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# Nonlinear shear-current dynamo and magnetic helicity transport in sheared turbulence

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The nonlinear mean-field dynamo due to a shear-current effect in a nonhelical homogeneous turbulence with a mean velocity shear is discussed. The transport of magnetic helicity as a dynamical nonlinearity is taken into account. The shear-current effect is associated with the $\mathbf{W} \times \mathbf{J}$ term in the mean electromotive force, where $\mathbf{W}$ is the mean vorticity due to the large-scale shear motions and $\mathbf{J}$ is the mean electric current. This effect causes the generation of large-scale magnetic field in a turbulence with large hydrodynamic and magnetic Reynolds numbers. The dynamo action due to the shear-current effect depends on the spatial scaling of the correlation time $\tau(k)$ of the background turbulence, where $k$ is the wave number. For Kolmogorov scaling, $\tau(k) \propto k^{-2/3}$, the dynamo instability occurs, while when $\tau(k) \propto k^{-2}$ (small hydrodynamic and magnetic Reynolds numbers) there is no the dynamo action in a sheared nonhelical turbulence. The magnetic helicity flux strongly affects the magnetic field dynamics in the nonlinear stage of the dynamo action. Numerical solutions of the nonlinear mean-field dynamo equations which take into account the shear-current effect, show that if the magnetic helicity flux is not small, the saturated level of the mean magnetic field is of the order of the equipartition field determined by the turbulent kinetic energy. Turbulence with a large-scale velocity shear is a universal feature in astrophysics, and the obtained results can be important for elucidation of origin of the large-scale magnetic fields in astrophysical sheared turbulence.

*Keywords:* Nonlinear shear-current dynamo; Sheared turbulent flow; Magnetic helicity transport

## 1. Introduction

It is generally believed that one of the main reasons for the generation of large-scale magnetic fields in turbulent flow is the $\alpha$ effect (see, e.g. books and reviews by Moffatt 1978, Parker 1979, Krause and Rädler 1980, Zeldovich *et al.* 1983, Ruzmaikin *et al.* 1988, Stix 1989, Roberts and Soward 1992, Ossendrijver 2003, Brandenburg and Subramanian 2005a). However, the $\alpha$ effect caused by the helical

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random motions of conducting fluid, requires rotating inhomogeneous or density stratified turbulence.

In a turbulence with a large-scale velocity shear and high hydrodynamic and magnetic Reynolds numbers there is a possibility for a mean-field dynamo (Rogachevskii and Kleeorin 2003, 2004). Turbulence with a large-scale velocity shear is a universal feature in astrophysical plasmas. The large-scale velocity shear creates anisotropic turbulence with a nonzero background mean vorticity $\mathbf{W}$. This can cause the $\mathbf{W} \times \mathbf{J}$ effect (or the shear-current effect), which creates the mean electric current along the original mean magnetic field and produces the large-scale dynamo even in a nonrotating and nonhelical homogeneous turbulence (Rogachevskii and Kleeorin 2003, 2004). Here $\mathbf{J}$ is the mean electric current.

The mean-field dynamo instability is saturated by the nonlinear effects. A dynamical nonlinearity in the mean-field dynamo which determines the evolution of small-scale magnetic helicity, is of great importance due to the conservation law for the total (large and small scales) magnetic helicity in turbulence with very large magnetic Reynolds numbers (see, e.g. Kleeorin and Ruzmaikin 1982, Gruzinov and Diamond 1994, 1996, Kleeorin *et al.* 1995, 2000, 2002, 2003a, 2003b, Kleeorin and Rogachevskii 1999, Blackman and Field 2000, Vishniac and Cho 2001, Blackman and Brandenburg 2002, Brandenburg and Subramanian 2005a, Zhang *et al.* 2006). On the other hand, the effect of the mean magnetic field on the motion of fluid and on the cross-helicity can cause quenching of the mean electromotive force which determines an algebraic nonlinearity. The combined effect of the dynamic and algebraic nonlinearities saturates the growth of the mean magnetic field.

The mean-field dynamo is essentially nonlinear due to the evolution of the small-scale magnetic helicity (Gruzinov and Diamond 1994, 1996). In particular, even for very small mean magnetic field the magnetic $\alpha$ effect is not small. This is a reason why we have to take into account the dynamical nonlinearity in the mean-field dynamo. When we ignore the dynamical nonlinearity due to evolution of small-scale magnetic helicity and take into account only algebraic nonlinearity caused by the nonlinear shear-current effect, we obtain the saturated level of mean magnetic field which is in several times larger than the equipartition field determined by the turbulent kinetic energy (Rogachevskii *et al.* 2006). This result can be important in view of astrophysical applications whereby the super-equipartition large-scale magnetic fields are observed, e.g. in the outer parts of a few galaxies (Beck 2004, 2005). Note that it is a problem to reach a super-equipartition level of the large-scale magnetic field in the $\alpha\Omega$ dynamo.

The goal of this study is to investigate the nonlinear mean-field dynamo due to the shear current effect. In this study, we have taken into account the dynamic nonlinearity caused by the evolution of the small-scale magnetic helicity. This article is organized as follows. In section 2, we elucidate the physics of the shear-current effect. In section 3, we consider kinematic dynamo problem due to this effect. In section 4, we describe the results of numerical solutions of the nonlinear mean-field dynamo equations which take into account the shear-current effect, the algebraic and dynamic nonlinearities. Finally, the discussion and conclusions are given in section 5. In Appendixes A, we compare the problem of generation of the mean magnetic field in a turbulence with a large-scale velocity shear with that of the generation of mean vorticity in a sheared turbulence.

## 2. The physics of the shear-current effect

In this section, we elucidate the mechanism of generation of large-scale magnetic field due to the shear-current effect. To this end we first discuss the physics of the $\alpha$ effect (see, e.g. Moffatt 1978, Parker 1979, Krause and Rädler 1980, Zeldovich et al. 1983, Ruzmaikin *et al.* 1988). The $\alpha$ term in the mean electromotive force $\boldsymbol{\mathcal{E}} = \langle \mathbf{u} \times \mathbf{b} \rangle$ in a rotating inhomogeneous turbulence can be written in the form $\boldsymbol{\mathcal{E}}^\alpha \equiv \alpha \mathbf{B} \propto -l_0^2 (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}^u) \mathbf{B}$ (see, e.g., Krause and Rädler 1980, Rädler *et al.* 2003), where $\mathbf{u}$ and $\mathbf{b}$ are fluctuations of the velocity and magnetic field, respectively, angular brackets denote ensemble averaging, $\boldsymbol{\Omega}$ is the angular velocity, the vector $\boldsymbol{\Lambda}^u = \boldsymbol{\nabla} \langle \mathbf{u}^2 \rangle / \langle \mathbf{u}^2 \rangle$ determines the inhomogeneity of the turbulence, $\mathbf{B}$ is the mean magnetic field and $l_0$ is the maximum scale of turbulent motions (the integral turbulent scale). The $\alpha$ effect is caused by the kinetic helicity $\chi_u \propto \eta_{_T} (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}^u)$ in an inhomogeneous rotating turbulence, where $\eta_{_T} \propto l_0 \, u_0$ is the turbulent magnetic diffusion and $u_0$ is the characteristic turbulent velocity in the maximum scale of turbulent motions $l_0$. The deformations of the magnetic field lines are caused by upward and downward rotating turbulent eddies (figure 1). The inhomogeneity of the turbulence breaks a symmetry between the upward and downward eddies. Therefore, the total effect of these eddies on the mean magnetic field does not vanish, and it creates the mean electric current along the original mean magnetic field due to the $\alpha$ effect.

The large-scale magnetic field can be generated even in a nonrotating and nonhelical turbulence with a mean velocity shear due to the shear-current effect (Rogachevskii and

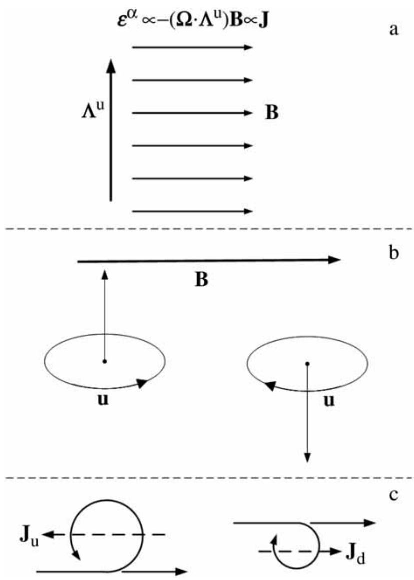


Figure 1. Mechanism for the $\alpha$ effect: (a) Interaction between uniform original mean magnetic field and inhomogeneous rotating turbulence; (b) The deformations of the original magnetic field lines are caused by the upward and downward turbulent eddies; (c) Formation of the mean electric current $\boldsymbol{\mathcal{E}}^\alpha \equiv \alpha \mathbf{B} \propto -l_0^2 (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}^u) \mathbf{B} \propto \mathbf{J}$ opposite to the original mean magnetic field (for $\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}^u > 0$). Here, $\mathbf{J}_u$ and $\mathbf{J}_d$ are the electric currents caused by the deformations of the original magnetic field lines by the upward and downward turbulent eddies, respectively, and $|\mathbf{J}_u| > |\mathbf{J}_d|$.

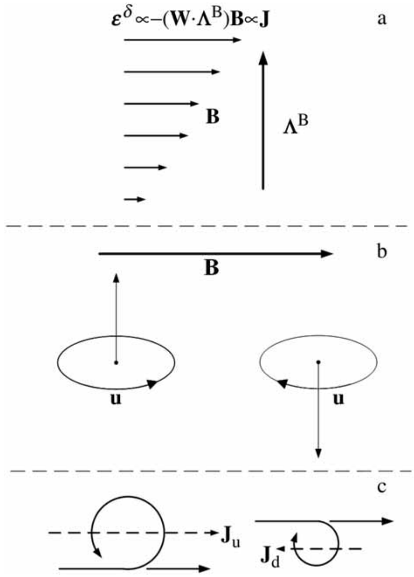


Figure 2. Mechanism for the $\mathbf{W} \times \mathbf{J}$ effect: (a) Interaction between nonuniform original mean magnetic field and homogeneous sheared turbulence; (b) The deformations of the original magnetic field lines are caused by the upward and downward turbulent eddies; (c) Formation of the mean electric current $\mathcal{E}^\delta \propto -l_0^2 \, \mathbf{W} \times (\nabla \times \mathbf{B}) \propto l_0^2 \, (\mathbf{W} \cdot \mathbf{\Lambda}^B)\mathbf{B} \propto \mathbf{J}$ along the original mean magnetic field (for $\mathbf{W} \cdot \mathbf{\Lambda}^B > 0$). Here, $\mathbf{J}_u$ and $\mathbf{J}_d$ are the electric currents caused by the deformations of the original magnetic field lines by the upward and downward turbulent eddies, respectively, and $|\mathbf{J}_u| > |\mathbf{J}_d|$.

Kleeorin 2003, 2004). This effect is related to the $\mathbf{W} \times \mathbf{J}$ term in the mean electromotive force, and it can be written in the form $\mathcal{E}^\delta \propto -l_0^2 \, \mathbf{W} \times (\nabla \times \mathbf{B}) \propto l_0^2 \, (\mathbf{W} \cdot \mathbf{\Lambda}^B)\mathbf{B}$, where the mean vorticity $\mathbf{W} = \nabla \times \mathbf{U}$ is caused by the mean velocity shear and $\mathbf{\Lambda}^B = \nabla \mathbf{B}^2/2\mathbf{B}^2$ determines the inhomogeneity of the mean original magnetic field. In a sheared turbulence the inhomogeneity of the original mean magnetic field breaks a symmetry between the influence of upward and downward turbulent eddies on the mean magnetic field. The deformations of the original magnetic field lines in the $\mathbf{W} \times \mathbf{J}$ effect are caused by the upward and downward turbulent eddies. This creates the mean electric current along the mean magnetic field and produces the magnetic dynamo (figure 2).

In order to demonstrate how the shear-current dynamo operates, let us consider a homogeneous turbulence with a mean velocity shear, $\mathbf{U} = (0, Sx, 0)$ and $\mathbf{W} = (0, 0, S)$. Let us assume that the mean magnetic field has a simple form $\mathbf{B} = (B_x(z), B_y(z), 0)$. The mean magnetic field in the kinematic approximation is determined by

$$\frac{\partial B_x}{\partial t} = -\sigma_{_B} \, S \, l_0^2 \, B_y'' + \eta_{_T} \, B_x'' \, , \tag{1}$$

$$\frac{\partial B_y}{\partial t} = S \, B_x + \eta_{_T} \, B_y'' \, , \tag{2}$$

(Rogachevskii and Kleeorin 2003, 2004), where $B_i'' = \partial^2 B_i / \partial z^2$, $\eta_{_T}$ is the coefficient of turbulent magnetic diffusion and the dimensionless parameter $\sigma_{_B}$ determines the $\mathbf{W} \times \mathbf{J}$ effect (see equation (3)). The first term $\propto SB_x$ in the right-hand side

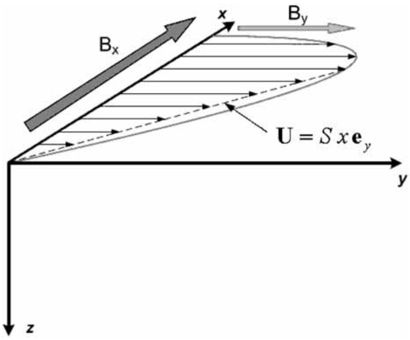


Figure 3. Mechanism for shear-induced generation of perturbations of the mean magnetic field $B_y$ by sheared stretching of the field $B_x$. This effect is determined by the first term ($\propto SB_x$) in the right hand side of equation (2), and it is similar to the differential rotation because $\nabla \times (\mathbf{U} \times \mathbf{B}) = SB_x \mathbf{e}_y$.

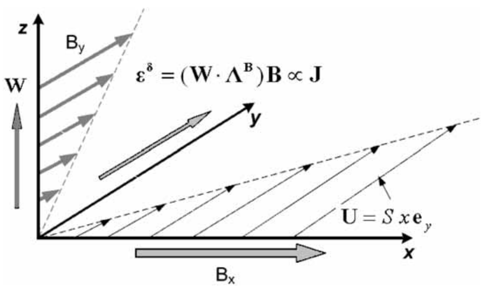


Figure 4. Mechanism for shear-current generation of perturbations of the mean magnetic field $B_x$ from the inhomogeneous magnetic field $B_y$. This effect is determined by the first term in right-hand side of equation (1).

of equation (2) determines the stretching of the magnetic field $B_x$ by the shear motions, which produces the field $B_y$ (figure 3). On the other hand, the interaction of the non-uniform magnetic field $B_y$ with the background vorticity $\mathbf{W}$ (caused by the large-scale shear) produces the electric current along the field $B_y$. This implies the generation of the field component $B_x$ (figure 4) due to the $\mathbf{W} \times \mathbf{J}$ effect, which is determined by the first term in the right-hand side of equation (1). This causes the dynamo instability.

## 3. Kinematic dynamo due to the shear-current effect

Let us consider the kinematic dynamo due to the shear-current effect. In this study, we have derived a more general form of the parameter $\sigma_B$ for arbitrary scaling of the correlation time $\tau(k)$ of turbulent velocity field. The generalized form of the parameter $\sigma_B$ defining the shear-current effect, is derived using equation (A44) given in the paper by Rogachevskii and Kleeorin (2004). The parameter $\sigma_B$ entering in equation (1) is given by

$$\sigma_B = \frac{4I_0}{15}\left[1 + \frac{I}{I_0} + 3\,\epsilon\right], \tag{3}$$

where

$$I = \int \tau'(k)\, k\, \tau(k)\, E(k)\, \mathrm{d}k, \quad I_0 = \int \tau^2(k)\, E(k)\, \mathrm{d}k\ , \tag{4}$$

$E(k)$ is the turbulent kinetic energy spectrum, $\tau(k)$ is the scale-dependent correlation time of turbulent velocity field, the parameter $\epsilon = E_m\, l_m / E_v\, l_0$, $E_m$ and $E_v$ are the magnetic and kinetic energies per unit mass in the background turbulence (with a zero mean magnetic field), $l_m$ is the scale of localization of the magnetic fluctuations generated by the small-scale dynamo in the background turbulence and $\tau'(k) = d\tau/dk$. Equations (3) and (4) are written in dimensionless form, where the turbulent time $\tau(k)$ is measured in the units of $\tau_0 = l_0/u_0$, the wave number $k$ is measured in the units of $l_0^{-1}$, and the turbulent kinetic energy spectrum $E(k)$ is measured in the units of $u_0^2\, l_0$.

The solution of equations (1) and (2) we seek for in the form $\propto \exp(\gamma\, t + \mathrm{i} K_z\, z)$, where the growth rate, $\gamma$, of the mean magnetic field due to the magnetic dynamo instability is given by

$$\gamma = S\, l_0\, \sqrt{\sigma_{B}}\, K_z - \eta_{T}\, K_z^2. \tag{5}$$

The necessary condition for the magnetic dynamo instability is $\sigma_{B} > 0$. The dynamo action due to the shear-current effect depends strongly on the spatial scaling of the correlation time $\tau(k)$ of the turbulent velocity field. In particular, when $\tau(k) \propto k^{-\mu}$, the ratio $I/I_0 = -\mu$, and the criterion for the dynamo instability reads

$$1 - \mu + 3\,\epsilon > 0. \tag{6}$$

For example, when $\tau(k) \propto k^{-2/3}$ (the Kolmogorov scaling), the parameter $\sigma_{B} = (4/135)\,(1 + 9\epsilon)$. This case was considered by Rogachevskii and Kleeorin (2003, 2004). When $\epsilon = 0$ (there are no magnetic fluctuations in the background turbulence due to the small-scale dynamo), the shear-current dynamo occurs for $\mu < 1$. The boundary $\mu = 1$ corresponds to the spatial scaling of the correlation time $\tau(k) \propto k^{-1}$. For the Kolmogorov's type turbulence (i.e. for a turbulence with a constant energy flux over the spectrum), the energy spectrum which corresponds to the correlation time $\tau(k) \propto k^{-1}$, is $E(k) = -\mathrm{d}\tau/\mathrm{d}k = k^{-2}$. This implies that the velocity is dominated by the large scales more strongly than for the turbulence with a purely Kolmogorov spectrum $E(k) \propto k^{-5/3}$.

For small hydrodynamic and magnetic Reynolds numbers, the turbulent time $\tau(k) \propto 1/(\nu k^2)$ or $\tau(k) \propto 1/(\eta k^2)$ depending on the magnetic Prandtl number, i.e. $\tau(k) \propto k^{-2}$. Then $\sigma_{B} = (4 I_0/15)\,(-1 + 3\,\epsilon)$, where $\mu = 2$, $\nu$ is the kinematic viscosity and $\eta$ is the magnetic diffusivity due to electrical conductivity of the fluid. When $\epsilon = 0$ the parameter $\sigma_{B} < 0$, and there is no dynamo action due to the shear-current effect in agreement with the recent studies by Rädler and Stepanov (2006) and Rüdiger and Kichatinov (2006). They have not found the dynamo action in nonrotating and nonhelical shear flows with $\epsilon = 0$ in the framework of the second order correlation approximation (SOCA). This approximation is valid for small hydrodynamic Reynolds numbers. Even in a highly conductivity limit (large magnetic Reynolds numbers), SOCA can be valid only for small Strouhal numbers, while for large hydrodynamic Reynolds numbers (i.e. for fully developed turbulence), the Strouhal number is 1.

When $\epsilon > 1/3$, the mean magnetic field can be generated due to the shear-current effect even for small hydrodynamic and magnetic Reynolds numbers. However, the latter case seems to be not realistic.

The effect of shear on the mean electromotive force and shear-current effect have been studied by Rogachevskii and Kleeorin (2003, 2004) for large hydrodynamic and magnetic Reynolds numbers using two different approaches: the spectral $\tau$ approximation (the third-order closure procedure) and the stochastic calculus, i.e., the Feynman–Kac path integral representation of the solution of the induction equation and Cameron–Martin–Girsanov theorem.

Note that Ruderman and Ruzmaikin (1984) formally constructed an example of an exponentially growing magnetic field in a fluid with shear and a homogeneous anisotropic magnetic diffusivity. An essential condition for generation is that the vector defining the anisotropy in this phenomenological model must be non-parallel and non-perpendicular to the velocity. However, equations (1) and (2) are different from those given by Ruderman and Ruzmaikin (1984) because they have not studied the effect of shear on the mean electromotive force. The first (although incorrect) attempt to determine the effect of shear on the mean electromotive force has been made by Urpin (1999a, 1999b) in the framework of SOCA.

In order to study the kinematic dynamo due to the shear current effect, let us rewrite equations (1) and (2) for the mean magnetic field in the dimensionless form

$$\frac{\partial A}{\partial t} = D\, B_y' + A'' \, , \tag{7}$$

$$\frac{\partial B_y}{\partial t} = -A' + B_y'' \, , \tag{8}$$

where the mean magnetic field is $\mathbf{B} = B_y(t,z)\,\mathbf{e}_y + S_*^{-1}\boldsymbol{\nabla}\times[A(z)\,\mathbf{e}_y]$, i.e. $B_x(t,z) = -S_*^{-1}\,A'(z)$, the parameter $S_* = S\,L^2/\eta_{_T}$ is the dimensionless shear number and $D = (l_0\,S_*/L)^2\,\sigma_{_B}$ is the dynamo number. We consider the following boundary conditions for a layer of thickness $2L$ in the $z$ direction:

$$B_y(t,|z|=1) = 0, \quad A'(t,|z|=1) = 0 \, , \tag{9}$$

i.e. $\mathbf{B}(t,|z|=1) = 0$. In dimensionless equations (7) and (8) the length is measured in units of $L$, the time is measured in units of the turbulent magnetic diffusion time $L^2/\eta_{_T}$, the mean magnetic field $\mathbf{B}$ is measured in units of the equipartition field $B_{\rm eq} = \sqrt{4\pi\rho}\,u_0$ determined by the turbulent kinetic energy and the turbulent magnetic diffusion coefficient is measured in units of the characteristic value of $\eta_{_T} = l_0\,u_0/3$. The solution of equations (7) and (8) reads

$$B_y(t,z) = B_0\,\exp(\gamma\,t)\,\cos(K_z z + \varphi) \, , \tag{10}$$

$$B_x(t,z) = \frac{l_0}{L}\,K_z\,\sqrt{\sigma_{_B}}\,B_0\,\exp(\gamma\,t)\,\cos(K_z z + \varphi). \tag{11}$$

The growth rate of the mean magnetic field in the dimensionless form is given by $\gamma = \sqrt{D}\,K_z - K_z^2$. The wave vector $K_z$ is measured in units of $L^{-1}$ and the growth rate $\gamma$ is measured in units of the inverse turbulent magnetic diffusion time $\eta_{_T}/L^2$.

For the symmetric mode the angle $\varphi = \pi n$, the wave number $K_z = (\pi/2)(2m+1)$ and the mean magnetic field is generated when the dynamo number $D > D_{\rm cr} = (\pi^2/4)(2m+1)^2$, where $n, m = 0, 1, 2, \ldots$. For this mode, the mean magnetic field is symmetric relative to the middle plane $z = 0$. For the antisymmetric mode, the angle $\varphi = (\pi/2)(2n+1)$ with $n = 0, 1, 2, \ldots$, the wave number $K_z = \pi m$ and the magnetic field is generated when the dynamo number $D > D_{\rm cr} = \pi^2 m^2$, where $m = 1, 2, 3, \ldots$ Note that for the shear-current dynamo, the ratio of the field components $B_x/B_y$ is small [i.e. $B_x/B_y \sim (l_0/L)\, K_z \sqrt{\sigma_B} \ll 1$ when $K_z$ is not very large], see equations (10) and (11). This feature is similar to that for the $\alpha\Omega$ dynamo, whereby the poloidal component of the mean magnetic field is much smaller than the toroidal field. The maximum growth rate of the mean magnetic field, $\gamma_{\rm max} = D/4$, is attained at $K_z = K_m = \sqrt{D}/2$. This corresponds to the characteristic scale of the mean magnetic field variations $L_B = 2\pi L/K_m = (4\pi/\sqrt{D})\, L$.

## 4. Nonlinear dynamo due to the shear-current effect

Kinematic dynamo models predict a field that grows without limit, and they give no estimate of the magnitude for the generated magnetic field. In order to find the magnitude of the field, the nonlinear effects which limit the field growth must be taken into account. The nonlinear theory of the shear-current effect was developed by Rogachevskii and Kleeorin (2004, 2006).

### 4.1. *The algebraic nonlinearity*

First, let us start with the algebraic nonlinearity which is determined by the effects of the mean magnetic field on the motion of fluid and on the cross-helicity. These effects cause quenching of the mean electromotive force.

Below we outline the procedure of the derivation of the equations for the nonlinear coefficients defining the mean electromotive force in a homogeneous turbulence with a mean velocity shear (for details, Rogachevskii and Kleeorin 2004). We use the momentum equation and the induction equation for the turbulent fields written in a Fourier space. We derive equations for the correlation functions of the velocity field $f_{ij} = \langle u_i u_j \rangle$, the magnetic field $h_{ij} = \langle b_i b_j \rangle$ and the cross-helicity $g_{ij} = \langle b_i u_j \rangle$, where the angular brackets denote ensemble averaging. We split the tensors $f_{ij}, h_{ij}$ and $g_{ij}$ into nonhelical, $h_{ij}$, and helical, $h_{ij}^{(H)}$, parts. The helical part of the tensor $h_{ij}^{(H)}$ for magnetic fluctuations depends on the magnetic helicity, and it is determined by the dynamic equation which follows from the magnetic helicity conservation arguments (section 4.2). Then we split the nonhelical parts of the correlation functions $f_{ij}, h_{ij}$ and $g_{ij}$ into symmetric and antisymmetric tensors with respect to the wave vector $\mathbf{k}$.

The second-moment equations include the first-order spatial differential operators $\hat{\mathcal{N}}$ applied to the third-order moments $M^{(III)}$. A problem arises how to close the system, i.e. how to express the set of the third-order terms $\hat{\mathcal{N}} M^{(III)}$ through the lower moments $M^{(II)}$ (see, e.g. Orszag 1970, Monin and Yaglom 1975, McComb 1990). Various approximate methods have been proposed in order to solve it. A widely used spectral $\tau$ approximation (see, e.g. Orszag 1970, Pouquet *et al.* 1976, Kleeorin *et al.* 1990, Kleeorin *et al.* 1996, Blackman and Field 2002, Rogachevskii and Kleeorin 2004, Brandenburg *et al.* 2004, Brandenburg and Subramanian 2005a) postulates that the deviations of the third-moment terms, $\hat{\mathcal{N}}M^{(III)}(\mathbf{k})$, from the contributions to these terms afforded by the background turbulence, $\hat{\mathcal{N}}M_0^{(III)}(\mathbf{k})$, are expressed through the similar deviations of the second moments, $M^{(II)}(\mathbf{k}) - M_0^{(II)}(\mathbf{k})$:

$$\hat{\mathcal{N}}M^{(III)}(\mathbf{k}) - \hat{\mathcal{N}}M_0^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_0^{(II)}(\mathbf{k})}{\tau(k)} \, , \tag{12}$$

where $\tau(k)$ is the characteristic relaxation time, which can be identified with the correlation time of the turbulent velocity field. The background turbulence is determined by the budget equations and the general structure of the moments is obtained by symmetry reasoning. In the background turbulence, the mean magnetic field is zero. We applied the spectral $\tau$-approximation only for the nonhelical part $h_{ij}$ of the tensor of magnetic fluctuations. We consider an intermediate nonlinearity which implies that the mean magnetic field is not enough strong in order to affect the correlation time of turbulent velocity field. The theory for a very strong mean magnetic field can be corrected after taking into account a dependence of the correlation time of the turbulent velocity field on the mean magnetic field.

We assume that the characteristic time of variation of the mean magnetic field $\mathbf{B}$ is substantially larger than the correlation time $\tau(k)$ for all turbulence scales (which corresponds to the mean-field approach). This allows us to get a stationary solution for the equations for the second moments $f_{ij}$, $h_{ij}$ and $g_{ij}$. For the integration in $\mathbf{k}$-space of these second moments we have to specify a model for the background turbulence (with $\mathbf{B} = 0$). We use a simple model for the background homogeneous and isotropic turbulence. Using the derived equations for the second moments $f_{ij}$, $h_{ij}$ and $g_{ij}$ we calculate the mean electromotive force $\mathcal{E}_i = \varepsilon_{imn} \int g_{nm}(\mathbf{k}) \, d\mathbf{k}$. This procedure allows us to derive equations for the nonlinear coefficients defining the mean electromotive force in a homogeneous turbulence with a mean velocity shear (for details, Rogachevskii and Kleeorin 2004, 2006).

For simplicity in this study, we do not take into account a quenching of the turbulent magnetic diffusion. This facet is discussed in details by Rogachevskii and Kleeorin (2004, 2006). We consider the nonlinear dynamo problem with the algebraic nonlinearity $\sigma_{_N}(B)$ which determines the nonlinear shear-current effect. The mean magnetic field is determined by the following nonlinear equations

$$\frac{\partial A}{\partial t} = D \, \sigma_{_N}(B) \, B'_y + A'' \, , \tag{13}$$

$$\frac{\partial B_y}{\partial t} = -A' + B''_y \, , \tag{14}$$

(Rogachevskii and Kleeorin 2004, Rogachevskii *et al.* 2006), where $B = |\mathbf{B}|$. The function $\sigma_{_N}(B)$ defining the nonlinear shear-current effect (which is normalized by $\sigma_{_B}$), is shown in figure 5 for different values of the parameter $\epsilon$. The asymptotic formulas for the nonlinear function $\sigma_{_N}(B)$ are given by $\sigma_{_N}(B) = 1$ for a weak mean magnetic field ($B \ll B_{\rm eq}/4$) and $\sigma_{_N}(B) = -11(1+\epsilon)/4(1+9\epsilon)$ for $B \gg B_{\rm eq}/4$ (figure 5). This implies that the nonlinear function $\sigma_{_N}(B)$ defining the shear-current effect changes its sign at some value of the mean magnetic field $B = B_*$. Here $B_* = 1.2B_{\rm eq}$ for $\epsilon = 0$ and $B_* = 1.4B_{\rm eq}$ for $\epsilon = 1$. However, there is no quenching

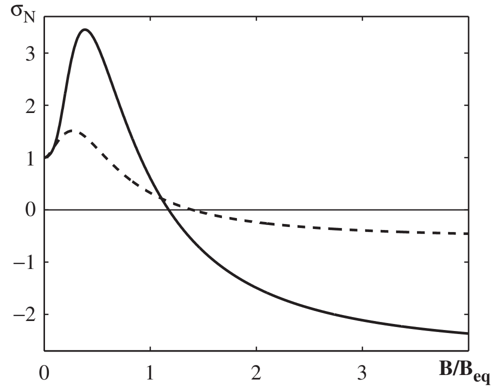


Figure 5. The dimensionless nonlinear coefficient $\sigma_{_N}(B)$ defining the shear-current effect for different values of the parameter $\epsilon$: $\epsilon = 0$ (solid); $\epsilon = 1$ (dashed).

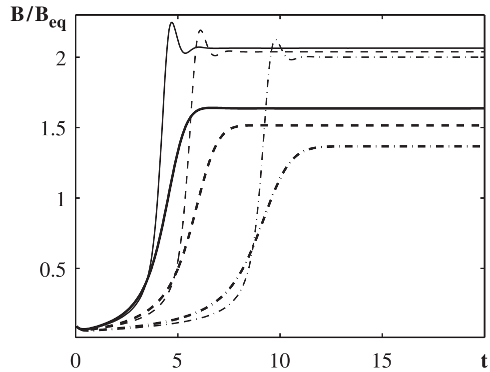


Figure 6. The nonlinear evolution of the mean magnetic field $B(t, z = 0)$ due to the shear-current effect with the algebraic nonlinearity for $\epsilon = 0$ (thin curve) and $\epsilon = 1$ (thick curve) and different near-threshold values of the dynamo number: $D = 1.45\,D_{\rm cr}$ (solid); $D = 1.3\,D_{\rm cr}$ (dashed); $D = 1.15\,D_{\rm cr}$ (dashed-dotted).

of the nonlinear shear-current effect contrary to the quenching of the nonlinear alpha effect, the nonlinear turbulent magnetic diffusion, etc. The background magnetic fluctuations (caused by the small-scale dynamo and described by the parameter $\epsilon$), affect the nonlinear function $\sigma_{_N}(\mathbf{B})$.

Numerical solutions of equations (13) and (14) for the nonlinear problem with the algebraic nonlinearity $\sigma_{_N}(\mathbf{B})$ are plotted in figures 6–8. In particular, these figures show the nonlinear evolution of the mean magnetic field $B(t, z = 0)$ due to the shear-current effect for different values of the dynamo number $D$ and the parameter $\epsilon$. The magnitude of the saturated mean magnetic field is several times larger than the equipartition field depending on the dynamo number. Inspection of figures 7 and 8 shows that there is a range in the dynamo number $D$ =22.8–59 where the nonlinear oscillations of mean magnetic field are observed at $\epsilon = 0$.

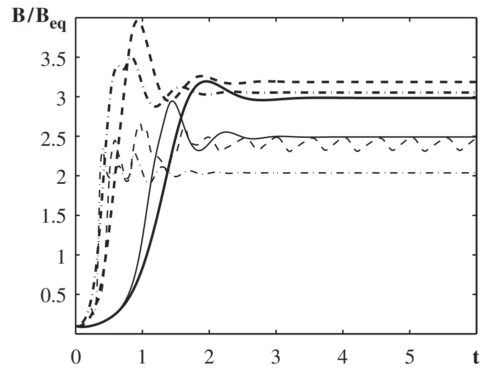


Figure 7. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic nonlinearity for $\epsilon=0$ (thin curve) and $\epsilon=1$ (thick curve) and different values of the dynamo number: $D=10$ (solid); $D=30$ (dashed); $D=50$ (dashed-dotted).

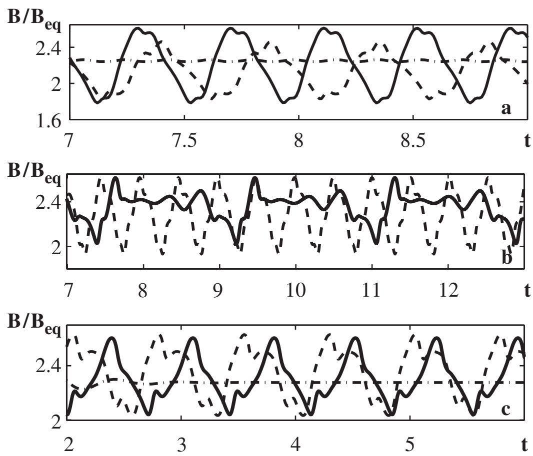


Figure 8. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic nonlinearity for $\epsilon=0$ and different values of the dynamo number: (a) $D=47$ (solid), $D=52$ (dashed), $D=59$ (dashed-dotted); (b) $D=31$ (solid), $D=35$ (dashed); (c) $D=28$ (solid); $D=22.77$ (dashed); $D=22.765$ (dashed-dotted).

## 4.2. *The algebraic and dynamic nonlinearities*

In this study, we consider nonhelical and nonrotating homogeneous turbulence. This implies that the kinetic helicity and the hydrodynamic $\alpha$ effect are zero. However, the magnetic $\alpha$ effect caused by the small-scale magnetic helicity is not zero even in nonhelical turbulence. In particular, the magnetic helicity conservation implies the growth of a magnetic alpha effect independent of whether kinetic helicity is driven into the system. In section 4.1, we have concentrated on the nonlinear shear-current effect (the algebraic nonlinearity) and have not discussed the effect of small-scale magnetic helicity (the dynamic nonlinearity) on the nonlinear saturation of the mean magnetic field. In this subsection, we study joint action of the algebraic and dynamic nonlinearities.

The small-scale magnetic helicity causes the magnetic $\alpha$ effect which is given by $\alpha_m = \Phi_{_N}(B)\, \chi_c(\mathbf{B})$, where $\Phi_{_N}(B)$ is the quenching function of the magnetic $\alpha$ effect given below by equation (19). The function $\chi_c(\mathbf{B}) \equiv (\tau_0/12\pi\rho)\langle \mathbf{b}\cdot(\nabla\times\mathbf{b})\rangle$ is related to the small-scale current helicity $\langle \mathbf{b}\cdot(\nabla\times\mathbf{b})\rangle$. For a weakly inhomogeneous turbulence ($l_0 \ll L$), the function $\chi_c$ is proportional to the small-scale magnetic helicity, $\chi_c = \chi_m/(18\pi\eta_{_T}\rho)$ (Kleeorin and Rogachevskii 1999), where $\chi_m = \langle \mathbf{a}\cdot\mathbf{b}\rangle$ is the small-scale magnetic helicity and **a** is the vector potential of small-scale magnetic field. The function $\chi_c(\mathbf{B})$ entering the magnetic $\alpha$ effect is determined by the following dynamical dimensionless equation (which is derived using arguments based on the magnetic helicity conservation law):

$$\frac{\partial \chi_c}{\partial t} + \nabla\cdot\mathbf{F} + \frac{\chi_c}{\tau_\chi} = -\left(\frac{2L}{l_0}\right)^2 (\boldsymbol{\mathcal{E}}\cdot\mathbf{B}), \tag{15}$$

(see, e.g. Kleeorin and Ruzmaikin 1982, Gruzinov and Diamond 1994, 1996, Kleeorin *et al.* 1995, 2000, 2002, 2003a, 2003b, Kleeorin and Rogachevskii 1999, Blackman and Field 2000, Blackman and Brandenburg 2002, Brandenburg and Subramanian 2005a, Zhang *et al.* 2006), where $\tau_\chi = (1/3)(l_0/L)^2 \mathrm{Rm}$ is the characteristic relaxation time of the small-scale magnetic helicity, Rm is the magnetic Reynolds number and **F** is related to the flux of the small-scale magnetic helicity.

The simplest form of the magnetic helicity flux is the turbulent diffusive flux of the magnetic helicity, $\mathbf{F} = -\kappa_{_T}\nabla\chi_c$ (Kleeorin *et al.* 2002, 2003b), where the turbulent diffusivity coefficient $\kappa_{_T}$ is measured in units of $\eta_{_T}$ and the function $\chi_c$ is measured in units of $\eta_{_T}/L$. In real systems, the flux of small-scale magnetic helicity can be accompanied by some flux of large-scale magnetic helicity. However, the flux of large-scale magnetic helicity does not explicitly enters in the dynamical equation (15) for the evolution of the small-scale magnetic helicity. This flux mostly affects the large-scale magnetic helicity. It can also introduce an additional anisotropy of turbulence, which can affect the dynamics of the mean magnetic field.

Equation (15) determines the dynamics of the small-scale magnetic helicity, i.e. its production, dissipation and transport. For very large magnetic Reynolds numbers (which are typical for many astrophysical situations), the relaxation term $\chi_c/\tau_\chi$ is very small, and it is very often dropped in equation (15) in spite of the fact that the small yet finite magnetic diffusion is required for the reconnection of magnetic field lines. In particular, the magnetic Reynolds number, Rm does not enter into the steady state solution of equation (15) in the limit of very large Rm due to the effect of the magnetic helicity flux (Kleeorin *et al.* 2003b).

The account for the dynamics of the small-scale magnetic helicity results in that the mean magnetic field is determined by the following dimensionless equations

$$\frac{\partial A(t,z)}{\partial t} = D\,\sigma_{_N}(B)\, B_y' + B_y\, \Phi_{_N}(B)\, \chi_c(t,z) + A'' , \tag{16}$$

$$\frac{\partial B_y(t,z)}{\partial t} = -A' + B_y'' , \tag{17}$$

$$\frac{\partial \chi_c(t,z)}{\partial t} - \kappa_{_T}\, \chi_c'' + \frac{\chi_c}{\tau_\chi} = C\left(A'\, B_y' - B_y\, [D\,\sigma_{_N}(B)\, B_y' + B_y\, \Phi_{_N}(B)\, \chi_c(t,z) + A'']\right), \tag{18}$$

where $\alpha_m = S_*^{-1}\,\Phi_{_N}(B)\,\chi_c(t,z)$ is the magnetic $\alpha$ effect and the parameter $C = (2L/l_0)^2$. In equations (17) and (18), we have neglected small terms $\sim O(S_*^{-2}) \ll 1$. The algebraic function $\Phi_{_N}(B)$ in these equations is given by

$$\Phi_{_N}(B) = \frac{3}{8B^2}\left[1 - \frac{\arctan(\sqrt{8}B)}{\sqrt{8}B}\right], \tag{19}$$

(see, e.g. Rogachevskii and Kleeorin 2000), where $\Phi_{_N}(B) = 1 - (24/5)B^2$ for $B \ll 1/\sqrt{8}$ and $\Phi_{_N}(B) = 3/(8B^2)$ for $B \gg 1/\sqrt{8}$. Here the mean magnetic field $B$ is measured in units of the equipartition field $B_{\rm eq}$ determined by the turbulent kinetic energy. In equations (16)–(18) there are four parameters: the dynamo number $D = 4\,S_*^2\,\sigma_{_B}/C$, the turbulent diffusivity of the small-scale magnetic helicity $\kappa_{_T}$ (measured in units of $\eta_{_T}$), the parameter $C = (2L/l_0)^2$ and the relaxation time of the small-scale magnetic helicity $\tau_\chi = (4/3){\rm Rm}/C$. Consider the simple boundary conditions for a layer of thickness $2L$ in the $z$ direction: $B_y(t, |z| = 1) = 0$, $A'(t, |z| = 1) = 0$ and $\chi_c(t, |z| = 1) = 0$, where $z$ is measured in units of $L$. The initial conditions for the symmetric mode are chosen in the form: $B_y(t = 0, z) = B_0\,\cos(\pi z/2)$, $A(t = 0, z) = 0$ and $\chi_c(t = 0, z) = 0$.

Numerical solutions of equations (16)–(18) for the nonlinear problem with the algebraic and dynamic nonlinearities are plotted in figures 9–13. In particular, the nonlinear evolution of the mean magnetic field $B(t, z = 0)$ for different values of the parameters $\kappa_{_T}$, $C$, the dynamo numbers $D$ and very large magnetic Reynolds numbers Rm is shown in figures 9–13. Inspection of figures 9–10 shows that the saturated level of the mean magnetic field depends strongly on the value of the turbulent diffusivity of the magnetic helicity $\kappa_{_T}$. The saturated level of the mean magnetic field changes from very small value for $\kappa_{_T} = 0.1$ to the super-equipartition field for $\kappa_{_T} = 10$. This is an indication of very important role of the transport of the magnetic helicity for the saturated level of the mean magnetic field. Indeed, during the generation of the mean magnetic field, the product $\boldsymbol{\mathcal{E}}\cdot\mathbf{B}$ is positive, and this produces negative contribution to the small-scale magnetic helicity (and negative magnetic $\alpha$ effect, see equation (15)). Therefore, this reduces the rate of generation of large-scale magnetic field because

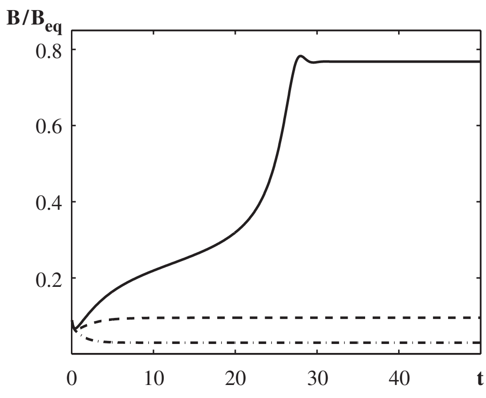


Figure 9. The nonlinear evolution of the mean magnetic field $B(t, z = 0)$ due to the shear-current effect with the algebraic and dynamic nonlinearities for $\epsilon = 0$, $D = 2\,D_{\rm cr}$, $C = 100$ and different values of the parameter $\kappa_{_T}$: $\kappa_{_T} = 0.5$ (solid); $\kappa_{_T} = 0.3$ (dashed); $\kappa_{_T} = 0.1$ (dashed-dotted).

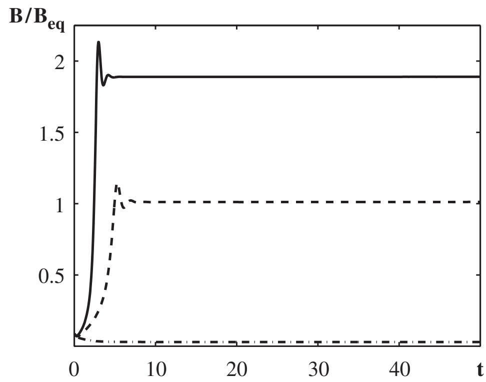


Figure 10. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic and dynamic nonlinearities for $\epsilon=0$, $D=2\,D_{\rm cr}$, $C=100$ and different values of the parameter $\kappa_{_T}$: $\kappa_{_T}=10$ (solid); $\kappa_{_T}=1$ (dashed); $\kappa_{_T}=0.1$ (dashed-dotted).

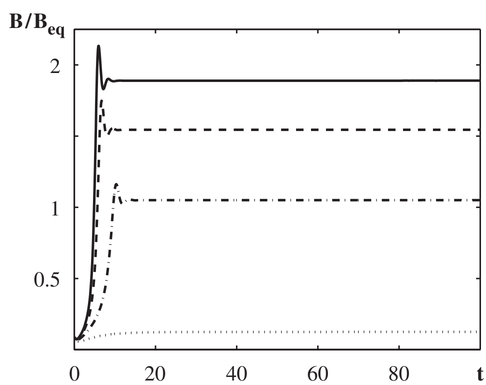


Figure 11. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic and dynamic nonlinearities for $\epsilon=0$, $\kappa_{_T}=10$, $D=2\,D_{\rm cr}$ and different values of the parameter $C$: $C=100$ (solid); $C=300$ (dashed); $C=900$ (dashed-dotted); $C=2700$ (dotted).

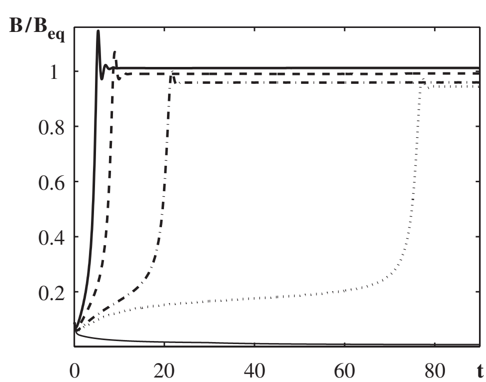


Figure 12. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic and dynamic nonlinearities for $\epsilon=0$, $\kappa_{_T}=1$, $C=100$ and different values of the dynamo number: (a) $D=2\,D_{\rm cr}$ (thick solid); $D=1.7\,D_{\rm cr}$ (dashed); $D=1.5\,D_{\rm cr}$ (dashed-dotted); $D=1.43\,D_{\rm cr}$ (dotted); $D=1.1\,D_{\rm cr}$ (thin solid).

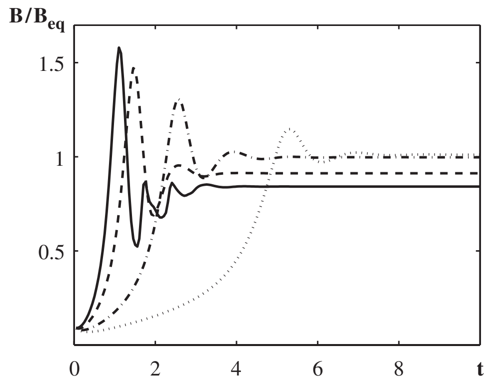


Figure 13. The nonlinear evolution of the mean magnetic field $B(t, z=0)$ due to the shear-current effect with the algebraic and dynamic nonlinearities for $\epsilon = 0$, $\kappa_T = 1$, $C = 100$ and different values of the dynamo number: (a) $D = 7\,D_{\rm cr}$ (solid); $D = 5\,D_{\rm cr}$ (dashed); $D = 3\,D_{\rm cr}$ (dashed-dotted); $D = 2\,D_{\rm cr}$ (dotted).

Table 1. The saturated mean magnetic field versus the magnetic Reynolds number.

| | $\kappa_T = 0.1$ | $\kappa_T = 0.3$ |
|---|---|---|
| Rm | $B/B_{\rm eq}$ | $B/B_{\rm eq}$ |
| 7.5 | 1.16 | 1.2 |
| 15 | 0.89 | 1 |
| 16.5 | 0.37 | 0.97 |
| 17 | 0.2 | 0.967 |
| 30 | 0.1 | 0.83 |
| 36 | 0.085 | 0.35 |
| 50 | 0.076 | 0.16 |
| $10^2$ | 0.05 | 0.12 |
| $10^6$ | 0.03 | 0.096 |

the first and the second terms in the right-hand side of equation (16) have opposite signs. The first term in equation (16) describes the shear-current effect, while the second term in equation (16) determines the magnetic $\alpha$ effect. If the magnetic helicity does not effectively transported out from the generation region, the mean magnetic field is saturated even at small value of the magnetic field. Increase of the magnetic helicity flux (by increasing the turbulent diffusivity $\kappa_T$ of magnetic helicity) results in increase of the saturated level of the mean magnetic field above the equipartition field (figures 9–10). Note that the increase in the parameter $C$ decreases the saturated level of the mean magnetic field (figure 11). Actually the ratio $\kappa_T/C$ determines the saturated level of the mean magnetic field in a steady-state.

For the cases shown in figures 9–13, we drop the small relaxation term $\chi_c/\tau_\chi$ in equation (15) due to very large magnetic Reynolds numbers. Now we consider moderate magnetic Reynolds numbers, when the relaxation term $\chi_c/\tau_\chi$ in equation (15) is not small but the flux of magnetic helicity is weak (e.g., $\kappa_T = 0.1 - 0.3$). In this case, the small-scale magnetic helicity does not effectively transported out from the generation region by the helicity flux. In table 1, we demonstrate the effect of the moderate magnetic Reynolds numbers on the saturated level of the mean magnetic field.

The decrease of the magnetic Reynolds numbers (and the relaxation time $\tau_\chi$) increases the saturated level of the mean magnetic field (table 1), because the relaxation term $\chi_c/\tau_\chi$ in equation (15) decreases the small-scale magnetic helicity in the generation region. On the other hand, for larger flux of small-scale magnetic helicity ($\kappa_{_T} \geq 0.5$), the effect of the magnetic Reynolds numbers on the saturated level of the mean magnetic field is very small. Note that the moderate magnetic Reynolds numbers $\mathrm{Rm} = 10$–$50$ are irrelevant for astrophysical applications, although they are of an interest for the direct numerical simulations.

Figures 12–13 show the nonlinear evolution of the mean magnetic field $B(t, z = 0)$ for different values of the dynamo numbers $D$. The saturated level of the mean magnetic field increases with the increase of the dynamo numbers $D$ within the range $D_{\rm cr} < D < 2\,D_{\rm cr}$, and it decreases with the increase of the dynamo number for $D > 2\,D_{\rm cr}$. This is a new feature in the nonlinear mean-field dynamo. For example, in the $\alpha\Omega$ dynamo the saturated level of the mean magnetic field usually increases with the increase the dynamo numbers.

Generation of the large-scale magnetic field in a nonhelical turbulence with an imposed mean velocity shear has been recently investigated by Brandenburg (2005) and Brandenburg *et al.* (2005) using direct numerical simulations. The results of these numerical simulations are in a good agreement with the numerical solutions of the nonlinear dynamo equations (16)–(18) discussed in section 4.

Now let us compare the results of the numerical solutions of the nonlinear dynamo equations (16)–(18) with the numerical study by Brandenburg and Subramanian (2005b) of the mean-field dynamo with a large-scale shear. In our study, we use the expression for the nonlinear electromotive force determined by Rogachevskii and Kleeorin (2004, 2006), which includes the nonlinear shear-current effect. On the other hand, Brandenburg and Subramanian (2005b) use very simplified form of the mean electromotive force, neglecting e.g., the $\kappa$ effect related to the symmetric parts of the gradient tensor of the mean magnetic field. This effect contributes to the shear-current effect (Rogachevskii and Kleeorin 2003, 2004). Brandenburg and Subramanian (2005b) have not taken into account the properties of the nonlinear shear-current effect found by Rogachevskii and Kleeorin (2004, 2006). In particular, there is no quenching of the nonlinear shear-current effect contrary to the quenching of the nonlinear alpha effect, the nonlinear turbulent magnetic diffusion, etc. During the nonlinear growth of the mean magnetic field, the shear-current effect only changes its sign at some value of the mean magnetic field which affects the level of the saturated mean magnetic field (Rogachevskii and Kleeorin 2004, 2006). In our study, we neglect small terms $\sim \mathrm{O}(S_*^{-2}) \ll 1$ in equations (16)–(18), i.e., we do not consider $\alpha_m^2$ effect because the parameter $S_*$ should be very large (section 3). In addition, we do not consider $\delta^2\,S$ effect because we neglected the small terms $\sim \mathrm{O}[(l_0/L)^2]$ in equation (17). Here, the parameter $\delta$ determines the $\mathbf{W} \times \mathbf{J}$ term in the mean electromotive force. In our numerical solutions of the nonlinear mean-field dynamo equations, we use the simplest form of the magnetic helicity flux (i.e., we use the turbulent diffusive flux of the magnetic helicity, $\mathbf{F} = -\kappa_{_T} \boldsymbol{\nabla} \chi_c$, where $\kappa_{_T}$ is not small), while Brandenburg and Subramanian (2005b) use the current helicity flux of Vishniac and Cho (2001) and a very small turbulent diffusive flux of the magnetic helicity (with $\kappa_{_T} = 10^{-2}$).

The parameter range in the study by Brandenburg and Subramanian (2005b) is different from that used in our study, and the maximum saturated level of the mean magnetic field obtained in the study by Brandenburg and Subramanian (2005b) is $B = 0.6B_{\rm eq}$ at $\rm Rm = 10^2$, which strongly decreases with increase the magnetic Reynolds number Rm (see table 4 of Brandenburg and Subramanian 2005b). On the other hand, in our numerical solutions of the nonlinear mean-field dynamo equations which take into account the shear-current effect, the saturated level of the mean magnetic field reaches the super-equipartition field. These are the reasons why our numerical results discussed in this section are different from those obtained by Brandenburg and Subramanian (2005b). Note, however, that increase of the saturated level of the mean magnetic field with the decrease of the magnetic Reynolds numbers in the case of very weak flux of small-scale magnetic helicity (found by Brandenburg and Subramanian 2005b), is confirmed by our numerical study (table 1).

## 5. Discussion

In this study, we show that in a sheared nonhelical homogeneous turbulence, the large-scale magnetic field can grow due to the shear-current effect from a very small seeding magnetic field. The shear-current dynamo strongly depends on the spatial scaling of the correlation time $\tau(k)$ of the background turbulence. In particular, for Kolmogorov scaling, $\tau(k) \propto k^{-2/3}$, the dynamo instability due to the shear-current effect occurs, while when $\tau(k) \propto k^{-2}$ (for small hydrodynamic and magnetic Reynolds numbers) there is no dynamo action in a sheared nonhelical turbulence. The dynamo instability is saturated by the nonlinear effects, and the dynamical nonlinearity due to the evolution of small-scale magnetic helicity, plays a crucial role in nonlinear saturation of the large-scale magnetic field. The magnetic helicity flux strongly affects the saturated level of the mean magnetic field in the nonlinear stage of the dynamo action. In particular, our numerical solutions of the nonlinear mean-field dynamo equations which take into account the shear-current effect, show that if the magnetic helicity flux is not small, the saturated level of the mean magnetic field is of the order of the equipartition field determined by the turbulent kinetic energy.

The shear-current dynamo acts also in inhomogeneous turbulence. However, in inhomogeneous turbulence with a large-scale velocity shear the kinetic helicity and the hydrodynamic $\alpha$ effect are not zero (Rogachevskii and Kleeorin 2003, 2006, Rädler and Stepanov 2006). In this case, the shear-current dynamo acts together with the $\alpha$-shear dynamo (which is similar to the $\alpha\Omega$ dynamo). The joint action of the shear-current and the $\alpha$-shear dynamo has been recently discussed by Rogachevskii and Kleeorin (2006) and Pipin (2006).

Turbulence with a large-scale velocity shear is a universal feature in astrophysics, and the obtained results can be important for explanation of the large-scale magnetic fields generated in astrophysical sheared turbulence. Rogachevskii *et al.* (2006) have suggested that the shear-current effect might be considered as an origin for the large-scale magnetic fields in colliding protogalactic clouds and in merging protostellar clouds.

Note that the problem of the generation of the mean magnetic field in a turbulence with large-scale velocity shear is similar to that for generation of mean vorticity in a sheared turbulence. The instability of the perturbations of the mean vorticity in a turbulence with a large-scale linear velocity shear was studied by Elperin *et al.* (2003). This instability is caused by a combined effect of the large-scale shear motions (skew-induced deflection of equilibrium mean vorticity due to the shear) and Reynolds-stress-induced generation of perturbations of mean vorticity. In Appendix A, we compare the problem of generation of the mean magnetic field in a turbulence with a large-scale velocity shear with that of the generation of mean vorticity in a sheared turbulence.

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## References

Beck, R., Magnetic fields in the Milky Way and other spiral galaxies. In *How Does the Galaxy Work?*, edited by E.J. Alfaro, E. Pérez and J. Franco, pp. 277–286, 2004 (Kluwer: Dordrecht).

Beck, R., Magnetic fields in galaxies. In *Cosmic Magnetic Fields*, edited by R. Wielebinski and R. Beck, pp. 41–68, 2005 (Springer: Berlin).

Blackman, E.G. and Brandenburg, A., Dynamic nonlinearity in large scale dynamos with shear. *Astrophys. J.*, 2002, **579**, 359–373.

Blackman, E.G. and Field, G., Constraints on the magnitude of alpha in dynamo theory. *Astrophys. J.*, 2000, **534**, 984–988.

Blackman, E.G. and Field, G., New dynamical mean-field dynamo theory and closure approach. *Phys. Rev. Lett.*, 2002, **89**, 265007(1–4).

Brandenburg, A., The case for a distributed solar dynamo shaped by near-surface shear. *Astrophys. J.*, 2005, **625**, 539–547.

Brandenburg, A. and Subramanian, K., Astrophysical magnetic fields and nonlinear dynamo theory. *Phys. Rept.*, 2005a, **417**, 1–209.

Brandenburg, A. and Subramanian, K., Strong mean field dynamos require supercritical helicity fluxes. *Astron. Nachr.*, 2005b, **326**, 400–408.

Brandenburg, A., Käpylä, P. and Mohammed, A., Non-Fickian diffusion and tau-approximation from numerical turbulence. *Phys. Fluids*, 2004, **16**, 1020–1027.

Brandenburg, A., Haugen, N.E.L., Käpylä, P.J. and Sandin, C., The problem of small and large scale fields in the solar dynamo. *Astron. Nachr.*, 2005, **326**, 174–185.

Elperin, T., Kleeorin, N. and Rogachevskii I., Generation of large-scale vorticity in a homogeneous turbulence with a mean velocity shear. *Phys. Rev. E*, 2003, **68**, 016311 (1–8).

Gruzinov, A. and Diamond, P.H., Self-consistent theory of mean-field electrodynamics. *Phys. Rev. Lett.*, 1994, **72**, 1651–1653.

Gruzinov, A. and Diamond, P.H., Nonlinear mean-field electrodynamics of turbulent dynamos. *Phys. Plasmas*, 1996, **3**, 1853–1857.

Kleeorin, N. and Rogachevskii, I., Magnetic helicity tensor for an anisotropic turbulence. *Phys. Rev. E*, 1999, **59**, 6724–6729.

Kleeorin, N. and Ruzmaikin, A., Dynamics of the averaged turbulent helicity in a magnetic field. *Magnetohydrodynamics*, 1982, **18**, 116–122. Translation from *Magnitnaya Gidrodinamika*, 1982, **2**, 17–24.

Kleeorin, N., Rogachevskii, I. and Ruzmaikin, A., Magnetic force reversal and instability in a plasma with advanced magnetohydrodynamic turbulence. *Sov. Phys. JETP*, 1990, **70**, 878-883. Translation from *Zh. Eksp. Teor. Fiz.*, 1990, **97**, 1555–1565.

Kleeorin, N., Rogachevskii, I. and Ruzmaikin, A., Magnitude of dynamo - generated magnetic field in solar - type convective zones. *Astron. Astrophys.*, 1995, **297**, 159–167.

Kleeorin, N., Mond, M. and Rogachevskii, I., Magnetohydrodynamic turbulence in the solar convective zone as a source of oscillations and sunspots formation. *Astron. Astrophys.*, 1996, **307**, 293–309.

Kleeorin, N., Moss, D., Rogachevskii, I. and Sokoloff, D., Helicity balance and steady-state strength of dynamo generated galactic magnetic field. *Astron. Astrophys.*, 2000, **361**, L5–L8.

Kleeorin N., Moss, D., Rogachevskii, I. and Sokoloff, D., The role of magnetic helicity transport in nonlinear galactic dynamos. *Astron. Astrophys.*, 2002, **387**, 453–462.

Kleeorin N., Kuzanyan, K., Moss, D., Rogachevskii, I., Sokoloff, D. and Zhang, H., Magnetic helicity evolution during the solar activity cycle: observations and dynamo theory. *Astron. Astrophys.*, 2003a, **409**, 1097–1105.

Kleeorin N., Moss, D., Rogachevskii, I. and Sokoloff, D., Nonlinear magnetic diffusion and magnetic helicity transport in galactic dynamos. *Astron. Astrophys.*, 2003b, **400**, 9–18.
Krause, F. and Rädler, K.-H., *Mean-Field Magnetohydrodynamics and Dynamo Theory*, 1980 (Pergamon: Oxford).
McComb, W.D., *The Physics of Fluid Turbulence*, 1990 (Clarendon: Oxford).
Moffatt, H.K., *Magnetic Field Generation in Electrically Conducting Fluids*, 1978 (Cambridge University Press: New York).
Monin, A.S. and Yaglom, A.M., *Statistical Fluid Mechanics*, 1975, Vol. 2 (MIT Press: Cambridge, Massachusetts).
Orszag, S.A., Analytical theories of turbulence. *J. Fluid Mech.*, 1970, **41**, 363–386.
Ossendrijver, M., The solar dynamo. *Astron. Astrophys. Rev.*, 2003, **11**, 287–367.
Parker, E., *Cosmical Magnetic Fields*, 1979 (Oxford University Press: New York).
Pipin, V.V., The mean electro-motive force, current- and cross-helicity under the influence of rotation, magnetic field and shear. *ArXiv: astro-ph/0606265*, 2006.
Pouquet, A., Frisch, U. and Leorat, J., Strong MHD turbulence and the nonlinear dynamo effect. *J. Fluid Mech.*, 1976, **77**, 321–354.
Rädler K.-H. and Stepanov R., Mean electromotive force due to turbulence of a conducting fluid in the presence of mean flow. *Phys. Rev. E*, 2006, **73**, 056311 (1–15).
Rädler, K.-H. Kleeorin, N. and Rogachevskii, I., The mean electromotive force for MHD turbulence: the case of a weak mean magnetic field and slow rotation. *Geophys. Astrophys. Fluid Dynam.*, 2003, **97**, 249–274.
Roberts, P.H. and Soward, A.M., Dynamo theory. *Annu. Rev. Fluid Mech.*, 1992, **24**, 459–512.
Rogachevskii, I. and Kleeorin, N., Electromotive force for an anisotropic turbulence: intermediate nonlinearity. *Phys. Rev. E*, 2000, **61**, 5202–5210.
Rogachevskii, I. and Kleeorin, N., Electromotive force and large-scale magnetic dynamo in a turbulent flow with a mean shear. *Phys. Rev. E*, 2003, **68**, 036301 (1–12).
Rogachevskii, I. and Kleeorin, N., Nonlinear theory of a "shear-current" effect and mean-field magnetic dynamos. *Phys. Rev. E*, 2004, **70**, 046310 (1–15).
Rogachevskii, I. and Kleeorin, N., Effects of differential and uniform rotation on nonlinear electromotive force in a turbulent flow. *ArXiv: astro-ph/0407375*, 2006, v3.
Rogachevskii, I., Kleeorin, N., Chernin, A. D. and Liverts, E., New mechanism of generation of large-scale magnetic fields in merging protogalactic and protostellar clouds. *Astron. Nachr.*, 2006, **327**, 591–594.
Ruderman, M.S. and Ruzmaikin, A.A., Magnetic field generation in an anisotropically conducting fluid. *Geophys. Astrophys. Fluid Dynam.*, 1984, **28**, 77–88.
Rüdiger, G. and Kichatinov, L.L., Do mean-field dynamos in nonrotating turbulent shear-flows exist? *Astron. Nachr.*, 2006, **327**, 298–303.
Ruzmaikin, A.A., Shukurov, A.M. and Sokoloff, D.D., *Magnetic Fields of Galaxies*, 1988 (Kluwer : Dordrecht).
Stix, M., *The Sun: An Introduction*, 1989 (Springer: Berlin and Heidelberg).
Urpin, V., Turbulent dynamo action in a shear flow, *Monthly Not. Roy. Astron. Soc.*, 1999a, **308**, 741–744.
Urpin, V., Mean electromotive force and dynamo action in a turbulent flow. *Astron. Astrophys.*, 1999b, **347**, L47–L50.
Vishniac, E.T. and Cho, J., Magnetic helicity conservation and astrophysical dynamos. *Astrophys. J.*, 2001, **550**, 752–760.
Zeldovich, Ya.B., Ruzmaikin, A.A. and Sokoloff, D.D., *Magnetic Fields in Astrophysics*, 1983 (Gordon and Breach: New York).
Zhang, H., Sokoloff, D., Rogachevskii, I., Moss, D., Lamburt, V., Kuzanyan, K. and Kleeorin, N., The Radial distribution of magnetic helicity in solar convective zone: observations and dynamo theory. *Monthly Not. Roy. Astron. Soc.*, 2006, **365**, 276–286.

## APPENDIX

### A. *Threshold for generation of mean vorticity in a sheared turbulence*

The problem of generation of the mean magnetic field in a turbulence with large-scale velocity shear is similar to that of generation of mean vorticity in a sheared turbulence. Indeed, let us discuss the generation of the mean vorticity in a turbulence with a large-scale linear velocity shear, $\mathbf{U} = (0, Sx, 0)$ and $\mathbf{W} = (0, 0, S)$.

Perturbations of the mean vorticity $\tilde{\mathbf{W}} = (\tilde{W}_x(z), \tilde{W}_y(z), 0)$ are determined by the following equations:

$$\frac{\partial \tilde{W}_x}{\partial t} = S\tilde{W}_y + \nu_{_T} \tilde{W}''_x \,, \tag{A.1}$$

$$\frac{\partial \tilde{W}_y}{\partial t} = -\sigma_{_W} \, S \, l_0^2 \, \tilde{W}''_x + \nu_{_T} \tilde{W}''_y \,, \tag{A.2}$$

(see for details Elperin *et al.* 2003), where $\nu_{_T}$ is the turbulent viscosity, $\tilde{W}''_i = \partial^2 \tilde{W}_i / \partial z^2$ and the parameter $\sigma_{_W}$ is given by equation (A.4) below. The first term, $S\tilde{W}_y$, in the right- hand side of equation (A.1) determines a skew-induced generation of perturbations of the mean vorticity $\tilde{W}_x$ by deflection of the equilibrium mean vorticity $\mathbf{W}$, where $\tilde{\mathbf{U}}$ are the perturbations of the mean velocity. In particular, the mean vorticity $\tilde{W}_x \mathbf{e}_x$ is generated from $\tilde{W}_y \mathbf{e}_y$ by equilibrium shear motions with the mean vorticity $\mathbf{W} = S\,\mathbf{e}_z$, i.e., $S\,\tilde{W}_y \mathbf{e}_x \propto (\mathbf{W} \cdot \boldsymbol{\nabla}) \tilde{U}_x \mathbf{e}_x \propto \tilde{W}_y \mathbf{e}_y \times \mathbf{W}$. Here $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors along $x$, $y$ and $z$ axis, respectively. On the other hand, the first term, $-\sigma_{_W} \, S \, l_0^2 \, \tilde{W}''_x$, in the right hand side of equation (A.2) determines a Reynolds-stress-induced generation of perturbations of the mean vorticity $\tilde{W}_y$ by turbulent Reynolds stresses. This implies that the mean vorticity $\tilde{W}_y \mathbf{e}_y$ is generated by an effective anisotropic viscous term $\propto -l_0^2 \, \Delta \, (\tilde{W}_x \mathbf{e}_x \cdot \boldsymbol{\nabla}) \, U(x) \mathbf{e}_y \propto -l_0^2 \, S \, \tilde{W}''_x \mathbf{e}_y$. This mechanism of the generation of perturbations of the mean vorticity $\tilde{W}_y \mathbf{e}_y$ can be interpreted as a stretching of the perturbations of the mean vorticity $\tilde{W}_x \mathbf{e}_x$ by the equilibrium shear motions $\mathbf{U} = S\,x\,\mathbf{e}_y$ during the turnover time of turbulent eddies (Elperin *et al.* 2003).

Note that equations (A.1) and (A.2) for the perturbations of the mean vorticity are very similar to equations (1)-(2) for the perturbations of the mean magnetic field in a sheared turbulence. The growth rate $\gamma$ of the instability of the perturbations of the mean vorticity is given by

$$\gamma = S \, l_0 \, K_z \, \sqrt{\sigma_{_W}} - \nu_{_T} \, K_z^2. \tag{A.3}$$

The form of the growth rate (A.3) of the perturbations of the mean vorticity is also very similar to the growth rate (5) of the mean magnetic field due to the shear-current effect. On the other hand, the magnetic dynamo instability is different from the instability of the perturbations of the mean vorticity although they are governed by similar equations. The mean vorticity $\tilde{\mathbf{W}} = \boldsymbol{\nabla} \times \tilde{\mathbf{U}}$ is directly determined by the velocity field $\tilde{\mathbf{U}}$, while the magnetic field depends on the velocity field through the induction equation and Navier–Stokes equation.

In the present study, we derived a more general form of the parameter $\sigma_{_W}$ for an arbitrary scaling of the correlation time $\tau(k)$ of the turbulent velocity field. The parameter $\sigma_{_W}$ is derived using equation (21) of the article by Elperin *et al.* (2003). It is given by

$$\sigma_{_W} = \frac{4 I_0}{45} \left[ 2 \, \frac{I^2}{I_0^2} + 43 \, \frac{I}{I_0} + 63 \right] , \tag{A.4}$$

where $I$ and $I_0$ are determined by equation (4). The instability depends on the correlation time $\tau(k)$. In particular, when $\tau(k) \propto k^{-\mu}$, the ratio $I/I_0 = -\mu$, and the criterion of the instability reads $2\mu^2 - 43\mu + 63 > 0$, i.e. the instability is excited when $0 \le \mu < 1.58$ and $\mu > 19.9$. Note that the condition $\mu > 19.9$ is not realistic.

When $\tau(k) \propto k^{1-q}$, we recover the result obtained by Elperin *et al*. (2003), i.e. $\sigma_W = 4(2q^2 - 47q + 108)/315$. In particular, for the Kolmogorov scaling, $\tau(k) \propto k^{-2/3}$ (i.e. for $q = 5/3$), we arrive at $\sigma_W \approx 0.45$.

For small hydrodynamic Reynolds numbers, the scaling $\tau(k) \sim 1/(\nu k^2)$, the ratio $I/I_0 = -2$ (i.e. $\mu = 2$), and the parameter $\sigma_W = -4/9 < 0$. This implies that the instability of the perturbations of the mean vorticity does not occur in agreement with the recent results by Rüdiger and Kichatinov (2006). They have not found the instability of the perturbations of the mean vorticity in a random flow with large-scale velocity shear using the second order correlation approximation (SOCA). Note that this approximation is valid only for small hydrodynamic Reynolds numbers [see discussion in section 3 after equation (5)].