Statistical Properties of Commodity Price Fluctuations

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Summary. Though the statistical properties of price fluctuations for stocks have been studied extensively since the last decade, not many studies have been done on commodity price fluctuations. Here, we perform a comparative study to test whether commodities are statistically similar to stocks with respect to (a) probability distribution and (b) correlation of price fluctuations. We analyze daily returns of spot prices for 29 commodities and daily returns of future prices for 13 commodities over a period exceeding 10 years and compare the results with a database of 2449 stocks over the same period.

1 Introduction

Until now much interest in the study of economic markets has been concentrated on stocks, where a number of empirical findings have been established such as [1, 2] (i) the distribution of price changes $P(x)$ being approximately symmetric and decaying with power law tails $P(x) \sim 1/x^{\alpha+1}$, with $\alpha + 1 \approx 4$; (ii) the price changes being exponentially (short-range) correlated, while the absolute values of price changes ("volatility") are power-law (long-range) correlated [3, 4].

Unlike stock and foreign exchange markets, commodity markets have received little attention. Recently it was found [5] that commodity markets have qualitative features similar to those of the stock market. This similarity is intriguing because of the special features unique to the commodity market such as: (i) most commodities require storage; (ii) most commodities require transportation to bring them to the market from where they are produced; and (iii) it is plausible that commodities may exhibit a slower response to change in demand because their price depends on the supply of the actual object.

Because of the stronger constraints, affecting commodity markets, mentioned above, one might surmise that commodity prices show larger fluctuations than stock prices. In fact, exponents of power law tails of probability distributions of the returns of spot prices [6] of commodities such as cotton and wheat have been reported [7] to be Lévy-stable i.e., $0 < \alpha < 2$, whereas the returns of future prices of commodities such as potatoes have been reported [8] to be outside the Lévy-stable domain i.e., $\alpha > 2$.

The multifractal (MF) spectrum reflects the n-point correlations providing more information about the temporal organization of price fluctuations than two-point correlations. Previous work reports a broad MF spectrum of stock indices and foreign exchange markets [9]. Two recent models [10] explain the observed MF properties by assuming that price changes are the product of two stochastic variables, one being uncorrelated and normally distributed and the other being correlated and log-normally distributed. The price changes predicted by these models do not have the power law probability distribution [2, 5] observed empirically, and thus destroying the temporal organization by shuffling the price changes significantly narrows the MF spectrum.

2 Probability distribution of price fluctuation

First we test whether the probability distribution function (PDF) of commodity price fluctuations is statistically distinguishable from that of stocks. To this end, we study the fluctuations in the spot price for 29 commodities and in the future price for 13 commodities and compare our results with the statistical properties of daily returns in stock markets. We define the normalized price fluctuation ("return") as $g(t) \equiv (\ln S(t+\Delta t) - \ln S(t))/\sigma$, where $\Delta t = 1$ day, $S(t)$ is the price, and σ is the standard deviation of $\ln S(t+\Delta t) - \ln S(t)$ over the duration of the time series (typically 15 years).

The probability distributions $P(g_i > x)$ of the returns follow power law forms $P(g_i > x) \sim \frac{1}{x^{\alpha_i}}$, where α_i is outside the Lévy-stable domain $0 < \alpha_i < 2$.

Figures 1(a), (b), (c), and (d) display Hill estimates [11] of α_i for the spot price of 29 commodities and future price of 13 commodities (see [5] for more detail). For the spot prices the average exponents are

$$\overline{\alpha}_{\text{spot}} \equiv \frac{1}{29}\sum_{i=1}^{29}\alpha_i = \begin{cases} 2.3 \pm 0.2 \text{ positive tail} \\ 2.2 \pm 0.1 \text{ negative tail.} \end{cases} \tag{1}$$

For the future prices the average exponents are

$$\overline{\alpha}_{\text{future}} \equiv \frac{1}{13}\sum_{i=1}^{13}\alpha_i = \begin{cases} 3.1 \pm 0.2 \text{ positive tail} \\ 3.3 \pm 0.2 \text{ negative tail.} \end{cases} \tag{2}$$

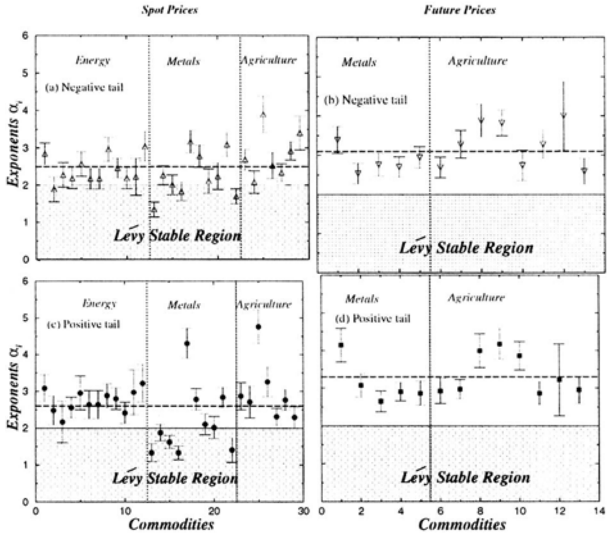

Fig. 1. Exponents α_i of the negative tail of commodity (a) spot prices and (b) future prices, where i indexes the 29 commodity spots and 13 commodity future prices analyzed. Exponents α_i of the positive tail for commodity (c) spots prices and (d) future prices. We employ Hill's method [11] to estimate the exponent α_i of each probability distribution in the range $x \geq x_{\text{cutoff}}$, with $x_{\text{cutoff}} = 2$. The dashed lines show the average values defined in Eq.(1), (2). Shaded regions indicate the range of Lévy-stable exponents, $0 < \alpha < 2$. Note that the mean exponent of spot prices is smaller than the mean exponent of the future prices, and that both are outside the Lévy-stable domain.

3 Correlations of price fluctuation

3.1 Two-point correlations

We next study the temporal correlations of the returns. The average autocorrelation function of commodities $\overline{C}(\tau) \equiv \frac{1}{N}\Sigma_i^N < g_i(t)g_i(t+\tau) >$, where N=29 for spot prices and N=13 for future prices decays exponentially as $e^{-\tau/\tau_c}$. We find that $\tau_c^{\text{spot}} = 2.3$ days and $\tau_c^{\text{future}} < 1$ day. To further quantify time correlations, we use the detrended fluctuation analysis (DFA) method [12]. The DFA exponent $\hat{\alpha}_{\text{DFA}}$ gives information about the correlations present. If $C(\tau) \sim \tau^{-\gamma}$, then $\hat{\alpha}_{\text{DFA}} = (2-\gamma)/2$, while if $C(\tau) \sim e^{-\tau/\tau_c}$, then $\hat{\alpha}_{\text{DFA}} = 1/2$ [12]. We find that $\hat{\alpha}_{\text{DFA}} = 0.51 \pm 0.05$ and $\hat{\alpha}_{\text{DFA}} = 0.50 \pm 0.05$ for spot and future prices respectively, consistent with the exponential decay of $\overline{C}(\tau)$. We also observe that $|g_i|$, the absolute value of returns (one measure of volatility), are power law correlated with

$$\hat{\alpha}_{\text{DFA}} = \begin{cases} 0.63 \pm 0.05 \text{ spot prices} \\ 0.60 \pm 0.05 \text{ future prices}, \end{cases} \tag{3}$$

which implies a power law decay of the autocorrelation of the absolute value of returns with

$$\gamma = \begin{cases} 0.74 \pm 0.1 \text{ spot prices} \\ 0.80 \pm 0.1 \text{ future prices.} \end{cases} \tag{4}$$

Note that γ for commodities is larger than γ for stocks [3].

3.2 Higher order correlations

We use the multifractal detrended fluctuation analysis (MF-DFA) technique [13] to study the MF properties and thus the different orders of temporal correlations, of the returns for stocks and commodities. The scaling function of moment q, $F_q(s)$ [13] follows the scaling law $F_q(s) \sim s^{\tau(q)}$.

First we perform a shuffling procedure on the time series of price fluctuations for stocks and commodities by randomly exchanging pairs. This shuffling procedure preserves the distribution of the returns but destroys any temporal correlations (see [14] for more details).

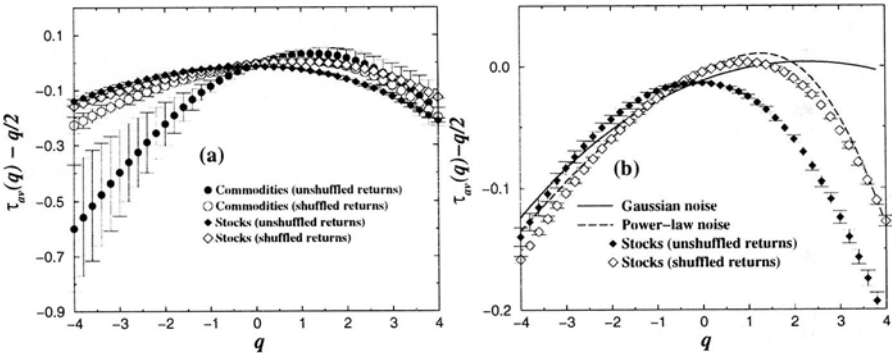

Fig. 2. (a) $\tau_{\text{av}}(q)$ for returns and shuffled returns for 29 commodities and 2449 stocks. To better visualize the results we plot $\tau_{\text{av}}(q) - q/2$ instead of $\tau_{\text{av}}(q)$. The exponents $\tau_{\text{av}}(q)$ are calculated for window scales of $10 - 100$ days. After shuffling, $\tau_{\text{av}}(q)$ are comparable for both stocks and commodities. (b) $\tau_{\text{av}}(q)$ spectrum of the returns and shuffled returns for stocks, compared with uncorrelated surrogate data with Gaussian probability distributions and power law probability distributions (with power law exponent $\alpha \approx 3$). After shuffling, $\tau_{\text{av}}(q)$ for stocks becomes comparable with $\tau_{\text{av}}(q)$ of the surrogate data obtained for the power law probability distribution.

Next, we analyze the MF properties of the returns of stocks and commodities before and after shuffling of the returns. Figure 2(a) displays the averages separately $\tau_{\text{av}}(q) \equiv \frac{1}{N}\sum_{i=1}^{N} \tau_i(q)$, for $N = 29$ commodities and $N = 2449$ stocks. Note that (i) the scaling exponents, $\tau_{\text{av}}(q)|_{q<0}$ significantly differ for commodities and stocks, whereas $\tau_{\text{av}}(q)|_{q>0}$ are similar, suggesting that commodities are similar to stocks for the large fluctuations and differ for the small fluctuations; (ii) we observe that after shuffling the returns, $\tau_{\rm av}(q)$ for stocks hardly changes for $q < 0$, but $\tau_{\rm av}(q)$ for commodities changes and becomes comparable to stocks for the entire range of q.

In order to study the contribution of the power law tails of the returns on the MF spectrum, we generate surrogate data sets (i) with a normal distribution and (ii) with power law tails with $\alpha \approx 3$ (as observed empirically [2, 5]). Figure 2(b) displays $\tau_{\rm av}(q)$ averaged over 2449 realizations of surrogate data, each with 3000 data points. The $\tau_{\rm av}(q)$ of the surrogate power law distributed data is very close to the $\tau_{\rm av}(q)$ of stocks after shuffling. This indicates that a significant part of the $\tau_{\rm av}(q)$ spectrum of stocks and commodities comes from the power law distribution of the returns. Note that there is a small difference in $\tau_{\rm av}(q)$ of stocks before and after shuffling, indicating that the power-law distribution of the returns is not the only source of multifractality, but that there is also a relatively small contribution due to the temporal organization of returns. For commodities, this temporal organization is more dominant.

4 Summary

In summary, we analyze spot prices for 29 commodities and future prices for 13 commodities. We find quantitative similarity between stock and future commodity markets, which strengthens the likelihood of a universal mechanism underlying both markets. We hypothesize that because nowadays a large fraction of the trading taking place at commodity markets, especially for futures, is for speculative purposes (i.e., with the intent of making a profit by buying low and selling high) we find similar values of α for commodities and stocks. Demand fluctuations drive price fluctuations and it is plausible that stocks respond more quickly than commodities to demand changes. Stochastic perturbations, together with the immediate price response to demand changes, may weaken the existing higher order temporal organization. Commodities, on the other hand, have a slower response. Thus, small or short-time perturbations are felt less by commodities than by stocks. We conjecture that the more homogeneous returns of stocks explain the difference between the MF properties of stocks and commodities.

We also find that commodities have a broader MF spectrum than stocks. A major contribution to multifractality is the power law tail of the probability distribution of the returns. Moreover, the MF spectra of stocks and commodities are partly related to the power law probability distribution of returns and partly to the different orders of temporal correlations present.

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