Geophysical and Astrophysical Fluid Dynamics,
Vol. 100, No. 3, June 2006, 243–263

Small-scale magnetic buoyancy and magnetic pumping effects in a turbulent convection

IGOR ROGACHEVSKII* and NATHAN KLEEORIN

Department of Mechanical Engineering, The Ben-Gurion University of the Negev, POB 653, Beer-Sheva 84105, Israel

(*Received 9 January 2006; in final form 15 May 2006*)

We determine the nonlinear drift velocities of the mean magnetic field and nonlinear turbulent magnetic diffusion in a turbulent convection. We show that the nonlinear drift velocities are caused by three kinds of the inhomogeneities; i.e., inhomogeneous turbulence, the nonuniform fluid density and the nonuniform turbulent heat flux. The inhomogeneous turbulence results in the well-known turbulent diamagnetic and paramagnetic velocities. The nonlinear drift velocities of the mean magnetic field cause the small-scale magnetic buoyancy and magnetic pumping effects in the turbulent convection. These phenomena are different from the large-scale magnetic buoyancy and magnetic pumping effects which are due to the effect of the mean magnetic field on the large-scale density stratified fluid flow. The small-scale magnetic buoyancy and magnetic pumping can be stronger than these large-scale effects when the mean magnetic field is smaller than the equipartition field. We discuss the small-scale magnetic buoyancy and magnetic pumping effects in the context of the solar and stellar turbulent convection. We demonstrate also that the nonlinear turbulent magnetic diffusion in the turbulent convection is anisotropic even for a weak mean magnetic field. In particular, it is enhanced in the radial direction. The magnetic fluctuations due to the small-scale dynamo increase the turbulent magnetic diffusion of the toroidal component of the mean magnetic field, while they do not affect the turbulent magnetic diffusion of the poloidal field.

Keywords: Turbulent convection; Magnetic buoyancy and magnetic pumping; Magnetic dynamos

1. Introduction

Magnetic fields observed in astrophysical plasma are strongly inhomogeneous (see, e.g., Moffatt 1978, Parker 1979, Krause and Rädler 1980, Zeldovich *et al.* 1983, Ruzmaikin *et al.* 1988, Stix 1989, Roberts and Soward 1992, Kulsrud 1999, and references therein). For instance, the sunspots and the solar active regions are related to the strongly inhomogeneous large-scale magnetic fields. The scales of the magnetic inhomogeneities,

*Corresponding author. Email: gary@bgu.ac.il

Geophysical and Astrophysical Fluid Dynamics
ISSN 0309-1929 print: ISSN 1029-0419 online
http://www.tandf.co.uk/journals
DOI: 10.1080/03091920600813516

e.g., in the Sun are much smaller than the radius of the Sun and usually much larger than the size of granules of the solar convection. One of the mechanisms of the formation of the magnetic inhomogeneities is associated with the magnetic buoyancy instability of stratified continuous magnetic field (see, e.g., Parker 1966, Gilman 1970, Priest 1982). The magnetic buoyancy instability is excited if the scale of variations of the initial magnetic field is less than the density stratification length. This mechanism does not include explicitly magnetic flux tubes. On the other hand, the buoyancy of the magnetic flux tubes as a mechanism of the formation of the magnetic structures was studied in a number of publications (see, e.g., Parker 1955, Spruit 1981, Spruit and van Ballegooijen 1982, Schüssler *et al.* 1994, Moreno-Insertis *et al.* 1996). Note also that the problem of the storage of magnetic fields and the formation of flux tubes in the overshoot layer near the bottom of the solar convective zone was investigated, e.g., by Spiegel and Weiss (1980), Tobias *et al.* (2001), Tobias and Hughes (2004), Brandenburg (2005).

Another universal mechanism of the formation of the nonuniform distribution of magnetic flux in flows of the conducting fluid is a magnetic flux expulsion. In particular, the expulsion of magnetic flux from two-dimensional flows (a single vortex and a grid of vortices) was demonstrated by Weiss (1966). In the context of solar and stellar convection, the topological asymmetry of stationary thermal convection plays a very important role in the magnetic field dynamics (Drobyshevski and Yuferev 1974). Fluid rises at the centers of the convective cells and falls at their peripheries. This results in the ascending fluid elements (contrary to the descending fluid elements) being disconnected from one another. This causes a topological magnetic pumping effect allowing downward transport of the mean horizontal magnetic field to the bottom of a cell but impeding its upward return (Drobyshevski and Yuferev 1974, Zeldovich *et al.* 1983, Galloway and Proctor 1983). The fine structure of a sunspot is determined by the local interaction between magnetic fields and turbulent convection near the Sun's surface. It was shown recently by Thomas *et al.* (2002) that a downward pumping of magnetic flux may cause filamentary structures in sunspot penumbrae. In particular, the magnetic field lines are kept submerged outside the spot by turbulent, compressible convection, which is dominated by strong, coherent, descending plumes.

Turbulence causes additional effects, e.g., the turbulent diamagnetic and paramagnetic drift velocities of the mean magnetic field (Zeldovich 1956, Krause and Rädler 1980, Vainshtein and Kichatinov 1983, Kichatinov 1991, Kichatinov and Rüdiger 1992, Kichatinov and Pipin 1993, Kleeorin and Rogachevskii 2003, Rädler *et al.* 2003, Rogachevskii and Kleeorin 2004). In particular, an inhomogeneity of the velocity fluctuations leads to a transport of mean magnetic flux from regions with high intensity of the velocity fluctuations (turbulent diamagnetism, see, e.g., Zeldovich 1956, Krause and Rädler 1980, Vainshtein and Kichatinov 1983, Kichatinov and Rüdiger 1992, Rädler *et al.* 2003). On the other hand, an inhomogeneity of magnetic fluctuations due to the small-scale dynamo causes turbulent paramagnetic velocity, i.e., the magnetic flux is pushed into regions with high intensity of the magnetic fluctuations (Vainshtein and Kichatinov 1983, Kichatinov 1991, Rädler *et al.* 2003). Other effects are the effective drift velocities of the mean magnetic field caused by inhomogeneities of the fluid density (Kichatinov 1991, Kichatinov and Rüdiger 1992) and pressure (Kichatinov and Pipin 1993). In a nonlinear stage of the magnetic field evolution, inhomogeneities of the mean magnetic field contribute to the diamagnetic or paramagnetic drift velocities depending on the level of magnetic fluctuations due to the small-scale dynamo and level of the mean magnetic field (Rogachevskii and Kleeorin 2004). The diamagnetic velocity causes a drift of the magnetic field components from the regions with a high intensity of the mean magnetic field.

The pumping of magnetic flux in three-dimensional compressible magnetoconvection has been studied in direct numerical simulations by Ossendrijver *et al.* (2002) (see also review by Ossendrijver 2003). The resulting magnetic pumping effects are isolated in the direct numerical simulations by calculating the turbulent diamagnetic and paramagnetic velocities. The pumping effect in the vertical direction is found as a predominating downward advection with a maximum speed in the turbulent convection of about 10% of the turbulent velocity (Ossendrijver *et al.* 2002).

The turbulent diamagnetic and paramagnetic velocities were determined analytically in previous studies only for purely hydrodynamic turbulence. A relation to the turbulent convection was made in some studies (see, e.g., Kichatinov 1991, Kichatinov and Pipin 1993) only phenomenologically, using the equation $\langle \mathbf{u}'^2 \rangle \propto g\tau_0 \langle u'_z s' \rangle$ which follows from the mixing-length theory. Here $\langle u'_z s' \rangle$ is the vertical turbulent heat flux, $\mathbf{u}'$ and s' are fluctuations of fluid velocity and entropy, $\mathbf{g}$ is the acceleration of gravity and τ_0 is the characteristic correlation time of turbulent velocity field. This relationship implies that the vertical turbulent heat flux plays a role of a stirring force for the turbulence. However, a more sophisticated approach implies a solution of a coupled system of dynamical equations which includes the equations for the Reynolds stresses $\langle u'_i u'_j \rangle$, the turbulent heat flux $\langle s' u'_i \rangle$, the entropy fluctuations $\langle s' s' \rangle$, the magnetic fluctuations $\langle b_i b_j \rangle$, the cross helicity tensor $\langle b_i u'_j \rangle$ and $\langle b_i s' \rangle$ in a turbulent convection. The latter has not been taken into account in the previous studies of the small-scale magnetic buoyancy and magnetic pumping effects caused by the turbulent diamagnetic and paramagnetic drift velocities. Note that the turbulent convection can strongly affect these phenomena.

The goal of this study is to determine the nonlinear drift velocities of the mean magnetic field in a turbulent convection. We demonstrate that the nonlinear drift velocities depend on the different kinds of inhomogeneities: (i) inhomogeneous turbulence; (ii) the nonuniform fluid density and (iii) the nonuniform turbulent heat flux. The inhomogeneous turbulence causes the well-known turbulent diamagnetic and paramagnetic velocities. In addition, the nonlinear drift velocities results in the small-scale magnetic buoyancy and magnetic pumping in the turbulent convection. These phenomena are different from the large-scale magnetic buoyancy and magnetic pumping effects. The large-scale phenomena are caused by the influence of the mean magnetic field on the large-scale fluid flow. Our study shows that these large-scale effects are stronger than the small-scale magnetic buoyancy and magnetic pumping only for a strong mean magnetic field (about equipartition field). We study the small-scale magnetic buoyancy and magnetic pumping effects in the context of the solar and stellar turbulent convection. In particular, we demonstrate that in the main part of the solar convective zone the small-scale magnetic pumping effect dominates, while near the solar surface the radial drift velocity of the weak mean magnetic field results in the small-scale magnetic buoyancy effect. We also investigate the anisotropic turbulent magnetic diffusion of the mean magnetic field in the turbulent convection.

This article is organized as follows. In section 2 we formulate the governing equations, the assumptions and the procedure of the derivations. In section 3 we consider the axisymmetric $\alpha\Omega$ dynamo problem and determine the nonlinear drift velocities of the mean magnetic field and nonlinear turbulent magnetic diffusion in a turbulent convection. In section 4 we discuss the small-scale magnetic buoyancy and magnetic pumping effects and make estimates for the solar and stellar turbulent convection. Finally, we draw conclusions in section 5. In appendix A we perform a detailed derivation of the nonlinear drift velocities of the mean magnetic field and nonlinear turbulent magnetic diffusion in a turbulent convection.

2. The governing equations

In this study we investigate the small-scale magnetic buoyancy and magnetic pumping effects in a turbulent convection. These phenomena are determined by the nonlinear drift velocities in the nonlinear electromotive force. In order to derive the nonlinear electromotive force in the turbulent convection, we use a mean field approach in which the magnetic and velocity fields, and entropy are decomposed into the mean and fluctuating parts, where the fluctuating parts have zero mean values. We assume that there exists a separation of scales, i.e., the maximum scale of turbulent motions l_0 is much smaller than the characteristic scale L of inhomogeneities of the mean fields. Here we adopt a procedure of the derivation of the nonlinear electromotive force which was applied previously by Rogachevskii and Kleeorin (2004) for the hydrodynamic incompressible turbulence. Let us outline here the procedure of the derivation of the nonlinear electromotive force for the turbulent convection (for details, see also appendix A). We consider a nonrotating turbulent convection with large Rayleigh numbers and large hydrodynamic and magnetic Reynolds numbers. The equations for fluctuations of the fluid velocity, entropy and the magnetic field are given by

$$\frac{1}{\sqrt{\rho_0}}\frac{\partial \mathbf{v}(\mathbf{x},t)}{\partial t} = -\boldsymbol{\nabla}\left(\frac{p}{\rho_0}\right) + \frac{1}{\sqrt{\rho_0}}\left[(\mathbf{b}\cdot\boldsymbol{\nabla})\mathbf{H} + (\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{b} + \frac{\Lambda_\rho}{2}[2\mathbf{e}(\mathbf{b}\cdot\mathbf{H}) - (\mathbf{b}\cdot\mathbf{e})\mathbf{H}]\right] - \frac{\mathbf{g}}{\sqrt{\rho_0}}\, s + \mathbf{v}^N, \tag{1}$$

$$\frac{\partial \mathbf{b}(\mathbf{x},t)}{\partial t} = (\mathbf{H}\cdot\boldsymbol{\nabla})\mathbf{v} - (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{H} + \frac{\Lambda_\rho}{2}[\mathbf{v}(\mathbf{H}\cdot\mathbf{e}) - \mathbf{H}(\mathbf{v}\cdot\mathbf{e})] + \mathbf{b}^N, \tag{2}$$

$$\frac{\partial s(\mathbf{x},t)}{\partial t} = -\frac{\Omega_b^2}{g}(\mathbf{v}\cdot\mathbf{e}) + s^N, \tag{3}$$

where we use new variables ($\mathbf{v}$, s, $\mathbf{H}$) for fluctuating fields $\mathbf{v} = \sqrt{\rho_0}\,\mathbf{u}'$ and $s = \sqrt{\rho_0}\, s'$, and also for the mean field $\mathbf{H} = \mathbf{B}/(\mu\sqrt{\rho_0})$. Here $\mathbf{B}$ is the mean magnetic field, ρ_0 is the fluid density, μ is the magnetic permeability of the fluid, $\mathbf{e}$ is the vertical unit vector, $\Omega_b^2 = -\mathbf{g}\cdot\boldsymbol{\nabla}S$ is the Brunt-Väisälä frequency, S is the mean entropy, $\mathbf{g}$ is the acceleration of gravity, $\mathbf{u}'$, $\mathbf{b}$ and s' are fluctuations of velocity, magnetic field and entropy (for simplicity of notations we omitted prime in $\mathbf{b}$ because we did not use new variables for magnetic fluctuations), $\mathbf{v}^N$, $\mathbf{b}^N$ and s^N are the nonlinear terms which include the molecular viscous and diffusion terms, $p = p' + \sqrt{\rho_0}\,(\mathbf{H}\cdot\mathbf{b})$ are the fluctuations of total pressure, p' are the fluctuations of fluid pressure. Equations (1)–(3) for fluctuations of fluid velocity, entropy and magnetic field are written in the anelastic approximation, which is a combination of the Boussinesq approximation and the condition $\boldsymbol{\nabla}\cdot(\rho_0\,\mathbf{u}') = 0$. The equation, $\boldsymbol{\nabla}\cdot\mathbf{u}' = \Lambda_\rho(\mathbf{u}'\cdot\mathbf{e})$, in the new variables reads: $\nabla \cdot \mathbf{v} = (\Lambda_\rho/2)(\mathbf{v} \cdot \mathbf{e})$, where $\nabla \rho_0/\rho_0 = -\Lambda_\rho \mathbf{e}$. The quantities with the subscript '0' correspond to the hydrostatic nearly isentropic basic reference state, i.e., $\nabla P_0 = \rho_0 \mathbf{g}$ and $\mathbf{g} \cdot [(\gamma P_0)^{-1} \nabla P_0 - \rho_0^{-1} \nabla \rho_0] \approx 0$, where γ is the specific heats ratio and P_0 is the fluid pressure in the basic reference state. The turbulent convection is regarded as a small deviation from a well-mixed adiabatic reference state.

Using equations (1)–(3) written in a Fourier space we derive equations for the two-point second-order correlation functions of the velocity fluctuations $\langle v_i \, v_j \rangle$, the magnetic fluctuations $\langle b_i \, b_j \rangle$, the entropy fluctuations $\langle s \, s \rangle$, the cross-helicity $\langle b_i \, v_j \rangle$, the turbulent heat flux $\langle s \, v_i \rangle$ and $\langle s \, b_i \rangle$. The equations for these correlation functions are given by equations (A.5)–(A.10) in appendix A. We split the tensor of magnetic fluctuations into nonhelical, $h_{ij} = \langle b_i \, b_j \rangle$, and helical, $h_{ij}^{(H)}$, parts. The helical part $h_{ij}^{(H)}$ depends on the magnetic helicity and is determined by a dynamic equation which follows from the magnetic helicity conservation arguments (see below). We also split all second-order correlation functions, $M^{(II)}$, into symmetric and antisymmetric parts with respect to the wave vector $\mathbf{k}$, e.g., $h_{ij} = h_{ij}^{(s)} + h_{ij}^{(a)}$, where the tensor $h_{ij}^{(s)} = [h_{ij}(\mathbf{k}) + h_{ij}(-\mathbf{k})]/2$ describes the symmetric part of the tensor and $h_{ij}^{(a)} = [h_{ij}(\mathbf{k}) - h_{ij}(-\mathbf{k})]/2$ determines the antisymmetric part of the tensor.

The second-moment equations include the first-order spatial differential operators $\hat{\mathcal{N}}$ applied to the third-order moments $M^{(III)}$. A problem arises – how to close the system, i.e., how to express the set of the third-order terms $\hat{\mathcal{N}} M^{(III)}$ through the lower moments $M^{(II)}$ (see, e.g., Orszag 1970, Monin and Yaglom 1975, McComb 1990). We use the spectral τ approximation which postulates that the deviations of the third-moment terms, $\hat{\mathcal{N}} M^{(III)}(\mathbf{k})$, from the contributions to these terms afforded by the background turbulent convection, $\hat{\mathcal{N}} M^{(III,0)}(\mathbf{k})$, are expressed through similar deviations of the second moments, $M^{(II)}(\mathbf{k}) - M^{(II,0)}(\mathbf{k})$:

$$\hat{\mathcal{N}} M^{(III)}(\mathbf{k}) - \hat{\mathcal{N}} M^{(III,0)}(\mathbf{k}) = -\frac{1}{\tau(k)} \, [M^{(II)}(\mathbf{k}) - M^{(II,0)}(\mathbf{k})], \tag{4}$$

(see, e.g., Orszag 1970, Pouquet *et al.* 1976, Kleeorin *et al.* 1990, Kleeorin *et al.* 1996, Blackman and Field 2002, Kleeorin and Rogachevskii 2003, Rogachevskii and Kleeorin 2004, Brandenburg *et al.* 2004, Brandenburg and Subramanian 2005b, Kleeorin and Rogachevskii 2006), where $\tau(k)$ is the scale-dependent relaxation time, which can be identified with the correlation time of the turbulent velocity field. In the background turbulent convection, the mean magnetic field is zero.

We apply the spectral τ approximation only for the nonhelical part h_{ij} of the tensor of magnetic fluctuations. The helical part $h_{ij}^{(H)}$ depends on the magnetic helicity, and it is determined by the dynamic equation which follows from the magnetic helicity conservation arguments (see, e.g., Kleeorin and Ruzmaikin 1982, Gruzinov and Diamond 1994, Kleeorin *et al.* 1995, Gruzinov and Diamond 1996, Kleeorin and Rogachevskii 1999, Kleeorin *et al.* 2000, Blackman and Field 2000, Kleeorin *et al.* 2002, Blackman and Brandenburg 2002, Kleeorin *et al.* 2003, Brandenburg and Subramanian 2005a, Zhang *et al.* 2006, and references therein). The characteristic time of evolution of the nonhelical part of the tensor h_{ij} is of the order of the turbulent time $\tau_0 = l_0/u_0$, while the relaxation time of the helical part of the tensor $h_{ij}^{(H)}$ is of the order of $\tau_0 \, \mathrm{Rm}$, where $\mathrm{Rm} = l_0 u_0/\eta$ is the magnetic Reynolds number (which is very large), u_0 is the characteristic turbulent velocity in the maximum scale of turbulent motions l_0 and

η is the magnetic diffusivity due to electrical conductivity of the fluid. In this study we consider an intermediate nonlinearity which implies that the mean magnetic field is not strong enough in order to affect the correlation time of turbulent velocity field. The theory for a very strong mean magnetic field can be corrected after taking into account a dependence of the correlation time of the turbulent velocity field on the mean magnetic field.

We assume also that the characteristic time of variation of the mean magnetic field **B** is substantially larger than the correlation time $\tau(k)$ for all turbulence scales. This allows us to get a stationary solution for the equations for the second-order moments, $M^{(II)}$. For the integration in **k** space of the second-order moments, we have to specify a model for the background turbulent convection which is determined by equations (A.32)–(A.34) in appendix A. This model takes into account the inhomogeneity of the turbulence described by the two parameters: $\Lambda_i^{(u)} = \nabla_i \langle \mathbf{u}'^2 \rangle^{(0)} / \langle \mathbf{u}'^2 \rangle^{(0)}$ and $\Lambda_i^{(b)} = \nabla_i \langle \mathbf{b}^2 \rangle^{(0)} / \langle \mathbf{b}^2 \rangle^{(0)}$. This model includes also the inhomogeneity of the turbulent heat flux, $\Lambda_i^{(F)} = \nabla_i \langle |\mathbf{u}'| \, s' \rangle^{(0)} / \langle |\mathbf{u}'| \, s' \rangle^{(0)}$, and the inhomogeneity of the fluid density described by the parameter Λ_ρ. The quantities with the superscript (0) correspond to the background turbulent convection with $\mathbf{B} = \mathbf{0}$. Using the solution of the derived second-moment equations, we determine the nonlinear electromotive force, $\mathcal{E}_i = \varepsilon_{imn} \int \langle b_n \, v_m \rangle_{\mathbf{k}} \, d\mathbf{k}$, in the turbulent convection (see appendix A), where ε_{ijk} is the fully antisymmetric Levi-Civita tensor. This allows us to determine the nonlinear drift velocities of the mean magnetic field and nonlinear turbulent magnetic diffusion, and to study the small-scale magnetic buoyancy and magnetic pumping effects in the turbulent convection.

3. The axisymmetric dynamo

Let us consider the axisymmetric $\alpha\Omega$ dynamo problem. The mean magnetic field in the local system of coordinate is $\mathbf{B} = B(x,z)\mathbf{e}_y + \nabla \times [A(x,z)\mathbf{e}_y]$, where $B(x,z)$ and $A(x,z)$ are determined by the dimensionless equations

$$\frac{\partial A}{\partial t} = \alpha(\mathbf{B}) B + \eta_{_A}^{(z)}(\mathbf{B}) \frac{\partial^2 A}{\partial z^2} + \eta_{_A}^{(x)}(\mathbf{B}) \frac{\partial^2 A}{\partial x^2} - (\mathbf{V}_A(\mathbf{B}) \cdot \nabla) A, \tag{5}$$

$$\frac{\partial B}{\partial t} = D \, [\nabla(\delta\Omega) \times \nabla A]_y + \nabla \cdot [\hat{\eta}_{_B}(\mathbf{B}) \nabla B - \mathbf{V}_B(\mathbf{B}) B], \tag{6}$$

$\delta\Omega$ determine the differential rotation, D is the dynamo number (see below), $\alpha(\mathbf{B})$ is the total (hydrodynamic + magnetic) nonlinear α effect (see, e.g., Kleeorin *et al.* 2000, Rogachevskii and Kleeorin 2000, and references therein), $\hat{\boldsymbol{\eta}}_{_B}$ is the diagonal tensor with the components $\eta_{_B}^{(z,x)}(\mathbf{B})$ of the nonlinear turbulent magnetic diffusion of toroidal field, $\eta_{_A}^{(z,x)}(\mathbf{B})$ are the nonlinear turbulent magnetic diffusion coefficients of the poloidal magnetic field, $\mathbf{V}_A(\mathbf{B})$ and $\mathbf{V}_B(\mathbf{B})$ are the nonlinear drift velocities (see below). The axis z of the local system of coordinate is directed opposite to the gravity acceleration **g** and the axis x is in meridional plane and directed to the equator, so that the spherical coordinates (r, θ, φ) translate to the local system of coordinate (z, x, y).

Here we adopt the dimensionless form of the mean dynamo equations; in particular, length is measured in units of L, time is measured in units of the turbulent magnetic diffusion time $L^2/\eta_{_T}$ and **B** is measured in units of the equipartition energy $B_{\rm eq} = \sqrt{\mu \rho_0} \, u_0$, α is measured in units of α_* (the maximum value of the hydrodynamic part of the α effect), the nonlinear turbulent magnetic diffusion coefficients are measured in units of $\eta_{_T} = l_0 u_0/3$, the nonlinear drift velocities $\mathbf{V}_{A,B}(B)$ are measured in the units of $\eta_{_T}/L$, the differential rotation $\delta\Omega$ is measured in units of $\delta\Omega_*$ and the dimensionless parameters $\Lambda^{(u)}, \Lambda^{(b)}$, Λ_ρ and $\Lambda^{(F)}$ are measured in the units of L^{-1}. We define $R_\alpha = L\alpha_*/\eta_{_T}$, $R_\omega = r\,(\mathrm{d}(\delta\Omega_*)/\mathrm{d}r)\,L^2/\eta_{_T}$, and the dynamo number $D = R_\omega R_\alpha$.

The derivation of equation for the nonlinear electromotive force allows us to determine the nonlinear turbulent magnetic diffusion coefficients and the nonlinear drift velocities of the mean magnetic field, which are given by

$$\begin{aligned} \eta^{(z)}_{_{A,B}}(\mathbf{B}) &= \eta^{(v)}_{_{A,B}}(\mathbf{B}) + a_* \eta^{(F,z)}_{_{A,B}}(\mathbf{B}), \\ \eta^{(x)}_{_{A,B}}(\mathbf{B}) &= \eta^{(v)}_{_{A,B}}(\mathbf{B}) + a_* \eta^{(F,x)}_{_{A,B}}(\mathbf{B}), \\ \mathbf{V}_{A,B}(\mathbf{B}) &= \mathbf{V}^{(v)}_{A,B}(\mathbf{B}) + a_* \mathbf{V}^{(F)}_{A,B}(\mathbf{B}). \end{aligned} \tag{7}$$

Here the superscript (v) corresponds to the contributions from the purely hydrodynamic turbulence and the superscript (F) corresponds to the contributions from the turbulent heat flux. These contributions are given by equations (A35)–(A47) in appendix A. The parameter a_* which is determined by the budget equation for the total energy, is given by

$$a_*^{-1} = 1 + \frac{\nu_{_T}(\nabla\mathbf{U})^2 + \eta_{_T}(\nabla B)^2/(\rho\mu)}{gF_*}, \tag{8}$$

where $\mathbf{U}$ is the mean velocity and $\nu_{_T}$ is the turbulent viscosity.

The asymptotic formulae for the nonlinear turbulent magnetic diffusion coefficients and the nonlinear drift velocities for the weak mean magnetic fields, $B \ll B_{\rm eq}/4$, are given by

$$\begin{aligned} &\eta^{(z)}_{_A}(B) = \eta^{(z)}_{_B}(B) = 1 + a_*, \\ &\eta^{(x)}_{_A}(B) = 1 + 0.1 a_*, \qquad \eta^{(x)}_{_B}(B) = 1, \\ &V^{(z)}_A(B) = V^{(z)}_B(B) = -\frac{1}{2}\left[\Lambda^{(u)}_z - \epsilon\Lambda^{(b)}_z - \epsilon\Lambda_\rho + \frac{9a_*}{5}\Lambda_\rho\right], \\ &V^{(x)}_A(B) = V^{(x)}_B(B) = -\frac{1}{2}[\Lambda^{(u)}_x - \epsilon\Lambda^{(b)}_x], \end{aligned} \tag{9}$$

where we neglect the terms $\sim\mathrm{O}(\beta^2)$. Here $\beta = \sqrt{8}\,B/B_{eq}$ and the parameter $\epsilon = \langle \mathbf{b}^2\rangle^{(0)}/\langle \mathbf{v}^2\rangle^{(0)}$. When $B \gg B_{eq}/4$ the nonlinear turbulent magnetic diffusion coefficients and the nonlinear drift velocities are given by

$$\begin{aligned} &\eta^{(z)}_{_A}(B) = \frac{a_*}{\beta}, \qquad \eta^{(x)}_{_A}(B) = \frac{2\,a_*}{5\beta}, \\ &\eta^{(z)}_{_B}(B) = \frac{2\,(1+\epsilon)}{3\beta} + \frac{a_*}{\beta}, \\ &\eta^{(x)}_{_B}(B) = \frac{2\,(1+\epsilon)}{3\beta}, \qquad \mathbf{V}_B(B) = -\frac{a_*}{\beta}\Lambda_\rho \mathbf{e}, \\ &\mathbf{V}_A(B) = -\frac{1+\epsilon}{3\beta}\boldsymbol{\Lambda}^{(B)} - \frac{a_*}{\beta}\Lambda_\rho \mathbf{e}, \end{aligned} \tag{10}$$

where we neglect the terms $\sim\mathrm{O}(\beta^{-2})$.

The nonlinear turbulent magnetic diffusion coefficients $\eta_{A,B}^{(z,x)}$ of the poloidal and toroidal components of the mean magnetic field in the vertical (along the z-axis) and horizontal (along the x-axis) directions are shown in figure 1 for the turbulent convection with $a_* = 0.8$. The magnetic fluctuations due to the small-scale dynamo (described by the parameter ϵ) increase the turbulent magnetic diffusion of the toroidal mean magnetic field (see figure 1b), and they do not affect the turbulent magnetic diffusion of the poloidal field. Note also that the nonlinear turbulent magnetic diffusion in a turbulent convection is anisotropic even for a weak mean magnetic field. In particular, it is enhanced in the vertical (radial) direction.

The vertical nonlinear drift velocities of poloidal and toroidal components of the mean magnetic field in the turbulent convection ($a_* = 0.8$) and in the nonconvective turbulence ($a_* = 0$) are shown in figure 2. The turbulent convection enhances the nonlinear drift velocities of the mean magnetic field in comparison with the case of a purely hydrodynamic turbulence (see figure 2). In the next section we discuss the nonlinear drift velocities of the mean magnetic field in the solar convective zone which cause the small-scale magnetic buoyancy and magnetic pumping effects.

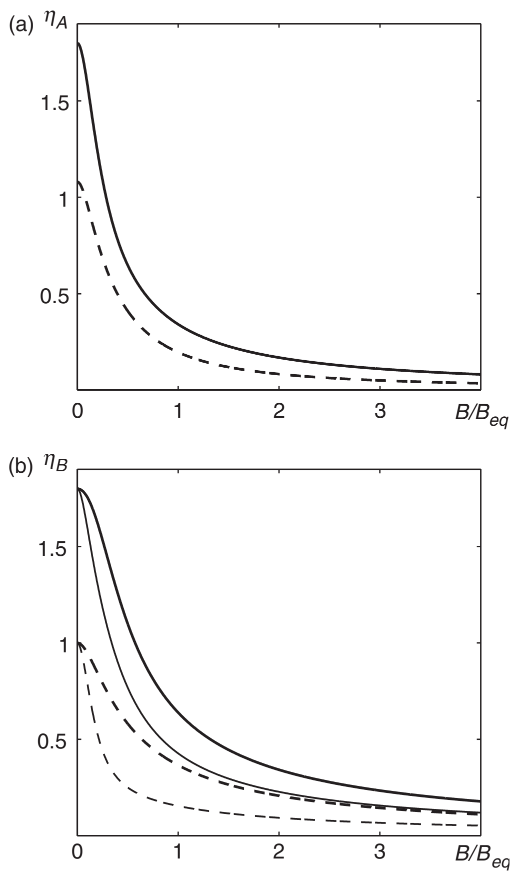

Figure 1. Nonlinear turbulent magnetic diffusion coefficients (a) η_A and (b) η_B in the vertical (solid) and horizontal (dashed) directions in a turbulent convection with $a_* = 0.8$. The thin curves in (b) correspond to $\epsilon = 0$ and thick curves to $\epsilon = 1$.

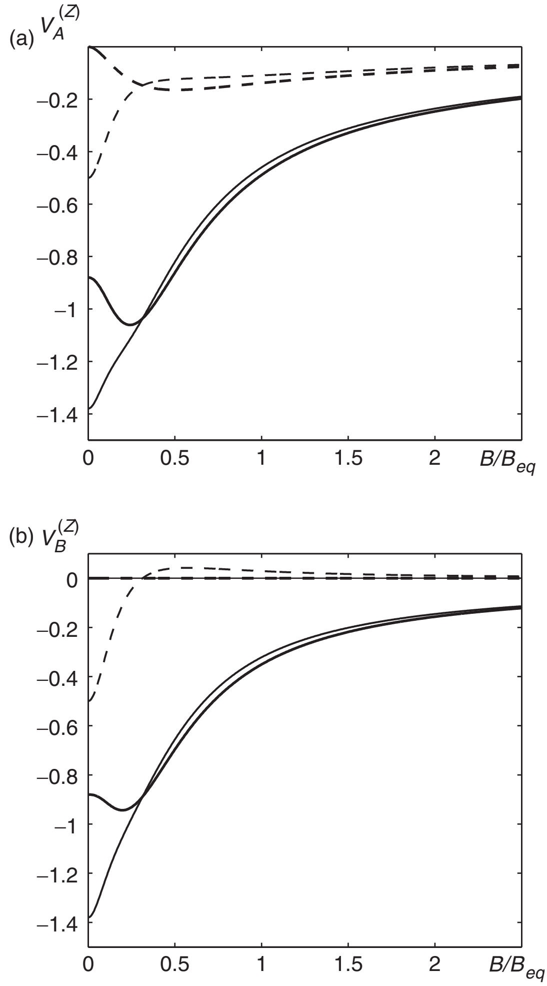

Figure 2. Vertical nonlinear drift velocities (a) $V_A^{(z)}$ and (b) $V_B^{(z)}$ in a turbulent convection with $a_* = 0.8$ (solid) and in a nonconvective turbulence, $a_* = 0$ (dashed) for $\Lambda_z^{(u)} = \Lambda_z^{(F)} = \Lambda_z^{(B)} = \Lambda_\rho = \Lambda_z^{(b)} = 1$. The thin curves correspond to $\epsilon = 0$ and thick curves to $\epsilon = 1$.

4. Discussion

Let us discuss the small-scale magnetic buoyancy and magnetic pumping effects. In figure 3 the vertical nonlinear drift velocities of the toroidal and poloidal magnetic fields are plotted for different depths h of the solar convective zone (measured from the solar surface): $h = 1.7 \times 10^7$ cm (figure 3a); $h = 3.7 \times 10^7$ cm (figure 3b), and $h = 1.9 \times 10^{10}$ cm (figure 3c). In order to estimate the governing parameters we use the models of the solar convective zone (see, e.g., Spruit 1974, Baker and Temesvary 1966). More modern treatments make little difference to these estimates.

In particular, in the upper part of the solar convective zone, say at the depth $h_* \sim 1.7 \times 10^7$ cm, the parameters are as follows: the characteristic turbulent velocity $u_0 \sim 2.2 \times 10^5$ cm s^{-1}; the maximum scale of turbulent motions $l_0 \sim 3.3 \times 10^7$ cm;

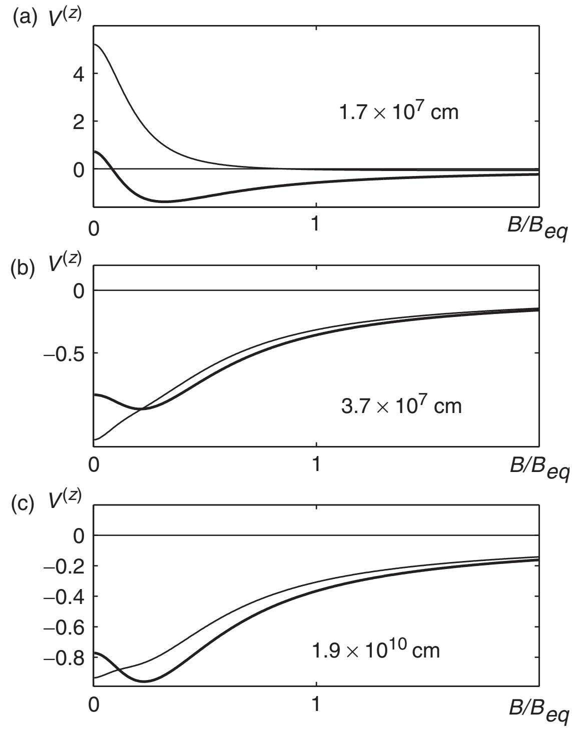

Figure 3. Vertical nonlinear drift velocities ($V_B^{(z)} = V_A^{(z)} \equiv V^{(z)}$) in a turbulent convection with $a_* = 0.8$ for $\Lambda_z^{(b)} = \Lambda_z^{(B)} = 0$, and for different depths h of the convective zone (from the solar surface): (a) $h = 1.7 \times 10^7$ cm; (b) $h = 3.7 \times 10^7$ cm; (c) $h = 1.9 \times 10^{10}$ cm. The thin curves correspond to $\epsilon = 0$ and thick curves to $\epsilon = 1$.

the fluid density $\rho \sim 4.6 \times 10^{-7}\,\mathrm{g\,cm^{-3}}$; the turbulent magnetic diffusion $\eta_{_T} \sim 2.4 \times 10^{12}\,\mathrm{cm^2\,s^{-1}}$; the density stratification scale $\Lambda_\rho^{-1} \sim 10^8$ cm and the characteristic scale of the inhomogeneity of the turbulent magnetic diffusion $\Lambda_\eta^{-1} = |\nabla_r \eta_{_T}/\eta_{_T}|^{-1} \sim 10^7$ cm.

At the depth $h_* \sim 3.7 \times 10^7$ cm, the parameters are $u_0 \sim 1.5 \times 10^5\,\mathrm{cm\,s^{-1}}$; $l_0 \sim 4.5 \times 10^7$ cm; $\rho \sim 8.3 \times 10^{-7}\,\mathrm{g\,cm^{-3}}$; $\eta_{_T} \sim 2.3 \times 10^{12}\,\mathrm{cm^2\,s^{-1}}$; $\Lambda_\rho^{-1} \sim 4 \times 10^7$ cm and $\Lambda_\eta^{-1} \sim 2.2 \times 10^8$ cm.

At the bottom of the solar convective zone, say at the depth $h_* \sim 1.9 \times 10^{10}$ cm, the parameters are $u_0 \sim 2 \times 10^3\,\mathrm{cm\,s^{-1}}$; $l_0 \sim 8.1 \times 10^9$ cm; $\rho \sim 2.1 \times 10^{-1}\,\mathrm{g\,cm^{-3}}$; $\eta_{_T} \sim 5.2 \times 10^{12}\,\mathrm{cm^2\,s^{-1}}$; $\Lambda_\rho^{-1} \sim 6.5 \times 10^9$ cm and $\Lambda_\eta^{-1} \sim 8 \times 10^{10}$ cm.

Figure 3 demonstrates that only near the solar surface the radial drift velocity for a weak mean magnetic field is directed upward to the surface of the Sun. This causes the small-scale magnetic buoyancy effect. However, in the main part of the solar convective zone the radial nonlinear drift velocities of the toroidal and poloidal mean magnetic fields are directed downward. This results in the small-scale magnetic pumping effect. These phenomena are determined by the nonlinear drift velocities in the nonlinear electromotive force, and they are different from the large-scale magnetic buoyancy and magnetic pumping effects. The large-scale phenomena are caused by the effect of the mean magnetic field on the large-scale density stratified fluid flow. These large-scale phenomena are stronger than the small-scale magnetic buoyancy and

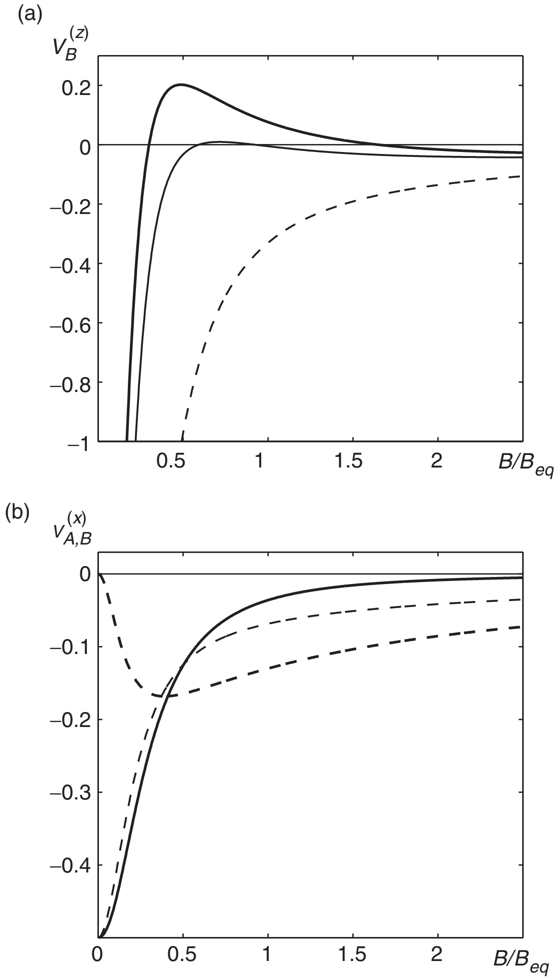

Figure 4. (a) Vertical nonlinear drift velocity $V_B^{(z)}$ of toroidal magnetic field in the overshoot layer with $a_* = 0.8$ for $\Lambda_z^{(u)} = \Lambda_z^{(F)} = 20,\ \Lambda_\rho = \Lambda_z^{(B)} = 1,\ \Lambda_z^{(b)} = \Lambda_z^{(u)} - \Lambda_\rho$. The thick curve corresponds to $\epsilon = 1$, the thin solid curve corresponds to $\epsilon = 0.9$ and thin dashed curve corresponds to $\epsilon = 0.5$. (b) Horizontal nonlinear effective drift velocities $V_{A,B}^{(x)}$ of toroidal (solid) and poloidal (dashed) magnetic fields in the turbulent convection with $a_* = 0.8$ for $\Lambda_x^{(u)} = \Lambda_x^{(b)} = \Lambda_x^{(F)} = \Lambda_x^{(B)} = 1$. The thin curves correspond to $\epsilon = 0$ and thick curves to $\epsilon = 1$.

magnetic pumping effects when the mean magnetic field is larger than the equipartition field. In particular, the ratio of the velocities which correspond to the large-scale and small-scale effects, is of the order of $(B/B_{eq})^2$.

In figure 4 the vertical (figure 4a) and horizontal (figure 4b) nonlinear drift velocities of the toroidal mean magnetic field are plotted for the overshoot layer located at the bottom of the solar convective zone. In this layer the turbulence and the turbulent heat flux are strongly inhomogeneous. The drift velocities in figures 2–4 are measured in the units of $\eta_T\,\Lambda_\rho$. Here we assume that $\Lambda_z^{(b)} = \Lambda_z^{(u)} - \Lambda_\rho$, which implies that $\Lambda_i^{(b)} = \nabla_i(\rho_0\,\langle \mathbf{u}'^2\rangle^{(0)})/(\rho_0\,\langle \mathbf{u}'^2\rangle^{(0)})$. Figure 4(a) demonstrates that the vertical nonlinear drift velocity of the toroidal mean magnetic field depends strongly on the level of the magnetic fluctuations caused by the small-scale dynamo (described by the parameter ϵ). If there is a small deviation from $\epsilon = 1$ (the equipartition between the kinetic and magnetic turbulent energies) there is only the magnetic pumping effect in the overshoot layer. On the other hand, the horizontal nonlinear drift velocity of the toroidal mean magnetic field in the overshoot layer is negative, i.e., it is directed to the solar polar regions (see figure 4b).

The magnetic pumping in three-dimensional compressible rotating magneto-convection has been studied by Ossendrijver *et al.* (2002) in direct numerical simulations (see also review by Ossendrijver 2003). The resulting pumping effects are isolated by calculating the effective drift velocities in turbulent convection. The pumping effects act differently on different components of the mean magnetic field (Ossendrijver *et al.* 2002). This result is in good agreement with our results [see figure 2 and equations (5), (6), (10), (A.37), (A.38)]. The pumping effect in the vertical direction is found to be equivalent to a predominating downward advection with a maximum drift velocity of the order of 10% of the turbulent velocity (Ossendrijver *et al.* 2002). This is in agreement with our theoretical findings (see, e.g., figures 2, 3 and 4a). Note that the effective drift velocity due to the inhomogeneity of the fluid density (see Kichatinov and Rüdiger 1992) also causes a predominating downward drift of the mean magnetic field.

The small-scale magnetic pumping and buoyancy effects were investigated in the present study for large hydrodynamic and magnetic Reynolds numbers using the spectral τ approximation (the third-order closure procedure). Previous analytical studies of the small-scale magnetic pumping and buoyancy effects (see Kichatinov 1991, Kichatinov and Rüdiger 1992, Kichatinov and Pipin 1993) were performed using the second-order correlation approximation (SOCA). This approximation is valid for small hydrodynamic Reynolds numbers. Indeed, even in a highly conductivity limit (large magnetic Reynolds numbers) SOCA is valid only for small Strouhal numbers, while for large hydrodynamic Reynolds numbers (fully developed turbulence) the Strouhal number is 1. In the present study we take into account the inhomogeneity of the fluid density assuming that $\langle \rho u_i' u_j' \rangle$ is weakly inhomogeneous (see equation (A32)). This is in agreement with the models of the solar convective zone (see, e.g., Baker and Temesvary 1966, Spruit 1974). On the other hand, in studies by Kichatinov (1991) and Kichatinov and Rüdiger (1992) it was assumed that $\langle \rho^2 u_i' u_j' \rangle$ is weakly inhomogeneous. Since the density in the solar convective zone varies over six orders of magnitude, the validity of the latter suggestion is questionable.

5. Conclusions

In summary, we study the nonlinear drift of the mean magnetic field in a turbulent convection. Three kinds of inhomogeneities determine the nonlinear drift velocities of the mean magnetic field: (i) the inhomogeneous turbulence; (ii) the nonuniform fluid density and (iii) the nonuniform turbulent heat flux. The inhomogeneous turbulence causes the well-known turbulent diamagnetic and paramagnetic velocities. The nonlinear drift velocities of the mean magnetic field result in the small-scale magnetic buoyancy and magnetic pumping effects in the turbulent convection. In the main part of the solar convective zone, the small-scale magnetic pumping effect dominates (i.e., the radial nonlinear drift velocity of the mean magnetic field is directed downward to the bottom of the convective zone), while near the solar surface the small-scale magnetic buoyancy effect is important when the mean magnetic field is weak. These small-scale phenomena can be stronger than the large-scale magnetic pumping and magnetic buoyancy which are caused by the influence of the mean magnetic field on the stratified fluid flow.

Acknowledgment

An important part of this work was performed during our visit to the Isaac Newton Institute for Mathematical Sciences (University of Cambridge) during the programme 'Magnetohydrodynamics of Stellar Interiors'.

References

Baker N. and Temesvary, S., *Tables of Convective Stellar Envelope Models*, 1966 (NASA: New York).

Blackman, E.G. and Brandenburg, A., Dynamic nonlinearity in large scale dynamos with shear. *Astrophys. J.*, 2002, **579**, 359–373.

Blackman, E.G. and Field, G., Constraints on the magnitude of alpha in dynamo theory. *Astrophys. J.*, 2000, **534**, 984–988.

Blackman, E.G. and Field, G., New dynamical mean-field dynamo theory and closure approach. *Phys. Rev. Lett.*, 2002, **89**, 265007 (1–4).

Brandenburg, A., The case for a distributed solar dynamo shaped by near-surface shear. *Astrophys. J.*, 2005, **625**, 539–547.

Brandenburg, A., Käpylä, P. and Mohammed, A., Non-Fickian diffusion and tau-approximation from numerical turbulence. *Phys. Fluids*, 2004, **16**, 1020–1027.

Brandenburg, A. and Subramanian, K., Astrophysical magnetic fields and nonlinear dynamo theory. *Phys. Rept.*, 2005a, **417**, 1–209.

Brandenburg, A. and Subramanian, K., Minimal tau approximation and simulations of the alpha effect. *Astron. Astrophys.*, 2005b, **439**, 835–843.

Drobyshevski, E.M. and Yuferev, V.S., Topological pumping of magnetic flux by three-dimensional convection. *J. Fluid Mech.*, 1974, **65**, 33–44.

Galloway, D.J. and Proctor, M.R.E., The kinematics of hexagonal magnetoconvection. *Geophys. Astrophys. Fluid Dyn.*, 1983, **24**, 109–136.

Gilman, P.A., Instability of magnetohydrostatic stellar interiors from magnetic buoyancy. *Astrophys. J.*, 1970, **162**, 1019–1029.

Gruzinov, A. and Diamond, P.H., Self-consistent theory of mean-field electrodynamics. *Phys. Rev. Lett.*, 1994, **72**, 1651–1653.

Gruzinov, A. and Diamond, P.H., Nonlinear mean-field electrodynamics of turbulent dynamos. *Phys. Plasmas*, 1996, **3**, 1853–1857.

Kichatinov, L.L., Turbulent transport of magnetic fields in a highly conducting rotating fluid and the solar cycle. *Astron. Astrophys.*, 1991, **243**, 483–491.

Kichatinov, L.L. and Pipin, V.V., Mean-field buoyancy. *Astron. Astrophys.*, 1993, **274**, 647–652.

Kichatinov, L.L. and Rüdiger, G., Magnetic-field advection in inhomogeneous turbulence. *Astron. Astrophys.*, 1992, **260**, 494–498.

Kleeorin, N., Kuzanyan, K., Moss, D., Rogachevskii, I., Sokoloff, D. and Zhang, H., Magnetic helicity evolution during the solar activity cycle: observations and dynamo theory. *Astron. Astrophys.*, 2003, **409**, 1097–1105.

Kleeorin, N., Mond, M. and Rogachevskii, I., Magnetohydrodynamic turbulence in the solar convective zone as a source of oscillations and sunspots formation. *Astron. Astrophys.*, 1996, **307**, 293–309.

Kleeorin, N., Moss, D., Rogachevskii, I. and Sokoloff, D., Helicity balance and steady-state strength of dynamo generated galactic magnetic field. *Astron. Astrophys.*, 2000, **361**, L5–L8.

Kleeorin, N., Moss, D., Rogachevskii, I. and Sokoloff, D., The role of magnetic helicity transport in nonlinear galactic dynamos. *Astron. Astrophys.*, 2002, **387**, 453–462.

Kleeorin, N. and Rogachevskii, I., Magnetic helicity tensor for an anisotropic turbulence. *Phys. Rev. E*, 1999, **59**, 6724–6729.

Kleeorin, N. and Rogachevskii, I., Effect of rotation on a developed turbulent stratified convection: the hydrodynamic helicity, the alpha-effect and the effective drift velocity. *Phys. Rev. E*, 2003, **67**, 026321 (1–19).
Kleeorin, N. and Rogachevskii, I., Effect of heat flux on differential rotation in turbulent convection. *Phys. Rev. E*, 2006, **73**, 046303 (1–9).
Kleeorin, N., Rogachevskii, I. and Ruzmaikin, A., Magnetic force reversal and instability in a plasma with advanced magnetohydrodynamic turbulence. *Sov. Phys. JETP*, 1990, **70**, 878–883. Translation from *Zh. Eksp. Teor. Fiz.*, 1990, **97**, 1555–1565.
Kleeorin, N., Rogachevskii, I. and Ruzmaikin, A., Magnitude of dynamo-generated magnetic field in solar-type convective zones. *Astron. Astrophys.*, 1995, **297**, 159–167.
Kleeorin, N. and Ruzmaikin, A., Dynamics of the averaged turbulent helicity in a magnetic field. *Magnetohydrodynamics*, 1982, **18**, 116–122. Translation from *Magnitnaya Gidrodinamika*, 1982, **2**, 17–24.
Krause, F. and Rädler, K.-H., *Mean-field Magnetohydrodynamics and Dynamo Theory*, 1980 (Pergamon: Oxford).
Kulsrud, R., A critical review of galactic dynamos. *Annu. Rev. Astron. Astrophys.*, 1999, **37**, 37–64.
McComb, W.D., *The Physics of Fluid Turbulence*, 1990 (Clarendon: Oxford).
Moffatt, H.K., *Magnetic Field Generation in Electrically Conducting Fluids*, 1978 (Cambridge University Press: New York).
Monin, A.S. and Yaglom, A.M., *Statistical Fluid Mechanics*, 1975, Vol. 2 (MIT Press: Cambridge, Massachusetts).
Moreno-Insertis, F., Schüssler, M. and Ferriz-Mas, A., Enhanced inertia of thin magnetic flux tubes. *Astron. Astrophys.*, 1996, **312**, 317–326.
Orszag, S.A., Analytical theories of turbulence. *J. Fluid Mech.*, 1970, **41**, 363–386.
Ossendrijver, M., Stix, M., Brandenburg, A. and Rüdiger, G., Magnetoconvection and dynamo coefficients. II. Field-direction dependent pumping of magnetic field. *Astron. Astrophys.*, 2002, **394**, 735–745.
Ossendrijver, M., The solar dynamo. *Astron. Astrophys. Rev.*, 2003, **11**, 287–367.
Parker, E., The formation of sunspots from the solar toroidal field. *Astrophys. J.*, 1955, **121**, 491–507.
Parker, E., The dynamical state of the interstellar gas and fluids. *Astrophys. J.*, 1966, **145**, 811–833.
Parker, E., *Cosmical Magnetic Fields*, 1979 (Oxford University Press: New York).
Pouquet, A., Frisch, U. and Leorat, J., Strong MHD turbulence and the nonlinear dynamo effect. *J. Fluid Mech.*, 1976, **77**, 321–354.
Priest, E.R., *Solar Magnetohydrodynamics*, 1982 (D. Reidel Publ. Co.: Dordrecht).
Rädler, K.-H., Kleeorin, N. and Rogachevskii, I., The mean electromotive force for MHD turbulence: the case of a weak mean magnetic field and slow rotation. *Geophys. Astrophys. Fluid Dynam.*, 2003, **97**, 249–274.
Roberts, P.H. and Soward, A.M., A unified approach to mean field electrodynamics. *Astron. Nachr.*, 1975, **296**, 49–64.
Roberts, P.H. and Soward, A.M., Dynamo theory. *Annu. Rev. Fluid Mech.*, 1992, **24**, 459–512.
Rogachevskii, I. and Kleeorin, N., Electromotive force for an anisotropic turbulence: intermediate nonlinearity. *Phys. Rev. E*, 2000, **61**, 5202–5210.
Rogachevskii, I. and Kleeorin, N., Nonlinear theory of a 'shear-current' effect and mean-field magnetic dynamos. *Phys. Rev. E*, 2004, **70**, 046310 (1–15).
Ruzmaikin, A.A., Shukurov, A.M. and Sokoloff, D.D., *Magnetic Fields of Galaxies*, 1988 (Kluwer Academic: Dordrecht).
Schüssler, M., Caligari, P., Ferriz-Mas, A. and Moreno-Insertis, F., Instability and eruption of magnetic flux tubes in the solar convection zone. *Astron. Astrophys.*, 1994, **281**, L69–L72.
Spiegel, E.A. and Weiss, N.O., Magnetic activity and variations in solar luminosity. *Nature*, 1980, **287**, 616–617.
Spruit, H.C., A model of solar convective zone. *Solar Phys.*, 1974, **34**, 277–290.
Spruit, H.C., Motion of magnetic flux tubes in the solar convection zone and chromosphere. *Astron. Astrophys.*, 1981, **98**, 155–160.
Spruit, H.C. and van Ballegooijen, A.A., Stability of toroidal flux tubes in stars. *Astron. Astrophys.*, 1982, **106**, 58–66.
Stix, M., *The Sun: An Introduction*, 1989 (Springer: Berlin and Heidelberg).
Thomas, J.H., Weiss, N.O., Tobias, S.M. and Brummell, N.H., Downward pumping of magnetic flux as the cause of filamentary structures in sunspot penumbrae. *Nature*, 2002, **420**, 390–393.
Tobias, S.M., Brummell, N.H., Clune T.L. and Toomre, J., Transport and storage of magnetic field by overshooting turbulent compressible convection. *Astrophys. J.*, 2001, **549**, 1183–1203.
Tobias, S.M. and Hughes, D.W., The influence of velocity shear on magnetic buoyancy instability in the solar tachocline. *Astrophys. J.*, 2004, **603**, 785–802.
Vainshtein, S.I. and Kichatinov, L.L., The macroscopic magnetohydrodynamics of inhomogeneously turbulent cosmic plasmas. *Geophys. Astrophys. Fluid Dynam.*, 1983, **24**, 273–298.
Weiss, N.O., The expulsion of magnetic flux by eddies. *Proc. R. Soc. Lond.*, 1966, **A293**, 310–328.

Zeldovich, Ya.B., The magnetic field in the two-dimensional motion of a conducting turbulent fluid. *Sov. Phys. JETP*, 1957, **4**, 460–462. Translation from *Zh. Eksp. Teor. Fiz.*, 1956, **31**, 154–156.
Zeldovich, Ya.B., Ruzmaikin, A.A. and Sokoloff, D.D., *Magnetic Fields in Astrophysics*, 1983 (Gordon and Breach: New York).
Zhang, H., Sokoloff, D., Rogachevskii, I., Moss, D., Lamburt, V., Kuzanyan, K. and Kleeorin N., The radial distribution of magnetic helicity in solar convective zone: observations and dynamo theory. *Mon. Not. R. Astron. Soc.*, 2006, **365**, 276–286.

Appendix A: the nonlinear electromotive force in turbulent convection

Let us derive equations for the second-order moments in a turbulent convection. For this purpose we rewrite equations (1)–(3) in a Fourier space. In particular,

$$\frac{\mathrm{d}v_i(\mathbf{k},t)}{\mathrm{d}t} = E_{im}\,\hat{S}^{(b)}_m(b;H) + g\,e_n\,P_{in}(k)\,s(\mathbf{k},t) + \Lambda_\rho\,D_{imn}(k)\,\hat{S}^{(a)}_{mn}(b;H) + \frac{\mathrm{i}\,\Lambda_\rho}{2k^2}\,g\,k_m\,P_{im}(e)\,s(\mathbf{k},t) + v_i^N, \tag{A.1}$$

$$\frac{\mathrm{d}b_i(\mathbf{k},t)}{\mathrm{d}t} = \frac{\Lambda_\rho}{2}\,R_{imn}\,\hat{S}^{(a)}_{mn}(v;H) + \mathrm{i}\,k_m\,\hat{S}^{(a)}_{mi}(v;H) - \hat{S}^{(c)}_i(v;H) + b_i^N, \tag{A.2}$$

where we multiply equation (1) written in **k**-space by $P_{ij}(\mathbf{k}) = \delta_{ij} - k_{ij}$ in order to exclude the pressure term from the equation of motion, and

$$\hat{S}^{(a)}_{ij}(a;A) = \int a_j(\mathbf{k}-\mathbf{Q})A_i(\mathbf{Q})\mathrm{d}\mathbf{Q},$$
$$\hat{S}^{(b)}_i(a;A) = (2P_{in}(k) - \delta_{in})\hat{S}^{(c)}_n(a;A) + ik_n\hat{S}^{(a)}_{ni}(a;A),$$
$$\hat{S}^{(c)}_i(a;A) = \mathrm{i}\int a_p(\mathbf{k}-\mathbf{Q})Q_pA_i(\mathbf{Q})\mathrm{d}\mathbf{Q},$$
$$E_{ij} = \delta_{ij} - (\mathrm{i}\,\Lambda_\rho/k^2)(k_ie_j - \delta_{ij}(\mathbf{k}\cdot\mathbf{e})),$$
$$D_{imn} = e_pP_{ip}(k)\delta_{mn} + e_pk_{mp}\delta_{in} - (\tfrac{1}{2})e_n\delta_{im},$$
$$R_{imn} = e_m\delta_{in} - e_n\delta_{im},$$

$P_{ij}(k) = \delta_{ij} - k_{ij}$, $P_{ij}(e) = \delta_{ij} - e_{ij}$, δ_{ij} is the Kronecker tensor, $k_{ij} = k_ik_j/k^2$ and $e_{ij} = e_ie_j$. Here we neglect terms $\sim\mathrm{O}(\Lambda_\rho^2)$. We use the mean-field approach, and the two-point correlation function of the velocity fluctuations is given by

$$\langle v_i(\mathbf{x})v_j(\mathbf{y})\rangle = \int f_{ij}(\mathbf{k},\mathbf{R})\exp(\mathrm{i}\,\mathbf{k}\cdot\mathbf{r})\mathrm{d}\mathbf{k},$$

where hereafter we omit argument t in the correlation functions, $f_{ij}(\mathbf{k},\mathbf{R}) = \hat{L}(v_i;v_j)$, and

$$\hat{L}(a;c) = \int\big\langle a(t,\mathbf{k}+\mathbf{K}/2)c(t,\,-\mathbf{k}+\mathbf{K}/2)\big\rangle\exp(\mathrm{i}\,\mathbf{K}\cdot\mathbf{R})\mathrm{d}\mathbf{K}, \tag{A.3}$$

(see Roberts and Soward 1975). Here $\mathbf{R} = (\mathbf{x}+\mathbf{y})/2$, $\mathbf{r} = \mathbf{x}-\mathbf{y}$. Note that **R** and **K** correspond to the large scales, and **r** and **k** to the small ones.

Using equations (3), (A.1)–(A.2) we derive equations for the following correlation functions:

$$\begin{aligned} f_{ij}(\mathbf{k}) &= \hat{L}(v_i; v_j), \quad h_{ij}(\mathbf{k}) = \hat{L}(b_i; b_j), \quad g_{ij}(\mathbf{k}) = \hat{L}(b_i; v_j), \\ F_i(\mathbf{k}) &= \hat{L}(s; v_i), \quad G_i(\mathbf{k}) = \hat{L}(s; b_i), \quad \Theta(\mathbf{k}) = \hat{L}(s; s). \end{aligned} \tag{A.4}$$

These equations are given by

$$\frac{\partial f_{ij}(\mathbf{k})}{\partial t} = \mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\Phi_{ij} + I_{ij}^f + \hat{\mathcal{N}} f_{ij}, \tag{A.5}$$

$$\frac{\partial h_{ij}(\mathbf{k})}{\partial t} = -\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\Phi_{ij} + I_{ij}^h + \hat{\mathcal{N}} h_{ij}, \tag{A.6}$$

$$\frac{\partial g_{ij}(\mathbf{k})}{\partial t} = \mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})[f_{ij}(\mathbf{k}) - h_{ij}(\mathbf{k})] + g e_n P_{jn}(k) G_i(-\mathbf{k}) + I_{ij}^g + \hat{\mathcal{N}} g_{ij}, \tag{A.7}$$

$$\frac{\partial F_i(\mathbf{k})}{\partial t} = -\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H}) G_i(\mathbf{k}) + g e_n P_{in}(k)\Theta(\mathbf{k}) + I_i^F + \hat{\mathcal{N}} F_i, \tag{A.8}$$

$$\frac{\partial G_i(\mathbf{k})}{\partial t} = -\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H}) F_i(\mathbf{k}) + I_i^G + \hat{\mathcal{N}} G_i, \tag{A.9}$$

$$\frac{\partial \Theta(\mathbf{k})}{\partial t} = -\frac{\Omega_b^2}{g} F_z(\mathbf{k}) + \hat{\mathcal{N}}\Theta, \tag{A.10}$$

where hereafter we also omit argument $\mathbf{R}$ in the correlation functions. Here $\Phi_{ij}(\mathbf{k}) = g_{ij}(\mathbf{k}) - g_{ji}(-\mathbf{k})$, and

$$\begin{aligned} I_{ij}^f &= \tilde{I}_{ij}^f(\mathbf{k}) + \tilde{I}_{ji}^f(-\mathbf{k}), \ \ I_{ij}^h = \tilde{I}_{ij}^h(\mathbf{k}) + \tilde{I}_{ji}^h(-\mathbf{k}), \\ \tilde{I}_{ij}^f(\mathbf{k}) &= N_{in}^f g_{nj}(\mathbf{k}) + M_i F_j(\mathbf{k}), \ \ \tilde{I}_{ij}^h(\mathbf{k}) = N_{in}^h g_{jn}(-\mathbf{k}), \\ I_{ij}^g &= N_{in}^h f_{nj}(\mathbf{k}) + N_{jn}^f h_{in}(\mathbf{k}) + M_j G_i(-\mathbf{k}), \\ I_i^F &= N_{in}^f G_n(\mathbf{k}) - M_i\Theta(\mathbf{k}), \ \ I_i^G = N_{in}^h F_n(\mathbf{k}), \\ N_{ij}^f &= \Lambda_\rho (D_{imj} + k_{im} e_j - \delta_{ij} k_{nm} e_n) H_m + (2P_{im}(k) - \delta_{im}) H_{m,j} \\ &\quad + \frac{1}{2}\delta_{ij}\left(\mathbf{H}\cdot\boldsymbol{\nabla} - H_{n,q}\, k_n \,\frac{\partial}{\partial k_q}\right), \\ N_{ij}^h &= \frac{1}{2}\left[\Lambda_\rho R_{imj} H_m + \delta_{ij}\left(\mathbf{H}\cdot\boldsymbol{\nabla} - H_{n,q}\, k_n\,\frac{\partial}{\partial k_q}\right)\right] - H_{i,j}, \\ M_i &= \frac{\mathrm{i}\,g}{2k^2}[e_m(P_{mn}(k) k_i + P_{in}(k) k_m)\nabla_n - \Lambda_\rho P_{in}(e) k_n], \end{aligned}$$

$\boldsymbol{\nabla} = \partial/\partial\mathbf{R}$ and $H_{i,j} = \nabla_j H_i$, $\hat{\mathcal{N}} f_{ij} = g e_n [P_{in}(k) F_j(\mathbf{k}) + P_{jn}(k) F_i(-\mathbf{k})] + \hat{\mathcal{N}}\tilde{f}_{ij}$, and $\hat{\mathcal{N}}\tilde{f}_{ij}$, $\hat{\mathcal{N}} h_{ij}$, $\hat{\mathcal{N}} g_{ij}$, $\hat{\mathcal{N}} F_i$, $\hat{\mathcal{N}} G_i$ and $\hat{\mathcal{N}}\Theta$ are the third-order moment terms appearing due to the nonlinear terms. The terms $\sim F_i$ in the tensor $\hat{\mathcal{N}} f_{ij}$ can be considered as a stirring force for the turbulent convection. Note that a stirring force in the Navier–Stokes turbulence is an external parameter.

For the derivation of equations (A.5)–(A.10) we use an identity for the function $Z_{ij}(\mathbf{k}, \mathbf{R})$:

$$Z_{ij}(\mathbf{k}, \mathbf{R}) = \mathrm{i} \int (k_p + K_p/2) H_p(\mathbf{Q}) \exp(i\mathbf{K} \cdot \mathbf{R}) \langle v_i(\mathbf{k} + \mathbf{K}/2 - \mathbf{Q}) v_j(-\mathbf{k} + \mathbf{K}/2) \rangle \mathrm{d}\mathbf{K}\, \mathrm{d}\mathbf{Q}.$$

The identity reads

$$Z_{ij}(\mathbf{k}, \mathbf{R}) \simeq [\mathrm{i}\,(\mathbf{k} \cdot \mathbf{H}) + \tfrac{1}{2}(\mathbf{H} \cdot \nabla)] f_{ij}(\mathbf{k}, \mathbf{R}) - \frac{1}{2} k_p \frac{\partial f_{ij}(\mathbf{k})}{\partial k_s} H_{p,s}, \tag{A.11}$$

(see Rogachevskii and Kleeorin 2004), and similarly for other second-order moments. We take into account that in equation (A7) the terms with symmetric tensors with respect to the indexes 'i' and 'j' do not contribute to the nonlinear electromotive force. In equations (A.5)–(A.10) we neglect the second-order and high-order spatial derivatives with respect to the large-scale variable $\mathbf{R}$.

Let us solve equations (A.5)–(A.10) neglecting the sources $I_{ij}^f, I_{ij}^h, I_{ij}^g, \ldots$ with the large-scale spatial derivatives. Then we take into account the terms with the large-scale spatial derivatives by perturbations. Thus, subtracting equations (A.5)–(A.10) written for background turbulent convection (i.e., for $\mathbf{B} = \mathbf{0}$) from those for $\mathbf{B} \neq 0$, using the spectral τ approximation [which is determined by equation (4)], neglecting the terms with the large-scale spatial derivatives, assuming that $\eta k^2 \ll \tau^{-1}$ and $\nu k^2 \ll \tau^{-1}$ for the inertial range of turbulent fluid flow, and assuming that the characteristic time of variation of the mean magnetic field $\mathbf{B}$ is substantially larger than the correlation time $\tau(k)$ for all turbulence scales, we arrive at the following steady-state solution of the obtained equations:

$$\hat{f}_{ij}(\mathbf{k}) \approx f_{ij}^{(0)}(\mathbf{k}) + \mathrm{i}\,\tau(\mathbf{k} \cdot \mathbf{H}) \hat{\Phi}_{ij}(\mathbf{k}), \tag{A.12}$$

$$\hat{h}_{ij}(\mathbf{k}) \approx h_{ij}^{(0)}(\mathbf{k}) - \mathrm{i}\,\tau(\mathbf{k} \cdot \mathbf{H}) \hat{\Phi}_{ij}(\mathbf{k}), \tag{A.13}$$

$$\hat{g}_{ij}(\mathbf{k}) \approx \mathrm{i}\,\tau(\mathbf{k} \cdot \mathbf{H})[\hat{f}_{ij}(\mathbf{k}) - \hat{h}_{ij}(\mathbf{k})] - \tau g e_n P_{jn}(k) \hat{G}_i(\mathbf{k}), \tag{A.14}$$

$$\hat{F}_i(\mathbf{k}) \approx F_i^{(0)}(\mathbf{k}) - \mathrm{i}\,\tau(\mathbf{k} \cdot \mathbf{H}) \hat{G}_i(\mathbf{k}) + \tau g e_n P_{in}(k)[\hat{\Theta}(\mathbf{k}) - \Theta^{(0)}(\mathbf{k})], \tag{A.15}$$

$$\hat{G}_i(\mathbf{k}) \approx -\mathrm{i}\,\tau(\mathbf{k} \cdot \mathbf{H}) \hat{F}_i(\mathbf{k}), \tag{A.16}$$

$$\hat{\Theta}(\mathbf{k}) \approx \Theta^{(0)}(\mathbf{k}) + O(\Omega_b^2), \tag{A.17}$$

where $\hat{f}_{ij}, \hat{h}_{ij}, \ldots, \hat{\Theta}$ are the solutions without the sources $I_{ij}^f, I_{ij}^h, \ldots, I_i^G$ and $\hat{\Phi}_{ij}(\mathbf{k}) = \hat{g}_{ij}(\mathbf{k}) - \hat{g}_{ji}(-\mathbf{k})$. The quantities with the superscript (0) in equations (A.12)–(A.17) correspond to the background turbulent convection. Here we take into account that for the background turbulent convection $g_{ij}^{(0)}(\mathbf{k}) = 0$ and $G_i^{(0)}(\mathbf{k}) = 0$.

Now we split all second-order correlation functions into symmetric and antisymmetric parts with respect to the wave vector $\mathbf{k}$, i.e., $f_{ij} = f_{ij}^{(s)} + f_{ij}^{(a)}$, where

$f_{ij}^{(s)} = [f_{ij}(\mathbf{k}) + f_{ij}(-\mathbf{k})]/2$ and $f_{ij}^{(a)} = [f_{ij}(\mathbf{k}) - f_{ij}(-\mathbf{k})]/2$. Thus, equations (A.12)–(A.17) yield

$$\hat{f}_{ij}^{(s)}(\mathbf{k}) \approx \frac{1}{1+2\psi}\Big[(1+\psi)f_{ij}^{(0s)}(\mathbf{k}) + \psi h_{ij}^{(0s)}(\mathbf{k}) - 2\psi\tau g e_n P_{in}(k)\hat{F}_j^{(s)}(\mathbf{k})\Big], \qquad \text{(A.18)}$$

$$\hat{h}_{ij}^{(s)}(\mathbf{k}) \approx \frac{1}{1+2\psi}\Big[\psi f_{ij}^{(0s)}(\mathbf{k}) + (1+\psi)h_{ij}^{(0s)}(\mathbf{k}) + \psi\tau g e_n P_{in}(k)\hat{F}_j^{(s)}(\mathbf{k})\Big], \qquad \text{(A.19)}$$

$$\hat{g}_{ij}^{(a)}(\mathbf{k}) \approx \frac{\mathrm{i}\,\tau(\mathbf{k}\cdot\mathbf{H})}{1+2\psi}\Big[f_{ij}^{(0s)}(\mathbf{k}) - h_{ij}^{(0s)}(\mathbf{k}) + \tau g e_n P_{in}(k)\hat{F}_j^{(s)}(\mathbf{k})\Big], \qquad \text{(A.20)}$$

$$\hat{F}_i^{(s)}(\mathbf{k}) \approx \frac{F_i^{(0s)}(\mathbf{k})}{1+\psi/2}, \qquad \text{(A.21)}$$

$$\hat{G}_i^{(a)}(\mathbf{k}) \approx -\mathrm{i}\,\tau(\mathbf{k}\cdot\mathbf{H})\hat{F}_i^{(s)}(\mathbf{k}), \qquad \text{(A.22)}$$

where $\psi(\mathbf{k}) = 2(\tau\,\mathbf{k}\cdot\mathbf{H})^2$ and we neglect the terms $\sim\mathrm{O}(\Omega_b^2)$ in equations (A.17). The correlation functions $\hat{f}_{ij}^{(a)}$, $\hat{h}_{ij}^{(a)}, \hat{g}_{ij}^{(s)}$ $\hat{F}_i^{(a)}$ and $\hat{G}_i^{(s)}$ vanish if we neglect the large-scale spatial derivatives, i.e., they are proportional to the first-order spatial derivatives.

Now we take into account the large-scale spatial derivatives in equations (A.5)–(A.10) by perturbations. Their effects determine the following steady-state equations for the second moments:

$$\tilde{f}_{ij}^{(a)}(\mathbf{k}) = f_{ij}^{(0a)}(\mathbf{k}) + \mathrm{i}\,\tau(\mathbf{k}\cdot\mathbf{H})\tilde{\Phi}_{ij}^{(s)}(\mathbf{k}) + \tau I_{ij}^{f}, \qquad \text{(A.23)}$$

$$\tilde{h}_{ij}^{(a)}(\mathbf{k}) = h_{ij}^{(0a)}(\mathbf{k}) - \mathrm{i}\,\tau(\mathbf{k}\cdot\mathbf{H})\tilde{\Phi}_{ij}^{(s)}(\mathbf{k}) + \tau I_{ij}^{h}, \qquad \text{(A.24)}$$

$$\tilde{g}_{ij}^{(s)}(\mathbf{k}) = \tau\Big[\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\Big(\tilde{f}_{ij}^{(a)}(\mathbf{k}) - \tilde{h}_{ij}^{(a)}(\mathbf{k})\Big) + g e_n P_{jn}(k)\tilde{G}_i^{(s)}(-\mathbf{k}) + I_{ij}^{g}\Big], \qquad \text{(A.25)}$$

$$\tilde{G}_i^{(s)}(\mathbf{k}) = -\tau\Big[\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\tilde{F}_i^{(a)}(\mathbf{k}) - I_i^{G}\Big], \qquad \text{(A.26)}$$

$$\tilde{F}_i^{(a)}(\mathbf{k}) = F_i^{(0a)}(\mathbf{k}) - \tau\Big[\mathrm{i}(\mathbf{k}\cdot\mathbf{H})\tilde{G}_i^{(s)}(\mathbf{k}) - I_i^{F}\Big], \qquad \text{(A.27)}$$

where the second moments $\tilde{f}_{ij}, \tilde{h}_{ij}, \tilde{g}_{ij}, \ldots$ determine the effect of the large-scale derivatives and $\tilde{\Phi}_{ij}^{(s)}(\mathbf{k}) = \tilde{g}_{ij}^{(s)}(\mathbf{k}) - \tilde{g}_{ji}^{(s)}(-\mathbf{k})$. The correlation functions of the background turbulent convection $f_{ij}^{(0a)}(\mathbf{k})$, $h_{ij}^{(0a)}(\mathbf{k})$ and $F_i^{(0a)}(\mathbf{k})$ are determined by the inhomogeneities of turbulence, the fluid density and the turbulent heat flux [see equations (A.32)–(A.34) below]. Equations (A.26) and (A.27) yield

$$\tilde{F}_i^{(a)}(\mathbf{k}) = \frac{1}{1+\psi/2}\Big[F_i^{(0a)} - \mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\tau I_i^{G} + \tau I_i^{F}\Big], \qquad \text{(A.28)}$$

$$\tilde{G}_i^{(s)}(\mathbf{k}) = -\frac{\tau}{1+\psi/2}\Big[\mathrm{i}(\mathbf{k}\cdot\mathbf{H})(F_i^{(0a)} + \tau I_i^{F}) - I_i^{G}\Big]. \qquad \text{(A.29)}$$

Our goal is to calculate the mean electromotive force $\mathcal{E}_i(\mathbf{r}=0) = (1/2\sqrt{\rho_0})\,\varepsilon_{inm}\int\tilde{\Phi}^{(s)}_{mn}(\mathbf{k})\mathrm{d}\mathbf{k}$. Solution of system of equations (A.23)–(A.25) allow us to get the expression for $\tilde{\Phi}^{(s)}_{mn}(\mathbf{k})$ which yields the mean electromotive force:

$$\mathcal{E}_i = \int \frac{\tau\varepsilon_{inm}}{1+2\psi}\Big[\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\big[f^{(0a)}_{mn} - h^{(0a)}_{mn} + \tau(I^f_{mn} - I^h_{mn})\big] + I^g_{mn} + \frac{\tau g e_p P_{mp}(k)}{1+\psi/2}\big[\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\big(F^{(0a)}_n + \tau I^F_n\big) - I^G_n\big]\Big]\mathrm{d}\mathbf{k}, \tag{A.30}$$

where we use equations (A.28) and (A.29). Equation (A.30) can be rewritten in the form:

$$\mathcal{E}_i = \int \frac{\tau\varepsilon_{inm}}{1+2\psi}\Big\{\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})\Big\{f^{(0a)}_{mn} - h^{(0a)}_{mn} + 2\tau\Big[\Big(N^f_{mp}+N^h_{mp}\Big)\hat{g}_{pn} - M_n\Big(\hat{F}_m - F^{(0s)}_m\Big)\Big]\Big\} + N^h_{mp}\hat{f}_{pn} - N^f_{mp}\hat{h}_{pn} + \mathrm{i}\,\tau M_n(\mathbf{k}\cdot\mathbf{H})\hat{F}_m + \frac{\tau g e_p P_{mp}(k)}{1+\psi/2}\Big[\mathrm{i}\,(\mathbf{k}\cdot\mathbf{H})F^{(0a)}_n + \Big(\frac{\psi}{2}N^f_{nq} - N^h_{nq}\Big)\hat{F}_q\Big]\Big\}\mathrm{d}\mathbf{k}. \tag{A.31}$$

For the integration in **k**-space in equation (A.31) we specify a model for the background turbulent convection (i.e., the turbulence with zero mean magnetic field, $\mathbf{B}=\mathbf{0}$), which is determined by

$$f^{(0)}_{ij}(\mathbf{k}) = f_*\,W(k)\left[P_{ij}(\mathbf{k}) + \frac{\mathrm{i}}{2k^2}\left(k_i\Lambda^{(v)}_j - k_j\Lambda^{(v)}_i\right)\right], \tag{A.32}$$

$$h^{(0)}_{ij}(\mathbf{k}) = h_*\,W(k)\left[P_{ij}(\mathbf{k}) + \frac{\mathrm{i}}{2k^2}\left(k_i\Lambda^{(b)}_j - k_j\Lambda^{(b)}_i\right)\right], \tag{A.33}$$

$$F^{(0)}_i(\mathbf{k}) = 3F_*\,W(k)\,e_j\left[P_{ij}(\mathbf{k}) - \frac{\mathrm{i}}{2k^2}\left(P_{jm}(\mathbf{k})k_i + P_{im}(\mathbf{k})k_j\right)\tilde{\Lambda}^{(F)}_m\right], \tag{A.34}$$

$\Theta^{(0)}(\mathbf{k}) = 2\Theta_*W(k)$, $g^{(0)}_{ij}(\mathbf{k}) = 0$ and $G^{(0)}_i(\mathbf{k}) = 0$, where $P_{ij}(\mathbf{k}) = \delta_{ij} - k_{ij}$, $k_{ij} = k_ik_j/k^2$, $W(k) = E(k)/8\pi k^2$, $\tau(k) = 2\tau_0\bar{\tau}(k)$, $E(k) = -\mathrm{d}\bar{\tau}(k)/\mathrm{d}k$, $\bar{\tau}(k) = (k/k_0)^{1-q}$, $1<q<3$ is the exponent of the kinetic energy spectrum (e.g., $q=5/3$ for Kolmogorov spectrum), $k_0 = 1/l_0$ and $\tau_0 = l_0/u_0$. Here $\Lambda^{(v)}_i = \Lambda^{(u)}_i - 2\Lambda_\rho e_i$ and $\tilde{\Lambda}^{(F)}_i = \Lambda^{(F)}_i - 2\Lambda_\rho e_i$. These imply that

$$\Lambda^{(v)}_i = \frac{\nabla_i(\rho_0^2\,\langle\mathbf{u}'^2\rangle^{(0)})}{\rho_0^2\,\langle\mathbf{u}'^2\rangle^{(0)}}, \quad \tilde{\Lambda}^{(F)}_i = \frac{\nabla_i(\rho_0^2\,\langle|\mathbf{u}'|\,s'\rangle^{(0)})}{\rho_0^2\,\langle|\mathbf{u}'|\,s'\rangle^{(0)}},$$

where

$$\Lambda^{(u)}_i = \frac{\nabla_i\langle\mathbf{u}'^2\rangle^{(0)}}{\langle\mathbf{u}'^2\rangle^{(0)}}, \quad \Lambda^{(b)}_i = \frac{\nabla_i\langle\mathbf{b}^2\rangle^{(0)}}{\langle\mathbf{b}^2\rangle^{(0)}}, \quad \Lambda^{(F)}_i = \frac{\nabla_i\langle|\mathbf{u}'|\,s'\rangle^{(0)}}{\langle|\mathbf{u}'|\,s'\rangle^{(0)}}$$

and $\int F^{(0)}_i(\mathbf{k})\mathrm{d}\mathbf{k} = F_*\,e_i$, $\int f^{(0)}_{ij}(\mathbf{k})\mathrm{d}\mathbf{k} = (f_*/3)\,\delta_{ij}$, $\int h^{(0)}_{ij}(\mathbf{k})\mathrm{d}\mathbf{k} = (h_*/3)\,\delta_{ij}$ and $\int\Theta^{(0)}(\mathbf{k})\mathrm{d}\mathbf{k} = \Theta_*$.

After the integration in **k** space in equation (A.31) we obtain the nonlinear electromotive force. This yields the nonlinear turbulent magnetic diffusion coefficients and the nonlinear drift velocities of the mean magnetic field in the axisymmetric case, which are given by equation (7), where the contributions from the purely hydrodynamic turbulence are given by

$$\eta_{_A}^{(v)}(B) = A_1^{(1)}(4B) + A_2^{(1)}(4B), \tag{A.35}$$

$$\eta_{_B}^{(v)}(B) = A_1^{(1)}(4B) + 3(1-\epsilon)\left[A_2^{(1)}(4B) - \frac{1}{2\pi}\bar{A}_2(16B^2)\right], \tag{A.36}$$

$$\mathbf{V}_A^{(v)}(B) = -\frac{1}{2}\eta_{_A}^{(v)}(B)(\mathbf{\Lambda}^{(u)} - \epsilon\mathbf{\Lambda}^{(b)}) + \mathbf{V}^{(u,\rho)} - \frac{\mathbf{\Lambda}^{(B)}}{2}\Big[(2-3\epsilon)A_2^{(1)}(4B) - \frac{3(1-\epsilon)}{2\pi}\bar{A}_2(16B^2)\Big], \tag{A.37}$$

$$\mathbf{V}_B^{(v)}(B) = -\frac{1}{2}\eta_{_A}^{(v)}(B)(\mathbf{\Lambda}^{(u)} - \epsilon\mathbf{\Lambda}^{(b)}) + \mathbf{V}^{(u,\rho)}, \tag{A.38}$$

$$\mathbf{V}^{(u,\rho)} = \frac{1}{2}\Lambda_\rho\mathbf{e}\left[\epsilon A_1^{(1)}(4B) - (5-6\epsilon)A_2^{(1)}(4B) + \frac{3(1-\epsilon)}{2\pi}\bar{A}_2(16B^2)\right], \tag{A.39}$$

and the contributions caused by the turbulent heat flux are

$$\eta_{_A}^{(F,z)}(B) = \frac{3}{4}[2\Psi_1\{A_2 - 3A_1 + 3C_1\} + 4\Psi_2\{A_1 - C_1\} + 3\Psi_3\{A_1 + C_1\}], \tag{A.40}$$

$$\eta_{_A}^{(F,x)}(B) = \frac{3}{4}[-2\Psi_1\{A_1 + C_1\} + 4\Psi_2\{C_1\} + 3\Psi_3\{A_1 - 2C_1\}], \tag{A.41}$$

$$\eta_{_B}^{(F,z)}(B) = \frac{3}{4}[(-6\Psi_1 + 4\Psi_2 + 3\Psi_3)\{A_1 + A_2 - C_1 - C_3\} + 2\Psi_1\{C_3\} + 6\Psi_3\{C_1\}], \tag{A.42}$$

$$\eta_{_B}^{(F,x)}(B) = 0, \tag{A.43}$$

$$\mathbf{V}_A^{(F)}(B) = \mathbf{V}_B^{(F)}(B) = \mathbf{V}^{(F)} + \mathbf{V}^{(F,\rho)}, \tag{A.44}$$

$$V_z^{(F)}(B) = -\frac{3}{4}\Lambda_z^{(F)}\Big[\Psi_4\{A_1 + A_2 - C_1 - C_3\} - A_1^{(2)}(4B) - A_2^{(2)}(4B) + C_1^{(2)}(4B) + C_3^{(2)}(4B)\Big], \tag{A.45}$$

$$V_x^{(F)}(B) = \frac{3}{4}\Lambda_x^{(F)}\Big[\Psi_4\{A_1 + A_2 - 5C_1 - 5C_3\} - A_1^{(2)}(4B) - A_2^{(2)}(4B) + 5C_1^{(2)}(4B) + 5C_3^{(2)}(4B)\Big], \tag{A.46}$$

$$\mathbf{V}^{(F,\rho)} = \frac{3}{8}\Lambda_\rho\mathbf{e}\Big[\Psi_1\{17A_1 + 17A_2 + 17C_1 - 7C_3\} - \Psi_2\{6A_1 + 6A_2 + 10C_1 - 2C_3\} - \Psi_3\{9A_1 + 9A_2 + 15C_1 - 3C_3\} - 4A_1^{(2)}(4B) - 4A_2^{(2)}(4B)\Big], \tag{A.47}$$

where $\mathbf{\Lambda}^{(B)} = (\nabla \mathbf{B}^2)/\mathbf{B}^2$, the parameter $\epsilon = \langle \mathbf{b}^2 \rangle^{(0)} / \langle \mathbf{v}^2 \rangle^{(0)}$, the parameter a_* is given by equation (8), and

$$
\begin{aligned}
\Psi_1\{X\} &= \frac{1}{3}[4X^{(2)}(4B) - X^{(2)}(2B)], \\
\Psi_2\{X\} &= \frac{1}{9}\left[16X^{(2)}(4B) - 16X^{(2)}(2B) + \frac{9}{2\pi}\bar{X}(4B^2)\right], \\
\Psi_3\{X\} &= -\frac{1}{9}\left[4X^{(2)}(4B) - 49X^{(2)}(2B) + \frac{18}{\pi}\bar{X}(4B^2)\right], \\
\Psi_4\{X\} &= 3X^{(1)}(4B) - \frac{3}{2\pi}\bar{X}(16B^2).
\end{aligned}
\tag{A.48}
$$

Note that $\Psi_4\{A_1\} = A_1^{(1)} + (1/2)A_2^{(1)}$. The functions $A_m^{(n)}(\beta)$, $C_m^{(n)}(\beta)$ for $n = 1; 2$ and the functions $\bar{A}_m(\beta^2)$, $\bar{C}_m(\beta^2)$ are given in Rogachevskii and Kleeorin (2004, in appendices B, C and D). Asymptotic formulae for the nonlinear turbulent magnetic diffusion coefficients and the nonlinear drift velocities of the mean magnetic field in the axisymmetric case are given by equations (9) and (10).