

# Magnetohydrodynamic turbulence in the solar convective zone as a source of oscillations and sunspots formation

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**Abstract.** As was previously shown (Kleeorin et al. 1990; Kleeorin & Rogachevskii 1994a), small-scale magnetohydrodynamic (MHD) turbulence results in modification of the Ampere law, by significantly reducing the elasticity of the magnetic field lines. As a result, the ‘effective’ magnetic pressure is reduced and may even reverse its sign. We show here that this modification can be related to a variety of phenomena observed in the Sun and in particular the following four are investigated: the 11-year variations of the solar radius, the torsional oscillations and the meridional flows, the solar short time oscillations and the large-scale magnetic flux ropes formation in convective zones of the Sun (as well as in stars and spiral galaxies).

**Key words:** turbulence – magnetohydrodynamics (MHD) – instabilities – Sun: oscillations – sunspots

## 1. Introduction

Under consideration are the 11-year variations of the solar radius, the torsional oscillations and the meridional flows, the solar short-time (1–100 minutes) oscillations and the large-scale magnetic flux ropes formation in convective zones of the Sun.

These different phenomena are assumed to be related to the reduction of the elasticity of the large-scale regular (mean) magnetic field by the developed magnetohydrodynamic (MHD) turbulence of the solar convective zone. It was found (see Kleeorin et al. 1990, Kleeorin & Rogachevskii 1994a) that the effective mean Ampere force in the presence of small-scale developed MHD turbulence is strongly modified. In particular, the effective mean Ampere force is given by

$$\mathbf{F}_m^{eff} = -\nabla \left( \frac{Q_p}{8\pi} \mathbf{B}^2 \right) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) Q_s \mathbf{B}, \quad (1)$$

where the functions  $Q_p$  and  $Q_s$  in the case  $\varepsilon_B \ll 1$ ,  $(R_m^*)^{-\frac{1}{3}}$  are given by

$$Q_p \simeq 1 - \frac{4}{15} \ln(R_m^*) + \frac{4}{7} [(R_m^*)^{\frac{1}{3}} - 1] \varepsilon_B,$$

$$Q_s \simeq 1 - \frac{8}{45} \ln(R_m^*) + \frac{16}{35} [(R_m^*)^{\frac{1}{3}} - 1] \varepsilon_B$$

(see Kleeorin et al. 1990). Here  $\varepsilon_B = 4C_A^2/u_0^2$ ,  $C_A = B/\sqrt{4\pi\rho}$  is the Alfvén velocity,  $R_m^* = A_* R_m$ ,  $R_m = u_0 l_0 / \eta_m$  is the magnetic Reynolds number,  $A_* = 0.1 - 0.5$ ,  $l_0$  is the maximal scale of turbulent motions,  $u_0$  is the characteristic turbulent velocity,  $\eta_m$  is the molecular magnetic diffusion.

Before turning to the investigation of the solar oscillations and the flux ropes formation a qualitative discussion of the sign reversal of the ‘effective’ magnetic pressure is presented (Kleeorin et al. 1990). Let us consider fully developed MHD turbulence with  $Re \gg 1$  and  $R_m \gg 1$ , where  $Re = u_0 l_0 / \nu_0$  is the Reynolds number,  $\nu_0$  is the kinematic viscosity. For isotropic turbulence the equation of state is given by

$$p_T = \frac{1}{3} W_m + \frac{2}{3} W_k \quad (2)$$

(see, e.g., Landau & Lifshitz 1975; 1984a). Here  $p_T$  is the total (hydrodynamic plus magnetic) turbulent pressure,  $W_m = \langle h^2 \rangle / 8\pi$  is the energy density of the magnetic fluctuations,  $W_k = \langle \rho u^2 \rangle / 2$  is the energy density of the turbulent hydrodynamic motion,  $\mathbf{u}$  and  $\mathbf{h}$  are the random fluctuations of the hydrodynamic and magnetic fields, and  $\rho$  is the density of the conducting fluid. The angular brackets denote averaging over the ensemble of turbulent fluctuations.

The equation that describes the evolution of the total energy density  $W_T = W_k + W_m$  of the homogeneous turbulence with a mean magnetic field  $\mathbf{B}$  is given by (see Appendix A):

$$\frac{\partial W_T}{\partial t} = I_T - \frac{W_T}{\tau} + \eta_T \Delta \left( \frac{B^2}{8\pi} \right), \quad (3)$$

where  $\tau$  is the correlation time of the turbulence in the scale  $l_0$ . The second term in (3)  $W_T/\tau$  determines the dissipation of

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the turbulent energy. For a given time-independent source of turbulence  $I_T$  the solution of Eq. (3) is given by

$$W_T = \tau \left[ I_T + \Delta \left( \frac{B^2}{8\pi} \right) \right] \left( 1 - \exp\left\{ -\frac{t}{\tau} \right\} \right) + \tilde{W}_T \exp\left\{ -\frac{t}{\tau} \right\}$$

where  $\tilde{W}_T = W_T(t=0)$ . Note that a time-independent source of the turbulence exists, for example, in the Sun.

In the absence of the mean magnetic field and for  $t \gg \tau$  the solution of Eq. (3) yields

$$W_T \equiv \frac{\langle \rho(u^{(0)})^2 \rangle}{2} + \frac{\langle (h^{(0)})^2 \rangle}{8\pi} = I_T \tau,$$

where  $u_\alpha^{(0)}$  and  $h_\alpha^{(0)}$  are the turbulent velocity and the turbulent magnetic field in a medium with zero mean magnetic field. The mean nonuniform magnetic field yields an additional source of the density energy of the turbulence:  $\eta_T \Delta(B^2/8\pi)$ . Now we estimate the ratio of the two sources of turbulence

$$\frac{|\eta_T \Delta(B^2/8\pi)|}{I_T} \simeq \frac{1}{3} \left( \frac{l_0}{L_0} \right)^2 \left( \frac{B^2}{8\pi} / \frac{\langle \rho u_0^2 \rangle}{2} \right) \ll 1,$$

where  $L_0$  is the characteristic scale of the mean magnetic field. Since  $l_0 \ll L_0$  and  $B^2/8\pi \ll \langle \rho u_0^2 \rangle/2$  we can neglect the small magnetic source of the turbulence. Therefore, for  $t \gg \tau$  the total energy density of the turbulence reaches a stationary value  $W_T = \text{const} = I_T \tau$ . It depends very weakly on the mean magnetic field  $\mathbf{B}$ . Therefore, the total energy density  $W_T$  of the homogeneous turbulence with mean magnetic field can be regarded as conserved (the dissipation is compensated by a supply of energy), i.e.

$$W_k + W_m = \text{const}. \quad (4)$$

For a statistically homogeneous medium it is equivalent to the conservation of the total turbulent energy. Note that the uniform large-scale magnetic field performs no work on the turbulence. It can only redistribute the energy between hydrodynamic fluctuations and magnetic fluctuations.

Combining Eqs. (2) and (4) one can express the change of turbulent pressure  $\delta p_T$  in terms of the change of the magnetic energy density  $\delta W_m$ :  $\delta p_T = -\delta W_m/3$ . It follows hence that the turbulent pressure is reduced when magnetic fluctuations are generated (i.e.  $\delta W_m > 0$ ).

The total turbulent pressure is decreased also by the ‘tangling’ of the large-scale mean magnetic field  $\mathbf{B}$  by hydrodynamic fluctuations (see, e.g., Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983). The mean magnetic field, ‘entangled’ with the hydrodynamic fluctuations, generates supplementary small-scale magnetic fluctuations. In this case the density of the magnetic energy  $W_m$  depends on  $W_k$  and  $W_B$ , where  $W_B = B^2/8\pi$  is the energy density of the large-scale mean magnetic field  $\mathbf{B}$ . For weak mean magnetic

fields ( $W_B \ll W_k$ ), expanding the function  $W_m$  in series in  $W_B$ , one obtains

$$W_m = W_m^{(0)} + a_p(W_k) \frac{B^2}{8\pi} + \dots, \quad (5)$$

where  $W_m^{(0)}$  is the energy density of the magnetic fluctuations in the absence of a large-scale magnetic field. This expression yields the variation of the magnetic energy  $\delta W_m$ . The sign of  $a_p$  is positive when magnetic fluctuations are generated and negative when they are damped. In view of Eq. (5) the turbulent pressure takes the form

$$p_T = p_T^{(0)} - a_p B^2/24\pi$$

(see Kleeorin et al. 1989; 1990), where  $p_T^{(0)}$  is turbulent pressure in the absence of the mean magnetic field.

The turbulent magnetic pressure  $p_h$  as well as the turbulent hydrodynamic pressure  $p_u$  are given by

$$p_h \equiv \frac{1}{3} \left( \frac{\langle h^2 \rangle}{8\pi} \right) = p_h^{(0)} + q_h \frac{B^2}{8\pi}, \quad (6)$$

$$p_u \equiv \frac{2}{3} \left( \frac{\langle \rho u^2 \rangle}{2} \right) = p_u^{(0)} - q_u \frac{B^2}{8\pi} \quad (7)$$

Here  $p_\alpha^{(0)}$  is pressure in a medium with zero mean field. Generation of magnetic fluctuations at the expense of the energy of the hydrodynamic fluctuations corresponds to  $q_h > 0$  and  $q_u > 0$ . The total turbulent pressure is given by  $p_T = p_h + p_u = p_T^{(0)} - (q_u - q_h)B^2/8\pi$ , where  $a_p = 3(q_u - q_h)$ . The total pressure is  $p_{\text{tot}} = p_k + p_T + p_B$ , where  $p_k$  is the usual gasdynamic pressure of the plasma and  $p_B = B^2/8\pi$  is the magnetic pressure of the mean field.

Let us examine the part in  $p_{\text{tot}}$  that depends on the mean (regular) magnetic field  $\mathbf{B}$ :

$$\begin{aligned} p_m(B) &= p_B + (q_h - q_u) \frac{B^2}{8\pi} = (1 + q_h - q_u) \frac{B^2}{8\pi} \\ &\equiv Q_p \frac{B^2}{8\pi}, \end{aligned}$$

such that

$$p_{\text{tot}} = p + p_m(B) = p + Q_p \frac{B^2}{8\pi}, \quad (8)$$

where  $p = p_k + p_T^{(0)}$ . The pressure  $p_m(B)$  is called the *effective magnetic pressure*. It follows that in the presence of developed MHD turbulence it is possible to reverse the sign of the ‘effective’ magnetic pressure  $p_m(B) = Q_p B^2/8\pi$  if  $Q_p < 0$  (i.e.  $1 + q_h < q_u$ ). Note that both the hydrodynamic fluctuations and the magnetic fluctuations contribute to the mean effective magnetic pressure. However, the gain in the turbulent magnetic pressure  $p_h$  is not as large as the reduction of the turbulent hydrodynamic pressure  $p_u$  by the mean magnetic field  $\mathbf{B}$ . It is due to different coefficients in the equation of state (2) before  $W_m$  and  $W_k$ . Therefore, it results in negative contribution of the MHD turbulence to the mean magnetic force.

We consider the case when  $p \gg B^2/8\pi$ , so that the total pressure  $p_{tot}$  is always positive. Only the effective magnetic pressure  $p_m(B)$  may be negative (when  $Q_p < 0$ ) while the pressure  $p_B$  as well as the values  $p$ ,  $p_u$ ,  $p_h$ ,  $p_T$  are positive.

Note that  $Q_p = 1 - a_p/3$ . When a mean magnetic field  $\mathbf{B}$  is superimposed on an isotropic turbulence, the isotropy breaks down. Nevertheless Eq. (8) remains valid; only the relationship between  $Q_p$  and  $a_p$  changes. Note that we use the conservation law for the total turbulent energy only for the demonstration of the principle of the effect, but we have not employed this law to develop the theory of this effect (see Kleeorin et al. 1990; Kleeorin & Rogachevskii 1994a). The high order closure procedure (Kleeorin et al. 1990) and the renormalization group method (Kleeorin & Rogachevskii 1994a) were employed for the investigation of the MHD turbulence at large magnetic Reynolds number  $R_m \gg 1$ . It was found that the effective mean magnetic force is given by (1).

In order to calculate  $Q_p$  in different way we use in this paper a spectrum  $M(k)$  of magnetic fluctuations generated in the presence of a mean magnetic field  $\mathbf{B}$ . The spectrum is given by

$$M(k) \propto k^{-1} B^2, \quad (9)$$

where

$$M(k) = \int \langle h_m(\mathbf{k}) h_m(-\mathbf{k}) \rangle k^2 \sin \theta d\theta d\varphi,$$

and  $l_0^{-1} < k < l_0^{-1} R_m^{1/2}$ . Here  $M(k)$  is the spectral density of magnetic fluctuations,  $(k, \theta, \varphi)$  denotes the spherical coordinates in  $\mathbf{k}$ -space.

The  $k^{-1}$  spectrum of the magnetic fluctuations was first obtained by means of dimensional analysis by Ruzmaikin & Shukurov (1982) (see also Ruzmaikin et al. 1988). Indeed, comparison of the terms

$$|(\mathbf{B} \cdot \nabla) \mathbf{u}| \sim \eta |\Delta \mathbf{h}|$$

in the induction equation [see Eq. (A2)] yields the spectrum of the magnetic fluctuations:

$$M(k) \sim \frac{B^2}{\eta^2} k^{-2} W(k), \quad (10)$$

where  $W(k) \approx k^{-1} u^2(k)$  is the spectrum of the kinetic energy of MHD turbulence. Now we take into account that the turbulent magnetic diffusion  $\eta \sim u(k)/k$ . Therefore Eq. (10) is reduced to (9) (see Ruzmaikin & Shukurov 1982; Ruzmaikin et al. 1988).

The spectrum (9) of the magnetic fluctuations was found also by Kleeorin and Rogachevskii (1994) by means of the renormalization group method. It was shown that the  $k^{-1}$  spectrum of magnetic fluctuations is universal; it is independent of the exponent of the spectrum of the turbulent velocity field. The  $k^{-1}$  spectrum of the magnetic fluctuations can also be obtained by means of the high-order closure procedure (see Appendix B). Direct three-dimensional numerical simulations by Brandenburg et al. 1993; 1994 of the magnetic dynamo in hydrodynamic

convection also reveal the  $k^{-1}$  spectrum of the magnetic fluctuations in the presence of the generated mean magnetic field.

The energy of the magnetic fluctuations can be calculated from (9):

$$\frac{\langle h_B^2 \rangle}{8\pi} = \int_{l_0^{-1}}^{l_0^{-1} R_m^{1/2}} M(k) dk \simeq \frac{1}{2} \ln(R_m) \frac{B^2}{8\pi}, \quad (11)$$

The total energy of magnetic fluctuations is  $\langle h^2 \rangle = \langle (h^{(0)})^2 \rangle + \langle h_B^2 \rangle$ . It follows from Eq. (4) that

$$\frac{\langle h^2 \rangle}{8\pi} + \frac{\rho \langle u^2 \rangle}{2} = \frac{\langle (h^{(0)})^2 \rangle}{8\pi} + \frac{\rho \langle (u^{(0)})^2 \rangle}{2}. \quad (12)$$

Equation (12) yields

$$\langle u^2 \rangle = \langle (u^{(0)})^2 \rangle - \frac{\langle h_B^2 \rangle}{4\pi\rho}. \quad (13)$$

Using Eqs. (11)-(13) we calculate the turbulent magnetic pressure  $p_h$  and the turbulent hydrodynamic pressure  $p_u$

$$p_h \equiv \frac{1}{3} \left( \frac{\langle h^2 \rangle}{8\pi} \right) = \frac{1}{3} \left( \frac{\langle (h^{(0)})^2 \rangle}{8\pi} \right) + \frac{1}{6} \ln(R_m) \frac{B^2}{8\pi}, \quad (14)$$

$$p_u \equiv \frac{2}{3} \left( \frac{\langle \rho u^2 \rangle}{2} \right) = \frac{2}{3} \left( \frac{\langle \rho (u^{(0)})^2 \rangle}{2} \right) - \frac{1}{3} \ln(R_m) \frac{B^2}{8\pi} \quad (15)$$

Comparing Eq. (14) with (6) and Eq. (15) with (7) yield

$$q_h = \frac{1}{6} \ln(R_m), \quad q_u = \frac{1}{3} \ln(R_m), \\ Q_p \equiv 1 + q_h - q_u = 1 - \frac{1}{6} \ln(R_m). \quad (16)$$

It is seen from (16) that if  $R_m > 403$  the effective negative magnetic pressure is negative.

Therefore, this simple calculations allow us to show that the effective magnetic pressure can change sign in a developed MHD turbulence. The difference in coefficient multiplying  $\ln(R_m)$  in the expression for  $Q_p$  obtained here and in the paper by Kleeorin et al. (1990) is due to the fact that we do not take into account here the anisotropy of the magnetic fluctuations  $\langle h_B^2 \rangle$  caused by the presence of the mean magnetic field.

## 2. The governing equations and models

Consider the processes developing in the solar convective zone on the time scale  $\sim 11$  years (see Sects. 3 and 5). The basic equations of the problem are the mean field equations:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla \left( p + \frac{Q_p}{8\pi} \mathbf{B}^2 \right) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) Q_s \mathbf{B} + \rho \mathbf{g} \\ + 2\rho \mathbf{v} \times \boldsymbol{\Omega} + \mathbf{F}(\mathbf{v}) + \mathbf{F}_\nu(\mathbf{v}) - \rho \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}), \quad (17)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} + \alpha \mathbf{B}) + \eta \Delta \mathbf{B}, \quad (18)$$

$$\frac{\partial \alpha}{\partial t} = f(\mathbf{B}) - \frac{\alpha - \alpha_0}{\tau_*}, \quad (19)$$

$$\nabla \cdot (\rho \mathbf{v}) = 0, \quad (20)$$

where  $\alpha_0 = -(\tau/3)\langle \mathbf{u} \cdot (\nabla \times \mathbf{u}) \rangle$  is the hydrodynamic part of the  $\alpha$ -effect,  $\mathbf{g}$  is the free-fall acceleration,  $\boldsymbol{\Omega}$  is the angular velocity,  $\mathbf{F}_\nu(\mathbf{v})$  is the dissipation force,  $\eta$  is the magnetic diffusion,  $p$  is the gas pressure determined by the equation of state,  $\mathbf{F}(\mathbf{v})$  is the additional force which depends on the velocity. This force, e. g., describes a source of the differential rotation and is related to the anisotropy of the turbulent convection (Rüdiger 1989). The force  $\mathbf{F}(\mathbf{v})$  may also include a component that is determined by the  $\Lambda$ -effect (see, e. g., Kichatinov & Rüdiger 1993). The nonlinear equation (19) determines the evolution of the  $\alpha$ -effect,  $\tau_*$  is the relaxation time of the  $\alpha$ -effect. Equations (17)-(19) are written in a frame rotating with angular velocity  $\boldsymbol{\Omega}$ . The velocity is given by  $\mathbf{v} = \mathbf{v}_\Omega + \mathbf{U}(\mathbf{B})$ , where  $\mathbf{v}_\Omega$  describes the differential rotation, while  $\mathbf{U}(\mathbf{B})$  corresponds to motions under the influence of the mean magnetic field (the torsional oscillations and the meridional flows in the solar convective zone). As follows from estimations and observations,  $|\mathbf{v}_\Omega| \gg |\mathbf{U}(\mathbf{B})|$  (see, e. g., Rüdiger 1989; Küker et al. 1993). Therefore in first approximation we can replace in the induction equation (18) the velocity  $\mathbf{v}$  by  $\mathbf{v}_\Omega$ . The equation for the velocity  $\mathbf{v}_\Omega$  coincides with Eq. (17) for  $\mathbf{B} = 0$  (see below). Therefore Eqs. (18)-(19) are decoupled from Eq. (17) and are regarded as the equations for the mean-field dynamo problem (see, e. g., Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983). The function  $f(\mathbf{B})$  depends on the model of the nonlinear dynamo. For example,

$$f(\mathbf{B}) = \frac{\mu}{4\pi\rho} \left( \mathbf{B} \cdot (\nabla \times \mathbf{B}) - \frac{\alpha \mathbf{B}^2}{\eta_T} \right),$$

where  $\mu \approx 0.1$ ,  $\tau_* = l_0^2/8\pi^2\eta_m$  (see, e. g., Kleeorin et al. 1994; 1995). When  $\tau_* \ll T_c$  and  $|\mathbf{B} \cdot (\nabla \times \mathbf{B})| \ll \alpha_0/(\xi\eta_T)$  we get the well-known result for the total  $\alpha$ -effect

$$\alpha = \frac{\alpha_0}{1 + \xi_* B^2}$$

(see, e. g., Rüdiger 1974; Roberts & Soward 1975), where  $\xi_* = \mu\tau_*/(4\pi\rho\eta_T)$ ,  $T_c$  is a period of the cyclic activity.

Comparison of the magnetic, kinetic and gravitation energy in the solar convective zone yields:

$$\frac{B^2}{8\pi} \ll \frac{\rho v_\Omega^2}{2} \ll \rho g R_\odot$$

(see, e. g., Priest 1982), where  $R_\odot$  is the solar radius. It follows that in first approximation the equilibrium for the momentum equation is given by

$$\nabla p_0 = \rho_0 \mathbf{g}, \quad (21)$$

where  $p_0$  and  $\rho_0$  are the unperturbed pressure and density, respectively. We consider axisymmetric case. The equation for  $\mathbf{v}_\Omega$  is given by

$$\rho_0 \frac{d\mathbf{v}_\Omega}{dt} = -\nabla p_1 + \rho_1 \mathbf{g} + 2\rho_0 \mathbf{v}_\Omega \times \boldsymbol{\Omega} - \rho_0 \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) + \mathbf{F}(\mathbf{v}_\Omega) + \mathbf{F}_\nu(\mathbf{v}_\Omega), \quad (22)$$

where  $p_1$  and  $\rho_1$  are the perturbed pressure and density, respectively, the force  $\mathbf{F}(\mathbf{v}_\Omega)$  describes a source of the differential

rotation and is related to the anisotropy of the turbulent convection. This equation determines the evolution of the differential rotation (see, e. g., Rüdiger 1989; Besnovatyi-Kogan 1990). In this paper we assume that the differential rotation and the mean magnetic field are given. Subtracting Eq. (22) and Eq. (21) from Eq. (17) yields an equation for the velocity  $\mathbf{U}(\mathbf{B})$ :

$$\rho_0 \frac{d\mathbf{U}}{dt} = -\nabla \left( p_2 + \frac{Q_p}{8\pi} B^2 \right) + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) Q_s \mathbf{B} + 2\rho_0 \mathbf{U} \times \boldsymbol{\Omega} + \mathbf{F}_\nu(\mathbf{U}) + \rho_2 \mathbf{g}, \quad (23)$$

where  $p = p_0 + p_1 + p_2$ ,  $\rho = \rho_0 + \rho_1 + \rho_2$ ,  $p_2 \ll p_1 \ll p_0$ ,  $\rho_2 \ll \rho_1 \ll \rho_0$  and

$$\rho_2 g R_\odot \ll \frac{\rho_0 U^2}{2} \ll \frac{B^2}{8\pi}$$

The force  $\mathbf{F}(\mathbf{U})$  is negligible. Equation (23) determines the evolution of the solar torsional oscillations and the meridional motions. This process will be considered in Sect. 5. In this paper we also study the short-time phenomena (Sect. 4) developed on time scales from several minutes to several hours (Alfvén time). The governing equations for these processes follow from Eqs. (17)-(18) except for the absence of the  $\alpha$ -effect term. This system should be supplemented by the thermal equation (see Sect. 4). This analysis is important for investigations of the short-period solar oscillations and the flux tubes formation in the solar convective zone.

### 3. 11-year oscillations of the solar radius

Measurements of the solar radius demonstrate 11-year periodicity: during the period of maximum solar activity the solar radius is minimal, while during the minimum of the activity it is maximal (Delache et al. 1993; Gavryusev et al. 1994). This anomalous phase shift between the oscillations and the magnetic activity is yet unexplained. In this section a mechanism of the 11-year oscillations of the solar radius is considered.

We start by estimating the radius of a star. The distribution of the mass of the star is described by

$$\frac{dM}{dr} = \Psi(M), \quad (24)$$

where  $\Psi(M) = \int_\sigma \rho(\mathbf{r}) d\sigma$  is the mass of spherical shell of unit thickness,  $d\sigma = r^2 \sin\theta d\theta d\varphi$  is the element of the surface in spherical coordinates  $r$ ,  $\theta$ ,  $\varphi$ ,  $dM$  is the mass of the spherical shell of thickness  $dr$ ,  $\rho$  is the density. Integration of Eq. (24) yields the radius of the star

$$R_* = \int_0^M \frac{dM}{\Psi(M)}, \quad (25)$$

Variation of the total pressure  $p = p_k + p_T^{(0)} + Q_p(B)B^2/8\pi$  vanishes over the time interval which considerably exceeds both acoustic and alfvénic times. Therefore the variation of the gas pressure  $\delta p_k$  is given by

$$\delta p_k \sim -\delta \left( Q_p(B) \frac{B^2}{8\pi} \right). \quad (26)$$

The variation of the pressure on the time scale  $\sim 11$ -year is determined only by the variation of the density  $\delta\rho$  due to the large turbulent thermal conductivity of the solar convective zone. Therefore, Eq. (26) is reduced to

$$\frac{\delta\rho}{\rho} \sim -\frac{1}{p_k} \delta \left( Q_p(B) \frac{B^2}{8\pi} \right). \quad (27)$$

Hence, the variation of the radius  $\delta R$  of a star with a spherical symmetric steady state is now given by

$$\delta R \sim \frac{1}{32\pi^2} \int_{R_*-L_0}^{R_*} \left( \frac{dr}{r^2 p_k} \right) \int_{\sigma} \delta \left( Q_p(B) B^2 \right) d\sigma. \quad (28)$$

Here  $L_0$  is the thickness of the convective zone. In Eq. (25) we assume that the mass  $M$  of the star is time-independent. The sign of  $\delta R$  depends on the sign of  $Q_p$ . In the absence of developed MHD turbulence ( $Q_p = 1$ ) the amplification of the magnetic field results in increase of the radius of the star. On the other hand, in developed MHD turbulence the magnetic coefficient  $Q_p < 0$ . Therefore, the growth of the magnetic field during the period of the solar activity leads to decrease of the radius of the Sun ( $\delta R < 0$ ) in agreement with the observations (Delache et al. 1993; Gavryusev et al. 1994). Note that the oscillations of the solar radius are regarded as oscillations of the equilibrium configuration. By contrast to these oscillations the solar torsional oscillations and meridional motions are non-potential ones and they are not accompanied by oscillations of the density.

Now we estimate the value  $\delta R/R$ . The distribution of pressure  $p_k$  is given by

$$p_k \simeq p_0 \exp\left(-\frac{r-R}{\Lambda_p}\right),$$

where  $p_0 = p_k(r \simeq R) \sim 3.2 \times 10^5$  d / cm<sup>2</sup> (the pressure  $p_0$  is chosen at depth  $H \sim 200$  km from the sun's surface),  $\Lambda_p \sim 2 \times 10^7$  cm is the scale height of the pressure. The surface integral is estimated as

$$\int_{\sigma} \delta \left( Q_p(B) B^2 \right) d\sigma \propto 4\pi r^2 Q_p B_0^2,$$

where  $B_0$  is the characteristic value of the mean magnetic field in the vicinity of the surface of the sun. Therefore the value  $\delta R/R$  is given by

$$\frac{\delta R}{R} \propto \frac{\Lambda_p}{p_0 R} |Q_p| \frac{B_0^2}{8\pi} \simeq (1.4 - 3.4) \times 10^{-6}, \quad (29)$$

where  $B_0 \sim 200 - 300$  G,  $Q_p \sim 1.1$  at depth  $H \sim 200$  km from the sun's surface. Note that the solar radius oscillations are determined by equations for mean fields. The equation for the mean fields can be used in regions with developed turbulence. According to models for solar convective zone (see, e. g., Spruit 1974) the developed turbulence exists at depth  $H \geq 2 \cdot 10^7$  cm (from the sun's surface). The value used for toroidal magnetic field  $B_0$  is in agreement with models of solar magnetic field (see, e. g., Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983; Kleeorin et al. 1994; 1995).

Now we compare (29) with observed value. To do this we use the solar radius measurements spectral density that are presented in Fig. 1 by Gavryusev et al. (1994). The first peak in the spectrum corresponds to oscillations with period  $\approx 11.4$  years. The area under this peak is square of the amplitude of the solar radius oscillations  $(\delta R)^2 \sim 4 \times 10^{-5}$  arcsec<sup>2</sup>. Here we take into account that a level of noise in the spectrum of the oscillations of the square of the radius is  $\sim 5 \times 10^{-3}$  arcsec<sup>2</sup>. Therefore, the observed value is  $\delta R/R \sim 6.6 \times 10^{-6}$ . The estimation (29) is thus in satisfactory agreement with the observed value.

#### 4. Short-time solar oscillations and large-scale magnetic ropes formations

The magnetic fields of solar active regions have a highly nonuniform structure: a configuration of flux ropes is developed by some mechanism. It has been suggested (see, e.g., Parker 1979; Priest 1982) that the magnetic flux ropes might originate from the prevailing large-scale field in the solar convective zone when magnetic buoyancy triggers instabilities there. However, in order for such an instability to set in, the initial magnetic field would have to be strongly nonuniform in the direction of gravity: the scale of field variation is smaller than the density height variations. Magnetic fields as nonuniform as we observe could be excited only by powerful, localized generators - sources. This situation, however, is not typical for the solar convective zone (see, e.g., Moffatt 1978; Zeldovich et al. 1983).

In addition, the source of the solar short-time oscillations and some of its features remain poorly understood. In particular, it is difficult to explain the observed correlation of the frequency and amplitude of the oscillations with the phase of the 11-year cycle of activity.

We suggest here that these phenomena are due to the modification of the mean magnetic force in the turbulent convective zone (see Sect. 1). This effect is a consequence of a generation of magnetic fluctuations at the expense of hydrodynamic fluctuations. It leads to a decrease of the elasticity of the large-scale (mean) magnetic field, so that under certain conditions, the 'effective' mean magnetic pressure can change sign (Kleeorin et al. 1990; Kleeorin & Rogachevskii 1994a).

This modification of the mean magnetic force results in the excitation of large-scale MHD instabilities. The instabilities cause the formation of inhomogeneities of the regular mean magnetic field. The energy for these processes is supplied by the small-scale turbulent fluctuations of the convective zone. This effect develops even in an initially uniform magnetic field.

Now we explain the essence of the effect qualitatively. We first consider the properties of magnetic buoyancy in the presence of small-scale turbulence. Let the  $x$  axis of a Cartesian coordinate system be directed along the gravitational field, and let the  $z$  axis lie along the mean magnetic field  $\mathbf{B}_0$ . We consider for simplicity an isothermal plasma in the absence of dissipative processes. A nonisothermal plasma will be considered in this section within the general treatment [see Eqs. (36)-(39)]. The isothermal model serves just as an heuristic argument in order to gain some insight into the physical processes.

We examine a magnetic flux tube located along the  $x$  axis at level  $a$ , say, where the density is  $\rho_a$  and the magnetic field  $B_a$ . Now we gradually move the flux tube as a whole upward (opposite the gravitational field) from level  $a$  to level  $b$ , at which the ambient medium has corresponding parameters  $\rho_b, B_b$ . If, after equalization of the total pressure inside and outside the magnetic tube, the density  $\rho_b^*$  within the tube (in position  $b$ ) turns out to be lower than the density  $\rho_b$  of the surrounding plasma, the flux tube will continue to float upward due to Archimedes forces. The criterion for balance between the total pressure inside and outside the flux tube (at level  $b$ ) is given by

$$C_s^2 \rho_b + Q_s \frac{B_b^2}{8\pi} = C_s^2 \rho_b^* + Q_s \frac{(B_b^*)^2}{8\pi} \quad (30)$$

where  $C_s$  is the sound speed,  $B_b^*$  is the magnitude of the magnetic field within the tube at the point  $x_b$ . The sound speed is assumed to be the same since the thermal conductivity of the convective zone is very high and hence the temperature is almost constant. Assuming the displacement  $\zeta = x_b - x_a$  to be small, we shall write the density  $\rho_b$  and the magnetic field  $B_b$  as

$$\rho_b = \rho_a(1 + \zeta/\Lambda_\rho), \quad B_b = B_a(1 + \zeta/\Lambda_B) \quad (31)$$

where  $\Lambda_\rho^{-1} = \rho'_0/\rho_0 \simeq C_s^2/g$  and  $\Lambda_B^{-1} = B'_0/B_0$  are the scale heights of the density and the magnetic field. Here  $f'$  denotes a derivative with respect to  $x$ . By use the laws of conservation of the mass and magnetic flux inside the tube one obtains:  $B_b^*/\rho_b^* = B_a/\rho_a$ . Combination (30)-(31) yields the density excess  $\Delta\rho = \rho_b^* - \rho_b$ :

$$\Delta\rho = -\frac{Q_p B_a^2 (\Lambda_B - \Lambda_\rho) \zeta}{4\pi C_s^2 \Lambda_B \Lambda_\rho}. \quad (32)$$

The flux tube becomes buoyant, that is, instability can set in, if

$$Q_p \left( \frac{\Lambda_B - \Lambda_\rho}{\Lambda_B \Lambda_\rho} \right) < 0. \quad (33)$$

In the case of weak turbulence with a relatively small magnetic Reynolds number (the quantity  $Q_p \approx 1$ ), the small-scale turbulence will not affect large-scale processes. Then in view of the condition (33), the criterion for instability due to buoyancy will take the form  $\Lambda_B < \Lambda_\rho$ . In other words, instability will develop only if the scale for change in the initial magnetic field is less than the density scale-height (Parker 1979; Priest 1982).

The situation will be radically different, however, in a medium with developed MHD turbulence. Negative contribution of the MHD turbulence to the mean magnetic force results in decrease of the elasticity of the field, so that  $Q_p$  can be negative. This means that conventional magnetic buoyancy in a highly nonuniform magnetic field will no longer exist [see the inequality (33)]. On the other hand, when  $\Lambda_B > \Lambda_\rho$  and  $Q_p < 0$  instability will be excited in the large-scale magnetic field. It is seen from condition (33) that instability will develop even in a large-scale field that is initially uniform.

Let us estimate the growth rate of this instability. Neglecting dissipative processes for simplicity's sake, we shall retain

only the Archimedes force in the momentum equation of the magnetic flux tube

$$\frac{d^2 \zeta}{dt^2} = - \left( \frac{C_A}{C_s} \right)^2 \frac{g Q_p (\Lambda_B - \Lambda_\rho) \zeta}{\Lambda_B \Lambda_\rho} \quad (34)$$

where  $C_A = B_a/\sqrt{4\pi\rho_a}$  is the Alfvén velocity. We seek a solution to Eq. (34) of the form  $\zeta \sim \exp(\Gamma t)$ . The growth rate of this instability is given by

$$\Gamma \simeq \frac{C_A}{\Lambda_\rho} \sqrt{Q_p \left( \frac{\Lambda_\rho}{\Lambda_B} - 1 \right)}. \quad (35)$$

Here  $\Lambda_\rho \simeq C_s^2/g$ .

As will be shown now, estimation (35) of the growth rate of the instability is in agreement with a more rigorous theory of this effect. Here we consider nonisothermal plasma.

Note that magnetic buoyancy applies to two different situations (see Priest 1982). The first corresponds to a problem described by Parker (1966; 1979) and Gilman (1970). They considered instability of stratified continuous magnetic field. They do not use a magnetic flux tube. The other situation was considered by Parker (1955); Spruit (1981); Spruit and van Ballegoijen (1982); Ferriz-Mas and Schüssler (1993); Schüssler et al. (1994). They study buoyancy of horizontal magnetic flux tubes. In our paper we investigate the first situation. Therefore we do not consider buoyancy of magnetic flux tubes, and we study instability of continuous magnetic field in small-scale turbulent flow. In such situation we take into account the dissipation due to turbulent magnetic diffusion separately from the analysis of the magnetic instability. This is possible because the dissipative term is determined by the second spatial derivative, whereas the term that determines the instability is independent of spatial derivatives. More rigorous study can be carry out by means of numerical simulation. This is the subject of a separate paper.

The equations for the large-scale fields in the absence of dissipation processes have the following form

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla \left( p + \frac{Q_p}{8\pi} \mathbf{B}^2 \right) + \frac{Q_s}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} + \rho \mathbf{g}, \quad (36)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (37)$$

where  $\mathbf{v}$  and  $\mathbf{B}$  are the velocity and magnetic field respectively,  $p$  is the pressure. In this section we consider the case when the angular velocity  $\Omega = 0$ . This is justified since we study effects with time scale of the order that ranges from several minutes to hours while the rotation of the Sun is essentially on a time scale longer than a month.

Equations (36)-(37) should be supplemented by the continuity equation and the equation for entropy  $S = \ln(p\rho^{-\gamma})/\gamma$ :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (38)$$

$$\rho T \left( \frac{\partial S}{\partial t} + (\mathbf{v} \cdot \nabla) S \right) = 0, \quad (39)$$

where  $\gamma$  is the ratio of the specific heats.

We now linearize Eqs. (36)-(39) about the equilibrium state, and denote the perturbed quantities by subscript 1 while equilibrium quantities are denoted by subscript 0. It is convenient to express all perturbed quantities in terms of the Lagrangian displacement vector  $\xi(\mathbf{r}, t)$ , where  $\mathbf{v}_1 = \partial\xi/\partial t$ .

We search for a solution of Eqs. (36)-(39) within the framework of the WKB approximation:

$$\xi = e^{i\chi/\varepsilon}(\xi^{(0)} + \varepsilon\xi^{(1)} + \varepsilon^2\xi^{(2)} + \dots),$$

where  $\chi = \chi(\mathbf{r}_\perp)$  and  $\mathbf{r}_\perp$  is a position vector perpendicular to  $\mathbf{B}_0$ . The parameter  $\varepsilon \ll 1$  is a measure of the fast variation of the phase across magnetic field lines. The functions  $\chi$  and  $\xi^{(j)}$  are considered to be of order 1. We use a method developed for investigations of interchange and ballooning mode (see, Hameiri 1983; Hameiri et al. 1991). It should be noted that the variations of the perturbation occur on length scales that are on one hand much smaller than that of the equilibrium state while on the other hand much larger than those turbulent fluctuations that contribute to  $Q_s$  and  $Q_p$ . To lowest order in  $\varepsilon$  Eqs. (36)-(39) yield

$$\xi^{(0)} \cdot \nabla\chi = 0,$$

which results in the following representation of  $\xi^{(0)}$ :

$$\xi^{(0)} = \xi_N \hat{\mathbf{e}}_N + \xi_B \hat{\mathbf{e}}_B,$$

where

$$\hat{\mathbf{e}}_B = \mathbf{B}_0/|\mathbf{B}_0|, \quad \hat{\mathbf{e}}_N = \hat{\chi} \times \hat{\mathbf{e}}_B, \quad \hat{\chi} = \nabla\chi/|\nabla\chi|.$$

As the magnetic field lines are considered here to be infinite, the following traveling solution is used:  $\xi = \xi_0 \exp\{-i(\omega t - \mathbf{k}_B \cdot \mathbf{r})\}$ , where  $\mathbf{k}_B$  is parallel to  $\mathbf{B}_0$ . The equations for the component of vector  $\xi^{(0)} = \xi_N \hat{\mathbf{e}}_N + \xi_B \hat{\mathbf{e}}_B$  are given by

$$(\omega^2 + \lambda^2 \sigma^2 - Q_s k_B^2) \xi_N = ik_B \lambda a \frac{Q_p}{K(Q_p)} \xi_B, \quad (40)$$

$$ik_B \lambda a \frac{Q_s}{K(Q_p)} \xi_N = -\left(\omega^2 - \frac{Q_s k_B^2}{K(Q_p)}\right) \xi_B \quad (41)$$

(Kleeorin et al. 1993) where  $\omega$  and  $k_B$  are the nondimensional frequency and wave number,  $\omega$  is measured in units of  $C_A/\Lambda_\rho$  and  $k_B$  is in units of  $\Lambda_\rho^{-1}$ ,  $C_A = B_0/\sqrt{4\pi\rho_0}$  is the Alfvén speed,  $\Lambda_\rho^{-1} = \rho'_0/\rho_0$ ,  $a = g\Lambda_\rho/C_s^2$ . Here and below  $f'$  is a derivative with respect to  $x$ , the  $x$  axis is directed along the gravitational field. The parameter  $\lambda = (\xi \cdot \mathbf{g})/g\xi_N$  is connected to the polarization parameter of the wave. The parameter  $\sigma$  is

$$\sigma^2 = -K^{-1}(Q_p) \left( \Omega_b^2 + Q_p a \left( 1 - \frac{\Lambda_\rho}{\Lambda_B} \right) \right),$$

where  $\Lambda_B^{-1} = B'_0/B_0$  and  $\Omega_b^2 = -\mathbf{g} \cdot \mathbf{N}_b \Lambda_\rho^2 / C_A^2$  is the nondimensional Brunt-Vaisala frequency, where

$$\mathbf{N}_b = \frac{\nabla p_0}{\gamma p_0} - \frac{\nabla \rho_0}{\rho_0}, \quad K(Q_p) = 1 + \frac{2Q_p}{\gamma\beta}, \quad \beta = \frac{8\pi p_0}{\mathbf{B}_0^2},$$

$C_s = \sqrt{\gamma p_0/\rho_0}$  is the sound speed.

Consider first the case  $k_B = 0$ . This corresponds to the interchange mode. In this case one can see from Eqs. (40)-(41) that  $\xi_B = 0$  and  $\omega^2 = -\lambda^2 \sigma^2$ . The stability is determined by the sign of  $\sigma^2$ . In the case of weak magnetic field  $K(Q_p)$  is close to unity and  $\Omega_b^2$  is much bigger than the other term in the definition of  $\sigma^2$ . Hence, if  $\Omega_b^2 < 0$  the classical convective instability develops with growth rate given by Schwarzschild (1958):

$$\Gamma^c = \lambda g^{1/2} \left( \frac{p'_0}{\gamma p_0} - \frac{\rho'_0}{\rho_0} \right)^{1/2}. \quad (42)$$

For the general case, the criterion for the interchange instability to occur is:

$$\Omega_b^2 < -Q_p a \left( 1 - \frac{\Lambda_\rho}{\Lambda_B} \right) \left( \frac{C_A}{\Lambda_\rho} \right)^2 \quad (43)$$

(Kleeorin et al. 1993). We first notice that for a uniform magnetic field, i.e.  $\Lambda_B \rightarrow \infty$ , and in the nonturbulent case condition (43) coincides with the criterion given by Tserkovnikov (1960). Examination of (43) reveals that in nonturbulent media (i.e.  $Q_p = 1$ ) the magnetic field stabilizes the system if  $\Lambda_B > \Lambda_\rho$ . If the latter is not satisfied, instability may occur for which Parker's instability (see, e. g., Parker 1979; Priest 1982), i.e. the case  $\Omega_b = 0$ , is a particular case.

In turbulent media the criterion for instability is significantly changed. Now, since  $Q_p$  may become negative, an instability may occur even if  $\Lambda_B > \Lambda_\rho$ . The source of free energy of the new type of instability is provided by the small-scale turbulent fluctuations. In contrast, the free energy in Parker's instability is drawn from the gravitational field. In this sense, it is analogous to the Rayleigh-Taylor instability. The growth rate of the MHD instability due to the developed small-scale MHD turbulence is given by

$$\Gamma = \frac{C_A}{\Lambda_\rho} \sqrt{Q_p a \left( \frac{\Lambda_\rho}{\Lambda_B} - 1 \right) \left( 1 + \frac{2Q_p}{\gamma\beta} \right)^{-1}} \quad (44)$$

(Kleeorin et al. 1993). The criterion of this instability for the case of the isothermal plasma and for  $\beta \gg 1$  coincides with the one given by Kleeorin & Rogachevskii (1990). The geometric optics approximation was not used there. Also (44) for the case of the isothermal plasma and for  $\beta \gg 1$  is in agreement with the estimation (35).

The instability mechanism consists of the following. An isolated tube of magnetic field lines, moving upwards, turns out to be lighter than the surrounding plasma, since the decrease of the magnetic field in it, due to expansion of the tube, is accompanied by an increase of the magnetic pressure inside the tube. This increase, due to the fact that the 'effective' magnetic pressure is negative, leads to a decrease of the density inside the tube

and to appearance of a buoyant force. It results in the upwards floating of the magnetic flux tube, i.e. it causes the development of the instability.

We turn now to the case where  $k_B \neq 0$ . The dispersion equation for this case is given by

$$\omega^2 = -\frac{1}{2} \left( \lambda^2 \sigma^2 - \frac{K+1}{K} Q_s k_B^2 \right) \pm \frac{1}{2} D^{1/2}, \quad (45)$$

where

$$D = \left( \lambda^2 \sigma^2 - \frac{K-1}{K} Q_s k_B^2 \right)^2 + \frac{4Q_s Q_p}{K^2} (k_B a \lambda)^2.$$

Generally, Eq. (45) describes alfvénic and magneto-gravitational modes. In order to separate the effects due to the classical convective instability (i.e.  $\Omega_b^2 < 0$ ) from the pure MHD instability we consider the case of very small  $\Omega_b^2$ :  $|\Omega_b^2| \leq \beta^{-1}$ , where  $\beta \gg 1$ . For example, this condition is valid in the case of developed turbulent convection (Priest 1982; Spruit 1974).

For  $\beta \gg 1$  ( $K \sim 1$ ) Eq. (45) is given by

$$\tilde{\Omega}^2 = (1 - \kappa^2 \pm \sqrt{1 - 2\tilde{\alpha}\kappa^2})/2, \quad (46)$$

where

$$\kappa^2 = \frac{2Q_s}{\lambda^2 \sigma^2} k_B^2 \simeq \frac{2Q_s}{a \lambda^2 |Q_p|} k_B^2, \\ \tilde{\alpha} = \frac{a^2 |Q_p|}{\sigma^2} \simeq a, \quad \tilde{\Omega}^2 = -\frac{\omega^2}{\lambda^2 \sigma^2}.$$

In the interval  $0 < \kappa \leq \kappa_0 = (2\tilde{\alpha})^{-1/2}$  (i.e.  $D \geq 0$ ) the value  $\tilde{\Omega}^2$  is real, hence the modes are either purely growing or purely oscillatory.

For  $\kappa > \kappa_0$ ,  $\tilde{\Omega}^2$  become complex and hence oscillatory modes with growing amplitudes exist. As was discussed before, the growth of the unstable modes is at the expense of the energy of the MHD turbulence. We now examine the dependence of the spectrum given by (46) on the single parameter  $\tilde{\alpha}$ . For  $\tilde{\alpha} \leq 1$  the value  $\kappa_* = \sqrt{2(1 - \tilde{\alpha})}$  lies in the interval  $0 < \kappa \leq \kappa_0$ . The interval  $0 < \kappa \leq \kappa_0$  provides a gap for the growth rate spectrum (the imaginary part of  $\tilde{\Omega}$ ) or for the frequency spectrum. This can be seen in Figs. 1-2 for which the plus and minus signs in (46) was used respectively. For  $\tilde{\alpha} > 1$  no such gap exists.

In the limit  $\kappa \gg 1$  the frequency tends to  $\omega_0 \simeq \kappa/\sqrt{2}$  while the growth rate is close to  $\Gamma_0 \simeq \sqrt{\tilde{\alpha}}/2$ . For the case  $\sigma = 0$  the frequency is given by  $\omega_R \simeq k_B C_A \sqrt{|Q_p|}$  while the growth rate is given by  $\gamma \simeq (C_A/2\Lambda_\rho) a \lambda \sqrt{|Q_p|} \ll \omega_R$  where now  $\omega_R$  and  $\gamma$  are dimensioned variables.

The obtained results are important for the problem of a source of the solar short-time oscillations and the sunspots formation. The oscillations can be excited by the MHD instability in the upper layers of the turbulent convective zone located under the visible surface of the Sun. In this region convective cells (granules) are created and annihilated, a large-scale regular magnetic field is generated, and fine-structure oscillations are excited. The growing oscillations in the interval  $\kappa > \kappa_0$

can be interpreted as a source of the observed short-time solar oscillations. In contrast to the previous models which relate the source to the convective noise (see, e.g., Priest 1982), a source of the short-time solar oscillations proposed here is coherent. The plasma in the solar convective zone has the following parameters (Spruit 1974):

a) at depth  $H \sim 2 \cdot 10^7$  cm (from the sun's surface):  $R_m \sim 10^5$ ,  $u_0 \sim 9.4 \cdot 10^4$  cm/s,  $l_0 \sim 2.6 \cdot 10^7$  cm,  $\rho_0 \sim 4.5 \cdot 10^{-7}$  g/cm<sup>3</sup>,  $B_0 \sim 10^2$  G,  $\Lambda_\rho \sim 3.6 \cdot 10^7$  cm. Here  $u_0$  is the characteristic turbulent velocity. By Eq. (1), the coefficient  $Q_p \sim -1.1$ .

b) at depth  $H \sim 10^9$  cm:  $R_m \sim 3 \cdot 10^7$ ,  $u_0 \sim 10^4$  cm/s,  $l_0 \sim 2.8 \cdot 10^8$  cm,  $\rho_0 \sim 5 \cdot 10^{-4}$  g/cm<sup>3</sup>,  $B_0 \sim 10^2$  G,  $\Lambda_\rho \sim 4.3 \cdot 10^8$  cm. We then have  $Q_p \sim -1.8$ .

For the parameters given above, the period of oscillations ranges between several minutes ( $H \simeq 200$  km) and several hours ( $H \simeq 10^4$  km). This is within the range of the observed oscillations in the Sun. The frequency and amplitude of the oscillations depend on the large-scale magnetic field. The field is changed with the 11-year cycle. It explains the observed correlation of the frequency and amplitude of the solar oscillations with a phase of the 11-year cycle of activity (see, e.g., Priest 1982).

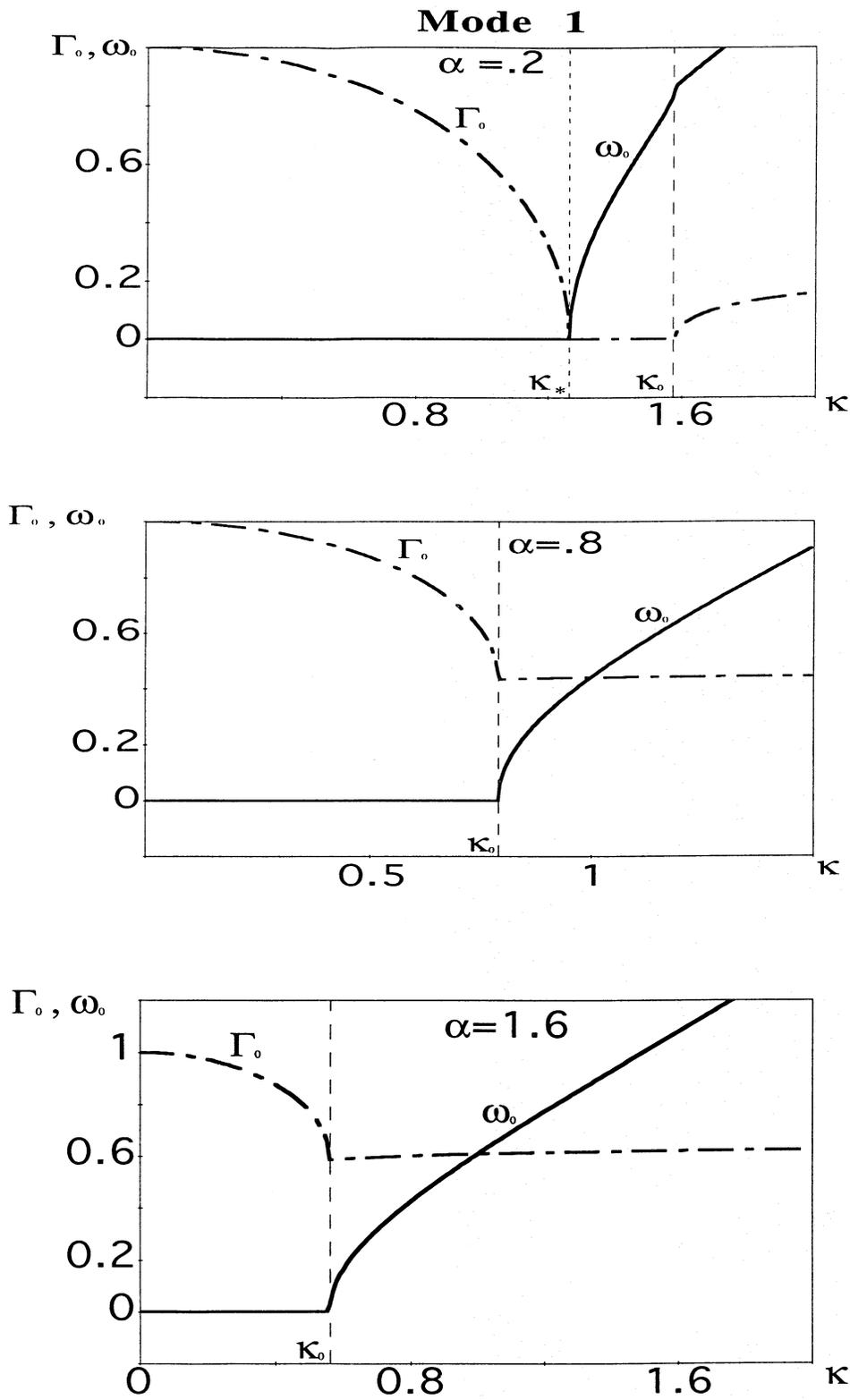
It should be noted that the considered oscillatory modes with growing amplitudes cannot be interpreted directly as the observed short-periodic solar oscillations. Conversion of the described modes into magneto-acoustic-gravitation modes inside of the solar resonance cavity (Priest 1982) results in a formation of the observed short-periodic solar oscillations. One of the main result of the present paper is that we have revealed a mechanism of the energy transfer from the small-scale turbulence to the deterministic large-scale wave motions.

The MHD instability due to 'effective' negative magnetic pressure in the interval  $\kappa < \kappa_0$  may also provide a mechanism of the large-scale magnetic ropes formation in the solar convective zone (see also Kleeorin et al. 1989, 1990; Kleeorin & Rogachevskii 1990). At a depth  $\sim 10^9$  cm (from the sun's surface) the magnetic coefficient  $Q_p \sim -1.8$  and the 'effective' magnetic pressure is negative. The magnetic instability develops on a time scale  $\tau_0 \sim 2.5 \cdot 10^5$  s. It apparently determines the formation of the magnetic flux tubes in the solar convective zone. These magnetic ropes float up from under the sun's surface leading to the onset of the observed sunspots.

As for the role of the dissipation processes, they serve either to weaken the instability or to stabilize it completely. The damping rate of the instability is of the order of  $\eta_T/\Lambda_\rho^2$ . As a consequence, the instability has a threshold character in the large-scale regular (mean) magnetic field  $\mathbf{B}$ , because the growth rate of the instability depends on the magnetic field. It follows that the instability occurs only if  $B > B_{cr}(\eta_T, R_m)$ , where  $B_{cr}$  is the instability threshold for the large-scale magnetic field. The threshold is determined from equation

$$\varepsilon_B(B_{cr}) Q_p(\varepsilon_B, R_m) + (l_0/3\Lambda_\rho)^2 = 0.$$

The dissipation determines the characteristic cross section of the magnetic flux tubes  $L_f \simeq 3\Lambda_\rho$ , the scale  $L_f$  corresponds to



**Fig. 1.** The growth rate and the frequency for the mode 1.

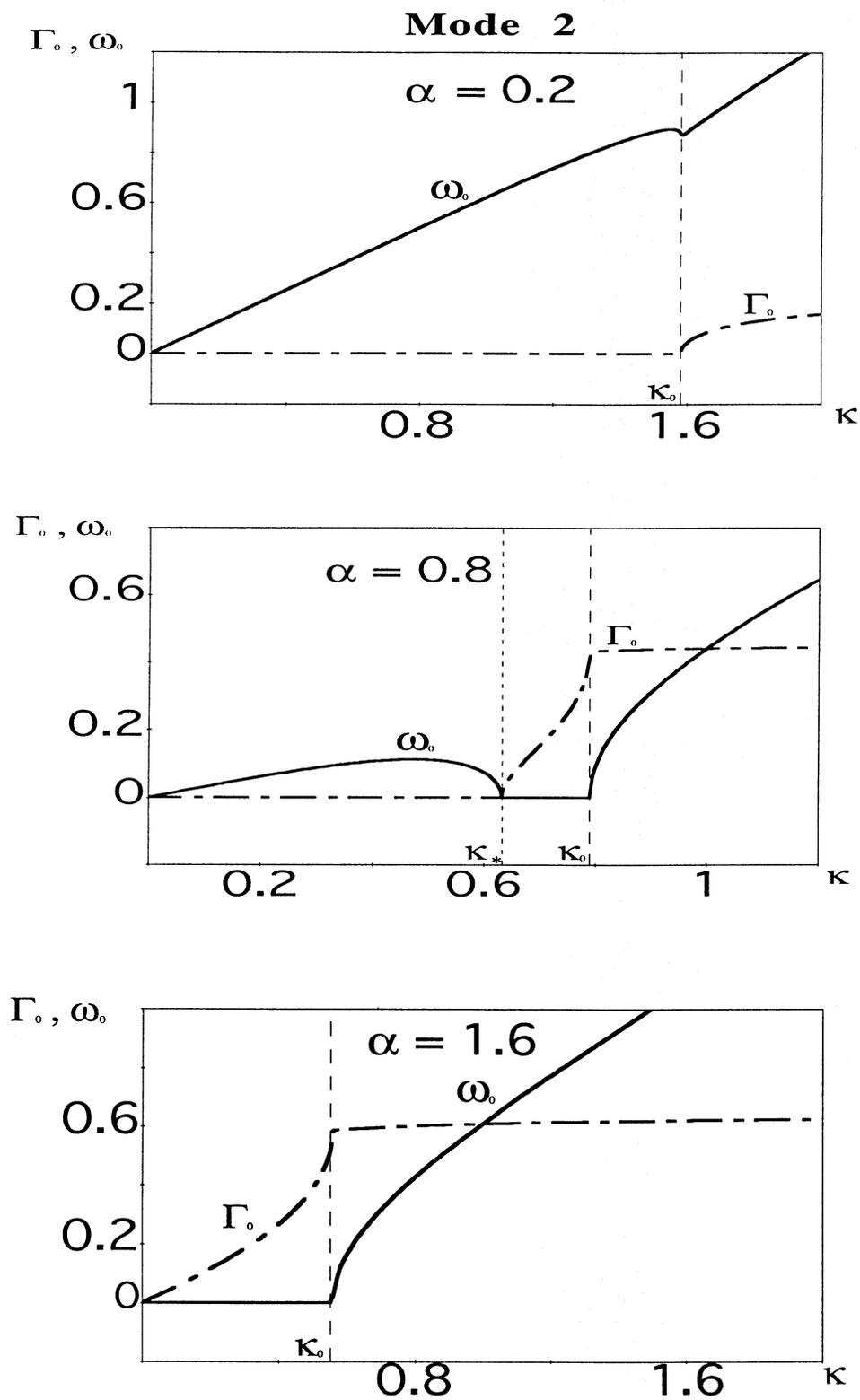


Fig. 2. The growth rate and the frequency for the mode 2.

the maximum growth rate of the instability. The cross section of the magnetic flux tubes  $L_f \sim 10^9 \text{cm}$  is comparable to the spot size.

The MHD instability may provide also a mechanism of the magnetic flux tubes in stars and spiral galaxies. For instance, large-scale magnetic ropes have been observed in the spiral galaxy IC 342 (Krause et al. 1989). In spiral galaxies the plasma typically has the following parameters (Ruzmaikin et al. 1988):  $R_m \sim 10^6$ ,  $l_0 \sim 100 \text{pc}$ ,  $u_0 \sim 10^6 \text{cm/s}$ ,  $\rho_0 \sim (1-3) \cdot 10^{-24} \text{g/cm}^3$ ,  $\Lambda_\rho \sim 400 \text{pc}$  and  $B_0 \sim 10^{-6} \text{G}$ . The magnetic coefficient is  $Q_p \sim -1.2$  and hence magnetic instability is excited on a time scale  $\tau_0 \sim 6 \cdot 10^{15} \text{s}$ . The characteristic cross section of the magnetic ropes  $L_f \sim 1.2 \text{kpc}$ . Radio observations of the spiral galaxy IC 342 reveal magnetic ropes of comparable thickness (Krause et al. 1989).

### 5. The torsional oscillations and meridional flows

Another type of 11-year solar oscillations, the torsional oscillations and meridional motions, have been studied by a number of authors experimentally (see, e.g., La Bonte and Howard 1982; Tuominen et al. 1983; Snodgrass 1985; 1987; Makarova & Solonsky 1989), numerically (see, e.g., Schüssler 1981; Yoshimura 1981; Rüdiger et al. 1986) and theoretically (see, e.g., Kleeorin & Ruzmaikin 1991). The following observed properties should be explained:

1. The torsional oscillations exist in form of traveling waves with constant amplitude rather than as standing waves.
2. The phase velocity of the wave of activity (for example, the dynamo wave) coincides with that of the torsional wave.
3. A phase shift exists between the activity and oscillations of the velocity of the torsional wave and meridional motions.
4. The amplitude of the torsional wave and the fine structure of the torsional oscillations and meridional motions.

The papers by Schüssler (1981); Yoshimura (1981) and Rüdiger et al. (1986) explain the first and the second properties of the torsional oscillations except for the fact of the constant amplitude of the traveling wave. A recent analytical model by Kleeorin & Ruzmaikin (1991) describes properties 1-3 of the torsional oscillations and meridional motions. In the *present* paper the observed fine structure of the torsional oscillations and meridional motions is explained.

The torsional oscillations and the meridional flows in the solar convective zone can be excited by magnetic dynamo waves. Here an analytical model for the zonal and meridional flows is presented. We consider the axisymmetric case. The large-scale mean magnetic field  $\mathbf{B}(r, \theta, t)$  generated by  $\alpha\Omega$ -dynamo mechanism (Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983) is assumed to be given. The magnetic field excites large-scale flows  $\mathbf{U}(r, \theta, t)$  in the convective zone. These flows are superimposed on the differential rotation  $\Omega(r, \theta)$  and include the torsional oscillations and meridional motions. The total velocity including the differential rotation, the meridional flows and the torsional oscillations is given by

$$\mathbf{v}(r, \theta, t) = \mathbf{v}_\Omega(r, \theta) + \mathbf{U}(r, \theta, t),$$

where  $\mathbf{v}_\Omega(r, \theta) = r\Omega(r, \theta)\mathbf{e}_\varphi$ . We consider a slow process with characteristic time of about 11 years. This time is much longer than the acoustic time. Therefore we can neglect the temporal variations of the density, and the velocities satisfy to  $\nabla \cdot (\rho\mathbf{v}) = \nabla \cdot (\rho\mathbf{U}) = 0$ . In the absence of a mean magnetic field  $\mathbf{B}(r, \theta, t)$  only the differential rotation  $\mathbf{v}_\Omega$  exists. The mean magnetic field  $\mathbf{B}$  generates the additional flow  $\mathbf{U}(\mathbf{B})$  (see Sect. 2). The momentum equation in linear approximation with respect to  $\mathbf{U}$  follows from Eq. (23):

$$\rho_0 \frac{\partial \mathbf{U}}{\partial t} = -\nabla \left( p_2 + \frac{Q_p}{8\pi} \mathbf{B}^2 \right) + \rho_2 \mathbf{g} + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) Q_s \mathbf{B} + \mathbf{F}_\nu(\mathbf{U}) + 2\rho_0 \mathbf{U} \times \boldsymbol{\Omega}, \quad (47)$$

Equation (47) is written in a frame rotating with the Sun. The linear inertial terms in Eq. (47) related to the differential rotation are included in the Coriolis force. Here  $p_2$  and  $\rho_2$  are the perturbed pressure and density, respectively,  $\mathbf{F}_\nu = \nabla \cdot \hat{\tau}$  is the force due to the eddy viscosity  $\nu_T$ ,  $\hat{\tau}$  is the tensor of turbulent viscous tensions (see Appendix C).

Now let us discuss the assumptions of the model:

1. The eddy viscosity  $\nu_T$  varies slightly with the depth in the solar convective zone  $L_0$ , while the density is drastically changed:  $\Lambda_\rho \ll L_0$  (see, e.g., Spruit 1974). Here  $\Lambda_\rho$  is the density height scale. As a result, in the momentum of the fluid  $\mathbf{P} = \rho_0 \mathbf{U}$  is slightly depends on the depth of the convective zone (see Kleeorin & Ruzmaikin 1991).

2. The characteristic time of the large-scale magnetic field variations is about 11 year, while the relaxation time of the perturbations of the velocity  $\mathbf{U}$  due to the turbulent viscosity varies from  $\approx 4$  days at the top of the convective zone to a year near its bottom. Therefore the momentum Eq. (47) can be reduced to the stationary equation.

3. We also ignore very small variations of the gravitational force  $\rho_2 \mathbf{g}$ , and take into account the fact that for the  $\alpha\Omega$ -dynamo  $B_\varphi \gg B_r, B_\theta$ .

4. The following boundary conditions should be satisfied:

$$\hat{\tau}_{r\varphi} + \hat{\tau}_{r\varphi}^M \simeq \nu_T \frac{P_\varphi}{\Lambda_\rho} + Q_s \frac{B_r B_\varphi}{4\pi} = 0, \quad (48)$$

$$\hat{\tau}_{r\theta} \simeq \nu_T \left( \frac{\partial P_\theta}{\partial r} + P_\theta \left( \frac{1}{\Lambda_\rho} - \frac{1}{r} \right) \right) = 0, \quad (49)$$

where  $\hat{\tau}_{r\varphi}^M = Q_s B_r B_\varphi / (4\pi)$  is the component of the magnetic tension. We assume that outside the solar convective zone  $B_\varphi \rightarrow 0$  and the viscosity is very small. This means that at the boundary of the convective zone the viscous and magnetic tensions vanish.

The boundary conditions (48)-(49) differ from that used in the numerical models (see Schüssler, 1981; Yoshimura, 1981; Rüdiger et al., 1986). By contrast to these models we take into account magnetic stresses in the boundary conditions and do not assume here the vacuum condition  $B_\varphi(r = R_\odot) = 0$ . In reality this component of the magnetic field vanishes only in the non-turbulent region  $r \rightarrow R_\odot$  (at the vicinity of the boundary between the convective zone and the photosphere). This region is electrically conductive rather than a perfect insulator. Note

that the use of the vacuum boundary condition in the numerical models by Schüssler (1981), Yoshimura (1981), Rüdiger et al. (1986) results in a small magnitude of the velocity compared with the observed ones.

Here we do not take into account an anisotropy of the eddy viscosity. It is due to the fact that we search for a solution of Eq. (47) near the surface of the Sun, where  $\Omega\tau \ll 1$ , and  $\tau$  is the turnover time of the turbulent eddies.

The solutions of the Eq. (47) for  $U_\theta$  and  $U_\varphi$  with the boundary conditions (48)-(49) at the near-surface layers are given by (see Appendix C):

$$U_\theta \simeq \frac{\Lambda_\rho^2}{4\pi\rho\nu_T} \frac{F(\theta)}{R_\odot}, \quad (50)$$

$$U_\varphi \simeq -\frac{\Lambda_\rho}{4\pi\rho} \frac{Q_s B_r B_\varphi}{\nu_T}, \quad (51)$$

where we neglect small terms of the order  $\sim O(L_0/R_\odot)$ , and

$$F(\theta) = \int_{R_\odot-L_0}^{R_\odot} \left( \frac{1}{r} \frac{\partial}{\partial \theta} - \cot \theta \frac{\partial}{\partial r} \right) (Q_s B_\varphi^2) dr.$$

For  $Q_s = 1$  Eqs. (50)-(51) coincide with that obtained by Kleeorin & Ruzmaikin (1991).

To compare with observations we need to know the form of the mean field  $\mathbf{B}$  and the function  $Q_s(B)$ . According to the linear mean-field dynamo (see, e.g., Parker 1979; Zeldovich et al. 1983; Ruzmaikin & Starchenko 1987; Brandenburg et al. 1989) the mean magnetic field is presented in form of dynamo waves

$$B_r \simeq b_r(r, \theta) \sin(\Phi - \delta), \quad B_\varphi \simeq b_\varphi(r, \theta) \cos \Phi, \quad (52)$$

where  $\Phi(t, r, \theta) = \omega_c t - g(r, \theta)$ ,  $\delta$  is a phase shift between the toroidal components of the magnetic field and the vector potential of the poloidal field,  $\omega_c = \pi/11 \text{ years}^{-1}$  is a frequency of the solar cycle, and a phase function

$$g(\theta) \simeq 2\pi(1 \pm \cos \theta). \quad (53)$$

Expression (53) is valid for  $15^\circ < \theta < 165^\circ$  (see Kleeorin & Ruzmaikin 1991). After substitution of (52) into Eqs. (50)-(51) we obtain time dependent part  $\tilde{\mathbf{U}}$  of the rotational velocity (see Appendix C). The obtained velocity field has a form of a rotational wave with an 11-year period.

Now let us compare results of the theory with the observed latitude dependencies of the zonal velocity (see Fig. 3). The thin curve in Fig. 3 shows the distribution of the zonal velocity  $\tilde{U}_\varphi$  with latitude obtained from observations by LaBonte & Howard (1982). The thick curve in Fig. 3 corresponds to the theoretical latitude distribution of the zonal velocity  $\tilde{U}_\varphi$  calculated by means of Eq. (C5) for  $\epsilon = 0.1$  and  $\delta = \pi/4$ . The zero latitude in Fig. 3 does not coincide with the equator and is chosen conditionally (see LaBonte & Howard 1982). The theoretical and experimental zonal velocities  $\tilde{U}_\varphi$  are normalized to the maximal velocity of the zonal flows.

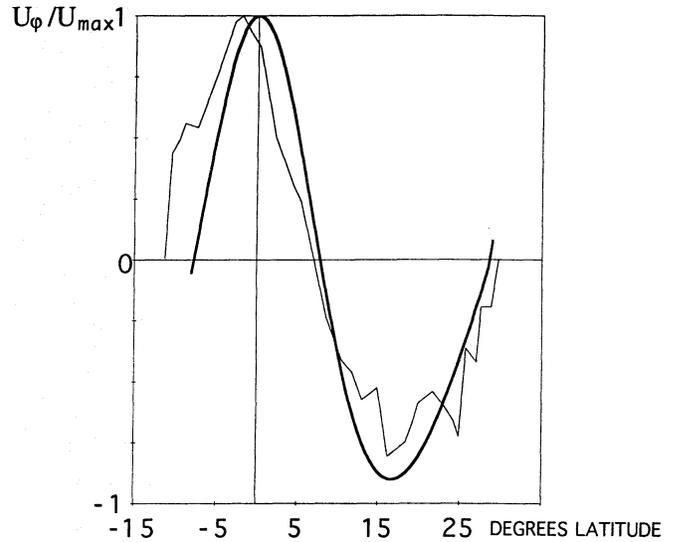


Fig. 3. Experimental (thin curve) and theoretical (thick curve) latitude distributions of the zonal velocity.

The effect of decreasing the elasticity of the mean magnetic field by the small-scale MHD turbulence of the solar convective zone results in asymmetry between the zones with rapid and slow rotations: the regions with slow rotation are wider and they have smaller amplitude of the velocity in comparison with the zones of the rapid rotation. It is in agreement with the observation (see Fig. 3).

The theory also predicts the latitude dependencies for the variable parts of the radial velocity: at a latitude higher (lower) than that of the maximum of magnetic field, the flow is directed towards the equator (the pole). It is in agreement with the observations (Snodgrass 1987). The matter descends ( $U_r < 0$ ) near the maximum of the activity and it rises near the minimum of activity. The effect of decreasing the elasticity of the mean magnetic field leads to weak reduction (expansion) of the regions with vertical (horizontal) motions of the matter.

Note that in the paper by Rüdiger et al. (1986) a possible modification of the torsional oscillations by change of the mean magnetic tension  $(\mathbf{B} \cdot \nabla)\mathbf{B}/4\pi$  due to small-scale turbulent fluctuations has been pointed out. Study of the interaction between the mean magnetic field and small-scale MHD turbulence at large magnetic Reynolds numbers (see Kleeorin et al. 1990; Kleeorin & Rogachevskii 1994a) show that both the mean magnetic pressure and the magnetic tension are significantly modified.

In the *present* paper we have found that the decrease of the magnetic tension affects only the fine structure of the torsional oscillations and the meridional motions. On the other hand, the strong modification of the effective magnetic pressure by the developed MHD turbulence of the convective zone may be the reason of the anomalous oscillations of the solar radius (see Sect. 3), and cause the large-scale magnetic flux tubes formation in the convective zones of the Sun, stars and spiral galaxies (see Sect. 4).

## 6. Conclusions

The following phenomena have been studied: the 11-year variations of the solar radius, the torsional oscillations and the meridional flows, the solar short-period oscillations and the large-scale magnetic flux ropes formation in convective zones of the Sun. It is shown that all these different phenomena are related to the reduction of the elasticity of the large-scale regular (mean) magnetic field by the developed magnetohydrodynamic (MHD) turbulence of the solar convective zone. The ‘effective’ magnetic pressure  $p_m = Q_p B^2 / 8\pi$  is negative (i.e.,  $Q_p < 0$ ) in the upper part of the convective zone. Due to this effect, the growth of the magnetic field during the period of the solar activity leads to decrease of the radius of the Sun in agreement with the observations (see Section 3). On the other hand, the reduction of the magnetic tension (i.e.,  $0 < Q_s < 1$ ) determines the fine structure of the torsional oscillations and the meridional motions (see Section 5). The negative effective magnetic pressure results in the excitation of large-scale MHD instabilities. The instability of the mode with  $k_B = 0$  causes the formation of inhomogeneities of the regular mean magnetic field. In particular, the MHD instability may provide a mechanism of the magnetic flux tubes in the convective zone of the sun, stars and spiral galaxies (see Section 4). The growing oscillations with  $k_B \neq 0$  can be interpreted as a source of the observed short-period solar oscillations. In contrast to the previous models which relate the source to the convective noise (see, e.g., Priest 1982), a source of the short-time solar oscillations proposed here is coherent.

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### Appendix A: evolutionary equation for the total turbulent energy density

In order to derive Eq. (3), the velocity  $\mathbf{v}(\mathbf{r}, \mathbf{t})$  and the magnetic field  $\mathbf{H}(\mathbf{r}, \mathbf{t})$  in the turbulent medium are represented in the form  $\mathbf{v} = \mathbf{V} + \mathbf{u}$  and  $\mathbf{H} = \mathbf{B} + \mathbf{h}$ , where  $\mathbf{V} = \langle \mathbf{v} \rangle$ ,  $\mathbf{B} = \langle \mathbf{H} \rangle$ . The fluctuations of the density are assumed to be weak. The momentum equation and the induction equation for the turbulent fields  $\mathbf{u}$  and  $\mathbf{h}$  in a frame moving with a local velocity of the large-scale flows  $\mathbf{V}$  are given by

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{V} - \frac{\nabla p_*}{\rho} - \frac{1}{4\pi\rho} [\mathbf{h} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{h})] + \mathbf{T} + \frac{\mathbf{F}_\nu + \mathbf{F}_r}{\rho}, \quad (\text{A1})$$

$$\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta_m \nabla \times \mathbf{h}) + (\mathbf{h} \cdot \nabla) \mathbf{V} - \mathbf{h}(\nabla \cdot \mathbf{V}) + \mathbf{G}, \quad (\text{A2})$$

$$\nabla \cdot \mathbf{u} = 0, \quad (\text{A3})$$

where  $p_*$  is the fluctuations of the hydrodynamic pressure,  $\mathbf{F}_\nu$  is the viscous force,  $\mathbf{F}_r$  is a random external force,  $\mathbf{T}$  and  $\mathbf{G}$  are terms nonlinear in the fluctuations and describe the energy

transport over the spectrum of MHD turbulence

$$\begin{aligned} \mathbf{T} &= \langle (\mathbf{u} \cdot \nabla) \mathbf{u} \rangle - (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{4\pi\rho} [\langle \mathbf{h} \times (\nabla \times \mathbf{h}) \rangle \\ &\quad - \mathbf{h} \times (\nabla \times \mathbf{h})], \\ \mathbf{G} &= \nabla \times (\mathbf{u} \times \mathbf{h} - \langle \mathbf{u} \times \mathbf{h} \rangle). \end{aligned}$$

The fluctuations are concentrated in small scales. Hence the derivatives of the large-scale fields are small in comparison with the derivatives of the turbulent fields. Let us now multiply Eq. (A1) by  $\mathbf{u}$ , Eq. (A2) by  $\mathbf{h}/(4\pi)$ , add them and average over the ensemble of turbulent fluctuations. The result is given by

$$\frac{\partial}{\partial t} \left( \frac{\rho \langle \mathbf{u}^2 \rangle}{2} + \frac{\langle \mathbf{h}^2 \rangle}{8\pi} \right) = -\nabla \cdot (\Phi_B + \Phi_u) - D + I_T, \quad (\text{A4})$$

Here

$$\begin{aligned} \Phi_B &= \frac{1}{4\pi} \langle \mathbf{h} \times (\mathbf{u} \times \mathbf{B}) \rangle, \\ \Phi_u &= \langle \mathbf{u} p_* \rangle + \langle \rho \mathbf{u} \cdot \frac{\mathbf{u}^2}{2} \rangle + \frac{1}{4\pi} \langle \mathbf{h} \times (\mathbf{u} \times \mathbf{h}) \rangle. \end{aligned}$$

The functions  $\Phi_B$  and  $\Phi_u$  describe energy fluxes of magnetic fields and flows in the turbulence with large-scale (mean) magnetic field  $\mathbf{B}$ , the values  $I_T = \langle \mathbf{u} \cdot \mathbf{F}_r \rangle$  and

$$D = \nu \rho \langle (\nabla \times \mathbf{u})^2 \rangle + \eta \langle (\nabla \times \mathbf{h})^2 \rangle$$

determine the power of the external source maintaining the turbulence and dissipation of the turbulent energy respectively.

To obtain Eq. (A4) a use was made of the following identities:

$$\begin{aligned} \rho \mathbf{u}(\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{\mathbf{u}^2}{2} \nabla \cdot (\rho \mathbf{u}) &= \nabla \cdot \left( \mathbf{u} \frac{\rho \mathbf{u}^2}{2} \right), \\ \nabla \cdot (\mathbf{h} \times (\mathbf{u} \times \mathbf{A})) &= -\mathbf{h} \cdot (\nabla \times (\mathbf{u} \times \mathbf{A})) \\ &\quad + (\mathbf{u} \times \mathbf{A}) \cdot (\nabla \times \mathbf{h}). \end{aligned}$$

Here we made calculations similar to those described in Landau & Lifshitz (1984b).

In a homogeneous turbulence with uniform large-scale magnetic field  $\mathbf{B}$  we obtain  $\nabla \cdot \Phi_B = \nabla \cdot \Phi_u = 0$ . Therefore the terms containing the uniform large-scale regular magnetic field  $\mathbf{B}$  are eliminated from Eq. (A4). This reflects the fact that the uniform large-scale magnetic field performs no work on the turbulence. It can only redistribute the energy between hydrodynamic and magnetic fluctuations.

If the mean magnetic field  $\mathbf{B}$  is nonuniform, the value  $\nabla \cdot \Phi_B \neq 0$ . The flux  $\Phi_B$  is given by

$$\langle \Phi_B \rangle_m = \frac{1}{4\pi} [\langle u_m h_n \rangle B_n - \langle u_n h_n \rangle B_m]. \quad (\text{A5})$$

The second moment  $\langle u_m h_n \rangle$  in a homogeneous and isotropic turbulence is given by

$$\langle u_m h_n \rangle = -\eta_T \frac{\partial B_n}{\partial x_m} \quad (\text{A6})$$

(see, e. g., Moffatt 1978; Krause & Rädler 1980; Zeldovich et al. 1983), where  $\eta_T$  is the turbulent magnetic diffusion. It is seen from here that  $\langle u_m h_m \rangle = 0$ , because  $\nabla \cdot \mathbf{B} = 0$ . Substitution (A6) into (A5) yields

$$\nabla \cdot \Phi_B = -\eta_T \Delta \left( \frac{B^2}{8\pi} \right). \quad (\text{A7})$$

Combination Eqs. (A4) and (A7) yields

$$\frac{\partial W_T}{\partial t} = I_T - \frac{W_T}{\tau} + \eta_T \Delta \left( \frac{B^2}{8\pi} \right), \quad (\text{A8})$$

where the total energy density is  $W_T = W_k + W_m$ ,  $\tau$  is the correlation time of the turbulence in the scale  $l_0$ . The second term in (A8)  $D = W_T/\tau$  determines the dissipation of the turbulent energy. This form of the dissipation results from the condition that the energy flows be constant over the spectrum.

### Appendix B: $k^{-1}$ spectrum of magnetic fluctuations

Now let us derive equations describing the evolution of the second moments. For this purpose we rewrite the MHD equations (A1)-(A3) in a Fourier representation and repeat twice the vector multiplication of Eq. (A1) by the wave vector  $\mathbf{k}$ . The result is given by

$$\begin{aligned} \frac{du_m(\mathbf{k}, t)}{dt} &= \frac{i(\mathbf{k} \cdot \mathbf{B})}{4\pi\rho} h_m(\mathbf{k}, t) - \tilde{T}_m(\mathbf{k}, t) \\ &\quad - \nu_0 k^2 u_m(\mathbf{k}, t), \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} \frac{dh_m(\mathbf{k}, t)}{dt} &= i(\mathbf{k} \cdot \mathbf{B}) u_m(\mathbf{k}, t) - G_m(\mathbf{k}, t) \\ &\quad - \eta_m k^2 h_m(\mathbf{k}, t), \end{aligned} \quad (\text{B2})$$

where  $\tilde{\mathbf{T}} = \mathbf{k} \times (\mathbf{k} \times \mathbf{T})/k^2$ . Recall that here  $k \gg l_0^{-1}$ , so that  $(\mathbf{k} \cdot \mathbf{B})$  is not equal to zero. Let us introduce the second moments and consider homogeneous turbulence. In this case, for example, dependence of the second moment  $f_{mn}(\mathbf{r}, \mathbf{R}, t) = \langle u_m(\mathbf{x}, t) u_n(\mathbf{y}, t) \rangle$  on  $\mathbf{R} = (\mathbf{x} + \mathbf{y})/2$  is not as strong as on  $\mathbf{r} = \mathbf{x} - \mathbf{y}$ . This means that

$$\begin{aligned} f_{mn}(\mathbf{k}, t) &= \langle u_m(\mathbf{k}, t) u_n(-\mathbf{k}, t) \rangle, \\ h_{mn}(\mathbf{k}, t) &= \langle h_m(\mathbf{k}, t) h_n(-\mathbf{k}, t) \rangle. \end{aligned}$$

Let us multiply Eqs. (B1) for  $u_m(\mathbf{k}, t)$  by  $u_n(-\mathbf{k}, t)$  and Eqs. (B1) written for  $u_n(-\mathbf{k}, t)$  by  $u_m(\mathbf{k}, t)$ , add them, and average over the ensemble of turbulent pulsations. We use the same procedure for other correlation functions. It results in the equations describing the evolution of the second moments (see Kleeorin et al. 1990):

$$\begin{aligned} \frac{df_{mn}}{dt} &= \frac{i(\mathbf{k} \cdot \mathbf{B})\phi_{mn}}{4\pi\rho} + A_{mn} + F_{mn} \\ &\quad - 2\nu_0 k^2 f_{mn}, \end{aligned} \quad (\text{B3})$$

$$\frac{dh_{mn}}{dt} = -i(\mathbf{k} \cdot \mathbf{B})\phi_{mn} + R_{mn} - 2\eta_m k^2 h_{mn}, \quad (\text{B4})$$

$$\begin{aligned} \frac{d\chi_{mn}}{dt} &= i(\mathbf{k} \cdot \mathbf{B}) \left( f_{mn} - \frac{h_{mn}}{4\pi\rho} \right) + C_{mn} \\ &\quad - (\nu_0 + \eta_m) k^2 \chi_{mn}. \end{aligned} \quad (\text{B5})$$

Here

$$\begin{aligned} \phi_{mn}(\mathbf{k}, t) &= \chi_{mn}(\mathbf{k}, t) - \chi_{nm}(-\mathbf{k}, t), \\ \chi_{mn}(\mathbf{k}, t) &= \langle h_m(\mathbf{k}, t) u_n(-\mathbf{k}, t) \rangle, \\ F_{mn}(\mathbf{k}, t) &= \langle \tilde{F}_m(\mathbf{k}, t) u_n(-\mathbf{k}, t) \rangle + \langle u_m(\mathbf{k}, t) \tilde{F}_n(-\mathbf{k}, t) \rangle, \\ \tilde{\mathbf{F}}(\mathbf{k}, t) &= \frac{\mathbf{k} \times (\mathbf{k} \times \mathbf{F}_r(\mathbf{k}, t))}{k^2 \rho}. \end{aligned}$$

The third moment is given by

$$A_{mn}(\mathbf{k}, t) = \langle \tilde{T}_m(\mathbf{k}, t) u_n(-\mathbf{k}, t) \rangle + \langle u_m(\mathbf{k}, t) \tilde{T}_n(-\mathbf{k}, t) \rangle.$$

The expressions for the remaining moments  $R_{mn}$  and  $C_{mn}$  are similar.

By means of Eqs. (B3) and (B4) we obtain equation for  $f_{mn} + h_{mn}/(4\pi\rho)$

$$\begin{aligned} \frac{d}{dt} \left( f_{mn} + \frac{h_{mn}}{4\pi\rho} \right) &= A_{mn} + F_{mn} \\ &\quad - 2\nu_0 k^2 f_{mn} + (R_{mn} - 2\eta_m k^2 h_{mn})/(4\pi\rho) \end{aligned}$$

It is seen from here that the second moment  $f_{mn} + h_{mn}/(4\pi\rho)$  is independent of the mean magnetic field.

Equations (B3)-(B5) describe evolution of the second moments. Equations of this type raise, as usual, a question of closing the equations for the higher moments. Various approximate methods have been proposed for the solution of problems of this type (see, for example, Orszag 1970, Monin and Yaglom 1975, McComb 1990). The simplest closure procedure is the  $\tau$  approximation, which is widely used in the theory of kinetic equations. As applied to MHD turbulence problems, this approximation was developed by Pouquet et al. (1976). In the simplest variant, it allows us to express the third moments in terms of the second moments:

$$A_{mn} - A_{mn}^{(0)} = -\frac{f_{mn} - f_{mn}^{(0)}}{\tau(k)}, \quad (\text{B6})$$

and similarly for the other correlation functions. The superscript (0) corresponds here to the background MHD turbulence (it is a turbulence without the mean magnetic field  $\mathbf{B}$ ), and  $\tau(k)$  is the characteristic relaxation time of the statistical moments.

The  $\tau$ -approximation is in general similar to Eddy Damped Quasi Normal Markovian (EDQNM) approximation. However some principle difference exists between these two approaches (see Orszag 1970; McComb 1990). The EDQNM closures do not relax to equilibrium, and this procedure does not describe properly the motions in the equilibrium state in contrast to the  $\tau$ -approximation. Within the EDQNM theory, there is no dynamically determined relaxation time, and no slightly perturbed steady state can be approached (Orszag 1970). In the  $\tau$ -approximation, the relaxation time for small departures from equilibrium is determined by the random motions in the equilibrium state, but not by the departure from equilibrium (Orszag 1970). We use the  $\tau$ -approximation, but not the EDQNM approximation because we consider a case with

$B^2/8\pi \ll \langle \rho u^2 \rangle / 2$ . The mean magnetic field slightly perturbs the background turbulence (the equilibrium state). As follows from the analysis by Orszag (1970) the  $\tau$ -approximation describes the relaxation to equilibrium state much more accurately than the EDQNM approach.

Assume that  $\nu_0 k^2 \ll \eta_m k^2 \ll \tau^{-1}$  for the main part of the spectrum. It is also natural to assume that the characteristic time of variation of the large-scale magnetic field  $\mathbf{B}$  is substantially longer than the correlation time  $\tau(k)$  for all turbulence scales. The stationary solution of the system (B3)-(B5) takes the form (Kleorin et al., 1990)

$$f_{mn} = f_{mn}^{(0)} - \frac{\psi}{2(1+\psi)} \left( f_{mn}^{(0)} - \frac{h_{mn}^{(0)}}{4\pi\rho} \right), \quad (\text{B7})$$

$$\frac{h_{mn} - h_{mn}^{(0)}}{4\pi\rho} = -(f_{mn} - f_{mn}^{(0)}), \quad (\text{B8})$$

$$\chi_{mn} = \chi_{mn}^{(0)} + i(\mathbf{k} \cdot \mathbf{B})\tau \left( f_{mn}^{(0)} - \frac{h_{mn}^{(0)}}{4\pi\rho} \right), \quad (\text{B9})$$

where  $\psi = (\mathbf{k} \cdot \mathbf{B}\tau)^2 / \pi\rho$ . It is seen from (B7)-(B9) that the equipartition state in which

$$\frac{\rho \langle u_m u_n \rangle^{(0)}}{2} = \frac{\langle h_m h_n \rangle^{(0)}}{8\pi} \quad (\text{B10})$$

is special. In this case there is no shift from the background turbulence level for any uniform field  $\mathbf{B}$ .

Now let us choose a spectrum of the MHD background turbulence and find the spectrum of the MHD turbulence in the presence of the large-scale regular magnetic field  $\mathbf{B}$ . Suppose that the hydrodynamic fluctuations of the background turbulence in the region  $k_m \gg k \geq k_0$  is substantially stronger than magnetic ones

$$\frac{\rho f_{mn}^{(0)}}{2} \gg \frac{h_{mn}^{(0)}}{8\pi},$$

where  $k_m = l_m^{-1}$  is determined by the characteristic scale of the magnetic fluctuations  $l_m \simeq l_0 R_m^{-1/2}$  (Zeldovich et al. 1990; Kleorin and Rogachevskii 1994b). We also take into account that  $\psi \ll 1$ . It follows from Eqs. (B7)-(B9) that for  $k_m \gg k \geq k_0$

$$h_{mn} = 2(\mathbf{k} \cdot \mathbf{B})^2 \tau^2 f_{mn}^{(0)}.$$

Note that here  $h_{mn} \gg h_{mn}^{(0)}$ . It means that the generation of the magnetic fluctuations in this region by the 'tangling' of the large-scale mean magnetic field by the hydrodynamic fluctuations is more substantial than the small-scale dynamo without mean field (Zeldovich et al. 1990; Kleorin and Rogachevskii 1994a; 1994b). Suppose that the background hydrodynamic spectrum is close to Kolmogorov's one

$$f_{mn}^{(0)} = \frac{u_0^2}{6\pi k^2} k^{-5/3} \left( \delta_{mn} - \frac{k_m k_n}{k^2} \right)$$

and for  $k > k_m$  there is equipartition (B10). Here the wave vector  $k$  is normalized to the value  $k_0 = l_0^{-1}$ . Therefore,

$$\langle h^2 \rangle = \int_1^{k_m/k_0} M(k) dk, \quad (\text{B11})$$

$$M(k) = \frac{8}{9} k^{-1} B^2.$$

Our results, obtained by means of the  $\tau$ -approximation, are in agreement with that of the numerical simulations by Brandenburg (1993).

### Appendix C: equations for the torsional oscillations and meridional motions

Let us define the force due to the eddy viscosity  $\nu_T$ ,  $\mathbf{F}_\nu = \nabla \cdot \hat{\tau}$ , where  $\hat{\tau}$  is a tensor of turbulent viscous tensions:

$$\hat{\tau}_{rr} = 2\eta \frac{\partial}{\partial r} \left( \frac{P_r}{\rho_0} \right),$$

$$\hat{\tau}_{\theta\theta} = \frac{2\eta}{r} \left( \frac{\partial}{\partial \theta} \left( \frac{P_\theta}{\rho_0} \right) + \frac{P_r}{\rho_0} \right),$$

$$\hat{\tau}_{\varphi\varphi} = \frac{2\eta}{r\rho_0} (P_r + P_\theta \cot \theta),$$

$$\hat{\tau}_{r\varphi} = \hat{\tau}_{\varphi r} = \eta r \frac{\partial}{\partial r} \left( \frac{P_\varphi}{r\rho_0} \right),$$

$$\hat{\tau}_{\theta\varphi} = \hat{\tau}_{\varphi\theta} = \frac{\eta}{r} \sin \theta \frac{\partial}{\partial \theta} \left( \frac{P_\varphi}{\rho_0 \sin \theta} \right),$$

$$\hat{\tau}_{r\theta} = \hat{\tau}_{\theta r} = \eta \left( \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{P_r}{\rho_0} \right) + r \frac{\partial}{\partial r} \left( \frac{P_\theta}{r\rho_0} \right) \right),$$

Here  $\mathbf{P} = \rho_0 \mathbf{U}$  is the momentum of fluid,  $\Lambda_\rho$  is the density height scale,  $\eta = \rho(r)\nu_T$ . The momentum equation (47) is reduced to the stationary equation:

$$\frac{\partial}{\partial r} \left( p_1 + \frac{Q_p}{8\pi} \mathbf{B}^2 \right) \simeq \frac{2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{P_r \nu_T}{\Lambda_\rho} \right) - Q_s \frac{B_\varphi^2}{4\pi r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{P_\theta \nu_T}{\Lambda_\rho} \right) + 2P_\varphi \Omega \sin \theta, \quad (\text{C1})$$

$$\frac{\partial}{\partial \theta} \left( p_1 + \frac{Q_p}{8\pi} \mathbf{B}^2 \right) \simeq \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^3 \frac{P_\theta \nu_T}{\Lambda_\rho} \right) - Q_s \frac{B_\varphi^2}{4\pi} \cot \theta + 2P_\varphi \Omega r \cos \theta, \quad (\text{C2})$$

$$0 \simeq \frac{1}{r^3} \frac{\partial}{\partial r} \left( r^3 \frac{P_\varphi \nu_T}{\Lambda_\rho} \right) + Q_s \frac{B_r B_\varphi}{4\pi} - 2\Omega (P_\theta \cos \theta + P_r \sin \theta) + \frac{1}{r \sin^2 \theta} \frac{\partial}{\partial \theta} \left( \sin^2 \theta \frac{Q_s B_\theta B_\varphi}{4\pi} \right). \quad (\text{C3})$$

The total pressure is excluded from the momentum equation by taking the 'curl' of this equation. Also we use the condition  $\nabla \cdot \mathbf{P} = 0$  and introduce the function  $\Psi$ :

$$P_r = \frac{1}{r^2 \sin \theta} \frac{\partial \Psi}{\partial \theta}, \quad P_\theta = -\frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial r}.$$

After neglecting of the slight dependence of  $\nu_T/\Lambda_\rho$  on  $r$  Eqs. (C1)-(C2) are reduced to

$$\frac{\partial^2 Y}{\partial X^2} + \frac{1}{9X^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (Y \sin \theta) \right) = f(X, \theta), \quad (C4)$$

where  $X = r^3$ ,  $Y = X P_\theta \nu_T / \Lambda_\rho$ ,

$$f(X, \theta) = \frac{1}{36\pi} \left( \frac{1}{X} \frac{\partial}{\partial \theta} - \frac{3}{\tan \theta} \frac{\partial}{\partial X} \right) (Q_s B_\varphi^2).$$

Here we take into account that the contribution of the Coriolis force into the function  $f(X, \theta)$  under condition of the slow rotation is small (see Kleeorin & Ruzmaikin 1991). The solution of the Eq. (C4) with the boundary condition (49) is given by Eq. (50), where we neglect small terms of the order  $\sim O(L_0/R_\odot)$ . The solution of Eq. (C3) with the boundary condition (48) is determined by Eq. (50). The components  $U_r$  can be found from  $\nabla \cdot (\rho \mathbf{U}) = 0$ .

To obtain the solutions for  $U_r$  and  $U_\theta$  inside the convective zone we neglect the small terms  $\sim O(\Lambda_\rho \partial P_\theta / \partial r / (P_\theta)) \simeq \Lambda_\rho / R_\odot$ . On the other hand, in the boundary condition (50) we should take into account these terms. It is due to the following:  $U_r, U_\theta \sim \tilde{\zeta} U_\varphi$ , where  $\tilde{\zeta} = \Lambda_\rho B_\varphi / R_\odot B_r$ . At the vicinity of the surface  $\tilde{\zeta} \simeq 1$ , while inside the convective zone  $\tilde{\zeta} \ll 1$  (see Kleeorin & Ruzmaikin 1991).

After substitution (52) into Eqs. (50)-(51) we obtain the time dependent part of the rotational velocity

$$\begin{aligned} \tilde{U}_\varphi &\simeq U_\varphi^{(0)} \{ -(1 + \epsilon) \sin(2\Phi - \delta) + \epsilon \sin \delta \cos(2\Phi) \\ &\quad - \frac{\epsilon}{2} \sin(4\Phi - \delta) \}, \end{aligned} \quad (C5)$$

$$\begin{aligned} \tilde{U}_\theta &\simeq \pm U_\theta^{(0)} \{ (1 + 2\epsilon) \sin(2\Phi + \Delta_1) \\ &\quad + \epsilon c_0 \sin(4\Phi + \Delta_2) \}, \end{aligned} \quad (C6)$$

where

$$U_\varphi^{(0)} \simeq \frac{\Lambda_\rho Q_s}{8\pi \rho \nu_T} b_r b_\varphi,$$

$$U_\theta^{(0)} \simeq \frac{\Lambda_\rho^2 Q_s}{4\pi \rho \nu_T R_\odot} b_\varphi^2 \mathcal{F} \left| \frac{\partial g}{\partial \theta} \right| \sqrt{1 + a_0^2 \left( \frac{\partial g(\theta)}{\partial \theta} \right)^4},$$

$$\epsilon = \frac{b_\varphi^2 Q'_s}{2Q_s}, \quad c_0 = \sqrt{\frac{1 + a_0^2}{1 + a_0^2/4}}, \quad a_0 = \frac{\beta \cot \theta}{2\partial g / \partial \theta},$$

$$\Delta_1 = -\arctan(a_0), \quad \Delta_2 = -\arctan(a_0/2).$$

The factor  $\mathcal{F}$  depends on the altitude profile of the toroidal magnetic field  $B_\varphi(r)$  in the convective zone. For the model by Ivanova & Ruzmaikin (1977)  $\mathcal{F} \simeq 1.64$ . The velocity has a form of a rotational wave with an 11-year period.

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