Magnetohydrodynamic instabilities in developed small-scale turbulence

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Modification of the magnetic force by a developed small-scale magnetohydrodynamic (MHD) turbulence can result in the sign reversal of the *effective* magnetic pressure. It is due to negative contribution of the MHD turbulence, to the large-scale magnetic force. It can significantly lower the elasticity of the large-scale magnetic field [Sov. Phys. JETP **70**, 878 (1990)]. This effect excites instabilities of the large-scale magnetic field due to the energy transfer from the turbulent pulsations to the latter. The nonturbulent stability criteria are modified due to the *effective* negative magnetic pressure. These instabilities may provide a mechanism of the large-scale magnetic ropes formation in the solar convective zone and spiral galaxies. In addition, the instabilities can excite the short-period solar oscillations.

I. INTRODUCTION

One of the problems of magnetohydrodynamics is a construction of inhomogeneities of magnetic field from an originally uniform field. The magnetic buoyancy instability is used for the explanation of formation of magnetic inhomogeneities.[1,2] However, for such instability to arise the original magnetic field must be strongly inhomogeneous along the direction of gravity so that the scale of field variation is smaller than the density height variations. This situation, however, is not typical for space plasmas.[3–5]

In this paper a new source of excitation of magnetohydrodynamic (MHD) instabilities in plasma is considered. It is a small-scale developed MHD turbulence, which results in modification of the mean magnetic force.[6,7] The phenomenon is due to the generation of magnetic fluctuations at the expense of hydrodynamic pulsations. It leads to a decrease of the elasticity of the large-scale magnetic field, so that under certain conditions, the *effective* magnetic pressure can change sign.

It results in an excitation of large-scale MHD instabilities. The instabilities cause a formation of inhomogeneities of the regular magnetic field on account of the energy transferred from the small-scale turbulent pulsations. This effect is developed even in an initially uniform magnetic field.

The onset of instabilities that are due to sign reversal of force of various types has been studied many times.[8,9] No studies, however, were made of the instabilities of a large-scale regular magnetic field in plasma with developed small-scale MHD turbulence for $\beta \gg 1$ (β is the ratio of the plasma pressure to that of the large-scale magnetic field).

Before turning to the investigation of the instabilities a qualitative discussion of the sign reversal of the *effective* magnetic pressure is presented.[6,10] For isotropic turbulence the equation of state is given by [11,12]

$$p_T = \frac{W_m}{3} + \frac{2W_k}{3}. \tag{1}$$

Here p_T is the total (hydrodynamic plus magnetic) turbulent pressure, $W_m = \langle h^2 \rangle / 8\pi$ is the energy density of the magnetic fluctuations, $W_k = \langle \rho u^2 \rangle /2$ is the energy density of the turbulent hydrodynamic motion, **u** and **h** are random pulsations of the hydrodynamic and magnetic fields, and ρ is the density of plasma. The angle brackets denote averaging over the ensemble of turbulent pulsations. Assume that the turbulence is maintained by an "inexhaustible" energy reservoir. The total energy of the turbulence is then conserved (the dissipation is compensated for by a supply of energy), i.e.,

$$W_k + W_m = \text{const.} \tag{2}$$

If, for example, a certain amount of the energy of the hydrodynamic pulsation, ΔW_m, is transferred into generated magnetic fluctuations, this process results in the following change in the total turbulent pressure [see Eqs. (1) and (2)]

$$\Delta p_T = -\frac{\Delta W_m}{3}. \tag{3}$$

It follows hence that the turbulent pressure is lowered when magnetic fluctuations are generated (when $\Delta W_m > 0$ holds).

The total turbulent pressure is decreased also by the "tangling" of the large-scale regular magnetic field by hydrodynamic pulsations.[3–5] The regular magnetic field, "entangled" with the hydrodynamic pulsations, generates supplementary small-scale magnetic fluctuations. In this case density of the magnetic energy W_m depends on W_k and W_B, where $W_B = B^2/8\pi$ is the energy density of the large-scale magnetic field **B**. For weak magnetic fields ($W_B \ll W_k$), expanding the function W_m in a series in W_B, one obtains

$$W_m = W_m^{(0)} + a_p(W_k)\frac{B^2}{8\pi} + \ldots, \tag{4}$$

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where $W_m^{(0)}$ is the energy density of the magnetic fluctuations in the absence of a large-scale magnetic field. This expression gives the change of the magnetic energy. Then the turbulent pressure reduces to

$$p_T = p_T^{(0)} - a_p \frac{B^2}{24\pi}. \quad (5)$$

The total pressure is $P = p_k + p_T + p_B$, where p_k is the usual gas-dynamic pressure of the plasma and $p_B = B^2/8\pi$ is the magnetic pressure of the large-scale field. With allowance for the expression for p_T, the total pressure is

$$P = p_k + p_T^{(0)} + Q_p \frac{B^2}{8\pi}, \quad (6)$$

where $Q_p = 1 - a_p/3$. The sign of a_p, as seen from analysis, is determined by the direction of energy transfer. It is positive when magnetic fluctuations are generated and negative when they are damped. It follows that in the presence of developed MHD turbulence it is possible to reverse the sign of the *effective* magnetic pressure

$$P_m^{\text{eff}} = Q_p \frac{B^2}{8\pi} \quad (7)$$

for $Q_p < 0$. We consider the case when $p_k \gg B^2/8\pi$. Hence the total pressure P is always positive.

The high order closure procedure[6] and modified renormalization group method[7] were employed for the investigation of the MHD turbulence at the large magnetic Reynolds number $R_m = u_0 l_0/\eta_0 \gg 1$. Here η_0 is the molecular magnetic diffusion, l_0 is the main scale of the turbulence, u_0 is the characteristic turbulent velocity. It was found that the *effective* large-scale regular magnetic force is given by

$$\mathbf{F}_m^{\text{eff}} = -\nabla\left(\frac{Q_p}{8\pi}\mathbf{B}^2\right) + \frac{Q_s}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}, \quad (8)$$

where

$$Q_p \simeq 1 - (4/15)\ln(Rm), \quad Q_s \simeq |1 - (8/45)\ln(Rm)|. \quad (9)$$

These asymptotic expressions for the magnetic coefficients Q_p and Q_s are for the case

$$W_B \ll W_k. \quad (10)$$

II. BASIC EQUATIONS AND ENERGY CONSERVATION LAW

In this section large-scale effects in the presence of a developed small-scale MHD turbulence are investigated. A general diagram of the energetic processes considered here is shown in Fig. 1. In the very small scales $l < l_d$ the molecular and atomic effects are important. The input of energy into the region is from an external thermal source I. The region $l_d < l < l_0$ corresponds to the MHD turbulence maintained by an external source I_T. The large-scale effects are significant for $l > L_0$. The energy of the large-scale hydrodynamic flow and magnetic field is dissipated into both the MHD turbulence and the molecular motions. The former dissipation process is described by the turbulent viscosity ν_T and the turbulent magnetic diffusion η_T, while

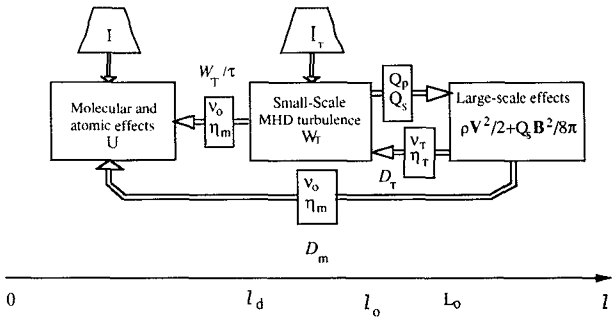

FIG. 1. A general diagram of the energetic processes.

the latter is governed by the molecular viscosity ν_0 and the molecular magnetic diffusion η_0. Generation of the magnetic fluctuations in the MHD turbulence results in a decrease of the elasticity of the large-scale magnetic field. The influence of the MHD turbulence on the large-scale magnetic force can be described by the turbulent magnetic coefficients Q_p and Q_s. The equations for the large-scale fields have the following form:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla\left(p + \frac{Q_p}{8\pi}\mathbf{B}^2\right) + \frac{Q_s}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B} + \mathbf{F}_\nu + \mathbf{F}_{\text{ext}}, \quad (11)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\Delta\mathbf{B}, \quad (12)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \quad (13)$$

$$\rho T\left(\frac{\partial S}{\partial t} + (\mathbf{v}\cdot\nabla)S\right) = I + D_m + \frac{\rho W_T}{\tau} - \nabla\cdot\mathbf{\Phi}, \quad (14)$$

where $\mathbf{v}$ and $\mathbf{B}$ are the velocity and magnetic field, respectively, $p = p_k + p_T$, $S = \ln(p\rho^{-\gamma})/\gamma$ is the entropy, γ is the ratio of the specific heats, $\mathbf{F}_{\text{ext}}$ is the external force (for example, the gravitational force $\mathbf{F}_{\text{ext}} = \rho\mathbf{g}$, $\mathbf{g}$ is the free-fall acceleration), $\mathbf{F}_d$ is the dissipation force due to the molecular and turbulent viscosities, η is the magnetic diffusion (molecular plus turbulent), I is the external source of the thermal energy, W_T is the density of the total energy of the MHD turbulence, τ is the characteristic time of the dissipation of the turbulent energy into the thermal one, D_m is the density of the power released due to the molecular dissipation, and $\mathbf{\Phi}$ is the total thermal flow. The turbulent diamagnetism and the α-effect are not included in Eq. (12)[3-5] since both α and the diamagnetic velocity are much smaller than the Alfvén velocity for the range investigated here and hence do not affect the perturbations discussed hereafter. For instance, a typical Alfvén velocity near the interface of the solar convective zone is of the order of magnitude of few hundreds m/sec while α is few tens cm/sec.

Consider now the energy conservation law. We multiply Eq. (11) by the velocity $\mathbf{v}$, Eq. (2) by $(Q_s/4\pi)\mathbf{B}$, Eq. (13) by $v^2/2$, and add them. The result is given by:

$$\frac{\partial}{\partial t}\left(\frac{\rho \mathbf{v}^2}{2}+Q_s\frac{\mathbf{B}^2}{8\pi}\right)$$
$$=-\nabla\cdot\left(\mathbf{v}\left(\frac{\rho\mathbf{v}^2}{2}+p\right)+\frac{Q_s}{4\pi}\mathbf{B}\times(\mathbf{v}\times\mathbf{B})+\frac{Q_p-Q_s}{8\pi}\mathbf{B}^2\right)$$
$$+(\mathbf{F}_{\text{ext}}\cdot\mathbf{v})-D_m-D_T+\left(p+\frac{Q_p-Q_s}{8\pi}\mathbf{B}^2\right)\nabla\cdot\mathbf{v}, \quad (15)$$

where D_T is the density of the power released due to the turbulent viscosity and the turbulent magnetic diffusion. A use was made of the following identities:

$$\rho\mathbf{v}(\mathbf{v}\cdot\nabla)\mathbf{v}+\frac{\mathbf{v}^2}{2}\nabla\cdot(\rho\mathbf{v})=\nabla\cdot\left(\mathbf{v}\frac{\rho\mathbf{v}^2}{2}\right),$$
$$\nabla\cdot(\mathbf{B}\times(\mathbf{v}\times\mathbf{B}))=\mathbf{B}\cdot\nabla\times(\mathbf{v}\times\mathbf{B})+\mathbf{v}\cdot(\nabla\times\mathbf{B}\times\mathbf{B}).$$

Equation (15) is the conservation law of the total energy of the large-scale flow and magnetic field $\rho\mathbf{v}^2/2 + Qs\mathbf{B}2/8\pi$.

The conservation law of the total energy after taking into account the MHD turbulence has the following form:

$$\frac{\partial}{\partial t}\left(\frac{\rho\mathbf{v}^2}{2}+Q_s\frac{\mathbf{B}^2}{8\pi}+\rho\epsilon\right)=-\nabla\cdot\mathbf{q}+(\mathbf{F}_{\text{ext}}\cdot\mathbf{v})+I+I_T, \quad (16)$$

where $\epsilon=U+W_T$ is the total internal energy. The total energy flux $\mathbf{q}$ is given by

$$\mathbf{q}=\rho\mathbf{v}\left(\frac{\mathbf{v}^2}{2}+\epsilon+\frac{p}{\rho}\right)+\frac{Q_s}{4\pi}\mathbf{B}\times(\mathbf{v}\times\mathbf{B})+\mathbf{v}\frac{Q_p-Q_s}{8\pi}\mathbf{B}^2+\mathbf{\Phi}. \quad (17)$$

An expression for the internal energy can be obtained from the first principle of thermodynamics

$$dU=TdS+\frac{p_k}{\rho^2}d\rho. \quad (18)$$

Using Eqs. (13), (14) and (18) we get the following energy equation:

$$\frac{\partial}{\partial t}(\rho U)=I+\frac{\rho W_T}{\tau}+D_m-p_k\nabla\cdot\mathbf{v}. \quad (19)$$

Subtracting Eqs. (15) and (19) from Eq. (16) yields the following conservation law of the turbulent energy W_T

$$\frac{\partial}{\partial t}(\rho W_T)=-\nabla\cdot(\rho W_T\mathbf{v})+D_T+I_T-\frac{\rho W_T}{\tau}$$
$$-\left(p_T+\frac{Q_p-Q_s}{8\pi}\mathbf{B}^2\right)\nabla\cdot\mathbf{v}. \quad (20)$$

If $Q_p\neq Q_s$ the MHD turbulence produces additional work. It is converted into the energy of the large-scale flow and magnetic field even in the absence of dissipation [see last term in Eq. (15)]. The terms D_T, I_T and $\rho W_T/\tau$ describe the sources and dissipation of the turbulence. Decrease of the elasticity of the large-scale magnetic field due to the generation of the magnetic fluctuations in the MHD turbulence is essential for systems with the large R_m. In this case the developed MHD turbulence can give rise to negative Q_p. Such values of Q_p (i.e., negative) result in excitations of the large-scale MHD instabilities which draw free energy from the turbulent motions and fields.[6,13] These MHD instabilities are described in the next section.

The large-scale processes in view of the conservation laws can be considered as an "open" system. Beside two dissipation channels D_T and D_m there is an additional energetic channel described by the magnetic turbulent coefficients Q_p and Q_s. This channel exists without dissipation.

III. THE LARGE-SCALE MHD INSTABILITY

For simplicity, the instabilities are investigated in this section in the absence of dissipation processes and for the case: $I+D_m+\rho W_T/\tau=\nabla\cdot\mathbf{\Phi}$.

Consider an equilibrium state without flow given by

$$\nabla\left(p_0+\frac{Q_p-Q_s}{8\pi}\mathbf{B}_0^2\right)=\mathbf{F}_{\text{ext}}^{(0)}+\frac{Q_s}{4\pi}(\nabla\times\mathbf{B}_0)\times\mathbf{B}_0. \quad (21)$$

We now linearize Eqs. (11)–(14) about the equilibrium state, and denote the perturbed quantities by subscript 1 while equilibrium quantities are denoted by subscript 0. It is convenient to express all perturbed quantities in terms of the Lagrangian displacement vector $\boldsymbol{\xi}(\mathbf{r},t)$, where $\mathbf{v}_1=\partial\boldsymbol{\xi}/\partial t$. Then the solution of Eqs. (12)–(14) for perturbed fields $\mathbf{B}_1$, ρ_1, S_1 and p_1 are given by[14,15]

$$\mathbf{B}_1=\mathbf{b}+\frac{\mathbf{B}_0}{\rho_0}\rho_1, \quad (22)$$
$$\rho_1=-\rho_0\nabla\cdot\boldsymbol{\xi}-(\boldsymbol{\xi}\cdot\nabla)\rho_0, \quad (23)$$
$$S_1=-(\boldsymbol{\xi}\cdot\nabla)S_0, \quad (24)$$
$$p_1=-(\boldsymbol{\xi}\cdot\nabla)p_0+\gamma p_0\nabla\cdot\boldsymbol{\xi}. \quad (25)$$

where

$$\mathbf{b}=(\mathbf{B}_0\cdot\nabla)\boldsymbol{\xi}-\rho_0(\boldsymbol{\xi}\cdot\nabla)\left(\frac{\mathbf{B}_0}{\rho_0}\right). \quad (26)$$

We introduce the perturbed total pressure (kinetic and magnetic):

$$p_*=p_1+\frac{Q_p}{4\pi}(\mathbf{B}_0\cdot\mathbf{B}_1). \quad (27)$$

After eliminating $\nabla\cdot\boldsymbol{\xi}$ from Eq. (25) the perturbed quantities are written in the following way:

$$\rho_1=\frac{1}{C_s^2K(Q_p)}p_*+\rho_\xi, \quad (28)$$
$$\mathbf{B}_1=\frac{\mathbf{B}_0/\rho_0}{C_s^2K(Q_p)}p_*+\mathbf{B}_\xi, \quad (29)$$

where

$$\rho_\xi=\frac{\gamma p_0(\boldsymbol{\xi}\cdot\mathbf{N}_b)-Q_p(\mathbf{B}_0\cdot\mathbf{b})/4\pi}{C_s^2K(Q_p)}, \quad (30)$$
$$\mathbf{B}_\xi=\mathbf{b}+\mathbf{B}_0\frac{\rho_\xi}{\rho_0}, \quad (31)$$

$$\mathbf{N}_b=\frac{\nabla p_0}{\gamma p_0}-\frac{\nabla \rho_0}{\rho_0},\quad K(Q_p)=1+\frac{2Q_p}{\gamma\beta},\quad \beta=\frac{8\pi p_0}{\mathbf{B}_0^2},$$

$C_s^2=\gamma p_0/\rho_0$ is the sound speed. Equations (27)–(29) are inserted into the linearized momentum equation that yields

$$\rho_0\frac{\partial^2 \xi}{\partial t^2}=-\mathbf{G}(p_*)-\mathbf{F}(\xi), \tag{32}$$

where

$$\mathbf{G}(p_*)=\nabla p_*-\frac{Q_s}{4\pi}(\mathbf{B}_0\cdot\nabla)\frac{\mathbf{B}_0/\rho_0}{C_s^2K(Q_p)}p_* -\frac{\nabla(p_0+Q_p\mathbf{B}_0^2/8\pi)}{\rho_0 C_s^2 K(Q_p)}p_*, \tag{33}$$

$$\mathbf{F}(\xi)=-\frac{Q_s}{4\pi}(\mathbf{B}_0\cdot\nabla)\mathbf{B}_\xi-\frac{Q_s}{4\pi}(\mathbf{B}_\xi\cdot\nabla)\mathbf{B}_0-\rho_\xi\mathbf{g}. \tag{34}$$

In the absence of MHD turbulence $Q_p=Q_s=1$, and without rotation and equilibrium flow the system (32)–(34) coincides with the one given by[14,15]

We search now for a solution of Eqs. (32)–(34) within the framework of the Wentzel–Kramers–Brillouin (WKB) approximation:

$$\hat{A}=e^{i\chi/\varepsilon}(\hat{A}^{(0)}+\varepsilon\hat{A}^{(1)}+\varepsilon^2\hat{A}^{(2)}+...), \tag{35}$$

where $\hat{A}=(\xi,p_*)$; $\chi=\chi(\mathbf{r}_\perp)$ and $\mathbf{r}_\perp$ is a position vector perpendicular to $\mathbf{B}_0$. The parameter $\varepsilon\ll 1$ is a measure of the fast variation of the phase across magnetic field lines. The functions χ and $\hat{A}^{(j)}$ are considered to be of order 1. It should be noted that the variations of the perturbation occur on length scales that are on one hand much smaller than that of the equilibrium state while on the other hand much larger than those turbulent fluctuations that contribute to Q_s and Q_p. Substituting expansion (35) for p_* in (33) yields

$$\mathbf{G}(p_*)=\frac{ip_*^{(0)}}{\varepsilon}\nabla\chi+ip_*^{(1)}\nabla\chi+O(\varepsilon).$$

Consequently, to lowest order in ε Eq. (32) results in

$$p_*^{(0)}=0. \tag{36}$$

In addition, Eqs. (25) and (27) yield

$$\xi^{(0)}\cdot\nabla\chi=0, \tag{37}$$

which results in the following representation of $\xi^{(0)}$:

$$\xi^{(0)}=\xi_N\hat{\mathbf{e}}_N+\xi_B\hat{\mathbf{e}}_B, \tag{38}$$

where

$$\hat{\mathbf{e}}_B=\mathbf{B}_0/|\mathbf{B}_0|,\quad \hat{\mathbf{e}}_N=\hat{\chi}\times\hat{\mathbf{e}}_B,\quad \hat{\chi}=\nabla\chi/|\nabla\chi|.$$

To zeroth order, Eq. (32) yields

$$\frac{\partial^2\xi^{(0)}}{\partial t^2}=-\mathbf{F}(\xi^{(0)})-ip_*^{(1)}\nabla\chi, \tag{39}$$

We consider now a horizontal magnetic field $\mathbf{B}_0$ which is perpendicular to the free-fall acceleration $\mathbf{g}$ (see Fig. 2). Note that the equilibrium variables ρ_0 and $\mathbf{B}_0$ depend only on r_g, where $r_g=(\mathbf{r}\cdot\mathbf{g})/g$. Two equations for ξ_N and ξ_B can now be obtained by the scalar multiplication of equation (29) by $\hat{\mathbf{e}}_N$ and $\hat{\mathbf{e}}_B$. Note that such multiplication eliminates the unknown $p_*^{(1)}\nabla\chi$ from Eq. (39). The resulting equations are

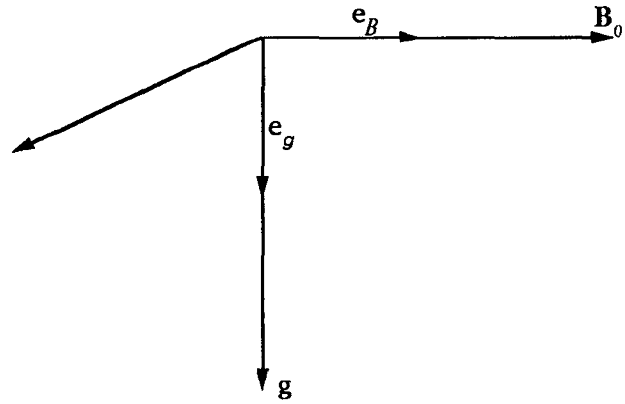

FIG. 2. The directions of the magnetic and gravitational fields.

$$\rho_0\frac{\partial^2\xi_N}{\partial t^2}=\rho_0\left(\lambda\sigma\frac{C_\mathrm{A}}{\Lambda_\rho}\right)^2\xi_N+\frac{Q_s}{4\pi}(\mathbf{B}_0\cdot\nabla)^2\xi_N -\frac{Q_p\lambda a B_0}{4\pi K(Q_p)\Lambda_\rho}(\mathbf{B}_0\cdot\nabla)\xi_B, \tag{40}$$

$$\rho_0\frac{\partial^2\xi_B}{\partial t^2}=\frac{Q_s}{4\pi K(Q_p)}(\mathbf{B}_0\cdot\nabla)^2\xi_B+\frac{Q_s\lambda a B_0}{4\pi K(Q_p)\Lambda_\rho}\times(\mathbf{B}_0\cdot\nabla)\xi_N, \tag{41}$$

where $C_\mathrm{A}=B_0/\sqrt{4\pi\rho_0}$ is the Alfvén speed, $\Lambda_\rho^{-1}=\rho'_0/\rho_0$ is the density height scale, $a=g\Lambda_\rho/C_s^2$. Here and below f' is a derivative with respect to r_g. The parameter $\lambda=(\xi\cdot\mathbf{g})/g\xi_N=(1+\lambda_0^2)^{-1/2}$ is connected to the polarization parameter of the wave,

$$\lambda_0=\frac{(\hat{\mathbf{e}}_g\cdot\nabla\chi)}{(\hat{\mathbf{e}}_\perp\cdot\nabla\chi)},\quad \hat{\mathbf{e}}_g=\frac{\mathbf{g}}{g},\quad \hat{\mathbf{e}}_\perp=\hat{\mathbf{e}}_g\times\hat{\mathbf{e}}_B.$$

The parameter σ is

$$\sigma^2=-K^{-1}(Q_p)\left(\Omega_b^2+Q_pa\left(1-\frac{\Lambda_\rho}{\Lambda_B}\right)\right),$$

where $\Lambda_B^{-1}=B'_0/B_0$ and $\Omega_b^2=-\mathbf{g}\cdot\mathbf{N}_b\Lambda_\rho^2/C_A^2$ is the nondimensional Brunt–Väisäläa frequency. As the magnetic field lined are considered here to be infinite, the following traveling solution is used:

$$\xi=\xi_0\exp[-i(\omega t-\mathbf{k}_B\cdot\mathbf{r})], \tag{42}$$

where $\mathbf{k}_B$ is parallel to $\mathbf{B}_0$. It should be noted that within the framework of our model the perturbations propagate along the average magnetic field. This is consequence of the fact that the characteristic time scales of the chaotic magnetic fluctuations are much shorter than those of the investigated perturbations. Substituting (42) into (41) yields

$$(\omega^2+\lambda^2\sigma^2-Q_s k_B^2)\xi_N = ik_B\lambda a \frac{Q_p}{K(Q_p)}\xi_B, \quad (43)$$

$$ik_B\lambda a \frac{Q_s}{K(Q_p)}\xi_N = -\left(\omega^2 - \frac{Q_s k_B^2}{K(Q_p)}\right)\xi_B, \quad (44)$$

where ω and k_B are the nondimensional frequency and wave number, ω is measured in the units of C_A/Λ_ρ and k_B is in units of Λ_ρ.

Consider first the case $k_B=0$. This corresponds to the interchange mode. In this case from Eqs. (43) and (44) one can see that $\xi_B=0$ and

$$\omega^2=-\lambda^2\sigma^2. \quad (45)$$

As can be seen in (45), the stability is determined by the sign of σ^2. In the case of weak magnetic field $K(Q_p)$ is close to unity and Ω_b^2 is much bigger than the other term in the definition of σ^2. Hence, if $\Omega_b^2<0$ the classical convective instability develops with growth rate given by[16,17]

$$\Gamma^c=\lambda g^{1/2}\left(\frac{p_0'}{\gamma p_0}-\frac{\rho_0'}{\rho_0}\right)^{1/2}, \quad (46)$$

For the general case, the criterion for the interchange instability to occur is

$$\Omega_b^2 < -Q_p a\left(1-\frac{\Lambda_\rho}{\Lambda_B}\right)\left(\frac{C_A}{\Lambda_\rho}\right)^2. \quad (47)$$

We first notice that for a uniform magnetic field, i.e., $\Lambda_B\to\infty$, and in the nonturbulent case condition (47) coincides with the criterion given by.[18] Further examination of (47) reveals that in nonturbulent media (i.e., $Q_p=1$) the magnetic field stabilizes the system if $\Lambda_B>\Lambda_\rho$. If the latter is not satisfied, instability may occur for which Parker's instability,[1,2] i.e., the case $\Omega_b=0$, is a particular case.

In turbulent media the criterion for instability is significantly changed. Now, since Q_p may become negative, an instability may occur even if $\Lambda_B>\Lambda_\rho$. The source of free energy of the new type of instability is provided by the small-scale turbulent pulsations. In contrast, the free energy in Parker's instability is drawn from the gravitational field. In this sense, it is analogous to the Rayleigh–Taylor instability. The growth rate of the MHD instability due to the developed small-scale MHD turbulence is given by

$$\Gamma=\frac{C_A}{\Lambda_\rho}\sqrt{Q_p a\left(\frac{\Lambda_\rho}{\Lambda_B}-1\right)\left(1+\frac{2Q_p}{\gamma\beta}\right)^{-1}}, \quad (48)$$

The criterion of this instability for the case of the isothermal plasma and for $\beta\gg 1$ coincides with the one given by.[13] The geometric optics approximation was not used there.

We turn now to the case where $k_B\neq 0$. The dispersion equation for this case is given by

$$\omega^2=-\frac{1}{2}\left(\lambda^2\sigma^2-\frac{K+1}{K}Q_s k_B^2\right)\pm\frac{1}{2}D^{1/2}, \quad (49)$$

where

$$D=\left(\lambda^2\sigma^2-\frac{K-1}{K}Q_s k_B^2\right)^2+\frac{4Q_sQ_p}{K^2}(k_B a\lambda)^2.$$

Generally Eq. (49) describes Alfvénic and magnetogravitational modes. In order to separate the effects due to the classical convective instability (i.e., $\Omega_b^2<0$) from the pure MHD instabilities we consider case very small Ω_b^2:

$$|\Omega_b^2|\leqslant\beta^{-1}, \quad (50)$$

where $\beta\gg 1$. For example, condition (50) is valid in the case of developed turbulent convection.[2,19]

For $\beta\gg 1$ ($K\sim 1$) Eq. (49) is given by

$$\Omega^2=(1-\kappa^2\pm\sqrt{1-2\alpha\kappa^2})/2, \quad (51)$$

where

$$\kappa^2=\frac{2Q_s}{\lambda^2\sigma^2}k_B^2\simeq\frac{2Q_s}{a\lambda^2|Q_p|}k_B^2,$$

$$\alpha=\frac{a^2|Q_p|}{\sigma^2}\simeq a, \quad \Omega^2=-\frac{\omega^2}{\lambda^2\sigma^2}.$$

In the interval $0<\kappa\leqslant\kappa_0=(2\alpha)^{-1/2}$ (i.e., $D\geqslant 0$) Ω^2 is real hence the modes are either purely growing or purely oscillatory.

For $\kappa>\kappa_0$ Ω^2 become complex and hence oscillatory modes with growing amplitudes exist. As was discussed before, the growth of the unstable modes is at the expanse of the energy of the MHD turbulence. For $\alpha<1$. We now examine the dependence of the spectrum given by (51) on the single parameter α. For $\alpha\leqslant 1$ the value $\kappa_*=\sqrt{2(1-\alpha)}$ lies in the interval $0<\kappa\leqslant\kappa_0$. The interval $0<\kappa\leqslant\kappa_0$ provides a gap for the growth rate spectrum (the imaginary part of Ω) or for the frequency spectrum. This can be seen in Figs. 3 and 4 for which the plus and minus signs in (51) was used, respectively. For $\alpha>1$ no such gap exists.

In the limit $\kappa\gg 1$ the frequency tends to $\omega_0\simeq\kappa/\sqrt{2}$ while the growth rate is close to $\Gamma_0\simeq\sqrt{\alpha}/2$. For the case $\sigma=0$ the frequency is given by $\omega_R\simeq k_B c_A\sqrt{|Q_p|}$ while the growth rate is given by $\gamma\simeq(c_A/2\Lambda_\rho)a\lambda\sqrt{|Q_p|}\ll\omega_R$ where now ω_R and γ are dimensioned variables.

We conclude this section by a comment about the general properties of the operator $H(\xi)=F(\xi)+G(p_*)$. In the absence of small-scale MHD turbulence the operator H is Hermitian, i.e., $(H\xi_1,\xi_2)=(\xi_1,H\xi_2)$ for every admissible test functions ξ_1 and ξ_2.[20] Here we use the inner product notation $(\xi_1,\xi_2)=\int\xi_1\cdot\xi_2^*\,d\mathbf{r}$. Equation (32) can be written as

$$\rho_0\frac{\partial^2\xi}{\partial t^2}+H(\xi)=0. \quad (52)$$

We search for solution of Eq. (52) in the form $\xi\sim\exp(-i\omega t)\xi(\mathbf{r})$. Then Eq. (52) is $-\rho_0\omega^2\xi+H(\xi)=0$. Since the operator H is Hermitian, its eigenvalues are real.

The modification of the large-scale Ampère force by the small-scale developed MHD turbulence results in the operator H not being Hermitian. For example, $(\mathbf{G}(p_*)\cdot\xi)$ is given by

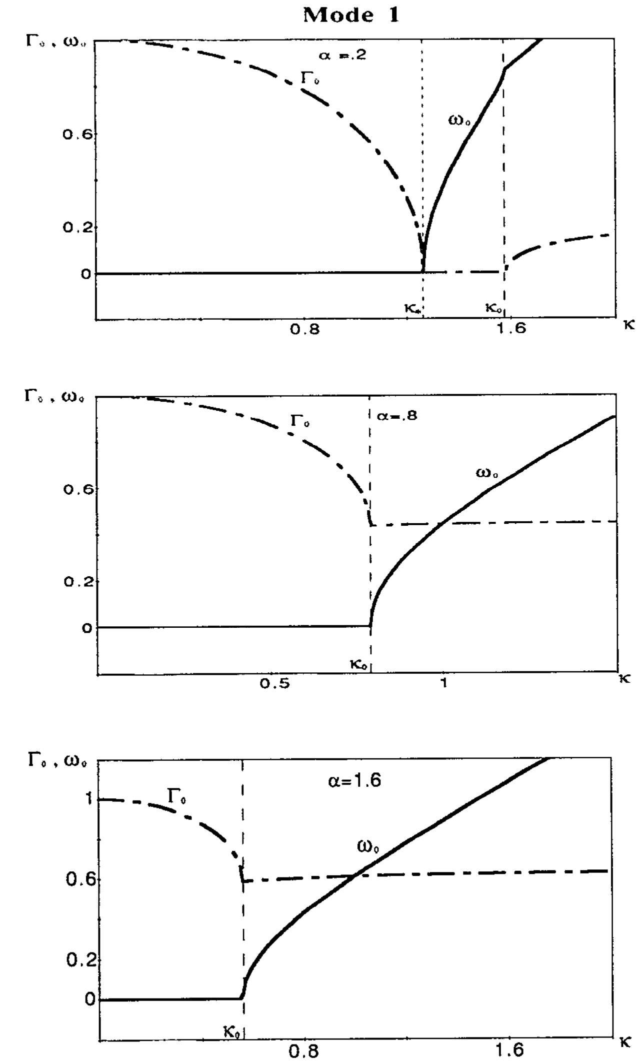

FIG. 3. The growth rate and the frequency for the mode 1 [the sign is "plus" in Eq. (51)].

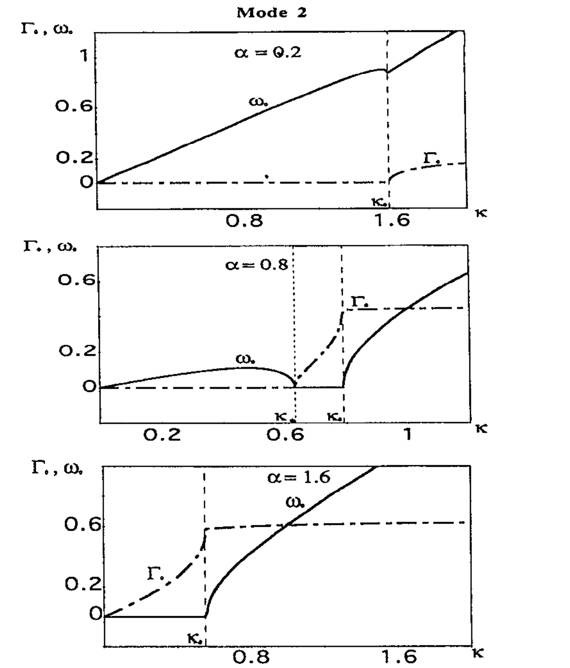

FIG. 4. The growth rate and the frequency for the mode 2 [the sign is "minus" in Eq. (51)].

$$(\mathbf{G}(p_*)\cdot\xi) = \int \mathbf{G}(p_*)\cdot\xi^*\, d^3r = \int \frac{|p_*|^2}{\gamma p_0 + (Q_p/4\pi)\mathbf{B}_0^2}\, d^3r - \frac{Q_s - Q_p}{4\pi}\int \frac{p_* \mathbf{B}_0(\mathbf{B}_0\cdot\nabla)\xi}{\gamma p_0 + Q_p \mathbf{B}_0^2/4\pi}\, d^3r. \quad (53)$$

It follows that for $Q_p \neq Q_s$ the operator $\mathbf{G}(p_*(\xi))$ is not a positive definite operator and may not be an Hermitian one. The occurrence of imaginary eigenvalues indeed indicates that F is not Hermitian and most probably that neither is H.

IV. DISCUSSIONS

The obtained results may be of interest for some applications to the solar and stellar physics. As an example we consider the problem of a source of the solar short-time oscillations and the sunspots formation. The oscillations can be excited by the MHD instability in the upper layers of the turbulent convective zone located under the visible surface of the Sun. In this region convective cells (granules) are created and annihilated, a large-scale regular magnetic field is generated, and fine-structure oscillations are excited. The growing oscillations in the interval $\kappa > \kappa_0$ can be interpreted as a source of the observed short-time solar oscillations. In contrast to the previous models which relate the source to the convective noise,[2] a source of the short-time solar oscillations proposed here is coherent. The plasma in the solar convective zone has the following parameters:[19]

(a) At depth $H \sim 2\cdot 10^7$ cm (from the Sun's surface): $R_m \sim 10^5$, $u_0 \sim 9.4\cdot 10^4$ cm/sec, $l_0 \sim 2.6\cdot 10^7$ cm, $\rho_0 \sim 4.5\cdot 10^{-7}$ g/cm^3, $B_0 \sim 10^2$ G, $\Lambda_p \sim 3.6\cdot 10^7$ cm. Here u_0 is the characteristic turbulent velocity. By Eq. (9), the coefficient $Q_p \sim -1.1$.

(b) At depth $H \sim 10^9$ cm: $R_m \sim 3\cdot 10^7$, $u_0 \sim 10^4$ cm/sec, $l_0 \sim 2.8\cdot 10^8$ cm, $\rho_0 \sim 5\cdot 10^{-4}$ g/cm^3, $B_0 \sim 10^2$ G, $\Lambda_p \sim 4.3\cdot 10^8$ cm. We then have $Q_p \sim -1.8$.

For the parameters given above, the period of oscillations ranges between several minutes ($H \simeq 200$ km) and several hours ($H \simeq 10^4$ km). This is within the range of the observed oscillations in the Sun. The frequency and amplitude of the oscillations depend on the large-scale magnetic field. The field is changed with the 11-year cycle. It explains the observed correlation of the frequency and amplitude of the solar oscillations with a phase of the 11-year cycle of activity.[2]

It should be noted that considered oscillatory modes with growing amplitudes can not be interpreted directly as the observed short-periodic solar oscillations. Conversion of the described modes into magneto-acoustic-gravitation modes inside of the solar resonance cavity[2] results in a formation of the observed short-periodic solar oscillations. One of the main results of the present paper is that we have revealed a mechanism of the energy transfer from the small-scale turbulence to the deterministic large-scale wave motions.

The MHD instability due to *effective* negative magnetic pressure in the interval $\kappa < \kappa_0$ may also provide a mechanism of the large-scale magnetic ropes formation in the solar convective zone.[6,10,13] These magnetic ropes float up from under the Sun's surface leading to the onset of the observed sunspots.

ACKNOWLEDGMENT

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[1]E. Parker, *Cosmical Magnetic Fields* (Oxford University Press, New York, 1978), and references therein.

[2]E. R. Priest, *Solar Magnetohydrodynamics* (Reidel, Dordrecht, 1982), and references therein.

[3]H. K. Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge University Press, New York, 1978), and references therein.

[4]F. Krause and K. H. Rädler, *Mean-Field Magnetohydrodynamics and Dynamo Theory* (Pergamon, Oxford, 1980), and references therein.

[5]Ya. B. Zeldovich, A. A. Ruzmaikin, and D. D. Sokoloff, *Magnetic Fields in Astrophysics* (Gordon and Breach, New York, 1983), and references therein.

[6]N. Kleeorin, I. Rogachevskii, and A. Ruzmaikin, Sov. Phys. JETP **70**, 878 (1990).

[7]N. Kleeorin and I. Rogachevskii, "Effective Ampère force in developed MHD turbulence," submitted to Phys. Rev. E.

[8]S. I. Vainshtein, Sov. Phys. JETP **47**, 1069 (1978).

[9]S. K. Zhadanov and B. A. Trubnikov, JETP Lett. **43**, 226 (1986), and references therein.

[10]N. Kleeorin, I. Rogachevskii, and A. Ruzmaikin, Sov. Astron. Lett. **15**, 274 (1989).

[11]L. D. Landau and E. M. Lifshitz, *Classical Theory of Fields* (Pergamon, Oxford, 1975).

[12]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, Oxford, 1984).

[13]N. Kleeorin and I. Rogachevskii, *Proceedings of the International Workshop on Plasma Astrophysics*, Telavi, Georgia, edited by T. D. Guyenne and J. J. Hunt (European Space Agency Publication Division, Noordwijk, 1990), p. 21.

[14]E. Hameiri, Phys. Fluids **26**, 230 (1983).

[15]E. Hameiri, P. Lawrence, and M. Mond, J. Geophys. Res. A **96**, 1513 (1991).

[16]M. Schwarzschild, *The Structure and Evolution of the Stars* (Dover, New York, 1958).

[17]W. A. Newcomb, Phys. Fluids **4**, 391 (1961).

[18]Y. A. Tserkovnikov, Sov. Phys. Dokl. **5**, 87 (1960).

[19]H. C. Spruit, Sol. Phys. **34**, 277 (1974).

[20]E. Frieman and M. Rotenberg, Rev. Mod. Phys. **32**, 898 (1960).