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# The nonlinear galactic dynamo and magnetic helicity transport

**N. Kleeorin**[1,⋆], **D. Moss**[2], **I. Rogachevskii**[1], and **D. Sokoloff**[3]

[1] Department of Mechanical Engineering, Ben-Gurion University of Negev, POB 653, 84105 Beer-Sheva, Israel
[2] Department of Mathematics, University of Manchester, Manchester M13 9PL, UK
[3] Department of Physics, Moscow State University, Moscow 119992, Russia

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We discuss nonlinear mean-field galactic dynamo models and associate a universal mechanism for saturation of the dynamo with the transport of magnetic helicity. The crucial point for saturation of the dynamo is the existence of a non-zero flux of magnetic helicity. We demonstrate that this saturation mechanism is quite insensitive to the form of this helicity flux. In that sense this is a robust mechanism which limits the growth of the mean magnetic field. Without this flux, the total magnetic helicity is conserved locally and the strength of the saturated mean magnetic field is very small compared to the equipartition strength. The inclusion of a flux of magnetic helicity means that the total magnetic helicity is not conserved locally because the magnetic helicity of small-scale magnetic fluctuations is redistributed by the flux. The equilibrium state is given by a balance between magnetic helicity production and magnetic helicity transport. The equilibrium value of the galactic large-scale magnetic field is given approximately by equipartition between the kinetic energy densities of the interstellar turbulence and the mean magnetic field. We also compare the action of algebraic and dynamic nonlinearities in the galactic dynamo. The algebraic $\alpha$ quenching saturates the dynamo, however a more realistic simultaneous quenching of the $\alpha$ effect and turbulent magnetic diffusion cannot saturate the growth of the mean magnetic field; this can only be achieved by the combined effects of algebraic and dynamic nonlinearities.



The main goal of this communication is a more detailed description of a nonlinear galactic dynamo which includes quenching of the turbulent magnetic diffusivity and effective drift velocity of the magnetic field in addition to the effects of evolution of magnetic helicity. We have modified the model of algebraic quenching of the nonlinear turbulent magnetic diffusion and nonlinear drift velocities used in Kleeorin et al. (2003). This modification is related to the improvement of the description of helical and nonhelical quantities in the derivation of the nonlinear electromotive force (for more details, see Rogachevskii & Kleeorin 2004).

In cylindrical coordinates $(r, \varphi, z)$ the axisymmetric mean magnetic field, $\boldsymbol{B} = B(r,z)\,\boldsymbol{e}_\varphi + \boldsymbol{\nabla}\times[A(r,z)\boldsymbol{e}_\varphi]$, is determined by the dimensionless equations

$$\frac{\partial A}{\partial t} = \alpha(\boldsymbol{B})B - \frac{1}{r}(\boldsymbol{V}^A(\boldsymbol{B})\cdot\boldsymbol{\nabla})(r\,A) + \eta_{_A}(\boldsymbol{B})\Delta_s A \quad (1)$$

$$\frac{\partial B}{\partial t} = D\,(\hat{\Omega}A) + r\,\boldsymbol{\nabla}\cdot\left[\frac{1}{r^2}[\eta_{_B}(\boldsymbol{B})\boldsymbol{\nabla} - \boldsymbol{V}^B(\boldsymbol{B})](r\,B)\right] \quad (2)$$

(Kleeorin et al. 2003; Rogachevskii & Kleeorin 2004). Here $\Delta_s = \Delta - 1/r^2$, $(\hat{\Omega}A) = \mathbf{e}_\varphi\cdot[\boldsymbol{\nabla}\Omega\times\boldsymbol{\nabla}A]$ determines the operator of the differential rotation, $\alpha(\boldsymbol{B})$ is the total nonlinear $\alpha$ effect, $\eta_{_A}(\boldsymbol{B})$ and $\eta_{_B}(\boldsymbol{B})$ are the nonlinear turbulent magnetic diffusion coefficients of poloidal and toroidal mean magnetic fields, $\boldsymbol{V}^A(\boldsymbol{B})$ and $\boldsymbol{V}^B(\boldsymbol{B})$ are the nonlinear drift velocities of poloidal and toroidal mean magnetic fields, and $B = |\boldsymbol{B}|$. The formulae for these nonlinearities are given in Rogachevskii & Kleeorin (2004). We adopt here the standard dimensionless form of the galactic dynamo equations. In particular, length is measured in units of the disc thickness $h$, time in units of $h^2/\eta_{_T}$ and $B$ is measured in units of the equipartition energy $B_{\rm eq} = \sqrt{4\pi\,\rho}\,u$, $u$ is the characteristic turbulent velocity in the maximum turbulent scale $l$, $\eta_{_T} = u\,l/3$, and $D = C_\omega C_\alpha$ is the dynamo number.

The total (hydrodynamic + magnetic) nonlinear $\alpha$ effect is given by $\alpha(\boldsymbol{B}) = \chi^v\phi_v(B) + \chi^c(\boldsymbol{B})\,\phi_m(B)$, where the hydrodynamic and magnetic parts of the $\alpha$ effect are determined by the corresponding helicities and quenching functions, $\phi_v(B)$ and $\phi_m(B)$, which are given in Kleeorin et al. (2002, 2003). The function $\chi^c(\boldsymbol{B})$ entering the magnetic part of the $\alpha$ effect is related to the magnetic helicity and determined by the dynamical equation

$$\frac{\partial\chi^c}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{F}} + \frac{\chi^c}{T} = -\left(\frac{2h}{l}\right)^2 \boldsymbol{\mathcal{E}}\cdot\boldsymbol{B} \quad (3)$$

(see for details, Kleeorin et al. 2000, 2002, 2003; a review by Brandenburg & Subramanian 2005, and references therein), where $\boldsymbol{\mathcal{F}}$ is the flux of the magnetic helicity, $T = (1/3)\,(l/h)^2\,{\rm Rm}$, and Rm is the magnetic Reynolds number.

The equations for the mean radial field $B_r = C_\alpha b_r$ and toroidal field $B_\varphi$ for the local thin-disc axisymmetric $\alpha\Omega$-

⋆ Corresponding author: nat@menix.bgu.ac.il

dynamo problem are given by

$$\frac{\partial b_r}{\partial t} = -(\alpha(\boldsymbol{B})B_\varphi)' + (\eta_{_A}(\boldsymbol{B})b_r')' - (V_z^A(\boldsymbol{B})b_r)' , \quad (4)$$

$$\frac{\partial B_\varphi}{\partial t} = Db_r + (\eta_{_B}(\boldsymbol{B})B_\varphi')' , \quad (5)$$

where a prime denotes $\partial/\partial z$, and $V_z^A(\boldsymbol{B})$ is the $z$-components of the nonlinear drift velocity of poloidal mean magnetic field. Since

$$\boldsymbol{\mathcal{E}}\cdot\boldsymbol{B} = \mathcal{E}_\varphi B - \mathcal{E}_r A' = B\frac{\partial A}{\partial t} + \mathcal{E}_r \frac{1}{D}\frac{\partial B}{\partial t} - \frac{1}{2D}(\mathcal{E}_r^2)' ,$$

then in steady state Eq. (3) reads $2(h/l)^2\mathcal{E}_r^2 - D\mathcal{F}_z = \text{const}$, where $\mathcal{E}_r = \eta_{_B} B'$. In a steady-state for fields of even parity with respect to the disc plane, we obtain

$$\frac{h}{l}\int_0^B \frac{\eta_{_B}(\tilde{B})}{\sqrt{|\mathcal{F}_B(\tilde{B})|}}\,\mathrm{d}\tilde{B} = \sqrt{\frac{|C\,D|}{2}}\int_{|z|}^1 \sqrt{|\chi^v(\tilde{z})|}\,\mathrm{d}\tilde{z} , \quad (6)$$

where $\mathcal{F}_z = C|\boldsymbol{\mathcal{F}}_B(B)||\chi^v(z)|$ and $C\,D > 0$. The crucial point for the dynamo saturation is a non-zero flux of magnetic helicity. It follows from Eq. (6) that this saturation mechanism is nearly independent of the form of the flux of magnetic helicity. In that sense this is a robust mechanism which limits growth of the mean magnetic field. If we assume that $|\boldsymbol{\mathcal{F}}_B(B)| \sim B^{-2\gamma}$, we obtain that the saturated mean magnetic field is

$$B_\varphi = |C\,D|^{\frac{1}{2\gamma}} \left[\int_{|z|}^1 \sqrt{|\chi^v(\tilde{z})|}\,\mathrm{d}\tilde{z}\right]^{\frac{1}{\gamma}} B_{\mathrm{eq}} , \quad (7)$$

where we have redefined the constant $C$, taking into account that $\eta_{_B}(B) \propto 1/B$ for $4B \gg 1$, and have restored the dimensional factor $B_{\mathrm{eq}}$. Note that the nonadvective flux of the magnetic helicity was chosen in Kleeorin et al. (2000, 2002, 2003) in the form $\boldsymbol{\mathcal{F}} = C\chi^v\phi_v(B)\boldsymbol{B}^2\eta_{_A}(B)(\boldsymbol{\nabla}\rho)/\rho$. This corresponds to $\gamma = 1$ in the function $|\boldsymbol{\mathcal{F}}_B(B)|$. For the specific choice of the profile $|\chi^v(z)| = \sin^2(\pi z/2)$ we obtain

$$B_\varphi \approx \frac{4}{1+\epsilon}\sqrt{|C\,D|}\,\bar{B}_{\mathrm{eq}}\cos\left(\frac{\pi z}{2}\right) , \quad (8)$$

$$B_r \approx -\frac{1+\epsilon}{4|R_\omega|}\bar{B}_{\mathrm{eq}}\tan\left(\frac{\pi z}{2}\right) . \quad (9)$$

The boundary conditions for $B_\varphi$ are $B_\varphi(z=1) = 0$, $B_\varphi'(z=0) = 0$, and for $B_r$ are $B_r(z=1) = 0$, $B_r'(z=0) = 0$. Note, however, that this asymptotic analysis performed for $B \gg B_{\mathrm{eq}}/4$ is not valid in the vicinity of the point $z = 1$ because $B(z=1) = 0$.

Our studies (Kleeorin et al. 2003, 2006) show that the model leads to results that are comparable with observations. These results are similar to those obtained from conventional galactic dynamo models, with large-scale magnetic fields typically of equipartition strength and with plausible values of the pitch angles. Our approach is based on first principles, as far as possible in the framework of mean-field dynamo theory, and results in the conclusion that the self-consistent form of dynamo saturation is much more complicated than suggested in conventional models for a galactic dynamo.

We have demonstrated the important role of two types of nonlinearity (algebraic and dynamic) in the mean-field galactic dynamo. The algebraic nonlinearity is determined by a nonlinear dependence of the mean electromotive force on the mean magnetic field. The dynamic nonlinearity is determined by a differential equation for the magnetic part of the $\alpha$ effect. This equation is a consequence of the conservation of the total magnetic helicity. We have taken into account the algebraic quenching of both the $\alpha$ effect and the turbulent magnetic diffusion, and also dynamical nonlinearities. Since the quenching of the $\alpha$ effect and the turbulent magnetic diffusion have the same origin, they cannot in general be taken into account separately. This implies that there is no reason to include $\alpha$ quenching and to ignore the quenching of the turbulent magnetic diffusion, or vice versa. We have also verified that the algebraic nonlinearity alone (i.e. quenching of both the $\alpha$ effect and turbulent magnetic diffusion) cannot saturate the growth of the mean magnetic field. The situation changes when the dynamic nonlinearity is taken into account. The crucial point is that the dynamical equation for the magnetic part of the $\alpha$ effect (i.e. the dynamic nonlinearity) includes the flux of magnetic helicity.

Without this flux, the strength of the saturated mean field is very small compared to the equipartition strength. The magnetic helicity flux results in that the combined effect of algebraic and dynamic nonlinearities limits the growth of the mean magnetic field and results in an equilibrium strength of the mean magnetic field which is of order the equipartition strength, in agreement with observations of galactic magnetic fields. We found that the saturation mechanism due to the dynamic nonlinearity is quite insensitive to the form of the magnetic helicity flux. Therefore, this is a robust mechanism which limits the growth of the mean magnetic field.

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