]). Equations (13)–(16) satisfy the boundary conditions

$$Q_s(k = k_d) = Q_p(k = k_d) = 1 \ , \ \nu(k = k_d) = \nu_0 \ ,$$
$$\eta(k = k_d) = \eta_m \ ,$$

where $k_d = l_d^{-1}$.

Let us make a comment. For the sake of simplicity the background turbulence is assumed to be homogeneous and isotropic. The background turbulence, by definition, is the turbulence without the mean fields. Application of the mean magnetic field **B** upsets the isotropy of the turbulence. We consider the case $\varepsilon \equiv W_B / W_k \ll 1$. Therefore the anisotropy of the turbulence is very weak ($\sim \varepsilon$). This means that the eddy viscosity ν and η can be regarded as scalars if we ignore small corrections $\sim O(\varepsilon)$. As to the turbulent magnetic coefficients Q_p and Q_s, the contribution of the turbulence to the mean magnetic force is at least of the order of $B^2/8\pi$ (i.e., of the same order of magnitude of the mean magnetic force). Therefore, by contrast to ν and η, the mean magnetic force is strongly modified in spite of small parameter ε. We also for simplicity consider a nonhelical turbulence: $\langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle = 0$ and $\langle \mathbf{h} \cdot (\nabla \times \mathbf{h}) \rangle = 0$. In this case the mean magnetic field **B** is not generated and assumed to be given. It is done to reveal the pure effect of the reduction in the elasticity of the mean magnetic field by turbulence and to make the calculations more transparent for the reader.

Note that a state in which

$$W_0(k) = \frac{Q_s}{4\pi\rho} \left[\frac{1+3P_m}{3+P_m} \right] M_0(k) \tag{17}$$

is special [see Eq. (15)]. In this case

$$Q_s(k) = \text{const} \ ,$$

$$Q_p(k) = \exp\left[-\int_{k_d}^{k} \frac{W_0(k) P_m (5+3P_m)(P_m - 1)}{3(\nu k)^2 (1+P_m)^2 (1+3P_m)} dk \right] > 0 \ .$$

This means that the effective magnetic pressure in this case is positive. Equation (17) can correspond to the effective energy equipartition. It is different from the regular equipartition state:

$$W_0(k) = \frac{M_0(k)}{4\pi\rho} \tag{18}$$

(see [1–4]). Comparison of Eqs. (17) and (18) shows that the level of the magnetic fluctuations can exceed that obtained from the usual equipartition assumption (18) if $Q_s < 1$ and $P_m \le 1$.

The system of Eqs. (13)–(16) is nonlinear. In general, it can be studied numerically for a given spectrum of the background turbulence. This system can be simplified if $W_0 \gg M_0$. It corresponds to a weak level of the magnetic fluctuations in the background turbulence. Note that in this case there is a generation of the magnetic fluctuations by the "tangling" of the large-scale mean magnetic field with the hydrodynamic pulsations (see [1–4]). The system of Eqs. (13)–(16) for this model of the background turbulence is reduced to

$$\frac{d\nu}{d\kappa} = \frac{7}{20\nu} \ , \tag{19}$$

$$\frac{d\eta}{d\kappa} = \frac{P_m}{\nu(1+P_m)} \ , \tag{20}$$

$$\frac{dQ_s}{d\kappa} = -Q_s \frac{P_m^2 (3+P_m)}{5\nu^2 (1+P_m)^2} \ , \tag{21}$$

$$\frac{dQ_p}{d\kappa} = 4\frac{dQ_s}{d\kappa} - Q_p \frac{P_m}{\nu^2 (1+P_m)} \ , \tag{22}$$

where

$$\kappa = -\int \frac{W_0}{3k^2} dk = \frac{u_0^2}{3k_0^2} \left[\frac{k}{k_0} \right]^{-\beta_0 - 1} .$$

The equation for the magnetic Prandtl number is derived from (19) and (20). It is given by

$$\frac{dP_m}{d\kappa} = -\frac{P_m (P_m - a_1)(P_m + a_2)}{\nu^2 (1+P_m)} \ . \tag{23}$$

Here $a_1 \sim 0.79$ and $-a_2 \sim -0.44$ are the roots of the quadratic equation $20y^2 - 7y - 7 = 0$. The dependence of the turbulent viscosity ν on κ determined from (19) is given by

$$\nu^2(\kappa) = \nu_0^2 + \tfrac{7}{10} [\kappa(k) - \kappa_d] \ , \tag{24}$$

where $\kappa_d = \kappa(k - k_d)$. It follows from Eq. (23) that there is a special case when the magnetic Prandtl number is constant in all scales of the MHD turbulence. It is for $P_m(k) \equiv P_m^{\lim} = a_1 \simeq 0.79$. This value corresponds to the fixed point of Eq. (23). A one-parametric set of solutions of Eqs. (19)–(23) for the turbulent transport coefficients is shown in Fig. 2 at $P_d = 0.1$, where $P_d = \nu_0 / \eta_m$. These turbulent coefficients depend only on the magnetic Prandtl number P_m. Analytical expressions for these turbulent coefficients are unwieldy and are presented in Appendix B [see Eqs. (B31)–(B33)].

The turbulent coefficients at $k = 1$ describe the large-scale transport coefficients due to the presence of the MHD turbulence. These coefficients determine a contribution of the MHD turbulence to the large-scale effects. It is seen in Fig. 2 that when $P_m > 0.15$ the value Q_p is *negative*.

The large-scale effects, such as the eddy viscosity ν, turbulent magnetic diffusion η, and helicity α have been considered earlier [13,18,25] by means of the RNG method. Indeed, the equations for η and α have been derived in [13], while the equations for ν for hydrodynamic turbulence have been obtained by [18]. In the case $W_0 \gg M_0$ the equations for η and ν coincide with that obtained in the present paper [compare Eq. (11.24) in [13] with Eq. (B28) in Appendix B of our paper, and Eq. (2.21) in [18] with Eq. (13) in our paper; for comparison we take into account that the spectrum of the hydrodynamic pulsations $W_0 \equiv 2E(k)$, see [13], and also we express the spectrum of stirring force, see [18], in terms of the spectrum of the hydrodynamic pulsations W_0 of the back-

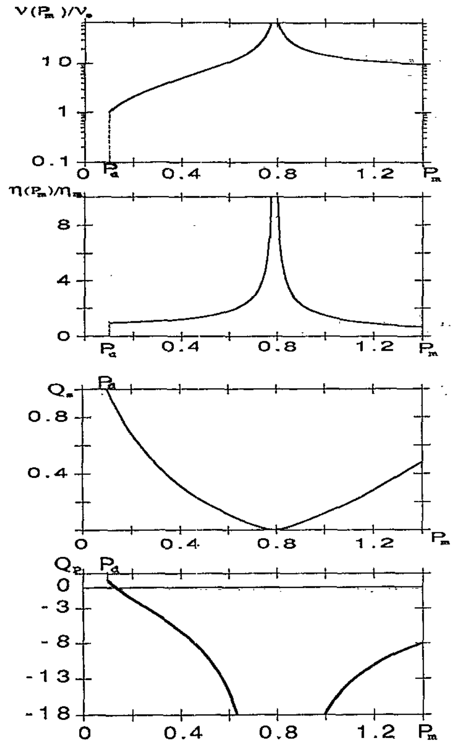

FIG. 2. The dependence of the turbulent viscosity ν, the turbulent magnetic diffusion η, and the turbulent magnetic coefficients Q_p and Q_s on the magnetic Prandtl number P_m at $P_d=0.1$.

ground turbulence]. Note that the correlation function $\langle u_m^{(0)} u_n^{(0)} \rangle$ for the background turbulence is chosen in [25] to be homogeneous and isotropic in the laboratory frame. Actually, by definition the background turbulence is the turbulence without the mean flows. Therefore the background turbulence can be homogeneous and isotropic only in the frame moving with a local mean flow $\mathbf{V}$, while the correlation function $\langle u_m^{(0)} u_n^{(0)} \rangle$ in the laboratory frame is anisotropic [13]. Therefore Eqs. (13) and (14) for ν and η (see also the corresponding equations in [13,18]) are different from that derived in [25].

The effects considered in [13,18,25] are linear in terms of the mean magnetic field $\mathbf{B}$. This means that the magnetic force in this approximation is not renormalized, i.e., $Q_p = Q_s = 1$. On the other hand, the nonlinear effects in terms of the mean magnetic field are studied in this section.

It follows from the results of this section that in the turbulence with mean magnetic field a universal spectrum of magnetic fluctuations exists. Let us discuss this question in more detail. These magnetic fluctuations are excited by the "tangling" of the mean magnetic field by hydrodynamic pulsations (see, e.g., [1–4]). We study the case of $\varepsilon \ll 1$ so that the energy of the mean magnetic field is much less than the energy of the hydrodynamic pulsations. This mechanism is different from that of generation of magnetic fluctuations with zero-mean magnetic field. The mechanism of excitation of the magnetic fluctuations with zero-mean magnetic field was proposed by Zeldovich (see, e.g., [6,26], and references therein): an original loop of magnetic field is stretched, twisted, and then folded. These nontrivial motions are three dimensional and result in an amplification of the magnetic fluctuations. These fluctuations correspond to that of the background turbulence with the magnetic energy M_0. Nonlinear theory of the magnetic fluctuations with zero-mean magnetic field was considered by [27,28].

Now let us obtain the spectrum of the magnetic fluctuations with large-scale mean magnetic field $\mathbf{B}$. The equation for the turbulent magnetic field $\mathbf{h}$ is given by

$$\frac{\partial \mathbf{h}}{\partial t} = (\mathbf{B}\cdot\nabla)\mathbf{u}^{(0)} + \eta \Delta \mathbf{h} \; . \tag{25}$$

In Eq. (25) we take into account the terms that are responsible for the generation of the magnetic fluctuations by the "tangling" of the mean magnetic field $\mathbf{B}$ with hydrodynamic pulsations $\mathbf{u}^{(0)}$. We consider the turbulence with uniform mean fields. The nonlinear terms in Eq. (25) are taken into account by means of the renormalized turbulent magnetic diffusion η [see Appendix B after Eq. (B6)]. Now we rewrite Eq. (25) in a Fourier representation and calculate the second moment $\langle h_m h_n \rangle$,

$$\langle h_m(\hat{k}) h_n(\hat{k}') \rangle = -(\mathbf{k}\cdot\mathbf{B})(\mathbf{k}'\cdot\mathbf{B}) G_\eta G'_\eta \times \langle u_m^{(0)}(\hat{k}) u_n^{(0)}(\hat{k}') \rangle \; , \tag{26}$$

where

$$\hat{k} = \begin{pmatrix} \mathbf{k} \\ \omega \end{pmatrix} , \quad G_\beta = [-i\omega + \beta(k) k^2]^{-1} \; ,$$

$$G'_\beta = [-i\omega' + \beta(k') k'^2]^{-1} \; ,$$

$\beta = \nu, \eta$. Recall that here $k \gg l_0^{-1}$, so that $(\mathbf{k}\cdot\mathbf{B})$ is not equal to zero. For the homogeneous and isotropic background turbulence

$$\langle u_m^{(0)}(\hat{k}) u_n^{(0)}(\hat{k}') \rangle = \frac{W_0(k) T(k,\omega)}{8\pi k^2} \left[\delta_{mn} - \frac{k_m k_n}{k^2} \right] \times \delta(\hat{k} + \hat{k}') \; , \tag{27}$$

where the frequency component of the spectrum $T(k,\omega)$ is given by [see Appendix B after Eq. (B30)]

$$T(k,\omega) = \left[\frac{\nu(k) k^2}{\pi} \right] \frac{1}{\omega^2 + \nu^2 k^4} \equiv \left[\frac{\nu(k) k^2}{\pi} \right] G_\nu G_\nu^* \; .$$

Substitute Eq. (27) into Eq. (26), integrate over ω space and over the angles in $\mathbf{k}$ space, and use Eq. (24). The equation for the trace of the resultant tensor is given by

$$M(k) = \langle h^2(k) \rangle = \frac{10}{7} (\beta_0 - 1) \frac{P_m^2}{1 + P_m} k^{-1} B^2 \; . \tag{28}$$

Here we consider the wave number $k \ll k_d$. Solution of Eq. (23) allows us to describe asymptotical behavior of the magnetic Prandtl number P_m for large Reynolds number $\mathrm{Re}(k) = \nu(k)/\nu_0 \gg 1$. For instance, for $P_d \ll 1$ we obtain [see Eq. (B31)]

$$P_m \approx a_1 - 0.68 R_m^{-1.56}(k) \; , \tag{29}$$

where $R_m(k)=\mathrm{Re}(k)P_d \gg 1$. This means that in most of the inertial range [where $R_m(k)=\eta(k)/\eta_m \gg 1$] the magnetic Prandtl number P_m tends to the ultimate value of $P_m^{\lim}\simeq 0.79$. Therefore in this case the magnetic Prandtl number P_m is independent of k.

Thus Eq. (28) describes the spectrum of the magnetic fluctuations $M(k)$ in the presence of the mean magnetic field **B**. Note that the magnetic spectrum is independent of the exponent of the spectrum of the turbulent velocity field. In that sense the k^{-1} spectrum of the magnetic fluctuations with mean magnetic field **B** is universal. This result is valid only for the case $\varepsilon \ll 1$.

The k^{-1} spectrum of the magnetic fluctuations was observed in the interplanetary magnetic field at 1 A.U. in the region of the solar wind (see [29,30]). This spectrum seems to exist in the galactic disk (see, e.g., [31], and references therein). In the first the k^{-1} spectrum of the magnetic fluctuations was obtained by means of the dimensional analysis in [32] (see also [31]). Indeed, comparison of the terms

$$|(\mathbf{B}\cdot\nabla)\mathbf{u}| \sim \eta|\Delta\mathbf{h}|$$

in Eq. (25) yields the spectrum of the magnetic fluctuations:

$$M(k) \sim \frac{B^2}{\eta^2} k^{-2} W(k) , \qquad (30)$$

where $W(k)\approx k^{-1}u^2(k)$ is the spectrum of the kinetic energy of MHD turbulence and $M(k)\approx k^{-1}h^2(k)$ is the spectrum of the magnetic energy. Now we take into account that the turbulent magnetic diffusion $\eta \sim u(k)/k$. Therefore Eq. (30) is reduced to

$$M(k) \sim k^{-1}B^2 \qquad (31)$$

(see [32,31]).

The spectrum (31) of the magnetic fluctuations in the presence of the hydrodynamic pulsations with the Kolmogorov spectrum $\propto k^{-5/3}$ was found also in [8] by means of the high-order closure problem. [Indeed, the trace of the tensor f_{ij} described by Eq. (3.9) in [8] yields the k^{-1} spectrum.] Direct three-dimensional numerical simulations [33,34] of the magnetic dynamo in hydrodynamic convection also reveal this spectrum of the magnetic fluctuation in the presence of the generated mean magnetic field.

IV. LARGE-SCALE EFFECTS AND ENERGY CONSERVATION LAW

In this section large-scale effects in the presence of the developed small-scale MHD turbulence are considered. A general diagram of the energetic processes described here is shown in Fig. 3. In the very small scales $l < l_d$ the molecular and atomic effects are important. The input of energy into the region is from an external thermal source I. The region $l_d < l < l_0$ corresponds to the MHD turbulence maintained by an external source I_T. The large-scale effects are significant for $l > L_0$. The energy of the large-scale hydrodynamic flow and magnetic field is dissipated into both the MHD turbulence and the molecular motions. The first dissipation process is described by turbulent viscosity $\nu_T=\nu(k=1)$ and turbulent magnetic diffusion $\eta_T=\eta(k=1)$, while the second is governed by the molecular viscosity ν_0 and the molecular magnetic diffusion η_m. Generation of the magnetic fluctuations in the MHD turbulence results in a decrease of both the effective magnetic pressure and magnetic tension. The influence of MHD turbulence on the large-scale magnetic force can be described by the turbulent magnetic coefficients $Q_p=Q_p(k=1)$ and $Q_s=Q_s(k=1)$. The equations for the large-scale fields (see also [10]) are given by

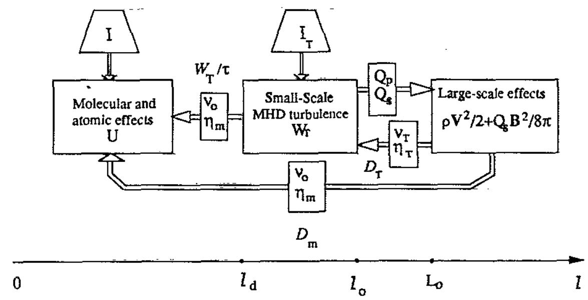

FIG. 3. A general diagram of the energetic processes.

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla\left[p+\frac{Q_p}{8\pi}B^2\right]+\frac{Q_s}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}+\mathbf{F}_d+\mathbf{F}_{\mathrm{ext}} , \qquad (32)$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B})+\eta_0\Delta\mathbf{B} , \qquad (33)$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{V})=0 , \qquad (34)$$

$$\rho T\left[\frac{\partial S}{\partial t}+(\mathbf{V}\cdot\nabla)S\right]=I+D_m+\frac{W_T}{\tau}-\nabla\cdot\mathbf{\Phi} , \qquad (35)$$

where **V** and **B** are the velocity and magnetic field, respectively, $S=\ln(p\rho^{-\gamma})/\gamma$ is the entropy, γ is the ratio of the specific heats, $\mathbf{F}_{\mathrm{ext}}$ is the external force (for example, the gravitational force $\mathbf{F}_{\mathrm{ext}}=\rho\mathbf{g}$, **g** is the free-fall acceleration), $\mathbf{F}_d$ is the dissipation force due to the molecular ν_0 and turbulent ν_T viscosities, $\eta_0=\eta_m+\eta_T$ is the total magnetic diffusion, I is the external source of the thermal energy, W_T is the density of the total energy of the MHD turbulence, τ is the characteristic time of the dissipation of the turbulent energy into the thermal one, D_m is the density of the power released due to the molecular dissipation, and $\mathbf{\Phi}$ is the total thermal flow. The turbulent diamagnetism and the α effect are not included in Eq. (33) (see, e.g., [1–4]).

Consider now the energy conservation law. We multiply Eq. (32) by the velocity **V**, Eq. (33) by $(Q_s/4\pi)\mathbf{B}$, Eq. (34) by $V^2/2$, and add them. The result is given by

$$\frac{\partial}{\partial t}\left[\frac{\rho V^2}{2}+Q_s\frac{B^2}{8\pi}\right]=-\nabla\cdot\left[\mathbf{V}\left[p+\frac{\rho V^2}{2}\right]+\frac{Q_s}{4\pi}\mathbf{B}\times(\mathbf{V}\times\mathbf{B})+\frac{Q_p-Q_s}{8\pi}\mathbf{V}B^2\right]$$
$$+(\mathbf{F}_{\rm ext}\cdot\mathbf{V})-D_m-D_T+\left[p+\frac{Q_p-Q_s}{8\pi}B^2\right]\nabla\cdot\mathbf{V}\ , \qquad (36)$$

where D_T is the density of the power released due to the turbulent viscosity and the turbulent magnetic diffusion. Equation (36) is the conservation law of the energy of the large-scale flow and magnetic field $\rho V^2/2+Q_sB^2/8\pi$.

On the other hand, the conservation law of the total energy after taking into account the MHD turbulence has the following form:

$$\frac{\partial}{\partial t}\left[\frac{\rho V^2}{2}+Q_S\frac{B^2}{8\pi}+\rho\epsilon\right]=-\nabla\cdot\mathbf{q}+(\mathbf{F}_{\rm ext}\cdot\mathbf{V})+I+I_T\ , \qquad (37)$$

where $\epsilon=U+W_T/\rho$ is the total internal energy. The total energy flux $\mathbf{q}$ is given by

$$\mathbf{q}=\rho\mathbf{V}\left[\frac{V^2}{2}+\epsilon+\frac{p}{\rho}\right]+\frac{Q_s}{4\pi}\mathbf{B}\times(\mathbf{V}\times\mathbf{B})+\mathbf{V}\frac{Q_p-Q_s}{8\pi}B^2+\Phi\ .$$

An expression for the internal energy can be obtained from the first principle of thermodynamics,

$$dU=TdS+\frac{P_k}{\rho^2}d\rho\ . \qquad (38)$$

Using Eqs. (34), (35), and (38) we get the following energy equation:

$$\frac{\partial}{\partial t}(\rho U)=I+\frac{W_T}{\tau}+D_m-P_k\nabla\cdot\mathbf{V}\ . \qquad (39)$$

Subtracting Eqs. (36) and (39) from Eq. (37) yields the conservation law of the turbulent energy W_T,

$$\frac{\partial}{\partial t}W_T=-\nabla\cdot(W_T\mathbf{V})+D_T+I_T-\frac{W_T}{\tau}-\left[P_T^{(0)}+\frac{Q_p-Q_s}{8\pi}\mathbf{B}^2\right](\nabla\cdot\mathbf{V})\ . \qquad (40)$$

In the case of a homogeneous turbulence with uniform large-scale fields Eq. (40) is in agreement with Eq. (2), obtained from the equations for the turbulent fields in a frame moving with a local velocity of mean flow $\mathbf{V}$ (see Appendix A).

If $Q_p\neq Q_s$ the MHD turbulence produces additional work. It is converted into the energy of the large-scale flow and magnetic field even in the absence of dissipation [see last term in Eq. (36)]. The terms D_T, I_T, and W_T/τ describe the sources and dissipation of the turbulence. The large-scale processes in view of the conservation laws can be considered as an "open" system. In addition to two dissipation channels D_T and D_m there is an additional energetic channel described by the magnetic turbulent coefficients Q_p and Q_s. This channel exists without dissipation.

V. DISCUSSION

In the present paper the interaction of the large-scale uniform magnetic field with developed background MHD turbulence is considered. This interaction results in modification of the mean Ampère force by the turbulence at large magnetic Reynolds numbers. It is found that the effective mean magnetic pressure is significantly reduced due to negative contribution of the MHD turbulence to the mean magnetic force. Under certain conditions the effective magnetic pressure can change sign. This effect is nonlinear in terms of the mean magnetic field.

The equations for the turbulent transport coefficients, turbulent viscosity, turbulent magnetic diffusion, and turbulent magnetic coefficients Q_p and Q_s are derived by means of the RNG method. The turbulent magnetic coefficients determine the influence of the MHD turbulence on the mean magnetic force. It is shown that in the turbulence with mean magnetic field there is the universal k^{-1} spectrum of magnetic fluctuations; this spectrum is independent of the exponent of the spectrum of the turbulent velocity field.

The effect of the *negative* magnetic pressure $p_m=Q_pB^2/(8\pi)$ should not be confused with the lowering of magnetic pressure by turbulent *diamagnetism* (see [36,37]). Recall that the nature of turbulent diamagnetism also differs significantly from diamagnetism in classical electrodynamics. The former is a kinematic effect that removes the magnetic field from a region of intense turbulent pulsations. The total magnetic energy does not depend here explicitly on the magnetic permeability. It follows that turbulent diamagnetism, in contrast to the "classical" case, does not modify the Ampère force. When a magnetic field is taken out of a turbulent region its strength is reduced, and the magnetic pressure $P_m=B^2/(8\pi)$ decreases in this region. The sign of the magnetic pressure, however, remains positive here. In contrast to turbulent diamagnetism, the reversal of the sign of the magnetic pressure is a dynamic effect. In this case the structure of the Ampère force is explicitly altered. Note also that this effect can amplify the magnetic field in a turbulent region.

Since the effective magnetic pressure p_m may become negative, a magnetic instability can be excited [7–10]. This leads to the formation of inhomogeneities of the large-scale regular magnetic field. Taking these effects into account one can describe the initial phase of sunspot formation in the solar convective zone and explain the origin of the magnetic flux tubes in stars and spiral galaxies (see, e.g., [35,38,7–9]). The phenomenon gives rise to the energy source for short-period (units of tens of minutes) solar oscillations and it explains 11-year oscillations of the solar radius (see, e.g., [35,39,40,10,11]). Decrease of the elasticity of the large-scale magnetic field ($0<Q_s<1$) affects the fine structure of the observed solar torsional oscillations and the solar meridional motions (see, e.g., [41,12]).

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APPENDIX A: EVOLUTIONARY EQUATION FOR THE TOTAL TURBULENT ENERGY DENSITY

Let us derive Eq. (2). The velocity $\mathbf{v}(\mathbf{r},t)$ and the magnetic field $\mathbf{H}(\mathbf{r},t)$ in the turbulent medium can be represented in the form $\mathbf{v}=\mathbf{V}+\mathbf{u}$ and $\mathbf{H}=\mathbf{B}+\mathbf{h}$, where $\mathbf{V}=\langle \mathbf{v}\rangle$, $\mathbf{B}=\langle \mathbf{H}\rangle$. The pulsations of the density are assumed to be weak. The momentum equation and the induction equation for the turbulent fields $\mathbf{u}$ and $\mathbf{h}$ in a frame moving with a local velocity of the large-scale flows $\mathbf{V}$ are given by

$$\frac{\partial \mathbf{u}}{\partial t}=-(\mathbf{u}\cdot\nabla)\mathbf{V}-\frac{\nabla P_*}{\rho}-\frac{\mathbf{h}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{h})}{4\pi\rho}+\mathbf{T}+\frac{\mathbf{F}_v+\mathbf{F}_r}{\rho}, \tag{A1}$$

$$\frac{\partial \mathbf{h}}{\partial t}=\nabla\times(\mathbf{u}\times\mathbf{B}-\eta_m\nabla\times\mathbf{h})+(\mathbf{h}\cdot\nabla)\mathbf{V}-\mathbf{h}(\nabla\cdot\mathbf{V})+\mathbf{G}, \tag{A2}$$

$$\nabla\cdot\mathbf{u}=0, \tag{A3}$$

where P_* is the pulsations of the hydrodynamic pressure, $\mathbf{F}_v$ is the viscous force, $\mathbf{F}_r$ is a random external force, and $\mathbf{T}$ and $\mathbf{G}$ are terms nonlinear in the pulsations and describe the energy transport over the spectrum of MHD turbulence

$$\mathbf{T}=\langle(\mathbf{u}\cdot\nabla)\mathbf{u}\rangle-(\mathbf{u}\cdot\nabla)\mathbf{u}+\frac{\langle\mathbf{h}\times(\nabla\times\mathbf{h})\rangle-\mathbf{h}\times(\nabla\times\mathbf{h})}{4\pi\rho},$$

$$\mathbf{G}=\nabla\times(\mathbf{u}\times\mathbf{h}-\langle\mathbf{u}\times\mathbf{h}\rangle).$$

The pulsations are concentrated in small scales. So the derivatives of the large-scale fields are small in comparison with the derivatives of the turbulent fields. Now let us derive equations describing the evolution of the second moments. For this purpose we rewrite the MHD equations (A1)–(A3) in a Fourier representation and repeat twice the vector multiplication of Eq. (A1) by the wave vector $\mathbf{k}$. The result is given by

$$\frac{du_m(\mathbf{k},t)}{dt}=\frac{i(\mathbf{k}\cdot\mathbf{B})}{4\pi\rho}h_m(\mathbf{k},t)-\tilde{T}_m(\mathbf{k},t)-\nu_0k^2u_m(\mathbf{k},t), \tag{A4}$$

$$\frac{dh_m(\mathbf{k},t)}{dt}=i(\mathbf{k}\cdot\mathbf{B})u_m(\mathbf{k},t)-G_m(\mathbf{k},t)-\eta_mk^2h_m(\mathbf{k},t), \tag{A5}$$

where $\tilde{\mathbf{T}}=\mathbf{k}\times(\mathbf{k}\times\mathbf{T})/k^2$. Recall that here $k\gg l_0^{-1}$, so that $(\mathbf{k}\cdot\mathbf{B})$ is not equal to zero. Let us introduce the second moments and consider quasihomogeneous turbulence. In this case, for example, dependence of the second moment $f_{mn}(\mathbf{r},\mathbf{R},t)=\langle u_m(\mathbf{x},t)u_n(\mathbf{y},t)\rangle$ on $\mathbf{R}=(\mathbf{x}+\mathbf{y})/2$ is not as strong as on $\mathbf{r}=\mathbf{x}-\mathbf{y}$. This means that

$$f_{mn}(\mathbf{k},t)=\langle u_m(\mathbf{k},t)u_n(-\mathbf{k},t)\rangle,$$

$$h_{mn}(\mathbf{k},t)=\langle h_m(\mathbf{k},t)h_n(-\mathbf{k},t)\rangle.$$

Let us multiply Eqs. (A4) for $u_m(\mathbf{k},t)$ by $u_n(-\mathbf{k},t)$ and Eqs. (A4) written for $u_n(-\mathbf{k},t)$ by $u_m(\mathbf{k},t)$, add them, and average over the ensemble of turbulent pulsations. We use the same procedure for other correlation functions. It results in the equations describing the evolution of the second moments (see [8]):

$$\frac{df_{mn}}{dt}=\frac{i(\mathbf{k}\cdot\mathbf{B})\phi_{mn}}{4\pi\rho}+A_{mn}-2\nu_0k^2f_{mn}+F_{mn}, \tag{A6}$$

$$\frac{dh_{mn}}{dt}=-i(\mathbf{k}\cdot\mathbf{B})\phi_{mn}+R_{mn}-2\eta_mk^2h_{mn}, \tag{A7}$$

$$\frac{d\chi_{mn}}{dt}=i(\mathbf{k}\cdot\mathbf{B})\left[f_{mn}-\frac{h_{mn}}{4\pi\rho}\right]+C_{mn}-(\nu_0+\eta_m)k^2\chi_{mn}. \tag{A8}$$

Here

$$\phi_{mn}(\mathbf{k},t)=\chi_{mn}(\mathbf{k},t)-\chi_{nm}(-\mathbf{k},t),$$

$$\chi_{mn}(\mathbf{k},t)=\langle h_m(\mathbf{k},t)u_n(-\mathbf{k},t)\rangle,$$

$$F_{mn}(\mathbf{k},t)=\langle\tilde{F}_m(\mathbf{k},t)u_n(-\mathbf{k},t)\rangle+\langle u_m(\mathbf{k},t)\tilde{F}_n(-\mathbf{k},t)\rangle,$$

$$\tilde{\mathbf{F}}(\mathbf{k},t)=\frac{\mathbf{k}\times[\mathbf{k}\times\mathbf{F}_r(\mathbf{k},t)]}{k^2\rho}.$$

The third moment is given by

$$A_{mn}(\mathbf{k},t)=\langle\tilde{T}_m(\mathbf{k},t)u_n(-\mathbf{k},t)\rangle+\langle u_m(\mathbf{k},t)\tilde{T}_n(-\mathbf{k},t)\rangle.$$

The expressions for the remaining moments R_{mn} and C_{mn} are similar.

We consider MHD turbulence with a uniform large-scale magnetic field. Let us multiply Eqs. (A6) and (A7) by $\rho/2$ and $(8\pi)^{-1}$, respectively, and add them. The equation for the trace of the resultant tensor is given by

$$\frac{d}{dt}\left[\frac{\rho\langle u^2\rangle}{2}+\frac{\langle h^2\rangle}{8\pi}\right]_k=L(k)+I_T(k)-D(k), \tag{A9}$$

where $I_T(k)=\rho F_{mm}/2$ is the spectral density of the power of the external source maintaining the turbulence, and

$$D(k)=2k^2\left[\nu_0\frac{\rho\langle u^2\rangle}{2}+\eta_m\frac{\langle h^2\rangle}{8\pi}\right]_k ,$$

$$L(k)=\frac{\rho A_{mm}}{2}+\frac{R_{mm}}{8\pi} .$$

Note that the terms containing the large-scale regular magnetic field **B** are eliminated from Eq. (A9). This reflects the fact that the uniform large-scale magnetic field performs no work on the turbulence. It can only redistribute the energy between hydrodynamic pulsations and magnetic fluctuations.

We change over to coordinate space in Eq. (A9). Calculations similar to those described in [42] yield

$$\int L(k)d^3k=-\nabla\cdot\langle\Phi_v\rangle=0 ,$$

where $\Phi_v=\mathbf{u}\rho u^2/2+\mathbf{h}\times(\mathbf{u}\times\mathbf{h})/4\pi$ is the energy flux of magnetic fields and flows in the homogeneous turbulence. In coordinate space Eq. (A9) is reduced to

$$\frac{\partial W_T}{\partial t}=I_T-\frac{W_T}{\tau} , \tag{A10}$$

where the total energy density is $W_T=W_k+W_m$ and τ is the correlation time of the turbulence in the scale l_0. The second term in (A10), W_T/τ, determines the dissipation of the turbulent energy. This form of the dissipation results from the condition that the energy flows be constant over the spectrum. Note that Eq. (A10) is in agreement with Eq. (40).

APPENDIX B: EQUATIONS FOR THE TURBULENT TRANSPORT COEFFICIENTS

Let us derive equations for the turbulent fields. Subtract Eq. (8) from Eq. (6) and Eq. (9) from Eq. (7), respectively, change to a frame moving with a local velocity of the "mean" flows **V**, and transform to the k and Ω spaces. The result is given by

$$u_j(\hat{k})+\frac{i}{2}P_{jmn}G_\nu\left[N_{mn}(u;u)-\frac{Q_s}{4\pi\rho}N_{mn}(h;h)\right]$$
$$=G_\nu f_j-iG_\nu\left[\bar{P}_{jmn}L_{mn}(V;u)-\frac{Q_s}{4\pi\rho}P_{jmn}L_{mn}(B;h)\right] , \tag{B1}$$

$$h_j(\hat{k})-iS_{jmn}G_\eta N_{mn}(u;h)$$
$$=G_\eta\varepsilon_j+iG_\eta[\bar{S}_{jmn}L_{mn}(V;h)+S_{jmn}L_{mn}(u;B)] , \tag{B2}$$

where

$$L_{mn}(a;b)=\int a_m(\hat{q})b_n(\hat{k}-\hat{q})d\hat{q} ,$$

$$N_{mn}(a;b)=L_{mn}(a;b)-\langle L_{mn}(a;b)\rangle ,$$

$$P_{jmn}=\Delta_{jm}k_n+\Delta_{jn}k_m , \quad \bar{P}_{jmn}=\Delta_{jm}k_n ,$$

$$\Delta_{jm}=\delta_{jm}-\frac{k_jk_m}{k^2} ,$$

$$S_{jmn}=\delta_{jm}k_n-\delta_{jn}k_m\equiv\Delta_{jm}k_n-\Delta_{jn}k_m ,$$

$$\bar{S}_{jmn}=\delta_{jm}k_n ,$$

$$\varepsilon_j=ic[\mathbf{k}\times\mathbf{E}(\hat{k})]_j , \quad G_\beta=(-i\omega+\beta k^2)^{-1} .$$

Here $\beta=\nu$, η, and the symbol $\hat{z}$ denotes a four-vector, i.e.,

$$\hat{k}=\begin{bmatrix}\mathbf{k}\\ \omega\end{bmatrix} , \quad \hat{q}=\begin{bmatrix}\mathbf{q}\\ \Omega\end{bmatrix} .$$

The total pressure can be excluded from the equation of motion by taking the "curl" of this equation. So we repeat twice the vector multiplication of Eqs. (6) and (8) written in k and Ω spaces by **k**.

The equations $\nabla\cdot\mathbf{u}=\nabla\cdot\mathbf{h}=0$ yield

$$\bar{P}_{jmn}L_{mn}(V;u)=\Delta_{jm}\int u_n(\hat{k}-\hat{Q})Q_nV_m(\hat{Q})d\hat{Q} ,$$

$$\bar{S}_{jmn}L_{mn}(V;h)=\delta_{jm}\int h_n(\hat{k}-\hat{Q})Q_nV_m(\hat{Q})d\hat{Q} .$$

This means that only the derivatives of the "large-scale" velocity **V** enter in Eqs. (B1) and (B2). Thus uniform "mean" flows do not affect the dynamics of the turbulence. The latter is a consequence of the Galilean invariance. On the other hand, the uniform magnetic fields **B** change properties of the MHD turbulence. This is a reason for the difference between homogeneous flows **V** and uniform magnetic fields **B**. The latter cannot be eliminated from the MHD equations.

Let us introduce a background MHD turbulence. It is the turbulence without the regular large-scale fields (**V**=**0** and **B**=**0**). The solutions $\mathbf{u}^{(0)}$ and $\mathbf{h}^{(0)}$ correspond to the background MHD turbulence. The background MHD turbulence is determined by the equations

$$u_j^{(0)}(\hat{k})+\frac{i}{2}P_{jmn}G_\nu\left[N_{mn}(u^{(0)};u^{(0)})-\frac{Q_s}{4\pi\rho}N_{mn}(h^{(0)};h^{(0)})\right]=G_\nu f_j , \tag{B3}$$

$$h_j^{(0)}(\hat{k})-iS_{jmn}G_\eta N_{mn}(u^{(0)};h^{(0)})=G_\eta\epsilon_j \tag{B4}$$

[see Eqs. (B1) and (B2)].

The momentum equation (B1) is different from that usually used in the RNG method (see, e.g., [18,19]). The "mean" fields in Eqs. (B1) and (B2) are explicitly separated from the turbulent fields. In addition the equations are written in a frame moving with a local "mean" flow. The correlation functions of the background turbulence have the most simple form (for example, they are homogeneous and isotropic) only in this frame, while these correlations in the laboratory frame are anisotropic [13].

The equations for the fields $\mathbf{u}^{(1)}=\mathbf{u}-\mathbf{u}^{(0)}$ and $\mathbf{h}^{(1)}=\mathbf{h}-\mathbf{h}^{(0)}$ are obtained from (B1) and (B2):

$$u_j^{(1)}(\hat{k})+\frac{i}{2}P_{jmn}G_v[N_{mn}(u^{(0)};u^{(1)})+N_{mn}(u^{(1)};u^{(0)})+N_{mn}(u^{(1)};u^{(1)})]$$
$$+iG_v\left[\bar{P}_{jmn}\cdot L_{mn}(V;u^{(0)})-\frac{Q_s}{4\pi\rho}P_{jmn}L_{mn}(B;h^{(0)})\right]=iG_v\left[\bar{P}_{jmn}L_{mn}(V;u^{(1)})-\frac{Q_s}{4\pi\rho}P_{jmn}L_{mn}(B;h^{(1)})\right], \quad \text{(B5)}$$
$$h_j^{(1)}(\hat{k})-iS_{jmn}G_\eta[N_{mn}(u^{(0)};h^{(1)})+N_{mn}(u^{(1)};h^{(0)})+N_{mn}(u^{(1)};h^{(1)})]$$
$$-iG_\eta[\bar{S}_{jmn}L_{mn}(V;h^{(0)})+S_{jmn}L_{mn}(u^{(0)};B)]=iG_v[\bar{S}_{jmn}L_{mn}(V;h^{(1)})+S_{jmn}L_{mn}(u^{(1)};B)] . \quad \text{(B6)}$$

These equations describe the shift from the background turbulence level due to the presence of the mean fields.

Let us estimate the nonlinear terms in Eqs. (B1)–(B6) determined by the functionals $N_{mn}(a;b)$. The domain of integration in $\mathbf{q}$ space is very small. Indeed, extreme points of the vectors $\mathbf{q}$, $\mathbf{k}-\mathbf{q}$, and $\mathbf{k}$ fall within a thin spherical shell of the thickness $|\Delta\mathbf{k}|$ [see Fig. 4(a)] due to the turbulent pulsations. Thus, for example,

$$|P_{jmn}N_{mn}(u^{(0)};u^{(1)})|\simeq\left|k_m\int u_m^{(0)}(\hat{q})u_j^{(1)}(\hat{k}-\hat{q})d\hat{q}\right|$$
$$\le k_*U_*\int u_*^{(0)}u_*^{(1)}d\Omega$$
$$=\pi k_*^2(\Delta k)^2\int u_*^{(0)}u_*^{(1)}d\Omega .$$

Here $U_*=\pi k_*(\Delta k)^2$ is the volume of the domain of integration [see Fig. 4(a)], $u_*^{(l)}=u^{(l)}(k=k_*)$. The other functionals $N_{mn}(a;b)$ containing the magnetic field $\mathbf{h}$ and velocity $\mathbf{u}$ are estimated similarly. They are proportional to $(\Delta k)^2$.

Now we find the linear terms in Eqs. (B1)–(B6) related to the "mean" fields $\mathbf{V}$ and $\mathbf{B}$. An equation of the surface which determines the domain of integration for the functionals $L_{mn}(V;b)$ and $L_{mn}(B;b)$ is given by $k_*^2=(\mathbf{k}-\mathbf{q})^2$. Because $|\mathbf{k}|=k_*$, the equation of this surface reduces to $|\mathbf{q}|=2k_*\cos\theta$, where θ is the angle between $\mathbf{k}$ and $\mathbf{q}$. The volume of the domain of integration is $U_0=\pi k_*^2|\Delta\mathbf{k}|$ [see Fig. 4(b)]. So the terms $L_{mn}(V;u)$, $L_{mn}(B;h)$, $L_{mn}(V;h)$, and $L_{mn}(u;B)$ in Eqs. (B1)–(B6) are of order $|\Delta\mathbf{k}|$. This means that the functionals $N_{mn}(a;b)\sim|\Delta\mathbf{k}|^2$ can be dropped out for small $|\Delta\mathbf{k}|\ll k_*$, where $k_*>k_0$. However, it does not mean that the nonlinear terms are dropped out in all $\mathbf{k}$ space. At the first step of the RNG method the MHD equations are averaged up to the scale k_*^{-1}. Therefore the nonlinear terms contribute to the turbulent transport coefficients ν, η, Q_p, and Q_s in all scales except for only a very small region of the spectrum: $k_*-|\Delta\mathbf{k}|<|\mathbf{k}|<k_*$. The equations for the fields $u_j^{(0)}(\hat{k})$ and $h_j^{(0)}(\hat{k})$ of the background MHD turbulence are reduced to

$$u_j^{(0)}(\hat{k})=G_vf_i , \quad h_j^{(0)}(\hat{k})=G_\eta\epsilon_j .$$

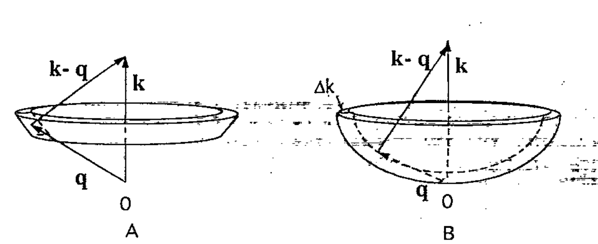

FIG. 4. Domain of integration in k space for the functionals N_{mn} (a) and L_{mn} (b).

The background MHD turbulence is assumed to be given. The problem of an origin of the effective external forces is not considered here. The goal of this paper is a study of the shift from the background turbulence level due to the presence of the regular large-scale fields. It is important for the investigation of the interaction of the mean magnetic field with the small-scale MHD turbulence.

In order to derive the equations for the turbulent coefficients ν, η, Q_p, and Q_s we have to find the dependence of the second moments $\langle h_ih_j\rangle$, $\langle u_iu_j\rangle$, and $\langle h_iu_j\rangle$ on the large-scale fields $\mathbf{V}$ and $\mathbf{B}$. Let us consider, for example, the correlation function $\langle u_iu_j\rangle$,

$$\langle u_i(\hat{x})u_j(\hat{y})\rangle=\int\langle u_i(\hat{k}_1)u_j\hat{k}_2)\rangle$$
$$\times\exp i(\hat{k}_1\hat{x}+\hat{k}_2\hat{y})d\hat{k}_1d\hat{k}_2$$
$$=\int F_{ij}(\hat{K},\hat{r})\exp(i\hat{K}\hat{R})d\hat{K} ,$$

where

$$F_{ij}(\hat{K},\hat{r})=\int\langle u_i(\hat{k}+\hat{K}/2)u_j(-\hat{k}+\hat{K}/2)\rangle\exp(i\hat{k}\hat{r})d\hat{k} ,$$
$$\hat{R}=\frac{1}{2}(\hat{x}+\hat{y}) , \quad \hat{r}=\hat{x}-\hat{y} , \quad \hat{K}=\hat{k}_1+\hat{k}_2 ,$$
$$\hat{k}=\frac{1}{2}(\hat{k}_1-\hat{k}_2) , \quad \hat{x}=\begin{bmatrix}\mathbf{x}\\-t_1\end{bmatrix} ,$$

$\hat{R}$ and $\hat{K}$ correspond to the large scales, and $\hat{r}$ and $\hat{k}$ to the small ones (see, for example, [43]). The other second moments have the same form. These correlation functions are calculated at $\hat{r}=0$ and $t_1=t_2$.

At first we have to solve the system of the equations (B5) and (B6). Let us consider here the effects that are quadratic in terms of the mean magnetic field $\mathbf{B}$ and are linear in the spatial derivatives of the mean fields $\mathbf{V}$ and $\mathbf{B}$. We use the method of iterations. The first iteration corresponds to the solution of Eqs. (B5) and (B6) when the right parts of these equations equal zero. These solutions are

$$u_j^{(I)}(\hat{k}) = -iG_\nu \left[\bar{P}_{jmn} L_{mn}(V;u^{(0)}) - \frac{Q_s}{4\pi\rho} P_{jmn} L_{mn}(B;h^{(0)}) \right], \quad (B7)$$

$$h_j^{(I)}(\hat{k}) = iG_\nu [\bar{S}_{jmn} L_{mn}(V;h^{(0)}) + S_{jmn} L_{mn}(u^{(0)};B)]. \quad (B8)$$

Note that the terms $N_{mn}(a;b) \sim (\Delta k)^2$ are dropped. These solutions in the explicit form are given by

$$u_j^{(I)}(\hat{k}_1) \simeq iG_\nu \int \left[k_j \frac{(\mathbf{k}\cdot\mathbf{V})(\mathbf{Q}\cdot\mathbf{u}^{(0)})}{k^2} - V_j(\mathbf{Q}\cdot\mathbf{u}^{(0)}) + \frac{Q_s}{4\pi\rho} \left[B_j(\mathbf{Q}\cdot\mathbf{h}^{(0)}) - k_j \frac{2(\mathbf{k}\cdot\mathbf{B})(\mathbf{Q}\cdot\mathbf{h}^{(0)})}{k^2} - \frac{G_\nu}{2}(\mathbf{k}\cdot\mathbf{B})\cdot[-i\Omega + 2\nu(\mathbf{k}\cdot\mathbf{K})]h_j^{(0)} + (\mathbf{k}_1\cdot\mathbf{B})h_j^{(0)} \right] \right] d\hat{Q}, \quad (B9)$$

$$h_j^{(I)}(\hat{k}_1) \simeq iG_\eta \int \left[V_j(\mathbf{Q}\cdot\mathbf{h}^{(0)}) - B_j(\mathbf{Q}\cdot\mathbf{u}^{(0)}) - \frac{G_\eta}{2}(\mathbf{k}\cdot\mathbf{B})\cdot[-i\Omega + 2\eta(\mathbf{k}\cdot\mathbf{K})]u_j^{(0)} + (\mathbf{k}_1\cdot\mathbf{B})u_j^{(0)} \right] d\hat{Q}, \quad (B10)$$

where $\hat{k}_1 = \hat{k} + \hat{K}/2$, the background fields $u^{(0)}$ and $h^{(0)}$ depend on $\hat{k}_1 - \hat{Q}$, and the large-scale fields $\mathbf{V}$ and $\mathbf{B}$ depend on $\hat{Q}$.

In order to obtain the second iteration we have to replace in (B7) and (B8) $u^{(0)}$ and $h^{(0)}$ with $u^{(I)}$ and $h^{(I)}$, respectively. The result is given by

$$u_j^{(II)}(\hat{k}_1) \simeq -\frac{Q_s}{4\pi\rho} G_\nu G_\eta \int [\mathbf{k}\cdot\mathbf{B}(\hat{Q})]\cdot[\mathbf{k}\cdot\mathbf{B}(\hat{Q}_1)]u_j^{(0)} \times d\hat{Q}_1 d\hat{Q}, \quad (B11)$$

$$h_j^{(II)}(\hat{k}_1) \simeq -\frac{Q_s}{4\pi\rho} G_\nu G_\eta \int [\mathbf{k}\cdot\mathbf{B}(\hat{Q})]\cdot[\mathbf{k}\cdot\mathbf{B}(\hat{Q}_1)]h_j^{(0)} \times d\hat{Q}_1 d\hat{Q}. \quad (B12)$$

Here the background fields $u^{(0)}$ and $h^{(0)}$ depend on $\hat{k}_1 - \hat{Q} - \hat{Q}_1$. The terms $\le O(K^2)$ are dropped in Eqs. (B11) and (B12). The solutions $u^{(1)}$ and $h^{(1)}$ are the sum over the first and the second iterations. The third iteration, $\sim \max(B^3;K^2B;K^2V)$, is neglected.

The second moments describing a shift of the turbulence from the background level can be obtained from Eqs. (B9)–(B12). For simplicity the background turbulence is assumed to be homogeneous and isotropic, i.e.,

$$\left\langle a_m^{(0)}\left[\hat{k} + \frac{\hat{K}}{2}\right] a_n^{(0)}\left[-\hat{k} + \frac{\hat{K}}{2}\right]\right\rangle = \frac{C_*(k,\omega)}{8\pi k^2}\left[\delta_{mn} - \frac{k_m k_n}{k^2}\right]\delta(\hat{K}), \quad (B13)$$

where $a = u$ and $C_* = W_*$, or $a = h$ and $C_* = M_*$; $\delta(y)$ is the Dirac delta function; $W_*(k,\omega)$ and $M_*(k,\omega)$ are the spectra of the hydrodynamic and magnetic pulsations of the background turbulence, respectively.

Integration in Eqs. (B9)–(B13) over the angles in $\mathbf{k}$ space results in the following expressions for the second moments:

$$\langle u_m^{(1)} u_n^{(0)} \rangle = -\frac{1}{5} V_{mn} \int W_* G_\nu dk d\omega - \frac{2}{15}\left[\frac{Q_s}{4\pi\rho}\right] b_{mn} \int W_* G_\nu G_\eta k^2 dk d\omega, \quad (B14)$$

$$\langle u_m^{(1)} u_n^{(1)} \rangle = \frac{2}{15}\left[\frac{Q_s}{4\pi\rho}\right]^2 b_{mn} \int M_* G_\nu G_\nu^* k^2 dk d\omega, \quad (B15)$$

$$\langle h_m^{(1)} h_n^{(0)} \rangle = \frac{1}{3}\frac{\partial V_m}{\partial R_n} \int M_* G_\eta dk d\omega - \frac{2}{15}\left[\frac{Q_s}{4\pi\rho}\right] b_{mn} \int M_* G_\nu G_\eta k^2 dk d\omega, \quad (B16)$$

$$\langle h_m^{(1)} h_n^{(1)} \rangle = \frac{2}{15} b_{mn} \int W_* G_\eta G_\eta^* k^2 dk d\omega, \quad (B17)$$

$$\langle u_m^{(1)} h_n^{(0)} \rangle = -\frac{Q_s}{120\pi\rho}(\partial_m B_n) \times \int M_* G_\nu (2 - G_\nu \nu k^2) dk d\omega, \quad (B18)$$

$$\langle u_m^{(0)} h_n^{(1)} \rangle = \frac{1}{30}(\partial_m B_n) \int W_* G_\eta^{*2}\cdot(\eta k^2) dk d\omega + \frac{1}{3}\frac{\partial B_n}{\partial R_m} \int W_* G_\eta^* dk d\omega, \quad (B19)$$

$$\langle u_m^{(1)} h_n^{(1)} \rangle = 0, \quad (B20)$$

where

$$(\partial_m B_n) = \frac{\partial B_n}{\partial R_m} + \frac{\partial B_m}{\partial R_n}, \quad b_{mn} = B^2\delta_{mn} - \tfrac{1}{2}B_m B_n,$$

$$G_\beta^* = (i\omega + \beta k^2)^{-1}, \quad V_{mn} = \frac{\partial V_m}{\partial R_n} + \frac{1}{6}\frac{\partial V_n}{\partial R_m}.$$

Integrations in (B14)–(B19) are over ω from $-\infty$ to ∞ and over $\mathbf{k}$ from $k_* - |\Delta\mathbf{k}|$ to k_*. The following integrals are used for the calculations of the second moments in (B14)–(B19):

$$\int (\mathbf{k}\cdot\mathbf{a})\frac{k_i}{k^2}\sin\theta d\theta d\varphi=\frac{4\pi}{3}a_i \ ,$$

$$\int \left[\delta_{ij}-\frac{k_ik_j}{k^2}\right]\sin\theta d\theta d\varphi=\frac{8\pi}{3}\delta_{ij} \ ,$$

$$\int (\mathbf{k}\cdot\mathbf{a})(\mathbf{k}\cdot\mathbf{b})\left[\delta_{ij}-\frac{k_ik_j}{k^2}\right]\sin\theta d\theta d\varphi =\frac{16\pi}{15}k^2[(\mathbf{a}\cdot\mathbf{b})\delta_{ij}-\tfrac{1}{4}(a_ib_j+a_jb_i)] \ .$$

Equations (B14)–(B19) are inserted into the expressions for the generalized Maxwell-stress tensor (11) and the effective electric field (12) that yields

$$\delta\sigma_{mn}=\rho(\partial_m V_n)\Delta\nu-\frac{B^2}{8\pi}\Delta Q_p\delta_{mn}+\frac{B_mB_n}{4\pi}\Delta Q_s+\delta\sigma_{mn}^{(0)} \ , \tag{B21}$$

$$\delta\mathbf{E}=-(\Delta\times\mathbf{B})\Delta\eta \ , \tag{B22}$$

where

$$\Delta\nu=\tfrac{7}{30}|\Delta k|\int\left[W_*G_\nu+\frac{5Q_s}{14\pi\rho}M_*G_\eta\right]d\omega \ , \tag{B23}$$

$$\Delta\eta=\tfrac{1}{3}|\Delta k|\int W_*G_\eta^*d\omega \ , \tag{B24}$$

$$\Delta Q_s=-Q_s\frac{2k^2}{15}|\Delta k|\int\left[W_*G_\eta\bar{G}_\nu-\frac{Q_s}{4\pi\rho}M_*G_\nu\bar{G}_\eta\right]d\omega \ , \tag{B25}$$

$$\Delta Q_p=4\Delta Q_s+Q_p\frac{k^2}{3}|\Delta k|\int G_\eta\left[W_*G_\eta^* -\frac{Q_s}{2\pi\rho}M_*G_\nu\right]d\omega \ , \tag{B26}$$

$\bar{G}_\nu=G_\nu+G_\eta^*/2$, $\bar{G}_\eta=G_\eta+G_\nu^*/2$. After substitution of (B21) and (B22) into Eqs. (8) and (9), it is seen that the form of these equations coincides with that of Eqs. (6) and (7). This means that these equations are invariant under the procedure of the reaveraging, i.e., invariant under the renormalization of the turbulent transport coefficients. The term $\delta\sigma_{mn}^{(0)}$ is dropped for the homogeneous background turbulence.

Now we divide Eqs. (B23)–(B26) by $\Delta k=-|\Delta k|$ and pass to the limit of small Δk. The sign "minus" arises because the procedure of the reaveraging is performed from small scales to the large ones. Note that the background turbulence is located in the region $k_0<k<k_d$, where $k_d=l_d^{-1}$ and $k_0=l_0^{-1}$. Therefore the MHD equations are not renormalized for $k<k_0$. The small values of Δk imply that $\Delta k \ll k_0$. Then the equations for the turbulent viscosity, turbulent magnetic diffusion, and turbulent magnetic coefficients are reduced to

$$\frac{d\nu}{dk}=-\frac{7}{30}\int\left[W_*G_\nu+\frac{5Q_s}{14\pi\rho}M_*G_\eta\right]d\omega \ , \tag{B27}$$

$$\frac{d\eta}{dk}=-\frac{1}{3}\int W_*G_\eta^*d\omega \ , \tag{B28}$$

$$\frac{dQ_s}{dk}=Q_s\frac{2k^2}{15}\int\left[W_*G_\eta\bar{G}_\nu-\frac{Q_s}{4\pi\rho}M_*G_\nu\bar{G}_\eta\right]d\omega \ , \tag{B29}$$

$$\frac{dQ_p}{dk}=4\frac{dQ_s}{dk}-Q_p\frac{k^2}{3}\int G_\eta\left[W_*G_\eta^*-\frac{Q_s}{2\pi\rho}M_*G_\nu\right]d\omega \ . \tag{B30}$$

The spectrum of the hydrodynamic pulsations of the background turbulence is given by

$$W_*(k,\omega)=T(k,\omega)W_0(k) \ .$$

The spatial component of the spectrum $W_0(k)$ in the inertial interval is

$$W_0(k)=(\beta_0-1)\left[\frac{u_0^2}{k_0}\right]\left[\frac{k}{k_0}\right]^{-\beta_0} .$$

The frequency component $T(k,\omega)$ is the Lorenz profile

$$T(k,\omega)=\left[\frac{\nu(k)k^2}{\pi}\right]\frac{1}{\omega^2+\nu^2k^4}\equiv\left[\frac{\nu(k)k^2}{\pi}\right]G_\nu G_\nu^* \ ,$$

$$\int_{-\infty}^{\infty}T(k,\omega)d\omega=1 \ .$$

Note that the time dependence of the correlation function $W(k,\tau)=\langle u(k,t)u(k,t+\tau)\rangle$ corresponds to a distribution: $W(k,\tau)=W_0(k)\exp[-\nu(k)k^2\tau]$. The spectrum of the magnetic fluctuations of the background turbulence is given by

$$M_*(k,\omega)=\mu(k,\omega)M_0(k) \ ,$$

where

$$\mu(k,\omega)=\left[\frac{\eta(k)k^2}{\pi}\right]\frac{1}{\omega^2+\eta^2k^4}=\left[\frac{\eta(k)k^2}{\pi}\right]G_\eta G_\eta^* \ ,$$

$$M_0(k)=(\beta_m-1)\left[\frac{h_0^2}{k_m}\right]\left[\frac{k}{k_m}\right]^{-\beta_m} ,$$

$k_m=l_m^{-1}$ is determined by the main scale of the magnetic fluctuations l_m. Integration in Eqs. (B27)–(B30) over ω space results in the equations for the turbulent coefficients (13)–(16). A use was made of integrals of the products of Green functions:

$$\int G_\alpha G_\alpha^*d\omega=\frac{\pi}{\alpha k^2} \ , \quad \int G_\alpha G_\alpha^*G_\beta^*d\omega=\frac{\pi}{\alpha k^4(\alpha+\beta)} \ ,$$

$$\int G_\alpha^2G_\alpha^*G_\beta d\omega=\frac{\pi}{\alpha k^6(\alpha+\beta)^2} \ ,$$

$$\int G_\alpha G_\alpha^*G_\beta G_\beta^*d\omega=\frac{\pi}{\alpha\beta k^6(\alpha+\beta)} \ .$$

The equations for the turbulent transport coefficients

(13)–(16) can be solved analytically in the case $W_0 \gg M_0$. The result is given by

$$\mathrm{Re}(P_m) \equiv \frac{\nu(P_m)}{\nu_0} = \frac{P_m}{P_d} \left| \frac{P_d - a_1}{P_m - a_1} \right|^{\alpha_1} \left| \frac{P_d + a_2}{P_m + a_2} \right|^{\alpha_2}, \tag{B31}$$

$$\eta(P_m) = \frac{\nu(P_m)}{P_m},$$

$$Q_s(P_m) = \left[\frac{P_m + 1}{P_d + 1} \right]^{\gamma_1} \left| \frac{P_m - a_1}{P_d - a_1} \right|^{\gamma_2} \left[\frac{P_d + a_2}{P_m + a_2} \right]^{\gamma_3}, \tag{B32}$$

$$Q_p(P_m) = -\left\{ 3 \frac{\Psi_d}{\Psi(P_m)} + 4\beta_1(a_1 + a_2) \times \int_{P_d}^{P_m} \frac{Q_s(z) \Psi^{1-\beta_1^{-1}}(z)}{(z + a_2)^2} dz - 4Q_s \right\}, \tag{B33}$$

where $P_d = \nu_0 / \eta_m \neq a_1$,

$$\Psi(P_m) = \left[\frac{P_m - a_1}{P_m + a_2} \right]^{\beta_1}, \quad \Psi_d = \Psi(P_m = P_d),$$

$$\alpha_1 = \frac{a_2(1 + a_1)}{a_1 + a_2} \sim 0.64, \quad \alpha_2 = \frac{a_1(1 - a_2)}{a_1 + a_2} \sim 0.36,$$

$$\gamma_1 = \frac{4}{(1 - a_2)(1 + a_1)} \sim 4.0,$$

$$\gamma_2 = \frac{a_1(3 + a_1)}{(1 + a_1)(a_1 + a_2)} \sim 1.36,$$

$$\gamma_3 = \frac{a_2(3 + a_2)}{(1 + a_2)(a_1 - a_2)} \sim 3.0, \quad \beta_1 = \frac{1}{a_1 + a_2} \sim 0.81.$$

Equation (B31) allows us to describe asymptotical behavior of the magnetic Prandtl number P_m for large Reynolds number $\mathrm{Re}(k) = \nu(k)/\nu_0 \gg 1$ [see Eq. (29)].

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