Nonlinearity and multifractality of climate change in the past 420,000 years

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[1] Evidence of past climate variations are stored in polar ice caps and indicate glacial-interglacial cycles of ∼100 kyr. Using advanced scaling techniques we study the long-range correlation properties of temperature proxy records of four ice cores from Antarctica and Greenland. These series are long-range correlated in the time scales of 1–100 kyr. We show that these time series are nonlinear for time scales of 1–100 kyr as expressed by temporal long-range correlations of magnitudes of temperature increments and by a broad multifractal spectrum. Our results suggest that temperature increments appear in clusters of big and small increments—a big (positive or negative) climate change is most likely followed by a big (positive or negative) climate change and a small climate change is most likely followed by a small climate change. *INDEX TERMS:* 3344 Meteorology and Atmospheric Dynamics: Paleoclimatology; 3220 Mathematical Geophysics: Nonlinear dynamics; 3250 Mathematical Geophysics: Fractals and multifractals; 3210 Mathematical Geophysics: Modeling. **Citation:** Ashkenazy, Y., D. R. Baker, H. Gildor, and S. Havlin, Nonlinearity and multifractality of climate change in the past 420,000 years, *Geophys. Res. Lett.*, *30*(22), 2146, doi:10.1029/2003GL018099, 2003.

1. Introduction

[2] Abundant geological evidence indicates that temperatures varied from the cold of ice ages to the warmth of interglacial periods through Earth's history. In the last 800,000 years (800 kyr) there is strong evidence for a dominant glacial-interglacial cycle of 100 kyr, with weaker secondary cycles of 40 kyr and 20 kyr [*Petit et al.*, 1999]. Each 100 kyr cycle consists of gradual cooling for ∼90 kyr followed by rapid warming during ∼10 kyr. "Milankovitch forcing", which refers to changes in insolation due to variations in the precession, obliquity, and eccentricity of Earth's orbit are thought to play an important role in glacial dynamics [*Imbrie et al.*, 1992]. These orbital variations are characterized by periods of 20 kyr, 40 kyr, and 100 kyr, respectively. The 20 kyr and 40 kyr periods in the climate records are generally believed to be a linear response of the climate system to insolation variations. In contrast, the weakness of the variations in solar radiation at the 100 kyr timescale has lead to the generally accepted conclusion that the glacial-interglacial oscillations at this timescale are most likely not a linear response of the climate system to external solar variations [*Imbrie et al.*, 1992].

[3] Many studies indicated that climate dynamics is *nonlinear*; [e.g., *Ghil and Tavantzis*, 1983; *Nicolis and Nicolis*, 1984; *Yiou et al.*, 1994; *King*, 1996]. The term nonlinearity is understood in different ways by different people. E.g., some would define nonlinearity according to the response of the system to external perturbation—if the response is linear/nonlinear then the system is linear/nonlinear. Others would define a system to be nonlinear according to its dynamical equations—if the system's dynamical equations contain nonlinear terms it is considered nonlinear. Others refer to nonlinear dynamics when they want to describe the complex behavior of systems such as chaotic systems.

[4] The understanding of climate dynamics is usually based upon reconstructed time series of proxies for climatic variables. These time series exhibit complex behavior while the underlying process is, in many cases, poorly known. Moreover, it is difficult to separate the system's dynamics and the perturbation added to it. In this case, the linearity or nonlinearity of the underlying process is subject to the model suggested to describe the process. Such an approach lacks the objectivity needed for classification of the underlying mechanics producing the time series.

[5] Generally, natural time series may be described by a set of dynamical equations that contains both deterministic and stochastic elements [*Hasselmann*, 1976]. The deterministic elements mimic a finite set of physical processes driving the system while the stochastic elements mimic processes that might perturb the system; these processes are approximated as a background noise. Auto regression moving average (ARMA) processes are processes that contain linear terms plus stochastic elements, i.e., $x_n = \sum_{i=1}^{M} a_i x_{n-i} + \sum_{i=0}^{L} b_i \eta_{n-i}$, where η is Gaussian white noise. The coefficients, a_i and b_i, fully specify the process, solely depend on the Fourier power spectrum, and are independent of the Fourier phases.

[6] *Schreiber and Schmitz* [2000] suggested defining the linearity/nonlinearity of a process with respect to the Fourier phases. If it is possible to reproduce the statistical properties of a process just from its power spectrum and its probability distribution, regardless of the Fourier phases, the process is defined as *linear*. This definition includes ARMA processes and fractional Brownian motion [*Ashkenazy et al.*, 2001]; the output, x_n, of these processes may undergo monotonic nonlinear transformations $s_n = s(x_n)$ and still be linear. Processes which do not fall under this category are defined as *nonlinear*. Following this definition, the Fourier phases of a series bear the nonlinearity of the series—randomizing the Fourier phases would destroy the nonlinearity of the

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series and "linearize" the series. Thus, the phase randomized series can be fully modeled by an ARMA linear model (plus monotonic linear/nonlinear transformation) since its coefficients are independent of the Fourier phases. On the other hand, if the original series is nonlinear it is impossible to find an ARMA model that has the same statistical characteristics as the original series.

[7] The nonlinearity of a time series can be assessed using the above definition. The NULL hypothesis is that the underlying process of a time series is linear. In order to reject the NULL hypothesis it is necessary to generate a surrogate time series with the same probability distribution and almost identical power spectrum, but with random Fourier phases. If a statistical measure of the original series is significantly different than that of the surrogate data, then the NULL hypothesis is rejected and the series is classified as nonlinear; the significantly different value for the statistical measure is due to special relations of the Fourier phases. Different versions of surrogate data tests are widely used in many branches of natural sciences including the climate system (see *Schreiber and Schmitz* [2000] and references therein). Measures for nonlinearity include, asymmetry of the time series [*King*, 1996; *Schreiber and Schmitz*, 2000], harmonics in the power spectrum [*King*, 1996], modulations of the series [*King*, 1996], phase relations between different frequencies [*King*, 1996], the presence of combination tones in the power spectrum [*Ghil and Treut*, 1981; *Yiou et al.*, 1994], and others [*Nicolis and Nicolis*, 1984; *Tsonis and Elsner*, 1992].

[8] Many deterministic theories have been developed to explain the glacial-interglacial 100 kyr variability; some suggested that the 100 kyr cycle is a result of nonlinear rectification of the very small eccentricity forcing while other studies suggested that the 100 kyr period is a result of self-sustained nonlinear mechanisms [e.g., *Saltzman*, 1990; *Imbrie et al.*, 1992; *Gildor and Tziperman*, 2000; *Tziperman and Gildor*, 2003]. Other studies proposed that climate variations are stochastic [*Hasselmann*, 1976; *Benzi et al.*, 1982] and follow scaling laws [e.g., *Kominz and Pisias*, 1979]. Importantly, the majority of the deterministic and stochastic mechanisms still assume that the variations on time scales below 100 kyr down to 10 kyr are linear.

[9] The objectives of the present study are to quantify the degree of nonlinearity of climate dynamics within the time scales of 1–100 kyr and to provide statistical characteristics of the proxy records which can serve as a test for distinguishing between existing climate models [*Wunsch*, 2002]. We study the correlation (scaling) properties of climate records of the past 420 kyr. We show that temperature variations are long-range correlated suggesting that the Milankovitch periods are indeed secondary and (contrary to common belief [*Imbrie et al.*, 1992]) that climate dynamics of all time scales below 100 kyr down to 1 kyr are highly nonlinear. In addition, we quantify the degree of nonlinearity in the climate records.

2. Ice-Core Data Analysis

[10] Our analysis is based on high resolution isotope records obtained from four ice cores, Vostok and Taylor Dome from Antarctica, and GISP (Greenland-Ice-Sheet-Project) and GRIP (Greenland-Ice-Project) from Greenland

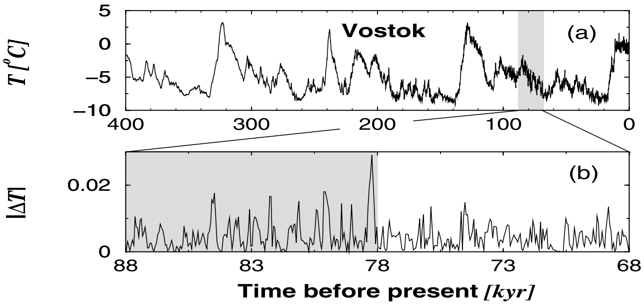

Figure 1. (a) Isotopic temperature record (calculated from the hydrogen isotope ratio) from the Vostok ice core [*Petit et al.*, 1999]; (b) A typical example for the clustering of the magnitude of temperature changes $|\Delta T_i|$.

(downloaded from www.ngdc.noaa.gov/paleo/). Measurements of oxygen and hydrogen isotope ratios ($\delta^{18}O$ and δD) of the ice at different depths provide a proxy record of temperature [*Petit et al.*, 1999] when the ice was formed (Figure 1a). These records extend back to 100–420 kyr.

[11] Fourier analysis is the standard method for studying temporal long-range correlations in time series. When the power spectrum follows scaling laws, $S(f) \sim 1/f^{\beta}$ ($\beta > 0$), the series is long-range correlated. However, the power spectrum might yield an inaccurate estimation of the scaling exponent due to constant or polynomial trends that are not necessarily related to the intrinsic dynamics. We therefore use detrended fluctuation analysis (DFA) [*Peng et al.*, 1994; *Bunde et al.*, 2000]; the mth order DFA eliminates polynomial trends of order $m - 1$ from the data. If the root mean square fluctuation function, $F(n)$, is proportional to n^{α}, where n is the window scale, the series is long range correlated ($\beta = 2\alpha - 1$). For a random series $\alpha = 0.5$ while for correlated (or anticorrelated) series $\alpha > 0.5$ (or $\alpha < 0.5$). Correlated series are dominated by low frequencies and thus the values of the series tends to persist. Anticorrelated series are dominated by high frequencies and thus values of the series tend to alternate.

[12] We begin our analysis with the Vostok ice core; we evenly sampled the data (0.1 kyr) before applying the scaling techniques. We find that temperature changes are highly correlated in the time range 1–100 kyr with a scaling exponent $\alpha \approx 1.5$ (Figure 2a), consistent with the previously reported power spectrum exponent $\beta = 2$ [*Kominz and Pisias*, 1979; *Pelletier*, 1997; *Wunsch*, 2002]. Random walk, i.e., the sum of white noise, also has exponent $\alpha = 1.5$. Consequently, one might conclude that the temperature increment time series is simple, linear, white noise [*Pelletier*, 1997]; we will show below that temperature increment series is a complex nonlinear series.

3. Nonlinearity of Ice-Core Data

[13] Next, we analyze the nonlinear properties of the ice core record. Long-range correlations in the temperature time series, T_i, reflect linear aspects of T_i. Long-range correlations in the magnitudes of temperature increments, $|\Delta T_i| \equiv |T_{i+1} - T_i|$ (Figure 1b) indicate nonlinearity of the underlying process [*Ashkenazy et al.*, 2001, 2003]. Linear series have uncorrelated $|\Delta T_i|$ series while nonlinear time series that follow a scaling law exhibit long-range correlations in

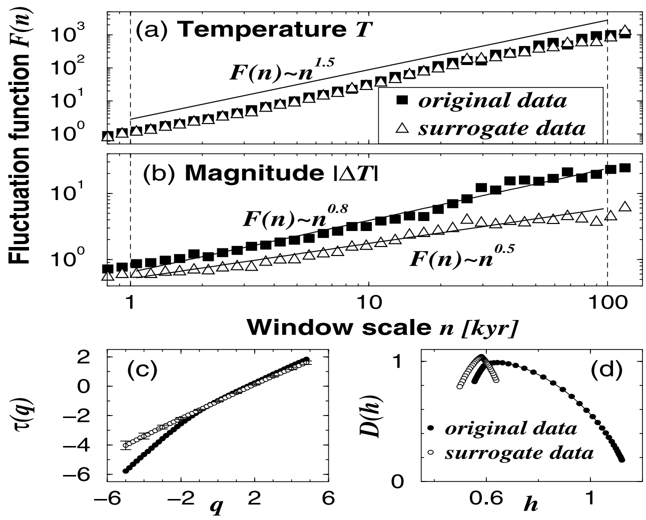

Figure 2. (a) The root mean square fluctuation $F(n)$ (2^{ND} order DFA) as a function of window scale n in kyr for the Vostok temperature proxy data indicates strong correlations (■). The surrogate series (△) exhibits almost identical scaling confirming that correlations in the T series are a linear measure. (b) $F(n)$ for the magnitude series, $|\Delta T_i|$, indicates strong correlations (■). The magnitude series of the surrogate data (triangles) is uncorrelated (with exponent 0.5) demonstrating the nonlinearity of the data. (c) The MF analysis uses the wavelet transform modulus maxima method [*Muzy et al.*, 1994], with the 8-tap Daubechies discrete wavelet transform [*Daubechies*, 1992]. The exponents $\tau(q)$ are calculated for window scales between 0.8 kyr and 25.6 kyr. The curvature in $\tau(q)$ reflects the multifractality of the temperature series (●). The $\tau(q)$ of the surrogate series (○) is linear (10 realizations; the average ±1 std is shown). (d) The MF spectrum, $D(h)$, is much broader for the original data (●) compared to the average $D(h)$ of the surrogate data (○) confirming that the underlying dynamics is nonlinear.

the magnitude series $|\Delta T_i|$. We find that $|\Delta T_i|$ is long-range correlated within the time range 1–100 kyr (Figure 2b) with exponent $\alpha \approx 0.8$. If the temperature series T_i was a simple random walk the corresponding $|\Delta T_i|$ series would be uncorrelated with exponent $\alpha \approx 0.5$. Thus, the underlying process is nonlinear. The value of the correlation exponent of $|\Delta T_i|$ quantifies the degree of nonlinearity in the ice core record. Correlations in $|\Delta T_i|$ indicate that the magnitude series is "clustered", i.e., a large magnitude is more likely to be followed by a large magnitude and a small magnitude is likely to be followed by a small magnitude, as can be seen in Figure 1b. As the correlations in $|\Delta T_i|$ increase, the clustering in $|\Delta T_i|$ are more pronounced; linear series have uncorrelated, homogeneous, magnitude series $|\Delta T_i|$ [*Ashkenazy et al.*, 2003].

[14] To demonstrate that the correlations in $|\Delta T_i|$ are related to the nonlinearity of the underlying process we apply a surrogate data test for nonlinearity that preserves both the power spectrum and the histogram of the temperature increment time series ΔT_i [*Schreiber and Schmitz*, 2000]. The surrogate series has random Fourier phases; the nonlinearities that are stored in the phases are destroyed. We find that the magnitude series obtained from the surrogate series is indeed uncorrelated (Figure 2b). We generate 10 surrogate series and measure the correlation exponent of $|\Delta T_i|$; we find that the mean exponent ±1 std is $\alpha = 0.5 \pm 0.05$, a value which is significantly different than the value of the original $|\Delta T_i|$ series exponent, $\alpha \approx 0.8$. This surrogate data test confirms that the original series is nonlinear within 1–100 kyr. We note that in some cases the asymmetry of a time series is an indication for nonlinearity and in other cases it is a result of a simple monotonic and static transformation of a symmetric distribution. Since we apply a surrogate data test that preserves the probability distribution of the series under consideration on the increment temperature series, the asymmetry in the surrogate series is the same as the asymmetry in the original increment temperature series. Thus the asymmetry of the ice-core data is not related to the nonlinearity we find in the ice-core data.

[15] An additional measure for nonlinearity is the multifractal (MF) spectrum [*Parisi and Frisch*, 1985]. A series x_i obeys scaling laws if $\langle |x_{i+n} - x_i|^q \rangle \sim n^{\tau(q)+1}$ ($\langle \cdot \rangle$ stands for average). When the exponents $\tau(q)$ are nonlinearly (or linearly) dependent on q the series x_n is *multifractal* (or *monofractal*). In many cases a MF (or monofractal) series has a nonlinear (or linear) underlying process. We use an advanced method of multifractality that accurately estimates the exponents of negative moments and is capable of removing polynomial trends from the data [*Muzy et al.*, 1994]. We calculate the exponents $\tau(q)$ of different moments q for the ice core data and find that $\tau(q)$ is a nonlinear function of q (Figure 2c), indicating that the temperature series is MF. Most of the multifractality observed in the ice-core data is due to the negative moments; i.e., the small fluctuations are more inhomogeneous than the big fluctuations. We also perform MF analysis on the surrogate data and find that its $\tau(q)$ is almost linear. The MF spectrum, $D(h) \equiv hq - \tau(q)$ ($h \equiv d\tau/dq$) (Figure 2d), is broad for the original data and narrower for the surrogate data. The broadness of the MF spectrum may also be used to quantify the degree of multifractality, and thus the degree of nonlinearity, in the data.

[16] We note that *Schmitt et al.* [1995] analyzed the GRIP ice-core data and found a broad MF spectrum; their study differs from ours because: (i) We exclude polynomial trends that exist in climate records and might cause inaccuracies in measuring the MF spectrum. (ii) We compute both positive and negative moments; for the Vostok ice-core, most of the broadness of the MF spectrum is due to the negative moments. (iii) We use a surrogate test for nonlinearity to verify whether the MF spectrum is indeed a measure of nonlinearity; linear series with broad (power-law) tails for the probability distribution may have a broad MF spectrum inspite of their linearity [*Kantelhardt et al.*, 2002]. (iv) We use longer ice-core time series (which became available recently) that leads to a more accurate MF spectrum. (v) We apply MF analysis to cores from both polar regions.

[17] We repeat the above analysis for the other three ice cores (Table 1). The DFA exponents of the original series are smaller for the Greenland cores ($\alpha \sim 1.2$) compared to the Antarctica cores ($\alpha \sim 1.5$). The smaller correlation exponents for Greenland cores may be related to the more variable climate of the Northern hemisphere

Table 1. Scaling Results of the Cores Under Consideration (0.1 kyr Sampling)

measure	GISP	GRIP	Taylor	Vostok
age	110 kyr	225 kyr	103 kyr	422 kyr
α_T	1.14	1.18	1.4	1.54
$\alpha_{\lvert\Delta T\rvert}$	0.77	0.82	0.8	0.78

The scaling regime starts at 1 kyr and is larger than the maximal spacing between consecutive values. The DFA exponents for the original (α_T) and magnitude ($\alpha_{\lvert\Delta T\rvert}$) series are obtained for window scales between 1 kyr and ~1/4 of the series length.

(expressed by Heinrich and Dansgaard-Oeschger events). The corresponding power spectrum exponent $\beta = 1.4$ for the Greenland cores is consistent with the previous studies [*Lovejoy and Schertzer*, 1986; *Schmitt et al.*, 1995]. Although the DFA exponents of the original series are smaller for the Greenland cores, the nonlinear measures of magnitude series exponents are almost the same for all cores. We also performed the MF analysis on the other three cores (GRIP, GISP, and Taylor Dome) and find a larger contribution of the positive moments than seen for the Vostok data. We note however that since these 3 additional records are shorter by more than a factor of two than the Vostok record their corresponding MF spectrum is less reliable.

[18] Based upon our DFA and MF spectra for all of the ice cores studied we conclude that climate dynamics is nonlinear for time scales of few thousands of years up to 100 kyr.

4. Summary

[19] We conclude that climate changes in the time range of 1–100 kyr are long-range correlated confirming the major role of stochasticity in climate. Moreover, our results suggest that the underlying dynamics in the time scales of 1–100 kyr is nonlinear. This nonlinearity is specified and quantified by strong long-range correlations in the magnitudes of temperature changes and in a broad MF spectrum. This nonlinearity is, most probably, a result of the tendency of temperature increments to appear in clusters of big or small increments rather than being randomly distributed.

[20] Climate models may be generally categorized into two main alternatives: (i) linear or nonlinear mechanisms that are driven by linear stochastic forcing [e.g., *Benzi et al.*, 1982; *Pelletier*, 1997; *Wunsch*, 2002], and (ii) nonlinear mechanisms without stochastic forcing [e.g., *Saltzman*, 1990; *Gildor and Tziperman*, 2000]. Our results suggest a third alternative—a mechanism that inherently involves nonlinear stochastic forcing. This nonlinear stochastic forcing may represent (i) interaction of a rapidly varying forcing (like the atmosphere) with a slowly varying forcing (like the ocean), (ii) interaction of noise with an intrinsic component of the glaciation process (like ice-volume), or (iii) interaction of noise with an external deterministic forcing (like insolation). Our results raise a new challenge for the many climate models, which ideally should reproduce the properties we have found in the ice core data, and may help guide development of better climate models, which include both periodic and stochastic elements of climate change.

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