Nonlinear Adiabatic Passage from Fermion Atoms to Boson Molecules

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We study the dynamics of an adiabatic sweep through a Feshbach resonance in a quantum gas of fermionic atoms. Analysis of the dynamical equations, supported by mean-field and many-body numerical results, shows that the dependence of the remaining atomic fraction Γ on the sweep rate α varies from exponential Landau-Zener behavior for a single pair of particles to a power-law dependence for large particle number N. The power law is linear, $\Gamma \propto \alpha$, when the initial molecular fraction is smaller than the $1/N$ quantum fluctuations, and $\Gamma \propto \alpha^{1/3}$ when it is larger. Experimental data agree well with a linear dependence, but do not conclusively rule out the Landau-Zener model.

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Adiabatic sweeps across an atom-molecule Feshbach resonance have recently been used to convert degenerate fermionic atomic gases containing two different internal spin states to bosonic dimer molecules [1–4]. Formation of a molecular condensate has also been observed using both adiabatic sweeps and three-body recombination processes [5]. In this Letter we show that for adiabatic Feshbach sweeps that convert degenerate fermionic atoms to diatomic molecules, the Landau-Zener behavior for a single pair of particles [6], which is a paradigm for modeling adiabatic evolution, can be significantly altered due to many-body effects. The fraction of unconverted atoms is shown to follow a power law in the sweep rate, rather than the exponential behavior predicted by an essentially single-particle, linear Landau-Zener model [6,7]. The exact power law is determined by quantum fluctuations. En route to this result we also find that, for a ladder of atomic states filled by fermionic atoms, the atom-molecule sweep efficiency is unaffected by atomic dispersion, and all fermionic atoms can go over to molecules, in contrast to the linear Landau-Zener model. Comparison with experimental results shows good agreement but does not rule out an exponential fit.

We consider the collisionless, single bosonic mode Hamiltonian [8–14]

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \mathcal{E}(t) b_0^{\dagger} b_0 + g\left(\sum_{\mathbf{k}} c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} b_0^{\dagger} + \text{H.c.}\right), \tag{1}$$

where $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ is the kinetic energy of an atom with mass m, and g is the atom-molecule coupling strength. The molecular energy $\mathcal{E}(t) = \alpha t$ is linearly swept at a rate α through resonance to induce adiabatic conversion of Fermi atoms to Bose molecules. The annihilation operators for the atoms, $c_{\mathbf{k},\sigma}$, obey fermionic anticommutation relations, whereas the molecule annihilation operator b_0 obeys a bosonic commutation relation.

We find that, provided that all atomic levels are swept through, the adiabatic conversion efficiency is completely insensitive to the details of the atomic dispersion. Figure 1 shows exact numerical results for the adiabatic conversion of five atom pairs into molecules, for different values of the atomic level spacing (and hence of the Fermi energy E_{F}). It is evident that, while the exact dynamics depends on E_{F}, levels are sequentially crossed, leading to the same final efficiency regardless of the atomic motional time scale. In particular, in the limit as $\alpha \to 0$, it is possible to convert *all* atom pairs into molecules. This is a unique feature of the nonlinear parametric coupling between atoms and molecules, which should be contrasted with a marginal conversion efficiency expected for linear coupling. Since the exact energies $\epsilon_{\mathbf{k}}$ do not affect the final fraction of molecules, we use a degenerate model [12–14] with $\epsilon_{\mathbf{k}} = \epsilon$ for all $\mathbf{k}$. In the spirit of Refs. [13,15], we define the operators:

$$J_- = \frac{b_0^{\dagger} \sum_{\mathbf{k}} c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow}}{(N/2)^{3/2}}, \qquad J_+ = \frac{\sum_{\mathbf{k}} c^{\dagger}_{-\mathbf{k},\downarrow} c^{\dagger}_{\mathbf{k},\uparrow} b_0}{(N/2)^{3/2}}, \qquad J_z = \frac{\sum_{\mathbf{k},\sigma} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} - 2 b_0^{\dagger} b_0}{N}, \tag{2}$$

where $N = 2b_0^{\dagger} b_0 + \sum_{\mathbf{k},\sigma} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$ is the conserved total

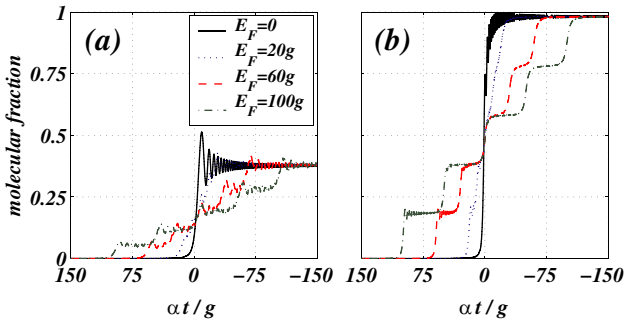

FIG. 1 (color online). Many-body collective dynamics of adiabatic passage from a fermionic atomic gas into a molecular BEC for five pairs of fermionic atoms. (a) Sweep rate $\alpha = 2g^2N$. (b) Sweep rate $\alpha = g^2N/4$. Overall efficiency is independent of atomic dispersion in both (a) and (b).

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number of particles. It is important to note that J_-, J_+, J_z do not span $SU(2)$ as $[J_+, J_-]$ is a quadratic polynomial in J_z. We also define $J_x = J_+ + J_-$ and $J_y = -i(J_+ - J_-)$. Up to a c-number term, Hamiltonian (1) takes the form

$$H = \frac{N}{2}\left(\Delta(t)J_z + g\sqrt{\frac{N}{2}}J_x\right), \tag{3}$$

where $\Delta(t) = 2\epsilon - \mathcal{E}(t)$. Defining a rescaled time $\tau = \sqrt{N}gt$, we obtain the Heisenberg equations of motion for the association of a quantum-degenerate gas of fermions,

$$\begin{aligned}
\frac{d}{d\tau}J_x &= \delta(\tau)J_y \\
\frac{d}{d\tau}J_y &= -\delta(\tau)J_x + \frac{3\sqrt{2}}{4}(J_z - 1)\left(J_z + \frac{1}{3}\right) - \frac{\sqrt{2}}{N}(1 + J_z), \\
\frac{d}{d\tau}J_z &= \sqrt{2}J_y,
\end{aligned} \tag{4}$$

which depend on the single parameter $\delta(\tau) = \Delta(t)/\sqrt{N}g = (\alpha/g^2N)\tau$. We note parenthetically that precisely the same set of equations, with $J_z \to -J_z$ and $g \to g/2$, is obtained for a two-mode atom-molecule Bose-Einstein condensate (BEC) [15], highlighting a mapping between the two systems [12–14].

We first consider the mean-field limit of Eqs. (4), replacing J_x, J_y, and J_z by their expectation values u, v, and w which correspond to the real and imaginary parts of the atom-molecule coherence and the atom-molecule population imbalance, respectively, and omitting the quantum-noise term $\sqrt{2}(1 + J_z)/N$. In this limit, the equations depict the motion of a generalized Bloch vector on a two-dimensional surface, determined by the conservation law,

$$u^2 + v^2 = \frac{1}{2}(w - 1)^2(w + 1). \tag{5}$$

Hamiltonian (3) is then replaced by the classical form

$$H(w, \theta; \Delta) = \frac{gN^{3/2}}{2}\left(\delta w + \sqrt{(1 + w)(1 - w^2)}\cos\theta\right), \tag{6}$$

with $\theta = \arctan(v/u)$.

To study the atom-molecule adiabatic passage, we closely follow the method of Ref. [16]. The eigenvalues of the atom-molecule system at any given value of δ correspond to the fixed points (u_0, v_0, w_0) of the classical Hamiltonian (6) or the mean-field limit of Eqs. (4):

$$v_0 = 0, \qquad \frac{\sqrt{2}}{4}(w_0 - 1)(3w_0 + 1) = \delta u_0. \tag{7}$$

The number of fixed points depends on the parameter δ. The point $u_0 = v_0 = 0$, $w_0 = 1$ is stationary for any value of δ. Using Eqs. (5) and (7), other fixed points satisfy

$$\frac{(3w_0 + 1)^2}{4(w_0 + 1)} = \delta^2. \tag{8}$$

In Fig. 2 we plot phase-space trajectories, corresponding to equal-energy contours of Hamiltonian (6), for different values of δ. As expected from (6), the plots have the symmetry $(w, \theta; \delta) \leftrightarrow (w, \theta + \pi; -\delta)$. For sufficiently large detuning, $|\delta| > \sqrt{2}$, Eq. (8) has only one solution in the range $-1 \le w_0 \le 1$. Therefore, there are only two (elliptic) fixed points, denoted by a red circle corresponding to the solution of Eq. (8) and a blue square at (0, 0, 1). As the detuning is changed, one of these fixed points (red circle) smoothly moves from all molecules towards the atomic mode. At detuning $\delta = -\sqrt{2}$ a homoclinic orbit appears through the point (0, 0, 1) which bifurcates into an unstable (hyperbolic) fixed point (black star) remaining on the atomic mode, and an elliptic fixed point (blue square) which starts moving towards the molecular mode. Consequently, in the regime $|\delta| < \sqrt{2}$ there are two elliptic fixed points and one hyperbolic fixed point, corresponding to the unstable all-atoms mode. Another crossing occurs at $\delta = \sqrt{2}$ when the fixed point that started near the molecular mode (red circle) coalesces with the all-atoms mode (black star).

The frequency of small periodic orbits around the fixed points, Ω_0, is found by linearization of the dynamical Eqs. (4) about (u_0, v_0, w_0) and using (8) to obtain

$$\frac{\Omega_0}{g\sqrt{N}} = \sqrt{\delta^2 + (1 - 3w_0)} = \sqrt{\frac{(1 - w_0)(3w_0 + 5)}{4(w_0 + 1)}}. \tag{9}$$

Hence, for $|\delta| < \sqrt{2}$ the period of the homoclinic trajectory beginning at (0, 0, 1) diverges.

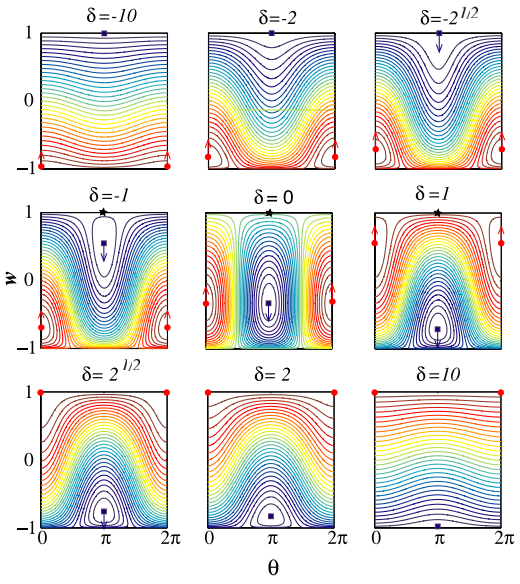

FIG. 2 (color online). Equal-energy contours of Hamiltonian (6) plotted as a function of w and θ for different detunings δ. $w = 1$ is all atoms and $w = -1$ is all molecules. The various fixed points corresponding to adiabatic eigenvectors are marked by (blue) squares, (red) circles, and (black) stars.