CHAOS AND DECOHERENCE IN A QUANTUM SYSTEM WITH A REGULAR CLASSICAL COUNTERPART

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We show that chaotic like behavior in a quantum system facilitates decoherence. It appears that the time scale on which decoherence takes place depends on the degree of complexity of the underlying quantum system, i.e., more complex systems decohere relatively faster than less complex ones.

Key words: chaos, decoherence, classical limit.

Bohr's correspondence principle states that quantum mechanical results become 'classical' when Planck's constant becomes very small. Ehrenfest's theorem, that the mean position of a quantum state will follow a classical trajectory, is generally taken as an argumentation for the correspondence principle, at least as far as dynamics is concerned. However, Ehrenfest's theorem is neither sufficient nor necessary to characterize the classical regime [1]. Ehrenfest's theorem breaks down sooner for a chaotic system than for a regular one [1,2]. Zurek and Paz showed [3] that a significantly non linear potential causes a rapid spread of the wave function implying that classical and quantum dynamics begin to differ.

In order to find a quantum mechanical paraphrase to the celebrated definition of classical chaos as being the exponential divergence in time of neighboring points in phase space we recognize the fast spreading of the wave function as an indication for chaotic-like behaviour. This is not a rigorous definition but rests on the fact that the wave function may be thought of as representing an ensemble of points in phase space and that its spread with time is a measure for the divergence of those points.

We have recently utilized a simple one dimensional model of a square barrier embedded in an infinite potential well to demonstrate that tunneling

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leads to very complex behaviour of the wave function [2]. There are many parallels to the well-known characteristics of classical chaos, e.g., an exponential decreasing correlation function of the peak-to-peak time series [2], a phase space plot of the expectation values $< x > (t)$ and its time derivative $< p > (t)$, revealing a phenomenon similar to period doubling and attractor-like behaviour of a double well chaotic system such as a driven Duffing oscillator [2] and level statistics for levels slightly above the barrier showing signs of Wigner statistics [4].

But most striking is the behaviour of the spatial entropy function $S(t) = -\int |\psi(x,t)|^2 \ln|\psi(x,t)|^2 dx$. This entropy function shows a rapid rise at early times to a non periodic fluctuating function around a smooth almost constant asymptotic value illustrating the early burst of chaotic behaviour.

By displacing the barrier in the double well system to the right or to the left certain positions are passed where the system becomes almost degenerated [5]. It is exactly for those positions that one may find significant tunneling.

Fig. 1 shows $S(t)$ for different positions of the barrier in the well. In Fig. 1a the barrier is placed at the centre of the double well where the highest degree of the 'almost degeneracy' for the levels is found. Changing the site of the barrier and thus going through positions of less and less 'almost degeneracy' a slower and slower approach to equilibrium and a faster and faster approach to recurrence of $S(t)$ is noted (see Fig. 1b-1d and also [5]).

A time dependent adiabatic modulation of the barrier which is tantamount to coupling of the system to the environment enhances the complex, chaotic-like behaviour. A first indication of this is rendered by an increased correspondence to Wigner statistics when an ensemble of slightly different barrier heights is considered.

Considering that chaos facilitates tunneling and vice versa [2,5] and that tunneling is enhanced by a high degree of 'almost degeneracy,' we conclude (see Fig. 1a-1d) that systems with a higher degree of complex, chaotic like, behaviour have a sharper increase of the entropy (higher entropy production) and a faster approach to equilibrium than systems with lower complexity.

One can use the model to illustrate some differences between classical and quantum dynamics. We start with a wave packet initially located on the left hand side of the square barrier placed in the infinite potential well. The parameters are as follows: The barrier height $V = 5$, the half-width of the well $\ell = 55$, the width of the Gaussian wave packet of the form $|\psi|^2 \cong c\exp[-(x-x_0)^2/2\sigma^2], \sigma = 5$, and the barrier width $2a = 2$, and the mass $m = 1/2$. The Gaussian wave packet is constructed from the first 30 energy levels and the standard deviation from an exact (normed) Gaussian is $8.81 \cdot 10^{-5}$. The computation preserves the norm to high accuracy, 0.999997. The problem is solved analytically as a function of time. Note that there is

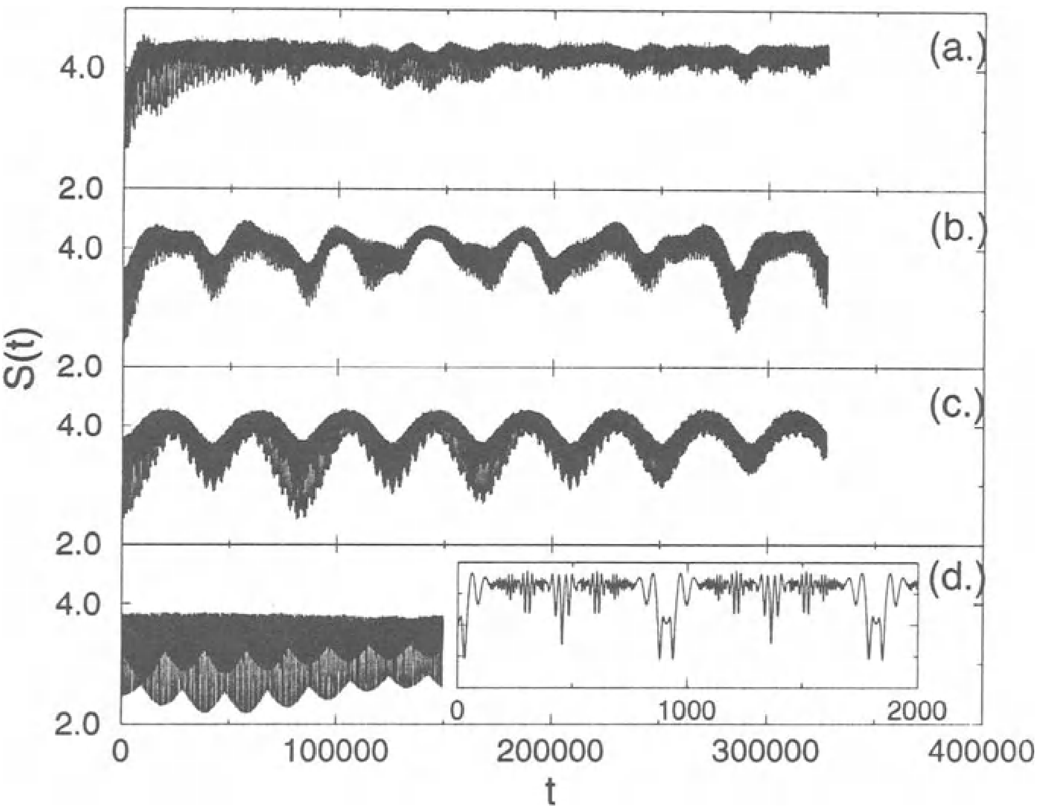

Fig. 1. The entropy function as a function of time for different locations in the well ($X \in [-55, 55]$). (a) the center of the barrier in the middle of the well X=0, (b) the center of the barrier in $X = -10,89$, (c) the center of the barrier in $X = -3,63$, (d) the center of the barrier in $X = -1$. (The inset shows an enlargement of typical periodic oscillations.)

no additional error for large t, since t enters in periodic exponential form.

The motion of this quantum state is compared to that of a classical ensemble whose initial position and momentum distributions are equal to those of the quantum state, the initial phase space distribution being the product of the position and the momentum distributions (this method is introduced and worked out in [1]).

In Figs. 2a-2d we have compared the time evolution of the spatial entropy function $S(t)$ for the quantum case and the classical case for decreasing values of the energy of the incoming particle high relative to the barrier height, $< E >= 5$. In Fig. 3a-3d the same comparison is performed for a low energy of the incoming particle (low relative to the barrier height), $< E >= 0.05$.

For both ratios $< E > /V$ it becomes obvious that decreasing $\hbar$ does not recover the classical behaviour of the entropy by merely letting $\hbar \to 0$, even though significant tunneling ($< E >\sim V$) leads to some similarity

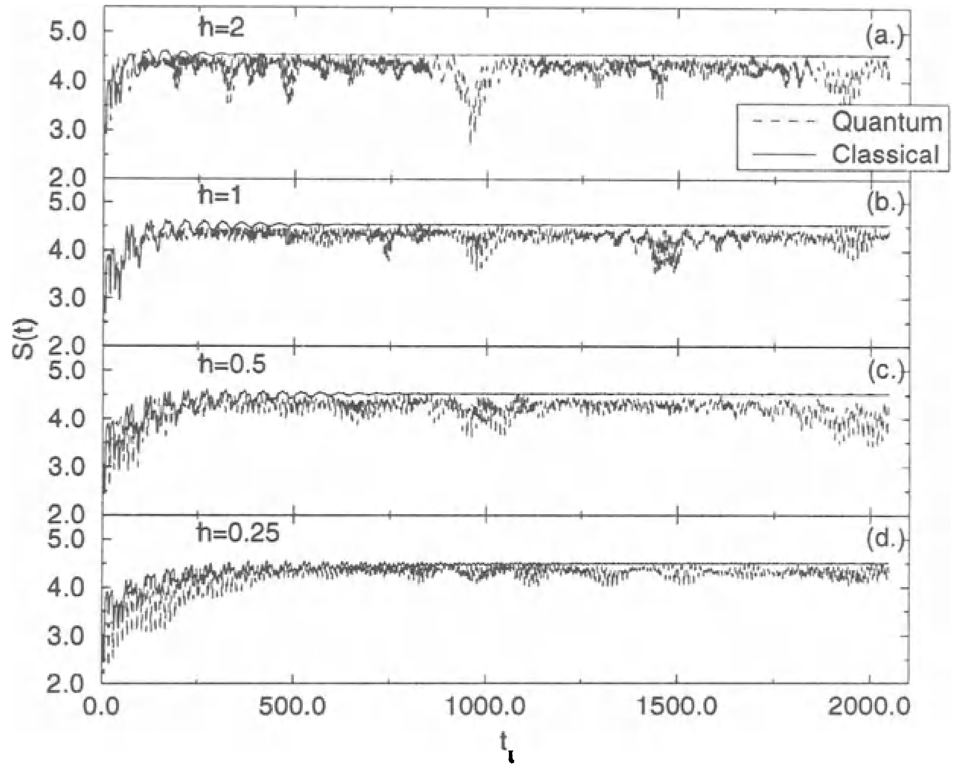

Fig. 2. The entropy function as a function of time for the classical case (solid line) and the quantum case (dotted), for different values of $\hbar$. The barrier is placed in the middle of the potential well and the mean energy of the incoming particle – $< E >$ – is equal to the barrier height $V, V = 5$. (a) $\hbar = 2$, (b) $\hbar = 1$, (c) $\hbar = 0.5$, (d) $\hbar = 0.25$.

between the classical and the quantum case.

Therefore, we stress that:

(1) $\hbar \to 0$ is a singular transition.
(2) Tunneling and thus chaotic-like behaviour weakens the differences between classical and quantum mechanics.

This second conclusion becomes even more apparent by comparing the classical case with the quantum case in connection with the position of the barrier inside the potential well. In [5] it is shown that placing the barrier at the center of the well results in a strong complex behaviour of the wave-function whereas placing the barrier off centre leads to a less complex, almost regular behaviour. Indeed, by comparing the two positions of the barrier, centered in Figs. 2a-2d and off center in Figs. 4a-4d, one observes that a similarity in the time development between the classical case and the quantum analogue is obtained much faster when the barrier is placed in the center, i.e., when there is a higher degree of 'almost degeneracy' and

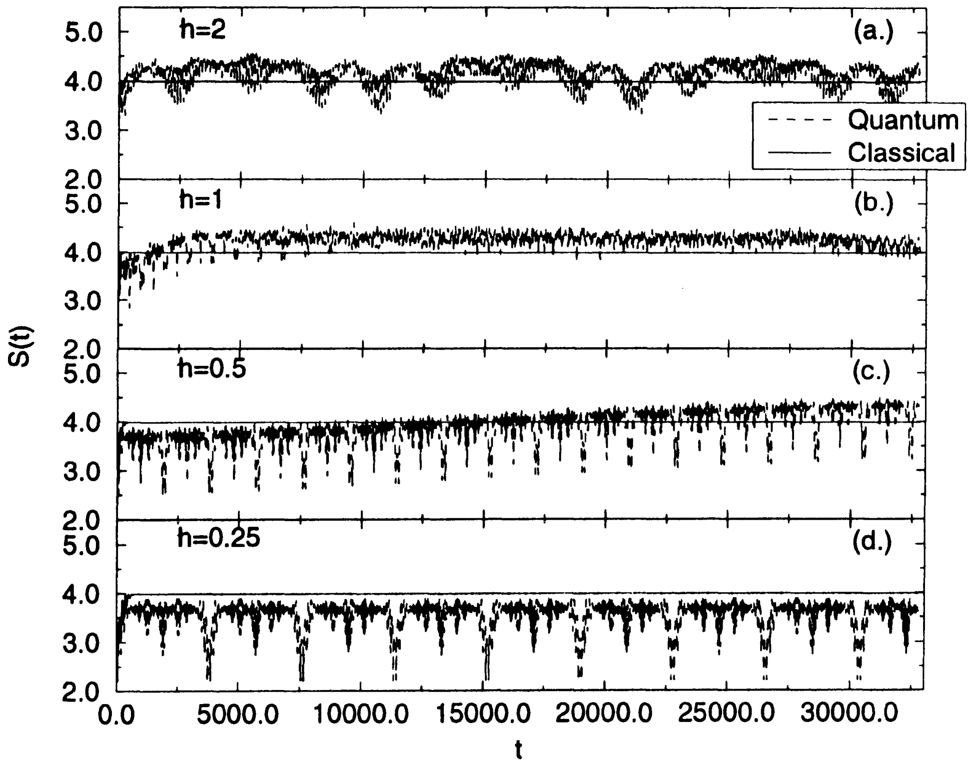

Fig. 3. The entropy function as a function of time for the classical case (solid line) and the quantum case (dotted), for different values of $\hbar$. The barrier is placed in the middle of the potential well and the mean energy of the incoming particle– $< E >= 0.05$– is low relative to the barrier height $V, V = 5$. (a) $\hbar = 2$, (b) $\hbar = 1$, (c) $\hbar = 0.5$, (d) $\hbar = 0.25$.

hence more chaotic like behaviour. We therefore conjecture that the time scale on which decoherence takes place depends on the degree of complexity of the underlying quantum mechanical system, i.e., more complex systems decohere relatively faster than less complex ones.

This is in accordance with the expectation that decoherence effects should suppress the possibility of interference and hence reduce the off diagonal elements of the reduced density matrix. The Schrödinger evolution cannot transform a pure state into a mixture and hence not make the off diagonal elements vanish completely (see also [6,7]). Quantum chaos on the other hand is expected to lead to irreversibility and hence also to manifest itself by implying that the non diagonal elements vanish (or become very small) [6-9], so it is understandable that quantum chaos will facilitate decoherence. This is illustrated in the present model by a fast increasing spatial entropy function (rapid entropy production) and is in accordance with previous results obtained in studies of decohering systems [6-8].

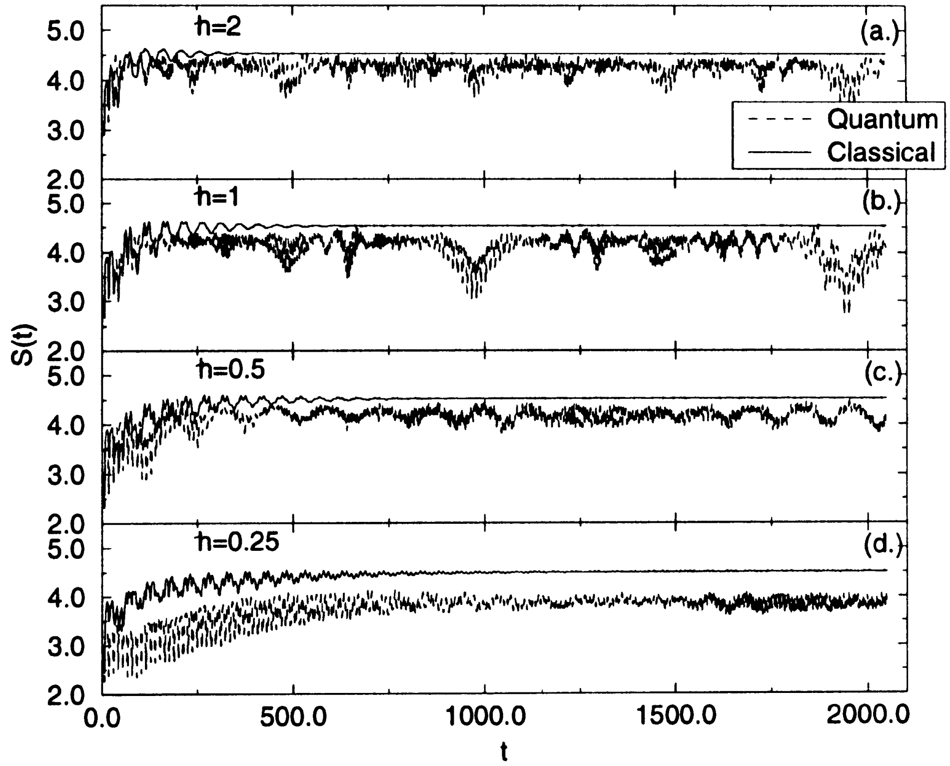

Fig. 4. The entropy function as a function of time for the classical case (solid line) and the quantum case (dotted) for different values of $\hbar$. The barrier is placed off center, $X = -1$, and the mean energy of the incoming particle – $< E >$ – is equal to the barrier height $V, V = 5$. (a) $\hbar = 2$, (b) $\hbar = 1$, (c) $\hbar = 0.5$, (d) $\hbar = 0.25$.

REFERENCES

1. L.E. Ballentine, Yumin Yang and J.P. Zibin, "Inadequacy of Ehrenfest's theorem to characterize the classical regime," *Phys. Rev. A* **50**, 2854-2859 (1994).
2. Y. Ashkenazy, L.P. Horwitz, J. Levitan, M. Lewkowicz, and Y. Rothschild, "Chaoticlike behavior in a quantum system without classical counterpart," *Phys. Rev. Lett.* **75**, 1070-1073 (1995).
3. W.H. Zurek and J.P. Paz, "Decoherence, chaos and the second law," *Phys. Rev. Lett.* **72**, 2508-2511 (1994).
4. R. Berkovits, Y. Ashkenazy, L.P. Horwitz, and J. Levitan, "Chaotic signatures in the spectrum of a quantum double well," *Physica A*, in press.
5. L.P. Horwitz, J. Levitan, and Y. Ashkenazy, "Complexity, tunneling and geometrical symmetry," *Phys. Rev. E*, in press.
6. R. Omnès, *The Interpretation of Quantum Mechanics* (Princeton University Press, Princeton, 1994), pp. 268-273.
7. W.H. Zurek and J.P. Paz, "Quantum chaos: a decoherent definition,"

Physica D **83**, 300-308 (1995).
8. J. Levitan, M. Lewkowicz, and Y. Ashkenazy, "Quantum chaos as a decoherence enhancing agent," *Found Phys.*, in press.
9. L. Bonci, R. Roncaglia, B.J. West, and P. Grigolini, "Quantum irreversibility and chaos," *Phys. Rev. Lett.* **67**, 2593-2596 (1991).