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New mechanism of generation of large-scale magnetic field in a sheared turbulent plasma

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Abstract

A review of recent studies on a new mechanism of generation of large-scale magnetic field in a sheared turbulent plasma is presented. This mechanism is associated with the shear-current effect which is related to the $\mathbf{W} \times \mathbf{J}$-term in the mean electromotive force. This effect causes the generation of the large-scale magnetic field even in a nonrotating and nonhelical homogeneous sheared turbulent convection whereby the α effect vanishes (where $\mathbf{W}$ is the mean vorticity due to the large-scale shear motions and $\mathbf{J}$ is the mean electric current). It is found that turbulent convection promotes the shear-current dynamo instability, i.e., the heat flux causes positive contribution to the shear-current effect. However, there is no dynamo action due to the shear-current effect for small hydrodynamic and magnetic Reynolds numbers even in a turbulent convection, if the spatial scaling for the turbulent correlation time is $\tau(k) \propto k^{-2}$, where k is the small-scale wave number. We discuss here also the nonlinear mean-field dynamo due to the shear-current effect and take into account the transport of magnetic helicity as a dynamical nonlinearity. The magnetic helicity flux strongly affects the magnetic field dynamics in the nonlinear stage of the dynamo action. When the magnetic helicity flux is not small, the saturated level of the mean magnetic field is of the order of the equipartition field determined by the turbulent kinetic energy. The obtained results are important for elucidation of origin of the large-scale magnetic fields in astrophysical and cosmic sheared turbulent plasma.

Keywords: Sheared turbulent flow; Nonlinear dynamo; Magnetic helicity transport

1. Introduction

Turbulence with a large-scale velocity shear is a universal feature in astrophysical plasmas. It has been recently recognized that in a sheared turbulent plasma with high hydrodynamic and magnetic Reynolds numbers a mean-field dynamo is possible even in a nonhelical and nonrotating homogeneous turbulence whereby a kinetic helicity and α effect vanish (see Rogachevskii and Kleeorin, 2003,2004,2007; Brandenburg, 2005; Brandenburg and Subramanian, 2005c; Rogachevskii et al., 2006a, b). The large-scale velocity shear produces anisotropy of turbulence with a nonzero background mean vorticity $\mathbf{W} = \nabla \times \mathbf{U}$, where $\mathbf{U}$ is the mean velocity. The dynamo instability in a sheared turbulent plasma is related to the $\mathbf{W} \times \mathbf{J}$-term in the mean electromotive force, and it can be written in the form $\mathcal{E}^{\delta} \propto -l_0^2 \mathbf{W} \times (\nabla \times \mathbf{B}) \propto l_0^2 (\mathbf{W} \cdot \mathbf{\Lambda}^B)\mathbf{B}$, where l_0 is the maximum scale of turbulent motions (the integral turbulent scale) and $\mathbf{\Lambda}^B = \nabla \mathbf{B}^2 / 2\mathbf{B}^2$ determines the inhomogeneity of the mean original magnetic field $\mathbf{B}$. In a sheared turbulent plasma the deformations of the original magnetic field lines are caused by the upward and downward turbulent eddies, and the inhomogeneity of the original mean magnetic field in the shear-current dynamo breaks a symmetry between the influence of upward and downward turbulent eddies on the mean magnetic field. This creates the mean electric current $\mathbf{J}$ along the mean magnetic field and produces the mean-field dynamo due to the shear-current effect.

The goal of this communication is to review recent studies on the new mechanism of generation of large-scale magnetic field due to the shear-current effect in a sheared turbulent plasma. The mean-field dynamo instability is saturated by the nonlinear effects. There are two types of the nonlinear effects caused by algebraic and dynamic

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nonlinearities. The effects of the mean magnetic field on the motion of fluid and on the cross-helicity result in quenching of the mean electromotive force which determines the algebraic nonlinearity. The dynamical nonlinearity in the mean-field dynamo is caused by the evolution of small-scale magnetic helicity, and it is of a great importance due to the conservation law for the magnetic helicity in turbulent plasma with very large magnetic Reynolds numbers (see, e.g., Kleeorin and Rogachevskii, 1999; Brandenburg and Subramanian, 2005a; Rogachevskii et al., 2006b, and references therein). The combined effect of the dynamic and algebraic nonlinearities saturates the growth of the mean magnetic field.

The shear-current effect has been studied by Rogachevskii and Kleeorin (2003,2004,2007) for large hydrodynamic and magnetic Reynolds numbers using two different approaches: the spectral τ approximation (the third-order closure procedure) and the stochastic calculus (the path integral approach in a turbulence with a finite correlation time). A justification of the τ approximation for different situations has been performed in numerical simulations and analytical studies by Blackman and Field (2002); Field and Blackman (2002); Brandenburg et al. (2004); Brandenburg and Subramanian (2005a,b); Sur et al. (2007).

2. The shear-current effect

Let us consider a nonhelical and nonrotating homogeneous turbulent plasma with a weak mean velocity shear, $\mathbf{U} = (0, Sx, 0)$ and mean vorticity $\mathbf{W} = (0, 0, S)$, where $(S\tau_0)^2 \ll 1$, $\tau_0 = l_0/\sqrt{\langle \mathbf{u}^2 \rangle}$ and $\mathbf{u}$ are the velocity fluctuations. The mean magnetic field $\mathbf{B}(t,z) = (B_x, B_y, 0)$ in the kinematic approximation is determined by the following equations

$$\frac{\partial B_x(t,z)}{\partial t} = -S l_0^2 \sigma_B B_y'' + \eta_T B_x'', \qquad (1)$$

$$\frac{\partial B_y(t,z)}{\partial t} = S B_x + \eta_T B_y'', \qquad (2)$$

where $B_i'' = \partial^2 B_i / \partial z^2$, η_T is the coefficient of turbulent magnetic diffusion and the dimensionless parameter σ_B determines the shear-current effect (Rogachevskii and Kleeorin 2003, 2004). In Eqs. (1) and (2) we have taken into account that $B_y \gg B_x$ since $(S\tau_0)^2 \ll 1$. The first term $\propto SB_x$ in Eq. (2) determines the stretching of the magnetic field B_x by the shear motions, which produces the field B_y. The interaction of the non-uniform magnetic field B_y with the background vorticity $\mathbf{W}$ produces the electric current along the field B_y. This implies generation the field component B_x due to the shear-current effect, which is determined by the first term $\propto -\sigma_B S l_0^2 B_y''$ in Eq. (1). This effect results in the large-scale dynamo instability. The solution of Eqs. (1) and (2) we seek for in the form $\propto \exp(\gamma t + \mathrm{i} K_z z)$, where the growth rate γ of the mean magnetic field due to the dynamo instability is given by $\gamma = S l_0 \sqrt{\sigma_B} K_z - \eta_T K_z^2$. The necessary condition for the dynamo instability is $\sigma_B > 0$.

The shear-current dynamo instability depends on the spatial scaling of the correlation time $\tau(k) \propto k^{-\mu}$ of the turbulent velocity field, where k is the small-scale wave number. In particular, the shear-current dynamo in a non-convective turbulence occurs when the exponent $\mu < 1$. For the Kolmogorov's type turbulent convection, the exponent $\mu = 2/3$ and $\sigma_B = (4/135)[1 + (6/7)a_*]$, where the convective contribution to the dynamo instability due to the shear-current effect depends on the parameter $a_* = 2g\tau_0 F^*/\langle \mathbf{u}^2 \rangle$. Here F^* is an imposed vertical heat flux which maintains the turbulent convection and $\mathbf{g}$ is the acceleration of gravity. For a turbulent convection with a scale-independent correlation time, the exponent $\mu = 0$ and the parameter σ_B is given by $\sigma_B = (1/15)[1 + (9/7) a_*(1 + 3\sin^2\phi)]$, where ϕ is the angle between the background mean vorticity $\mathbf{W}$ and $\mathbf{g}$. Note that the turbulent convection promotes the shear-current dynamo instability. In particular, the heat flux causes positive contribution to the shear-current effect when $2 + 3(2 - 3\mu)\sin^2\phi > 0$ (see Rogachevskii and Kleeorin, 2007).

However, for small hydrodynamic and magnetic Reynolds numbers, the turbulent correlation time is of the order of $\tau(k) \propto 1/(\nu k^2)$ or $\tau(k) \propto 1/(\eta k^2)$ depending on the magnetic Prandtl number, i.e., $\tau(k) \propto k^{-2}$, where ν is the kinematic viscosity and η is the magnetic diffusion due to the electrical conductivity of the plasma. In this case $\mu = 2$, and the parameter $\sigma_B < 0$ even in a turbulent convection. This implies that for small hydrodynamic and magnetic Reynolds numbers there is no dynamo action due to the shear-current effect. This result is in agreement with the recent studies by Rädler and Stepanov (2006) and Rüdiger and Kitchatinov (2006), where the dynamo action have not been found in non-helical and non-rotating sheared non-convective turbulent plasma in the framework of the second-order correlation approximation (SOCA) or the first-order smoothing approximation (FOSA). This approximation is valid only for small hydrodynamic Reynolds numbers. Even in a highly conductivity limit (large magnetic Reynolds numbers), SOCA can be valid only for small Strouhal numbers, while for large hydrodynamic Reynolds numbers (for a developed turbulence), the Strouhal number is 1.

Note that the standard approach (i.e., SOCA) cannot describe the situation in principle. The reason is that the shear-current dynamo requires a finite correlation time of turbulent velocity field, so the delta-correlated version of SOCA fails. The application of the path integral approach for the study of the shear-current dynamo requires a finite correlation time of turbulent velocity field. The shear-current dynamo is a phenomenon that results from the interaction of the energy-containing-scale of turbulence with large-scale shear, and the constraint is that the hydrodynamic and magnetic Reynolds numbers should be not small at least. Therefore, the SOCA-based approaches do not work properly to describe the shear-current dynamo. Probably, the hydrodynamic and magnetic Reynolds numbers can be of the order of unity and there is no need for a developed inertial range in order to maintain the shear-current dynamo.

3. Nonlinear effects

In order to find the magnitude of the magnetic field, the nonlinear effects must be taken into account. The nonlinear shear-current dynamo have been studied by Rogachevskii and Kleeorin (2004); Rogachevskii et al. (2006a,b). The mean magnetic field is determined by the following nonlinear equations

$$\frac{\partial B_x(t,z)}{\partial t} = -Sl_0^2[\sigma_B(B)B_y']' - [\alpha_m(B)B_y]' + \eta_T B_x'', \tag{3}$$

$$\frac{\partial B_y(t,z)}{\partial t} = SB_x + \eta_T B_y'', \tag{4}$$

$$\frac{\partial \chi_c(t,z)}{\partial t} - \kappa_T \chi_c'' + \frac{\chi_c}{\tau_\chi} = -\frac{1}{9\pi\rho\eta_T}\mathscr{E}\cdot\mathbf{B}, \tag{5}$$

where $B_i' = \partial B_i/\partial z$, $\mathscr{E} = \alpha_m \mathbf{B} + Sl_0^2 \sigma_B(B) B_y' \mathbf{e}_y - \eta_T(\nabla\times\mathbf{B})$ is the mean electromotive force, $\alpha_m = \chi_c(t,z)\Phi_N(B)$ is the magnetic α effect, $\Phi_N(B)$ is the quenching function of the magnetic α effect and ρ is the fluid density. The function $\chi_c(\mathbf{B})$ is related to the small-scale current helicity $\langle \mathbf{b}\cdot(\nabla\times\mathbf{b})\rangle$, where $\mathbf{b}$ are the magnetic fluctuations. For a weakly inhomogeneous turbulent plasma, the function χ_c is proportional to the small-scale magnetic helicity. In Eq. (5) we use the simplest form of the magnetic helicity flux, $\propto -\kappa_T \nabla\chi_c$, where κ_T is the coefficient of the turbulent diffusion of the magnetic helicity, $\tau_\chi = \tau_0\,\mathrm{Rm}$ is the characteristic relaxation time of the small-scale magnetic helicity and Rm is the magnetic Reynolds number. Eqs. (3) and (4) follow from the mean-field induction equation, while Eq. (5) is derived using arguments based on the magnetic helicity conservation law (see, e.g., Kleeorin and Rogachevskii, 1999; Brandenburg and Subramanian, 2005a; Rogachevskii et al., 2006b, and references therein). For large magnetic Reynolds numbers the relaxation term χ_c/τ_χ in Eq. (5) can be neglected. For moderate values of the magnetic Reynolds numbers this term has been taken into account by Brandenburg and Subramanian (2005c); Rogachevskii et al. (2006b). The quenching function of the magnetic α effect $\Phi_N(B)$ is given by $\Phi_N(B) = (3/8B^2)[1 - \arctan(\sqrt{8}B)/\sqrt{8}B]$, where the mean magnetic field $\mathbf{B}$ is measured in units of the equipartition field B_{eq} determined by the turbulent kinetic energy, $\Phi_N(B) = 1 - (24/5)B^2$ for $B \ll 1/4$ and $\Phi_N(B) = 3/(8B^2)$ for $B \gg 1/4$. The nonlinear function $\sigma_B(B)$ which is normalized by $\sigma_B(B=0)$, varies from 1 for $B \ll 1/4$ to $-11/4$ for $B \gg 1/4$ (see Rogachevskii and Kleeorin, 2004).

Let us consider the simple boundary conditions for a layer of the thickness $2L$ in the z-direction: $\mathbf{B}(t,|z|=L)=0$ and $\chi_c(t,|z|=L)=0$. We introduce the following non-dimensional parameters: $D = (l_0 S_*/L)^2 \sigma_B(B=0)$ is the dynamo number and the parameter $S_* = SL^2/\eta_T$ is the dimensionless shear number. In the kinematic dynamo, the mean magnetic field is generated when the dynamo number $D > D_{cr} = \pi^2/4$ for the symmetric mode (relative to the middle plane $z=0$) and when the dynamo number $D > D_{cr} = \pi^2$ for the antisymmetric mode. Numerical solutions of nonlinear equations (3)–(5) have been obtained by Rogachevskii et al. (2006b). The saturated level of the mean magnetic field depends strongly on the value of the turbulent diffusivity of the magnetic helicity κ_T. The mean magnetic field varies from very small value for $\kappa_T = 0.1\eta_T$ to the super-equipartition field for $\kappa_T = \eta_T$. This is an indication of very important role of the transport of the magnetic helicity. The generation of the mean magnetic field causes negative magnetic α effect, which reduces the growth rate of large-scale magnetic field. The reason is that the first and the second terms in the right hand side of Eq. (3) have opposite signs. The first term in Eq. (3) describes the shear-current effect, while the second-term in Eq. (3) determines the magnetic α effect. If the magnetic helicity does not effectively transported out from the generation region, the mean magnetic field is saturated even at small values of the magnetic field. Increase of the magnetic helicity flux by increasing of the turbulent diffusivity κ_T of magnetic helicity, results in increase of the saturated level of the mean magnetic field above the equipartition field. The magnitude of the saturated field increases also by the increase of the dynamo numbers D within the range $D_{cr} < D < 2\,D_{cr}$, and it decreases with the increase of the dynamo number for $D > 2\,D_{cr}$. This is a new feature in the nonlinear mean-field dynamo. For example, in the nonlinear $\alpha\Omega$ dynamo the saturated level of the mean magnetic field usually increases with the increase the dynamo numbers.

The generation of the large-scale magnetic field in a nonhelical sheared turbulent plasma has been recently investigated by Brandenburg (2005) using direct numerical simulations (DNS). In particular, in this DNS the non-convective turbulence is driven by a forcing that consists of eigenfunctions of the curl operator with the wavenumbers $4.5 < k_f < 5.5$ and of large-scale component with wavenumber $k_1 = 1$. The forcing produces the mean flow $U = U_0\cos(k_1 x)\cos(k_1 z)$. The numerical resolution in these simulations is $128\times512\times128$ meshpoints, and the parameters used in these simulations are as following: the magnetic Reynolds number $\mathrm{Rm} = u_{rms}/(\eta k_f) = 80$, the magnetic Prandtl number $\mathrm{Pr_m} = \nu/\eta = 1$ and $U_0/u_{rms} = 5$. This DNS clearly demonstrate the existence of the large-scale dynamo in the absence of mean kinetic helicity and alpha effect. The growth rate of the mean magnetic field is about $\gamma\tau_0 \approx 2\times10^{-2}$. This allows us to estimate the parameter σ_B characterizing the shear-current effect, $\sigma_B \approx 3.3\times10^{-2}$. On the other hand, our theory predicts $\sigma_B = (3-6)\times10^{-2}$ depending on the parameter μ. Note that in DNS by Brandenburg (2005) the shear is not small (i.e., the parameter $S\tau_0 \sim 1$), which explains some difference between the theoretical predictions and numerical simulations. The saturated level of the mean magnetic field in these numerical simulations is of the order of the equipartition field which is in a good agreement with the numerical solutions of the nonlinear dynamo equations (3)–(5) discussed here.

In summary, we show that in a sheared nonhelical homogeneous turbulent plasma whereby the kinetic α effect vanishes, the large-scale magnetic field can grow due to the shear-current effect from a very small seeding magnetic field. The dynamo instability is saturated by the nonlinear effects, and the dynamical nonlinearity due to the evolution of small-scale magnetic helicity, plays a crucial role in the nonlinear saturation of the large-scale magnetic field. Note that a sheared turbulence is a universal feature in astrophysical plasmas, and the obtained results can be important for elucidation of origin of the large-scale magnetic fields generated in astrophysical sheared turbulent plasmas, e.g., in merging protogalactic clouds or in merging protostellar clouds (Rogachevskii et al., 2006a).

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