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Magnitude of the dynamo-generated magnetic field in solar-type convective zones

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Abstract. Using a nonlinear dynamo model (in a single-mode approximation), an analytical expression is derived which gives the magnitude of the mean magnetic field as a function of rotation and other parameters for a solar-type convective zone. By means of this expression we find a power-law relation between the X-ray luminosity and stellar rotation. The exponent in this relation is in agreement with observations.

Key words: Sun: magnetic fields – stars: magnetic fields – Sun: X-rays – turbulence – MHD

1. Introduction

The magnetic field of the Sun and solar type stars is believed to be generated by a dynamo process in their convective zones (Parker 1979). Most dynamo models are kinematic and predict a field that grows without limit. Hence they give no estimate of the magnitude for the generated magnetic field. In order to find the magnitude of the field, the nonlinear effects which limit the field growth must be taken into account. However, most nonlinear dynamo models are numerical; in these models it is not possible to obtain information about the value of the resultant magnetic field as a function of stellar parameters. The first theoretical attempts to relate the magnitude of the magnetic field to the angular velocity and spectral type of the star were made by Durney & Latour (1978) and Robinson & Durney (1982). In these papers a simplified nonlinear dynamo model was used and crude scaling arguments were made.

In the present paper we obtain a new analytical expression for the magnitude of the mean magnetic field near the stellar surface as a function of the angular velocity of the star and the parameters of the convective zone. We use a more sophisticated type of nonlinearity in the dynamo (Kleeorin & Ruzmaikin 1982) which includes, in particular, the effect of delayed back action of the magnetic field on the magnetic part of the $\alpha-$ effect (Section 2).

As an application of this result (Sect. 3), we relate the amplitude of X-ray variability to stellar rotation. In doing this we used the fact that the observed correlation between the X-ray variability of solar type stars and their cyclic activity (Baliunas & Vaughan 1985) evidences a relation between the mean magnetic field and the X-ray variability. The theoretical relationship between the X-ray luminosity and the rotation agree with the observed relationship as given by Fleming et al. (1989).

2. A nonlinear mean-field dynamo model

We will use a mean field approach in which the magnetic, $\mathbf{H}$, and velocity, $\mathbf{v}$, fields are divided into the mean and fluctuating parts: $\mathbf{H} = \mathbf{B} + \mathbf{h}$, $\quad \mathbf{v} = \mathbf{V} + \mathbf{u}$, where the fluctuating parts have zero mean values. The mean velocity describes a differential rotation. The fluctuating velocity is parametrized by the turbulent diffusivity and mean helicity ($\alpha-$ effect). This parametrization is known as the $\alpha\Omega-$ dynamo (Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983). The dynamo is called kinematic if the back reaction of magnetic field on the differential rotation, $\alpha-$ effect , and turbulent diffusivity is neglected. The corresponding equation for the mean magnetic field is linear in the field so that it does not restrict its magnitude. Stationary solutions exist only for some special values of the dimensionless parameters entering the equation. The most important parameter is the dynamo number.

To obtain stationary solutions a nonlinear extension of the kinematic approach is needed. Here we consider a nonlinearity in which a back reaction of the magnetic field on the magnetic part of the $\alpha-$ effect is taken into account. The evolution of the mean magnetic field is described by the standard equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{V} \times \mathbf{B} + \alpha \mathbf{B} - (\eta_T + \eta_m) \nabla \times \mathbf{B}] \qquad (1)$$

(Moffatt 1978; Parker 1979; Krause & Rädler 1980; Zeldovich et al. 1983), where α determines the effect of the mean helic-

Send offprint requests to: N. Kleeorin ity of turbulent motions, $\mathbf{V}$ is a mean velocity (the differential rotation), η_T and η_m are the turbulent and molecular magnetic diffusion.

We now split the $\alpha-$ effect into two parts:

$$\alpha = \alpha_0 + \alpha_m,$$

where $\alpha_0 = -(\tau/3)\langle \mathbf{u}(\nabla\times\mathbf{u})\rangle$ is the hydrodynamic part of the $\alpha-$ effect, and $\alpha_m = (\tau/12\pi\rho)\langle \mathbf{h}(\nabla\times\mathbf{h})\rangle$ is the magnetic part of the $\alpha-$ effect, where $\mathbf{u}$ and $\mathbf{h}$ are the turbulent velocity and magnetic field, ρ is the density, $\tau \sim l_0^2/\eta_T$ is the lifetime of the turbulent eddy, l_0 is the characteristic scale of turbulent motions at the depth of the convective zone where the turbulent magnetic diffusion η_T is a maximum. This splitting of the total $\alpha-$ effect into the hydrodynamic and magnetic parts was first suggested by Frisch et al. (1975) (see also Pouquet & Patterson 1978; Zeldovich et al. 1983 for the other references). These velocity and magnetic field contributions to the $\alpha-$ effect are present in the kinematic approximation. However, in the kinematic case there is no reason to treat them separately since the contributions appear in a sum and this sum is considered as given. The situation is changed in the nonlinear dynamo because the growing magnetic field reacts differently on the hydrodynamic and the magnetic parts of the $\alpha-$ effect. The back reaction of the magnetic field on the hydrodynamic part of the $\alpha-$ effect is almost instantaneous (of the order of a characteristic turn-over time of the turbulence, τ). As a result, this part of the $\alpha-$ effect can be nonlinearized in the form of a quenching, i.e. by replacing α_0 with $\alpha_0 f(B)$, where $f(B)$ is a decreasing function of the mean magnetic field (see, e.g., Stix 1972; Jepps 1975; Ivanova & Ruzmaikin 1977; Yoshimura 1978; Brandenburg et al. 1989; Schmitt & Schüssler 1989). However, the characteristic time of the back action of the magnetic field on the magnetic part of the $\alpha-$ effect can be large or small depending on the magnetic Reynolds number, R_m, and the spectral properties of turbulence (Kleeorin & Ruzmaikin 1982). Thus the back reaction of the magnetic field on the magnetic part of the $\alpha-$ effect can not, in general, be reduced to a simple quenching but must be described by an evolutionary equation (see Appendix A):

$$\frac{\partial \alpha_m}{\partial t} = \frac{\mu}{4\pi\rho}\left(\mathbf{B}\cdot(\nabla\times\mathbf{B}) - \frac{\alpha\mathbf{B}^2}{\eta_T}\right) - \frac{\alpha_m}{T}. \qquad (2)$$

This differential equation was derived by Kleeorin & Ruzmaikin (1982). Here $T = l_s^2/8\pi^2\eta_m$, l_s is the characteristic scale of the turbulent motions near the top of the convective zone where α_m is a maximum, $\mu \approx 0.1$ (see Appendix A). Note that the scale l_s is close to the maximum scale of turbulent motions. The closed system of Eqs. (1) and (2) for $\mathbf{B}$ and α_m represents the nonlinear dynamo model under consideration.

In the case $T \ll T_c$ Eq. (2) yields

$$\alpha = \frac{\alpha_0 + \xi\eta_T\mathbf{B}\cdot(\nabla\times\mathbf{B})}{1+\xi B^2}, \qquad (2a)$$

where $\xi = \mu T/(4\pi\rho\eta_T)$, T_c is a period of the cyclic activity. If $|\mathbf{B}\cdot(\nabla\times\mathbf{B})| \ll \alpha_0/(\xi\eta_T)$ we get the well-known result for the total $\alpha-$ effect

$$\alpha = \frac{\alpha_0}{1+\xi B^2}$$

(see, e.g., Iroshnikov 1970; Rüdiger 1974; Roberts & Soward 1975; Noyes & Weiss 1984).

The large-scale flow (the differential rotation) is still assumed to be uninfluenced by the magnetic field. The back reaction of the magnetic field on the differential rotation can be neglected when $|\alpha_m/\alpha_0| \gg \delta\Omega/\Delta\Omega$, where $\delta\Omega$ is a variation of rotation due to the magnetic field, $\Delta\Omega$ is a characteristic value of the differential rotation (see below). This condition is valid, for example, for the solar convective zone under the assumption that the back reaction of the magnetic field on the differential rotation is provided by small amplitude torsional waves (Schüssler 1981; Yoshimura 1981; Kleeorin & Ruzmaikin 1991).

For the sake of simplicity we consider only the axisymmetric case. [Note that the axisymmetry refers only to the mean magnetic field, the fluctuating fields are basically non-axisymmetric so that there is no contradiction to the Cowling theorem.] Then the mean magnetic field can be represented by the poloidal, $\mathbf{B}_p = \nabla\times A(t,r,\theta)\mathbf{e}_\varphi$ and toroidal $\mathbf{B}_t = B(t,r,\theta)\mathbf{e}_\varphi$ components evolving according to Eq. (1),

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = (\hat{L}+\hat{N})\begin{pmatrix} A \\ B \end{pmatrix}, \qquad (3)$$

where r, θ, φ are the spherical coordinates, and

$$\hat{L} = \begin{pmatrix} \Delta_s & \alpha_0(r,\theta) \\ D\hat{\Omega} & \Delta_s \end{pmatrix}, \quad \hat{N} = \begin{pmatrix} 0 & \alpha_m(r,\theta) \\ 0 & 0 \end{pmatrix},$$

$$\hat{\Omega}A = \frac{1}{r}\frac{\partial(\Omega, Ar\sin\theta)}{\partial(r,\theta)}, \quad \alpha_0(r,\theta) = -\alpha_0(r,\pi-\theta),$$

$$\Delta_s = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}r\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\right) \equiv \Delta - \frac{1}{r^2\sin^2\theta}.$$

Note that Eq. (2) is written for the axisymmetric case. Combination of Eqs. (2) and (3) yields

$$\frac{\partial\alpha_m}{\partial t} + \frac{\alpha_m}{T} = -\frac{B}{\rho}\frac{\partial A}{\partial t} + \frac{\hat{M}(B,A)}{\rho}, \qquad (4)$$

where

$$\hat{M}(B,A) = 2B\Delta_s A + \frac{1}{r^2}\frac{\partial}{\partial r}(rA)\cdot\frac{\partial}{\partial r}(rB) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}(A\sin\theta)\cdot\frac{\partial}{\partial\theta}(B\sin\theta).$$

Equations (3, 4) are written in dimensionless variables: the coordinate r and time t are measured in the units R_* and R_*^2/η_T; the helicity is measured in the units α_*; the angular velocity $\Omega(r,\theta)$ is measured in the units Ω_*; the vector-potential of the poloidal field $A(t,r,\theta)$ and the toroidal magnetic field $B(t,r,\theta)$ are measured in units of $R_\alpha R_* B_*$ and B_*, the density $\rho(r,\theta)$

is measured in the units ρ_*. Here $R_\alpha = \alpha_* R_*/\eta_T$, $R_\Omega = \Omega_* R_*^2/\eta_T$, $D = R_\alpha R_\Omega$ is the dynamo number, R_* is the stellar radius, and

$$B_* = \left(\frac{4\pi\rho_*}{\mu}\right)^{1/2} \frac{\eta_T}{R_*}.$$

The terms $\sim O(R_\alpha/R_\Omega)$ are dropped in Eqs. (3, 4). This assumption corresponds to the usual assumptions of $\alpha\Omega-$ dynamos. Equations (3, 4) describe a closed nonlinear system.

The operator $\hat{L}$ describes the kinematic part of the dynamo. General properties of this operator are briefly presented in Appendix B. Eigenvalues of $\hat{L}$ are complex: $p_n = \gamma_n + i\omega_n$; γ_n is the growth (damping) rate of the magnetic field, and ω_n is the frequency of field oscillations. For the given functions $\alpha(r,\theta)$ and $\Omega(r,\theta)$ the solutions of Eqs. (3, 4) depend on a single parameter: the dynamo number D. The threshold for a self-excitation of mean magnetic field is determined by the equation: $\gamma_1(D = D_{cr}) = 0$. Here the mode $n = 1$ is the mode with the maximum growth rate.

Let us seek a solution of this nonlinear problem in the form

$$\begin{pmatrix} A \\ B \end{pmatrix} = \sum_{n=1}^{\infty} F^n(t)\mathbf{e}_n(r,\theta), \quad \mathbf{e}_n = \begin{pmatrix} a_n \\ b_n \end{pmatrix}, \qquad (5)$$

where $\mathbf{e}_n$ are the eigenvectors of the operator $\hat{L}$ for $D = D_{cr}$. Substituting (5) into (3) and taking into account the properties of the eigenvectors (see Eqs. (B6), (B9) in Appendix B), we obtain the following system of equations for the coefficients $F^m(t)$ of the expansion (5):

$$\frac{dF^m}{dt} - F^m p_m = D_{cr}\left(\frac{dp_m}{dD}\right)_{cr} \sum_{n=-\infty}^{\infty} F^n(t)(G_n^m + \alpha_n^m), \qquad (6)$$

where

$$G_n^m = \left(\frac{D}{D_{cr}} - 1\right)\frac{C_n^m}{C_m},$$

$$C_n^m = \int \alpha_0 (a^m)^* b_n \, d^3r + (p_n - p_m)\int (b^m)^* b_n \, d^3r,$$

$$\alpha_n^m = \frac{1}{C_m}\int \alpha_m (a^m)^* b_n \, d^3r, \quad C_m = C_n^m (m = n).$$

The coefficients $F^m(t)$ depend on the nonlinearity α_n^m. The equation for α_n^m can be derived from Eq. (4):

$$\frac{d\alpha_n^m}{dt} + \frac{\alpha_n^m}{T} = \sum_{k,s=-\infty}^{\infty} F^k(t)\left(F^s M_{ksn}^m - \frac{dF^s}{dt} S_{ksn}^m\right), \qquad (7)$$

where

$$M_{ksn}^m = \frac{1}{C_m}\int \frac{\hat{M}(b_k, a_s)}{\rho(r)}(a^m)^* b_n \, d^3r,$$

$$S_{ksn}^m = \frac{1}{C_m}\int \frac{1}{\rho(r)}(a^m)^* b_n b_k a_s \, d^3r.$$

It is assumed here that the relaxation time T of the helicity does not depend on $\mathbf{r}$.

Thus the problem reduces to the study of this infinite system of equations with coefficients determined by the eigenfunctions and eigenvalues of the linear problem for $D = D_{cr}$. Fortunately, the dynamo number in solar-type convective zones is not much larger than the critical dynamo number, so that only a few modes are expected to be excited.

Consider the simplest case, when only one mode (of a dipole type) is excited. This is sufficient to estimate the magnitude of the mean magnetic field in the stationary state. (The multi-mode regime could be considered similarly.) As follows from the analysis of this single-mode approximation presented in Appendix C, the dimensionless magnitude of the mean toroidal magnetic field near the stellar surface is

$$B_s \simeq D_{cr}^{\frac{1}{4}}\left(\frac{D}{D_{cr}} - 1\right)^{\frac{1}{2}}\left(\frac{L_0}{l_s}\right)\left(\frac{b_m}{b_s}\right)\left(\frac{\pi\rho_*\eta_m\eta_T}{L_0\Lambda_\rho\mu}\right)^{\frac{1}{2}}. \qquad (8)$$

Here L_0 is the depth of the convective zone, Λ_ρ is the density height scale, b_s and b_m are components of eigenvectors $\mathbf{e}_n$ of the linear dynamo problem corresponding to the toroidal magnetic fields near the surface and inside the convective zone, respectively. The ratio b_m/b_s depends only on D_{cr} and on the sources of generation (the differential rotation and $\alpha-$ effect). This values b_m and b_s can be determined, for example, from linear dynamo models by Ivanova & Ruzmaikin (1977); Brandenburg et al. (1989). The poloidal magnetic field is much smaller. The result (8) is in agreement with that obtained by means of a qualitative analysis (Kleeorin et al. 1994).

Note that, whereas the growth rate in the kinematic regime is independent of the molecular magnetic diffusivity η_m, the field magnitude in the stationary state is proportional to $\eta_m^{1/2}$. Therefore, in a perfectly conducting fluid, the magnitude B_s vanishes (see Appendix A). The value (8) for the magnetic field also correctly vanishes when the turbulent diffusivity η_T or the convective zone depth L_0 goes to zero.

In the present paper we consider a *nonlinear* model of the mean-field dynamo. The input parameters of this model are the parameters of the linear mean-field dynamo: D_{cr}, $d\gamma/dD$, $d\omega/dD$. The results of the nonlinear theory weakly depend on the details of the linear dynamo models. In particular, this nonlinear theory is independent of the details of the function $\alpha(\mathbf{r})$.

Now we specify the dependence of the dynamo number on the angular velocity and other parameters of the convective zone. We use a spatial distribution of the hydrodynamic part of the $\alpha-$ effect (α_0) of the form

$$\alpha_0 \simeq \begin{cases} l(z)\Omega_*(z) & \text{for } l\Omega_*/u \ll 1, \\ u(z) & \text{for } l\Omega_*/u \gg 1 \end{cases}$$

(Zeldovich et al. 1983). This function has a maximum at the depth $z = z_m$ determined by the condition $l_m(z_m) = u_0(z_m)/\Omega_*(z_m)$. The turbulent magnetic diffusivity is $\eta_T \simeq l_m(z_m)u_0(z_m)$. It follows then that $l_m(z_m) \simeq (\eta_T/\Omega_*)^{1/2}$. The maximal value of the the hydrodynamic part of the $\alpha-$ effect, α_*, is given by

$$\alpha_* \simeq u_0(z_m) \simeq \eta_T / l_m(z_m) \simeq (\eta_T \Omega_*)^{1/2}.$$

Now the dynamo number is

$$D = R_\alpha R_\Omega = \frac{\alpha_* \Omega_* R_*^3}{\eta_T^2} \simeq \left(\frac{\Omega_*}{\eta_T}\right)^{3/2} R_*^3, \tag{9}$$

and the magnitude of the mean magnetic field near the surface is

$$B_s \simeq (\Omega_* \tau)^{\frac{3}{4}} D_{cr}^{-\frac{1}{4}} \left(\frac{R_*}{l_0}\right)^{\frac{3}{2}} \left(\frac{L_0}{l_s}\right) \left(\frac{b_m}{b_s}\right) \left(\frac{\pi \rho_* \eta_m \eta_T}{L_0 \Lambda_\rho \mu}\right)^{\frac{1}{2}} \tag{10}$$

Here $\tau = l_0^2/\eta_T$ and it is assumed that $D/D_{cr} - 1 \sim D/D_{cr}$. The magnitude depends both on $\Omega_* \tau$ and the parameters of the stellar convective zone.

Let us estimate, as an example, the mean toroidal magnetic field near the surface of the Sun. The parameters of the solar convective zone at the depth $\sim 2 \cdot 10^7 cm$ are: $\eta_m \sim 4 \cdot 10^6 cm^2 s^{-1}$, $l_s \sim 2.6 \cdot 10^7 cm$, $\rho \sim 4.5 \cdot 10^{-7} g\, cm^{-3}$, $\Lambda_\rho \sim 3.6 \cdot 10^7 cm$ (Spruit 1974). We use here also $\eta_T \sim 10^{12} cm^2 s^{-1}$, $\mu \sim 0.1$, $b_m/b_s \sim 1 \div 2$, $L_0 = 0.3 R_\odot$, $D_{cr} \sim 10^4$, $D/D_{cr} \sim 2 \div 5$. This gives, according to (8), $B_s \sim (1 \div 3) \cdot 10^2 G$. This value is the same as the mean toroidal magnetic field usually estimated from solar observations (Parker 1979).

Note that the turbulent viscosity η_T and the differential rotation $|\nabla\Omega|$ generally depend on the Coriolis number $\Omega_c^* = 2\tau\Omega_*$. This dependence for the function $\eta_T(\Omega_c^*)$ can be found using the results by Kichatinov (1986); Rüdiger (1989); Küker et al. (1993) :

$$\eta(\Omega_c^*) \propto \eta_0(r) \Phi(\Omega_c^*), \quad \Phi(\Omega_c^*) = \frac{1}{\Omega_c^*} \arctan \Omega_c^*,$$

where $\eta_0(r)$ is a distribution of the turbulent viscosity in the nonrotation medium. The function $\Phi(\Omega_c^*)$ changes from 10^{-1} (at the bottom of the solar convective zone) to 1 (at the top of the convective zone). In the most part of the convective zone $\Phi(\Omega_c^*) \approx 1$ (see, e. g., Spruit 1974). The function $\eta_0(r)$ is maximal in the middle of the convective zone (see, e. g., Brandenburg & Tuominen 1988) where $\Phi(\Omega_c^*) \approx 1$. Therefore the maximum value η_{max} of the function $\eta_T(\Omega_c^*)$ does not change under the influence of rotation. For the nonlinear theory only η_{max} is essential, because it determines the dynamo number. The localization of the maximum η_{max} is slightly shifted to the top of the convective zone but this is important only for linear dynamo models.

As far as the dependence of the differential rotation $|\nabla\Omega|$ on the Coriolis number Ω_c^* is concern the current theories of the differential rotation cannot definitely answer to this question. The models (see, e. g., Rüdiger 1989; Bisnovatyi-Kogan 1990; Küker et al. 1993) yield that for slowly rotating stars $|\nabla\Omega| \sim \delta\Omega_*/L_0$, where $\delta\Omega_* \simeq \Omega_* \varphi_0$, where φ_0 is the function which depends on the spectral class of the star. The value $\delta\Omega_*$ which allow us to estimate the dynamo number is the most important parameter for the nonlinear dynamo model . The exact form the function $|\nabla\Omega(\Omega_c^*)|$ is essential only for linear dynamo models.

Let us discuss now the dependence of period of the activity cycle T_c on the stellar rotation. Nonlinear dynamo models which use the $\alpha-$ quenching in form $\alpha = \alpha_0/(1 + \xi B^2)$ result in the period of cycle $T_c = T_{cr}$ (see Noyes & Weiss 1984), where $T_{cr} = 2\pi/\omega_{cr}$, ω_{cr} is the frequency ω_l of the linear dynamo wave at $D = D_{cr}$ (see Appendix C). Thus these models predict the period of cycle T_c explicitly independent of the rotation of the star in contradiction with observations (see Noyes & Weiss 1984). Meanwhile, the quenching described by Eq. (2) yields the period of the activity cycle in form of

$$T_c \simeq T_l(\Omega_*)(1 + \sigma_0 B_s^2) \left[1 - d_1 \left(1 - \frac{T_l(\Omega_*)}{T_{cr}}\right)\right]^{-1}$$

(see Eq. (C9) in Appendix C). Here $T_l(\Omega_*) = 2\pi/\omega_l$, $d_1 \simeq 1/4$, $\sigma_0 B_s^2 \ll 1$. The dependence of the period of the linear dynamo wave $T_l(\Omega_*)$ on the rotation of the star is determined from the linear dynamo models. For instance, for slowly rotating stars $T_l(\Omega_*) \propto \Omega_*^{-1}$ whereas for the rapidly rotating stars $T_l(\Omega_*) \propto 1/\ln(\Omega_* \tau)$ (see Kleeorin et al. 1983). It follows from here that the period T_c in nonlinear regime is determined both by the period of the linear dynamo wave and T_{cr}. The dependence obtained from our nonlinear dynamo model is in agreement with observations discussed in Noyes & Weiss (1984).

In the next section we will use the theoretical dependence (10) for the magnitude of the mean magnetic field in estimations of the dependence of the stellar X-ray luminosity on rotation. We take the known theoretical dependence of the X-ray luminosity on the magnetic field and express the field using our formula (10) to obtain the luminosity dependence on the rotational velocity. The exponent of the resultant dependence is compared with the observed exponent.

3. X-ray luminosity

Observations of soft X-rays in stellar coronae show a correlation between the X-ray luminosity L_x and rotational velocity V_Ω :

$$L_x \simeq V_\Omega^\beta. \tag{11}$$

Fleming et al. (1989) analyze an X-ray-selected sample of 128 late-type (F-M) single stars and find the exponent $\beta \approx 1.05 \pm 0.08$.

Let us find the dependence of the X-ray luminosity on the rotational velocity. Theoretically, the X-ray luminosity is estimated by assuming that the necessary energy comes from reconnections of the magnetic fields in the coronae. Roughly, if the magnetic energy accumulated in a unit volume, $B^2/8\pi$, is released in a reconnection time τ_c, the rate of release is proportional to $B^2/(8\pi\tau_c)$. The accumulation of magnetic energy is due to electric currents generated by convective motions on the stellar surface (see, e.g., Priest 1982). More accurately, the rate of magnetic energy release in ergs per second, Q_x, is determined by a time averaged Poynting flux:

$$Q_x \simeq \frac{B_c^2 \tau_r}{8\pi l} \int \frac{d\omega}{1+(\omega\tau_r)^2} \int W(\omega)\, dS \qquad (12)$$

(Vekshtein 1987), where l is a characteristic size of the magnetic region, $W(\omega)$ is the spectrum of the hydrodynamic energy of the turbulent pulsations near the surface of the star, τ_r is a characteristic time describing the relaxation of the magnetic field to a minimal energy state. The time τ_r can be estimated as the time scale for the onset of the tearing-mode instability: $\tau_r \sim \tau_d^{1-\delta}\tau_a^{\delta}$, where δ varies from 0 to 1 depending on the regime of the tearing instability (see, e.g., White 1983), $\tau_a \sim l_c/C_A$ is the Alfven time, $\tau_d \sim l_c^2/\eta_c$ is the diffusive time, l_c is the thickness of the current layer in the reconnection region, $C_A = B_c/(4\pi\rho_c)^{1/2}$ is the Alfven speed. Parameters with the subscript "c" correspond to the corona and upper chromosphere. The spectrum $W(\omega)$ can be chosen in the form

$$W(\omega) = \int (q-1)\left(\frac{u_0^2}{k_0}\right)\cdot\left(\frac{k}{k_0}\right)^{-q}\left(\frac{\nu k^2}{\pi}\right)\frac{1}{\omega^2+\nu^2k^4}\, dk, \qquad (13)$$

where $\nu(k)$ is the turbulent viscosity in the scale k^{-1}, k is the wave number, k_0 and u_0 are the wave number and characteristic velocity in the maximum scale l_0 of the turbulent motions. For example, for the Kolmogorov spectrum of the hydrodynamic pulsations $q = 5/3$. We use here the Lorenz profile of the frequency component of the spectrum. Integration in $\omega-$ and $k-$ space in (12) yields

$$Q_x \simeq \frac{B_c^2 \tau_r}{8\pi l} \int u_0^2\, dS. \qquad (14)$$

Here we assume that $\nu(k)k^2 \sim 1/\tau_*(k)$ and $\tau_r < \tau_0$, where $\tau_*(k) = 2\tau_0(k/k_0)^{1-q}$, $\tau_0 \sim l_0/u_0$. It follows from (14) the rate of the released magnetic energy Q_x depends on magnetic field as

$$Q_x \propto B_c^{2-\delta},$$

where we take into account that $\tau_r \propto B_c^{-\delta}$. The X-ray luminosity L_x is defined as $L_x \simeq Q_x N_m$, where N_m is the number of magnetic regions in the corona. Therefore, the X-ray luminosity L_x is qiven by

$$L_x \propto N_m B_c^{2-\delta}.$$

We estimate the exponent β in the X-ray luminosity vs stellar rotation assuming that the basic contribution to the X-ray luminosity comes from the magnetic fields outside sunspots and active regions. In this case the number of magnetic regions N_m is independent of the mean magnetic field B_s and it is determined only by a number of convective cells at the photosphere. On the other hand, the rate of magnetic energy release in ergs per second, $Q_x \propto B_s^{2-\delta}$. This is due to the magnitude $B_c \sim k_c B_s$, $k_c \sim 10^{-1}$. Therefore, the dependence of the X-ray luminosity on the mean magnetic field is given by $L_x \propto B_s^{2-\delta}$. By use of the expression (10) for the field B_s near the surface we can express the X-ray luminosity in terms of the rotational velocity V_Ω. The result is given by $L_x \sim V_\Omega^{\beta}$, where $\beta = 3(2-\delta)/4$. Since $0 < \delta < 1$ we obtain that $3/4 < \beta < 3/2$ in satisfactory agreement with observations (see Fleming et al. 1989).

Note that within the present-day accuracy of observation of stellar activity the approximate "scaling" approach used here is sufficient to describe exponential factors controlling stellar activity. In our paper we show that the main theoretical uncertainty in the exponent of the X-ray luminosity is due to the parameter δ that is determined by the type of magnetic reconnection. On the other hand, the uncertainty caused by the approximate nonlinear dynamo model is shown to be much smaller.

4. Conclusions

By use of a nonlinear model of an axisymmetric $\alpha\Omega$-dynamo (in single-mode approximation) we have found an expression for the magnitude of the mean magnetic field as a function of the stellar rotation rate and other parameters of the solar-type convective zone. This expression predicts that the field varies as the 3/4 power of the rotation rate. The resulting theoretical relationship of the X-ray luminosity as functions of the angular velocity is in agreement with the observations.

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Appendix A: evolutionary equation for the magnetic part of the $\alpha-$ effect

Let us derive equation for the magnetic part α_m of the $\alpha-$ effect (for details see Kleeorin & Ruzmaikin 1982). The mean magnetic helicity of the turbulent field is given by

$$\chi \equiv \langle \mathbf{a}\cdot\mathbf{h}\rangle = \int \chi_*(k)\, dk$$

where k is the wave number, $\chi_*(k)$ is the spectral density of the magnetic helicity, $\mathbf{a}$ is the fluctuative part of the vector potential.

For $\nabla\cdot\mathbf{A} = \nabla\cdot\langle\mathbf{A}\rangle = \nabla\cdot\mathbf{a} = 0$ we obtain

$$\langle \mathbf{h}\cdot(\nabla\times\mathbf{h})\rangle = \int k^2\chi_*(k)\, dk, \qquad (A1)$$

$$\alpha_m \equiv \frac{\tau}{12\pi\rho}\langle \mathbf{h}\cdot(\nabla\times\mathbf{h})\rangle = \frac{1}{12\pi\rho}\int k^2\tau_*(k)\chi_*(k)\, dk, \qquad (A2)$$

where $\mathbf{A} = \langle\mathbf{A}\rangle + \mathbf{a}$ is the total vector potential. The induction equation for the total magnetic field $\mathbf{H} = \mathbf{B} + \mathbf{h}$ is given by

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla\times[\mathbf{u}\times\mathbf{H} - \eta_m\nabla\times\mathbf{H}], \qquad (A3)$$

where we consider the case with zero mean velocity. It follows from (A3) that the equation for the vector potential $\mathbf{A}$ is given by

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{H} - \eta_m \nabla \times (\nabla \times \mathbf{A}) + \nabla \tilde{\Phi} \,. \qquad (A4)$$

Here $\tilde{\Phi}$ is an arbitrary function. Let us multiply Eq. (A3) by **a** and Eq. (A4) by **h**, add them, and average over the ensemble of turbulent pulsations. The result is given by

$$\frac{\partial \chi}{\partial t} = -2\langle(\mathbf{u} \times \mathbf{h}) \cdot \mathbf{B}\rangle - 2\eta_m \langle \mathbf{h} \cdot (\nabla \times \mathbf{h})\rangle$$
$$-\langle \nabla \cdot [\mathbf{a} \times (\mathbf{u} \times \mathbf{h})]\rangle \,. \qquad (A5)$$

Here we omit the term $\sim \langle \mathbf{h} \cdot \nabla \tilde{\Phi}\rangle$ which is small in the medium with zero mean velocity. Taking into account the mean velocity **V** yields an additional term $\nabla \cdot (\chi \mathbf{V})$ in Eq. (A5). The effective electric field $\mathbf{E}_{eff} \equiv \langle \mathbf{u} \times \mathbf{h}\rangle$ is given by (Moffatt 1978; Parker 1979; Zeldovich et al. 1983)

$$\mathbf{E}_{eff} \equiv \langle \mathbf{u} \times \mathbf{h}\rangle = \alpha \mathbf{B} - \eta(\nabla \times \mathbf{B}) \,,$$

where $\alpha = \alpha_0 + \alpha_m$ and $\eta = \eta_m + \eta_T$ can be considered as scalars if $B^2/4\pi \ll \langle \rho u^2\rangle$. The term $\langle \nabla \cdot [\mathbf{a} \times (\mathbf{u} \times \mathbf{h})]\rangle$ in Eq. (A5) vanishes as a result of averaging of $\langle div \rangle$ over the volume. It follows from this that Eq. (A5) is reduced to

$$\frac{\partial \chi}{\partial t} = 2[\eta \mathbf{B} \cdot (\nabla \times \mathbf{B}) - \alpha \mathbf{B}^2 - \eta_m \langle \mathbf{h} \cdot (\nabla \times \mathbf{h})\rangle] \,. \qquad (A6)$$

Now let us assume that in the inertial range $k_0 < k < k_1$ the spectrum of the helicity $\chi_*(k)$ is given by

$$\chi_*(k) = \chi \frac{P}{k_0} \left(\frac{k}{k_0}\right)^{-q}, \quad P = (q-1)\left[1 - \left(\frac{k_0}{k_1}\right)^{q-1}\right]^{-1}, \qquad (A7)$$

where $|\chi| = |\langle \mathbf{a} \cdot \mathbf{h}\rangle| \sim B^2/k_0$, k_0^{-1} is the maximum scale of the turbulence, k_1^{-1} is the scale of the cutoff of the helicity spectrum. The parameter q is assumed to be known. For example, for Kolmogorov's spectrum $q = 5/3$ (developed hydrodynamic turbulence) and for Kraichnan's spectrum $q = 3/2$ (turbulence of interacting Alfven waves). Substitution (A6) into (A1) yields

$$\alpha_m = I\chi \,, \qquad (A8)$$

where $I = \mu/(4\pi\rho\eta_T)$,

$$\mu = \frac{1}{18}\frac{q-1}{2-q}\left[\left(\frac{k_1}{k_0}\right)^{4-2q} - 1\right]\left[1 - \left(\frac{k_0}{k_1}\right)^{q-1}\right]^{-1}.$$

Here we take into account that $\tau_*(k) = 2\tau_0(k/k_0)^{1-q}$. The turbulent magnetic diffusivity η_T for turbulence which is far from the equipartition of the energy of hydrodynamic pulsations and magnetic fluctuations is given by $\eta_T = (12\tau_0 k_0^2)^{-1}$.

Multiplying Eq. (A6) by I and using (A1) we obtain Eq. (2), where

$$\mu_* = \frac{3-q}{q-1}\left[1 - \left(\frac{k_0}{k_1}\right)^{q-1}\right]\left[\left(\frac{k_1}{k_0}\right)^{3-q} - 1\right]^{-1}$$

(see Kleeorin & Ruzmaikin 1982).

Spectral properties of the magnetic helicity $\chi_*(k)$ satisfy the realizability condition : $|\chi_*(k)| \le 2k^{-1}M(k)$ (Moffatt 1978), where $M(k)$ is the spectrum of the magnetic fluctuations. It follows that

$$\left(\frac{k}{k_0}\right)^{q_m+1-q} \le \frac{2}{P}. \qquad (A9)$$

Here we use a magnetic spectrum of the form

$$M(k) \simeq \frac{B^2}{k_0}\left(\frac{k}{k_0}\right)^{-q_m}$$

For the case $q_m = 1$ this spectrum was obtained using different techniques by Ruzmaikin & Shukurov (1982), Kleeorin et al. (1990), Kleeorin & Rogachevskii (1994), Brandenburg et al. (1993), Brandenburg et al. (1994). It is seen from (A9) that for $q_m \ge 1$ and $q < 2$ the wave number k and therefore k_1 is close to k_0. Indeed, k_1 is determined from equation

$$\left(\frac{k_1}{k_0}\right)^{q_m+1-q} = \frac{2}{P(k_1)}.$$

It follows from here that $P(k_1) \simeq 1$ and $k_1 \simeq k_0$. In this case which seems to be typical for the solar-type convective zones, $\mu \approx 1/9$ and $\mu_* \approx 1$, and $T \simeq \tau_0 R_m$.

Note that Eq. (2) can also be regarded as a consequence of the conservation of total magnetic helicity $\int \langle \mathbf{A} \cdot \mathbf{H}\rangle \, d^3r$ in the limit of $R_m \to \infty$.

If $\eta_m = 0$, the magnetic Reynolds number $R_m = \infty$. It follows from the induction Eq. (1) that in this case the magnetic field is frozen into plasma and the magnetic flux cannot increase. On the other hand, for $\eta_m = 0$ Eq. (2) for the nonlinearity is valid, and the nonlinear solution of Eqs. (1)-(2) for the magnitude of the mean magnetic field [see Eq. (8)] yields $B_s(\eta_m = 0) = 0$. Equation (2) describes quenching of the growth of the magnetic field due to the back reacton of the field on the magnetic part α_m of the $\alpha-$ effect. Evolution of the magnetic part of the $\alpha-$ effect depends on η_m. In particular, the characteristic time T of the relaxation of the α_m tends to ∞ when $\eta_m \to 0$. This means that when $\eta_m \to 0$ a very small mean magnetic field **B** generates large α_m :

$$\alpha_m \propto \eta_m^{-1}$$

which quenches any generation of the mean magnetic field and this field **B** tends to 0.

Appendix B: general properties of the linear operator L

Linear $\alpha\Omega-$ problem by substitution

$$\begin{pmatrix} A \\ B \end{pmatrix} = \sum_{n=1}^{\infty} c_n \exp(p_n t) \mathbf{e}_n(r, \theta), \qquad (B1)$$

can be reduced to the eigenvalue problem

$$\hat{L}\begin{pmatrix} a_n \\ b_n \end{pmatrix} = p_n \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \qquad (B2)$$

Since the coefficients of the operator $\hat{L}$ are real $p_n^* = \gamma_n - i\omega_n$ and $\mathbf{e}_n^*$ along with p_n and $\mathbf{e}_n$ are eigenvalues and eigenvectors of its spectrum. The combination of these two complex modes gives a real dynamo wave. In general, these vectors are not orthogonal to each other, because the operator $\hat{L}$ is not self-adjoint (see Kleeorin & Ruzmaikin 1984). The system of functions biorthogonal to $\mathbf{e}_n$ can be constructed from the eigenvectors $\mathbf{e}^n = (a^n b^n)$ of the adjoint operator

$$\hat{L}^+ = \begin{pmatrix} \Delta_s & \alpha_0(r,\theta) \\ -D\hat{\Omega} & \Delta_s \end{pmatrix},$$

whose eigenvalues are complex conjugate to p_n. Note that if $n \neq m$ then $(\mathbf{e}^m \mathbf{e}_n) = 0$. Eigenvalues problem for the $\hat{L}^+$ is reduced to

$$(a^n b^n)\hat{L}^+ = p_n^*(a^n b^n). \qquad (B3)$$

Now let us study some properties of the operator $\hat{L}$ which are useful for derivation of Eqs. (6)-(7). From (B2)-(B3) it follows

$$p_m(b^m)^* = \alpha_0(a^m)^* + \Delta_s(b^m)^*, \qquad (B4)$$

$$p_n b_n = D\hat{\Omega} a_n + \Delta_s b_n. \qquad (B5)$$

Multiply Eq. (B4) by b_n and Eq. (B5) by $(b^m)^*$, substract the first from the second equations and integrate over the space. The result is given by

$$D\int (b^m)^*\hat{\Omega} a_n\, d^3r = \int \alpha_0(a^m)^* b_n\, d^3r + $$
$$+(p_n - p_m)\int (b^m)^* b_n\, d^3r. \qquad (B6)$$

For $n = m$ Eq. (B6) is reduced to

$$D\int (b^n)^*\hat{\Omega} a_n\, d^3r = \int \alpha_0(a^n)^* b_n\, d^3r. \qquad (B7)$$

Consider the eigenvalue problem $p_n'\mathbf{e}_n = \hat{L}'\mathbf{e}_n$. Here

$$\hat{L}' = \hat{L} + \hat{\Omega}\delta D, \quad \hat{\Omega} = \begin{pmatrix} 0 & 0 \\ \hat{\Omega}(r,\theta) & 0 \end{pmatrix}.$$

According to the perturbation theory in the non-degenerate case we have for $\delta p_n = p_n' - p_n$

$$\delta p_n(\mathbf{e}^n \mathbf{e}_n) = (\mathbf{e}^n \hat{\Omega} \mathbf{e}_n)\delta D = \delta D \int (b^n)^* \hat{\Omega} a_n\, d^3r.$$

When $dp_n/dD \neq 0$ it follows

$$(\mathbf{e}^n \mathbf{e}_n) = \left(\frac{dp_n}{dD}\right)^{-1} \int (b^n)^* \hat{\Omega} a_n\, d^3r. \qquad (B8)$$

After substitution (B7) into Eq. (B8) we obtain

$$(\mathbf{e}^n \mathbf{e}_n) = \left(D\frac{dp_n}{dD}\right)^{-1} \int \alpha_0 (a^n)^* b_n\, d^3r. \qquad (B9)$$

When $dp_n/dD = 0$ Eq. (B9) is not valid. This case considered by Kleeorin & Ruzmaikin (1984). Solution of Eq. (B2) yields eigenvector $\mathbf{e}_n$. To obtain the adjoint eigenvector $\mathbf{e}_n$ it is sufficient to solve the linear problem for $D = -D$. The resulting B and A are associated with a^n and b^n respectively (see, Kleeorin & Ruzmaikin 1984).

Appendix C: single-mode approximation

Suppose that only one dynamo wave, the dipole type, is excited. In this case two complex conjugate modes of corresponding symmetry should be retained in (6). Since A and B are real, the amplitudes $F^m(t)$ of the complex conjugate eigenvectors in (5), are complex conjugate. Therefore we can consider a single complex equation for $F^1(t)$. [For the sake of simplicity we will omit the superscript, $F^1(t) = F$]. The equation for $F^{-1} = F^*$ may be obtained from the equation for F by complex conjugation.

The equations of the one-mode approximation then follow from (6) and (7):

$$\frac{dF}{dt} - pF = \left(F\left(\frac{D}{D_{cr}} - 1\right) + \alpha F + \tilde{\alpha}F^*\right)\Delta p \cdot \exp(i\beta_p), \qquad (C1)$$

$$\frac{d\alpha}{dt} + \frac{\alpha}{T} = (\zeta + \zeta^*)FF^* - \sigma F\frac{dF^*}{dt} - \sigma^* F^* \frac{dF}{dt}, \qquad (C2)$$

$$\frac{d\tilde{\alpha}}{dt} + \frac{\tilde{\alpha}}{T} = 3\zeta^* F^2 - \sigma^* F \frac{dF}{dt}, \qquad (C3)$$

where $\alpha = \alpha_1^1, \quad \tilde{\alpha} = \alpha_{-1}^1,$

$$\sigma = \frac{1}{C_1}\int \frac{1}{\rho(r)}(a^1)^* a_1^* b_1^2\, d^3r,$$

$$\zeta = \frac{1}{C_1}\int \frac{\hat{M}(b_1, a_1^*)}{\rho(r)}(a^1)^* b_1\, d^3r,$$

$$\Delta p \cdot \exp(i\beta_p) = D_{cr}\left(\frac{dp}{dD}\right)_{cr}.$$

We take into account that

$$a^m = \tilde{a}^m \exp(-ig^m), \quad a_m = a^m \exp(-i\delta_m)/\sqrt{D_{cr}},$$

$$b_m = \tilde{b}_m \exp(-ig_m), \quad |\tilde{a}^m| \sim \sqrt{D_{cr}}|\tilde{b}^m|,$$

$$|\tilde{a}_m| \sim |\tilde{b}_m|/\sqrt{D_{cr}}, \quad g^m = g_m.$$

The functions a^m and b_m describe the dynamo wave, g_m corresponds to a phase of the dynamo wave. The phase is changed from 0 at the pole to 2π at the equator (Parker 1979). This means that integrals of the rapidly oscillating functions are

$$\int_0^\pi \exp\{2ing(\theta)\}\sin\theta\, d\theta = 0,$$

where $n = 1, 2, 3, \ldots$.

Consider now the case $\tau \ll T \ll T_c$, where T_c is a period of the cyclic activity. This case is typical for solar type stars. For instance, for the Sun $T \sim$ several months, while $T_c \sim 22$

years. Therefore we can neglect the time derivatives of α and $\hat{\alpha}$ in Eqs. (C2) and (C3). This yields

$$\alpha \simeq T\left(2\zeta_R FF^* - \sigma F\frac{dF^*}{dt} - \sigma^* F^* \frac{dF}{dt}\right), \qquad (C4)$$

$$\tilde{\alpha} \simeq T\left(3\zeta^* F^2 - \sigma^* F\frac{dF}{dt}\right). \qquad (C5)$$

The solution of Eq. (C1) has the form

$$F(t) = f(t)\exp\{i\Psi(t)\}, \qquad (C6)$$

where the functions $f(t)$ and $\Psi(t)$ are real. Note that $\omega_c = d\Psi/dt$ corresponds to the frequency of the activity. Substitution of expressions (C4-C5) into Eq. (C1) yields

$$\frac{df}{dt} = \frac{f\Delta\gamma}{1 - f^2 T\sigma_* \Delta p}\left(\frac{D}{D_{cr}} - 1 + \right.$$
$$\left. + f^2 T(5\zeta_R + 3\zeta_I \tan\beta_p + c_q\omega_c\sigma_*)\right), \qquad (C7)$$

where $\Delta\gamma = D_{cr}(d\gamma/dD)_{cr}$,

$$c_p = \cos(\beta_p + \beta_\sigma) + 2\cos(\beta_p - \beta_\sigma),$$

$$c_q = [\sin(\beta_p + \beta_\sigma) + 2\sin(\beta_p - \beta_\sigma)]/\cos\beta_p.$$

$$\zeta = \zeta_R + i\zeta_I = -\zeta_* \exp(i\beta_\zeta), \qquad \sigma = \sigma_* \exp(-i\beta_\sigma)$$

$$\sigma_* \simeq D_{cr}^{-1/2}\left(\frac{\Lambda_\rho}{L_0}\right)\left(\frac{b_s^4}{b_m^2}\right),$$

$$\zeta_* \simeq \sigma_* \left(\frac{\partial g}{\partial\theta}\right)^2,$$

L_0 is the radial size of the stellar convective zone, Λ_ρ is the density height scale, b_s and b_m are components of eigenvectors $\mathbf{e}_n$ of the linear dynamo problem corresponding to the toroidal magnetic fields near the surface and inside the convective zone, respectively. To calculate the coefficients ζ we assume that $|\partial g/\partial\theta| \gg |\nabla b/b|$. This means that

$$\hat{M}(B, A) \simeq \frac{1}{r^2}\left(2B\frac{\partial^2 A}{\partial\theta^2} - \frac{\partial B}{\partial\theta}\frac{\partial A}{\partial\theta}\right).$$

The stationary solution of Eq. (C7) yields the magnitude of the mean magnetic field:

$$f_B^2 \simeq \frac{D - D_{cr}}{D_{cr}T(5|\zeta_R| + 3|\zeta_I|\tan\beta_p - c_q\omega_c\sigma_*)}. \qquad (C8)$$

We found here the steady state of the nonlinear system (C1-C3). As follows from analysis this steady state is stable in single-mode approximation. In the dimension variables the magnitude of the mean magnetic fields near the surface of the star is $B_s = f_B B_* b_s$ and given by Eq. (8), where we use that $g(\theta) \simeq 2\pi(1 \pm \cos\theta)$ (see, e.g., Kleeorin & Ruzmaikin 1991) and $\beta_\zeta \simeq \beta_p \sim \pi/4$, $c_q \sim 1$ (see, e.g., Ivanova & Ruzmaikin 1977).

The imaginary part of Eq. (C1) after taking into account of Eqs. (C4-C5) yields the nonlinear frequency $\omega_c = d\Psi/dt$ of the magnetic field oscillations (the frequency of the cycle activity):

$$\omega_c = \left(\omega_l - 2|\zeta_R|Tf_B^2 d_0\left(\frac{d\omega}{dD}\right)_{cr} D_{cr}\right)(1 + \tilde{\sigma}_0 f_B^2),$$

where $\tilde{\sigma}_0 = \sigma_*\cos(\beta_p - \beta_\sigma)$, $d_0 = 1 + 3\sin(\beta_p - \beta_\sigma)/\sin(2\beta_p)$, f_B is determined by Eq. (C8), the frequency ω_l corresponds to that of the linear dynamo wave. For the estimation we assume that

$$\left(\frac{d\omega}{dD}\right)_{cr} \sim \frac{\omega_l - \omega_{cr}}{D - D_{cr}}$$

and take into account that $\zeta_* \gg \sigma_*$ and $\tilde{\sigma}_0 f_B^2 \ll 1$. Here $\omega_{cr} = \omega_l(D - D_{cr})$. It follows from here the nonlinear frequency ω_c is given by

$$\omega_c = [\omega_l - d_1(\omega_l - \omega_{cr})](1 + \sigma_0 B_s^2)^{-1}, \qquad (C9)$$

where $d_1 = 2d_0/(5 + 3\tan\beta_p\tan\beta_\zeta)$, $\sigma_0 = \tilde{\sigma}_0(B_* b_s)^{-2}$. For the Sun $d_0 \simeq 1, d_1 \simeq 1/4$.

We have described the case $T \ll T_c$, where $T_c = 2\pi/\omega_c$. Now let us consider the case $T \geq T_c$. For the simplicity we take into account also that $\zeta \gg \sigma$. The solution of the system (C1-C3) we seek for in the form

$$F(t) = f(t)\exp\{i\Psi(t)\}, \quad \alpha(t) = \alpha(t),$$

$$\tilde{\alpha} = \hat{\alpha}(t)\exp\{i(2\Psi(t) + \delta)\},$$

where the functions $f(t), \alpha(t), \hat{\alpha}(t), \Psi(t), \delta$ are the real functions. Substitution the expressions for $F(t), \alpha(t), \tilde{\alpha}$ into Eqs. (C1-C3) yields

$$\frac{1}{f}\frac{df}{dt} = \Delta p\left(\frac{D}{D_{cr}} - 1 + \alpha\right)\cos\beta_p + \Delta p\hat{\alpha}\cos(\beta_p + \delta), \quad (C10)$$

$$\frac{d\alpha}{dt} + \frac{\alpha}{T} = 2f^2\zeta_R, \qquad (C11)$$

$$\frac{d\hat{\alpha}}{dt} + \frac{\hat{\alpha}}{T} = -3f^2\zeta_*\cos(\beta_\zeta - \delta), \qquad (C12)$$

$$\frac{d\Psi}{dt} = \omega + \Delta p\left(\frac{D}{D_{cr}} - 1 + \alpha\right)\sin\beta_p + \Delta p\hat{\alpha}\sin(\beta_p + \delta), \quad (C13)$$

$$2\hat{\alpha}\{\omega + \Delta p\left(\frac{D}{D_{cr}} - 1 + \alpha\right)\sin\beta_p + \Delta p\hat{\alpha}\sin(\beta_p + \delta)\} =$$
$$= -3f^2\zeta_*\sin(\beta_\zeta - \delta), \qquad (C14)$$

where $\zeta = -\zeta_*\exp(i\beta_\zeta)$.

Now we average Eqs. (C10)-(C14) over the time $\geq T$. In saturation of the growth of the magnetic field the mean values :

$$\frac{d\langle \ln(f)\rangle}{dt} = \frac{d\langle\alpha\rangle}{dt} = \frac{d\langle\hat{\alpha}\rangle}{dt} = 0 \qquad (C15)$$

and $\langle d\Psi/dt\rangle = \omega_c$, where the angular brackets denote the averaging, ω_c is the mean frequency of cycle. If the condition (C15) is violated the nonlinear solution is unstable. After averaging we obtain

$$\langle\alpha\rangle \simeq 2\langle f^2\rangle\zeta_R T, \qquad (C16)$$

$$\langle\hat{\alpha}\rangle \simeq -3\langle f^2\rangle\zeta_* T\cos(\beta_\zeta - \delta), \qquad (C17)$$

$$\left(\frac{D}{D_{cr}} - 1 + \langle\alpha\rangle\right)\cos\beta_p \simeq -\langle\hat{\alpha}\rangle\cos(\beta_p + \delta). \qquad (C18)$$

Substitution (C16-C17) into Eq. (C18) yields the magnitude of the mean magnetic field:

$$f_B^2 \simeq \langle f^2\rangle \simeq \frac{D - D_{cr}}{2D_{cr}T|\zeta_R|(1+3d)}, \qquad (C19)$$

which is in agreement with Eq. (C8). Here $d = \cos(\beta_p + \delta)\cos(\beta_\zeta - \delta)/(2\cos\beta_p\cos\beta_\zeta)$.

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