optimizing the performance of the interferometers at low frequencies, where both equations (4) and (6) become more significant. It appears natural to perform such studies in the quiet environment of space, perhaps through future refinements of LISA-type set-ups[27].

The above discussion of gravity-wave interferometers shows that the smallness of the Planck length does not preclude the possibility of direct investigations of space-time fuzziness. This complements the results of studies[28,29] which have shown that indirect evidence of quantum space-time fluctuations could be obtained by testing the predictions of theories consistent with a given picture of these fluctuations. Additional encouragement for experiment-driven progress in understanding the interplay between gravity and quantum mechanics comes from recent studies[30,31] in the area of gravitationally induced phases, the significance of which has been emphasized in refs 32 and 33. □

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Four-wave mixing with matter waves

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The advent of the laser as an intense source of coherent light gave rise to nonlinear optics, which now plays an important role in many areas of science and technology. One of the first applications of nonlinear optics was the multi-wave mixing[1,2] of several optical fields in a nonlinear medium (one in which the refractive index depends on the intensity of the field) to produce coherent light of a new frequency. The recent experimental realization of the matter-wave 'laser'[3,4]—based on the extraction of coherent atoms from a Bose–Einstein condensate[5]—opens the way for analogous experiments with intense sources of matter waves: nonlinear atom optics[6]. Here we report coherent four-wave mixing in which three sodium matter waves of differing momenta mix to produce, by means of nonlinear atom–atom interactions, a fourth wave with new momentum. We find a clear signature of a four-wave mixing process in the dependence of the generated matter wave on the densities of the input waves. Our results may ultimately facilitate the production and investigation of quantum correlations between matter waves.

The analogy between nonlinear optics with lasers and nonlinear atom optics with Bose–Einstein condensates can be seen in the similarities between the equations that govern each system. For a condensate of interacting bosons, in a trapping potential V, the macroscopic wavefunction Ψ satisfies a nonlinear Schrödinger equation[7]

$$i\hbar\frac{\partial\Psi}{\partial t}=\left(-\frac{\hbar^2}{2M}\nabla^2+V+U_0|\Psi|^2\right)\Psi \qquad (1)$$

where M is the atomic mass, U_0 describes the strength of the atom–atom interaction ($U_0>0$ for sodium atoms), and $|\Psi|^2$ is proportional to atomic number density. The nonlinear term $U_0|\Psi|^2\Psi$ in equation (1) is similar to the third order term $\chi^{(3)}|E|^2E$ in the wave equation for the electric field E describing optical four-wave mixing (4WM; where the susceptibility $\chi^{(3)}$ depends on the nonlinear medium). We therefore expected 4WM with coherent matter waves, analogous to optical 4WM. In contrast to optical 4WM, the nonlinearity in matter-wave 4WM comes from atom–atom interactions; there is no need for an additional nonlinear medium.

The first theoretical study of nonlinear atom optics was reported in 1993[6], and the idea of 4WM using condensates prepared in different electronic states to enhance the nonlinearity was discussed in 1995[8]. A recent calculation[9] showed that the nonlinearity associated with the interaction between ground-state atoms is large enough to observe 4WM with wavepackets created from existing Bose–Einstein condensates. To produce matter-wave mixing, we create three overlapping wavepackets with momenta $\mathbf{P}_n$ ($n=1,2,3$) and observe the creation of the 4WM wavepacket $\mathbf{P}_4$ that satisfies energy, momentum and particle-number conservation (Fig. 1).

In our experiment, we use Bragg diffraction of atoms from a moving optical standing wave[10] to create the necessary three wavepackets, starting from a Bose–Einstein condensate. Briefly, we first form a condensate of $\sim 2\times 10^6$ sodium atoms in the

$3S_{1/2}$, $F = 1$, $m = -1$ state using a combination of laser cooling and radio-frequency-induced evaporative cooling in a TOP (time-orbiting-potential) trap[11], without a discernible non-condensed fraction. We then adiabatically expand the potential[10] in 4 s by simultaneously reducing the magnetic field gradient and increasing the rotating bias field. This reduces the trap frequencies in the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ directions to 84, 59 and 42 Hz, respectively. The asymptotic r.m.s. momentum width of the released condensate after adiabatic expansion is measured to be $0.14(\pm 0.02)\hbar k$ (all uncertainties reported here are one standard deviation combined statistical and systematic uncertainties). Because this is small compared to $\sqrt{2}\hbar k$, the smallest momentum imparted to the condensate with the Bragg diffraction, the wavepackets will spatially separate as the system evolves.

After adiabatic expansion, we switch off the trap, wait 600 μs so that the trapping magnetic fields decay away and then apply a sequence of two Bragg pulses. Each 30-μs pulse is composed of two linearly polarized laser beams detuned from the $3S_{1/2}$, $F = 1 \rightarrow 3P_{3/2}$, $F' = 2$ transition by $\Delta/2\pi = -2$ GHz to suppress spontaneous emission. This large detuning makes negligible Bragg scattering of the optical waves by the atoms, which could lead to a spurious scattering of atoms into $\mathbf{P}_4$. The frequency difference between the two laser beams of a single Bragg pulse is chosen to fulfil a first-order Bragg diffraction condition that changes the momentum state of the atoms without changing their internal state[10]. The first Bragg pulse is composed of two mutually perpendicular laser beams of frequencies ν_1 and $\nu_2 = \nu_1 - 50$ kHz, and wavevectors $\mathbf{k}_1 = k\hat{\mathbf{x}}$ and $\mathbf{k}_2 = -k\hat{\mathbf{y}}$ ($k = 2\pi/\lambda$, $\lambda = 589$ nm). The maximum intensity of each beam is $\sim$10 mW cm^{-2}. The intensity was chosen so that roughly 1/3 of the condensate atoms acquire momentum $\mathbf{P}_2 = \hbar(\mathbf{k}_1 - \mathbf{k}_2) = \hbar k(\hat{\mathbf{x}} + \hat{\mathbf{y}})$. The second Bragg pulse is applied 20 μs after the end of the first Bragg pulse (well before the wavepackets are separated). This second Bragg pulse is composed of two counter-propagating laser beams with frequencies ν_1 and $\nu_3 = \nu_1 - 100$ kHz, and wavevectors $\mathbf{k}_1 = k\hat{\mathbf{x}}$ and $\mathbf{k}_3 = -k\hat{\mathbf{x}}$. The intensities of these laser beams were chosen to cause half of the remaining atoms in the momentum state $\mathbf{P}_1 = 0$ to acquire a momentum $\mathbf{P}_3 = \hbar(\mathbf{k}_1 - \mathbf{k}_3) = 2\hbar k\hat{\mathbf{x}}$ (atoms in $\mathbf{P}_2$ are not affected by this pulse because of the Doppler shift of the light). We chose this pulse sequence so that only $\mathbf{P}_2 = \hbar k(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ and $\mathbf{P}_3 = 2\hbar k\hat{\mathbf{x}}$ are produced from $\mathbf{P}_1 = 0$. Thus we create, nearly simultaneously, three overlapping wavepackets of the requisite momenta. Without the nonlinear term in equation (1), one would expect only to observe these three wavepackets after they have spatially separated. But as

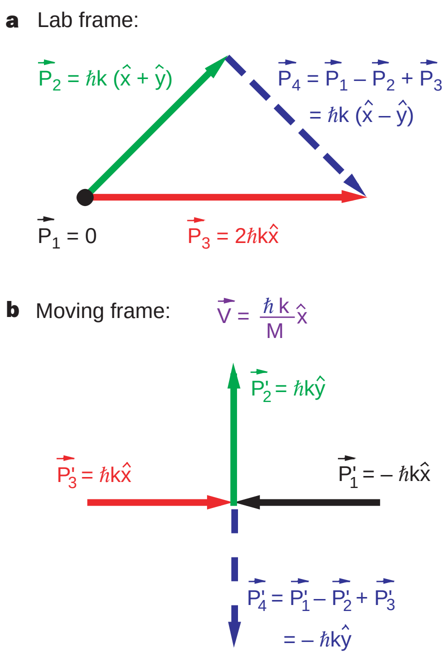

Figure 1 Momentum-energy conservation for 4WM and the bosonic stimulation viewpoint in a moving frame. **a**, Momentum conservation, $\mathbf{P}_4 = \mathbf{P}_1 - \mathbf{P}_2 + \mathbf{P}_3$ (equivalent to phase-matching in optical 4WM), in the laboratory frame. For clarity, over-arrows indicate vectors. Energy conservation requires $\mathbf{P}_4^2 = \mathbf{P}_1^2 - \mathbf{P}_2^2 + \mathbf{P}_3^2$. **b**, It is always possible to view matter 4WM in a frame moving with velocity **v** such that the three input momenta have the same magnitude, and two are counter-propagating. Then, in our case two atoms in momentum states $\mathbf{P}'_1 = -\hbar k\hat{\mathbf{x}}$ and $\mathbf{P}'_3 = -\hbar k\hat{\mathbf{x}}$ are bosonically stimulated by wavepacket $\mathbf{P}'_2 = -\hbar k\hat{\mathbf{y}}$ to scatter into momentum states $\mathbf{P}'_2$ and $\mathbf{P}'_4 = -\mathbf{P}'_2 = -\hbar k\hat{\mathbf{y}}$. We note that the energy and momentum conditions are satisfied independent of the direction of $\mathbf{P}'_2$. The 4WM wavepacket is a consequence of energy, momentum and particle-number conservation when atoms are stimulated into the momentum state $\mathbf{P}'_2$. Thus 4WM can be viewed as the annihilation of momentum states $\mathbf{P}'_1$ and $\mathbf{P}'_3$, and the creation of momentum states $\mathbf{P}'_2$ and $\mathbf{P}'_4$ (the minus signs in the energy and momentum conditions are attached to the sate that gains atoms). It is this bosonic stimulation of scattering that mimics the stimulated emission of photons from an optical nonlinear medium. Alternatively, by choosing a frame of reference in which $\mathbf{P}''_1 = -\mathbf{P}''_2$ (or $\mathbf{P}''_2 = -\mathbf{P}''_3$), 4WM can also be viewed as matter-wave Bragg diffraction of $\mathbf{P}''_3$ ($\mathbf{P}''_1$) from the grating produced by the interference of two others.

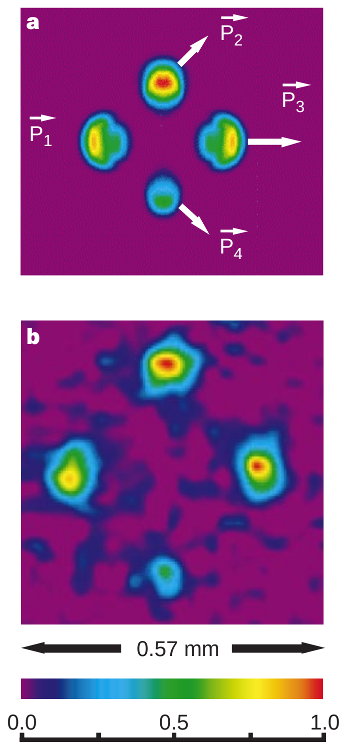

Figure 2 Numerical simulation and experimental results for 4WM. **a**, Calculated two-dimensional atomic distribution after 1.8 ms, showing the 4WM. The calculations were performed only until the wavepackets completely separated due to constraints on the simulation grid-size. The momenta are those of Fig. 1a. The field of view is 0.23 × 0.26 mm. We note that atoms are removed primarily from the back-end of the wavepackets because these regions overlap for the longest time. **b**, A false-colour image of the experimental atomic distribution showing the fourth (small) wavepacket generated by the 4WM process. The four wavepackets form a square measuring 0.26 × 0.26 mm, corresponding to the distance of 0.25 mm calculated using the experimental time of flight of 6.1 ms and the wavepacket momenta. We have verified that if we make initial wavepackets such that energy and momentum conservation cannot be simultaneously satisfied, no 4WM signal is observed. For instance, if we change the sign of the frequency difference between the two laser beams that comprise the second Bragg pulse, we will create a component with momentum $\mathbf{P}_3 = -2\hbar k\hat{\mathbf{x}}$ instead of $\mathbf{P}_3 = 2\hbar k\hat{\mathbf{x}}$. In this case there is no 4WM signal.

the three initial wavepackets separate, the nonlinear term will produce an additional wavepacket that satisfies the condition $\mathbf{P}_4 = \mathbf{P}_1 - \mathbf{P}_2 + \mathbf{P}_3 = \hbar k(\hat{\mathbf{x}} - \hat{\mathbf{y}})$, see Fig. 1a, as well as energy and particle-number conservation.

We have performed a two-dimensional numerical simulation of 4WM using equation (1) and the technique of ref. 9. The interaction energy (chemical potential) was chosen to be the same as it would be in three dimensions with a scattering length of 2.8 nm. The simulation releases 10^6 atoms from a trap with $\nu_x = 84$ Hz and $\nu_y = 59$ Hz. After 600 μs, the condensate was projected into the three initial momentum states. Figure 2a shows the atomic density 1.8 ms after this projection. The most important feature of Fig. 2a is the new wavepacket of atoms with momentum $\mathbf{P}_4 = \mathbf{P}_1 - \mathbf{P}_2 + \mathbf{P}_3$ generated by 4WM. The 4WM peak does not appear when the nonlinear term is absent.

Figure 2b is a false-colour image showing the results of the experiment. The atoms were imaged 6.1 ms after the second Bragg pulse by optically pumping the atoms to the $3S_{1/2}$, $F = 2$ state, and absorption-imaging[5] on the $3S_{1/2}$, $F = 2 \rightarrow 3P_{3/2}$, $F' = 3$ transition. The 4WM wavepacket is clearly visible. For this image, the numbers of atoms in each wavepacket were measured to be: $N_1 = 4.8(\pm 0.5) \times 10^5$, $N_2 = 5.3(\pm 0.5) \times 10^5$, $N_3 = 5.1(\pm 0.5) \times 10^5$ and $N_4 = 1.8(\pm 0.2) \times 10^5$, where the uncertainties are mainly due to uncertainties in background subtraction. The numbers of atoms in the three initial wavepackets N_1^0, N_2^0 and N_3^0 can be deduced using particle number conservation: $N_1^0 = N_1 + N_4$, $N_2^0 = N_2 - N_4$, and $N_3^0 = N_3 + N_4$. Defining the 4WM efficiency to be $\epsilon = N_4/N$, where $N = \Sigma_{j=1}^3 N_j^0 = \Sigma_{j=1}^4 N_j$, we obtain a conversion efficiency of 10.6(±0.13)%. This is the best we have observed, although under similar conditions we have also observed conversion efficiencies of only 6%. This difference suggests the influence of some uncontrolled experimental conditions, such as laser beam inhomogeneities, or non-zero average velocity of the released condensate. By comparison, the calculation of Fig. 2a gives an efficiency of 10%, albeit for only 10^6 atoms.

Equation (1) can be used to make a simple prediction about the expected nonlinear dependence of the 4WM signal on the numbers of atoms in the initial wavepackets. Substituting $\Psi = \Sigma_{j=1}^4 \Psi_j$ (where Ψ_j correspond to the individual momentum components) into equation (1), we find the initial rate of growth of the 4WM amplitude, $\partial\Psi_4/\partial t \propto \Psi_1\Psi_2^*\Psi_3$. We estimate the number of atoms in the fourth wave by multiplying this rate by a characteristic interaction time τ, proportional to the diameter of the condensate, squaring and integrating over space: $N_4 \propto n_1 n_2 n_3 V\tau^2$, where the density $n_j = N_j^0/V$. In the Thomas–Fermi limit[7], the volume of the condensate $V \propto N^{3/5}$, and $\tau \propto N^{1/5}$. Hence we expect $N_4/N \propto (N_1^0 N_2^0 N_3^0)N^{-9/5}$, a dependence which is supported by the numerical calculations. This nonlinear behaviour is clearly manifested in the initial linear growth seen in Fig. 3, where we vary the number of atoms in the original BEC and measure the number of atoms in the respective wavepackets. The data also show saturation at high N, as does the corresponding theory, although the maximum theoretical efficiency is somewhat higher.

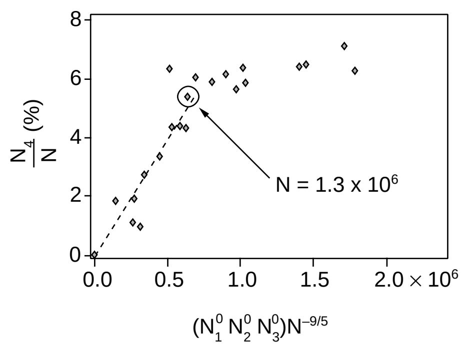

Figure 3 Measured conversion efficiency. Efficiency N_4/N is plotted as a function of $(N_1^0N_2^0N_3^0)N^{-9/5}$. The initial linear dependence is a signature of 4WM with matter waves. The dashed line is a fit to the first 12 points to guide the eye.

We now reconsider Fig. 1b. Here the process is seen as degenerate 4WM (where the magnitudes of all momenta are equal) in a geometry equivalent to phase-conjugation in optics[12]: indeed $\mathbf{P}'_4$ is the momentum conjugate of $\mathbf{P}'_2$. So this can also be considered as a demonstration of phase conjugation with matter waves. As in the case of optical phase conjugation, the process would work regardless of the angle between $\mathbf{P}'_1$ and $\mathbf{P}'_2$ (90° in the present case). If one alters the first Bragg pulse by changing only the angle of $\mathbf{k}_2$ (and appropriately changing ν_2) the magnitude and direction of $\mathbf{P}_2$ in the laboratory frame are changed, so that in the moving frame only the angle of $\mathbf{P}'_2$ is changed.

We emphasize that just as optical 4WM requires coherent light sources to coherently build up the generated wave, a condensate is also crucial for coherent generation of matter waves. If atoms are above the Bose–Einstein condensation temperature, the number density is necessarily low and the phase-matching condition is different for each velocity class. Both dramatically diminish the 4WM conversion efficiency.

In spite of the strong analogy between atom and optical 4WM, there are fundamental differences. In optical 4WM, the energy–momentum dispersion relation is $E = [c/n(k)]\hbar k$ (where $n(k)$ is the dispersive refractive index), whereas for massive particles (neglecting the matter-wave refractive index due to the atom–atom interaction energy) $E = P^2/2M$. Because atoms are neither created nor destroyed, the only 4WM processes allowed for matter waves conserve particle number. This is not the case for optical 4WM where, for example, in frequency tripling three photons are annihilated and one is created. Particle, energy and momentum conservation limit all matter 4WM processes to configurations that can be viewed as degenerate 4WM in an appropriate moving frame.

The present experiment used relatively large momenta. If we were to use momenta small enough to couple to phonons or other collective excitations of the condensate, we would be able to study these excitations and their nonlinear interactions with each other and with large-momentum excitations We could also change the internal states by using Raman transitions[4] to scatter atoms in one internal state from the matter-wave grating formed by atoms in a different internal state. It should even be possible to study 4WM between different isotopes or elements. Furthermore, just as nonlinear optics can create quantum correlations between photon beams, nonlinear atom optics may lead to the study of non-classical matter-wave fields. □

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