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A continuous time model of the bandwagon effect in collective action

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Abstract. The paper offers a complex and systematic model of the bandwagon effect in collective action using continuous time equations. The model treats the bandwagon effect as a process influenced by ratio between the mobilization efforts of social activists and the resources invested by the government to counteract this activity. The complex modeling approach makes it possible to identify the conditions for specific types of the bandwagon effect, and determines the scope of that effect. Relying on certain behavioral assumptions, these conditions are only indirectly connected to individual beliefs, so that a given bandwagon effect can be explained and planned without knowing the exact preferences of the players' preferences.

1 Introduction

Analyses of collective action dynamics often distinguish between spontaneous and non-spontaneous evolution of collective action (Taylor 1987; Sened 1991; Knight 1992). The second type is characterized by involvement of social activists who mobilize large numbers of people. While purely spontaneous collective action is rare, there are mechanisms that closely fit this characterization. One such mechanism is the "bandwagon" effect, also termed the "domino" or "snowball" effect, where individuals' decisions about whether or not to participate in collective action are influenced by the number of people currently involved in those efforts. Assuming that individuals recognize the need to protest, it follows that larger number of participants reduce the costs of action and increase the chances of success (Schelling 1978; Granovetter 1978; Karklins and Petersen 1993). When the government observes a large-scale movement it is less likely to take violent measures to counteract the movement but, rather, will tend to be sensitive to public demands. Thus, players join protest efforts when the number of participants reaches a certain subjective threshold meaning that the probability of a non-protester joining protest is proportional to the number of protesters. This creates an accumulation effect that may lead to mass mobilization.

Individuals have different subjective thresholds for participation depending on their beliefs and structural conditions. Given a certain distribution of thresholds, the accumulation effect may drag an entire society into mass protest or cease at a certain point. Granovetter (1978) and Kuran (1995) argue, for example, that since minor changes in individual thresholds may significantly affect the behavior of an entire society, models based solely on a simple aggregation of "beliefs" or "thresholds" have limited explanatory or predictive power. Indeed, the literature on the bandwagon effect focuses on explaining its basic rationale (Schelling 1960, 1978; Taylor 1987; Karklins and Petersen 1993; Colomer 1995) or its disadvantages and limitations (Granovetter 1978; Kuran 1995). Other studies explain the ways in which social or institutional changes evolve through informational cascades (Bikhchandani et al. 1992; Lohmann 1994) or belief cascades (Zeeman 1974; Denzau and North 1994). Nevertheless, these explanations refer to beliefs or thresholds that generate a kind of spontaneous bandwagon effect, but they do not analyze the bandwagon effect on the basis of specific structural parameters that influence these beliefs.

The paper offers a more complex and systematic model of the bandwagon effect, using continuous time equations as well as rationales derived from epidemic theory. Unlike most approaches, this model does not treat the bandwagon effect as a purely spontaneous process. Rather, it views the bandwagon effect as a process influenced by ratio between mobilization efforts of social activists and the resources invested by the government to counteract this activity. Furthermore, it is taken into account that while some people withdraw from collective action, others may join in. This complex modeling approach makes it possible to identify the conditions for specific types of the bandwagon effect. In keeping with the approach, a mathematical model for calculating the time that elapses until the dynamic reaches equilibrium is proposed. Since these conditions are only implicitly linked with individual preferences and beliefs, it is suggested that a given bandwagon effect can be explained and planned without knowing the exact preferences of the players. In other words, by aggregating individual beliefs and actions we are able to analyze the outcome of a certain bandwagon effect depending on a few structural variables rather than measuring individual beliefs and preferences in a given society.

Section 2 presents a model in which the population is divided into two groups – one consisting of citizens who participate in protest activities, and the other consisting of passive citizens – where there is movement to and from each group. Section 3 adds a third group to the model, consisting of citizens who die or are jailed during the protest activity or, alternatively, of those who start supporting the government. Based on epidemic theory, the conditions for two possible types of bandwagon effect are explained. Section 4 concludes the analysis.

2 A basic continuous time model of the bandwagon effect

The basic continuous time model developed here assumes that at time $t = 0$ there are P_0 protesters who participate in a mass protest activity, and C_0 passive citizens who do not protest but constitute potential participants. Denote by N the total population in a given country, such that at $t = 0$, $P_0 + C_0 = N$. Let the functions $P(t)$ and $C(t)$ represent the number of protesters and passive citizens, respectively, at a certain time, t. It is assumed that the size of the population is constant, such that for any $t > 0$, $P(t) + C(t) = N$.

Let us assume that for every point in time "t", the number of passive citizens who join the collective action and become protesters during the next time interval, $[t, t + \Delta t]$, is proportional to both the interval length, Δt, the number of protesters $P(t)$ and the number of citizens $C(t)$. That is, during the time interval $[t, t + \Delta t]$, the number of new protesters can be expressed as: $\beta P(t)C(t)\Delta t$, where the parameter β represents the rate of new protesters joining the collective activity. This rate is influenced by the preferences of passive citizens, and is also strongly affected by the mobilization efforts of social activists. Following Bikhchandani et. al. (1992), it is assumed that only a small number of individuals initiate an informational cascade, while the others in the society follow the choices of these decision leaders, free riding on their efforts. At the end of the time interval, the number of protesters is expressed by: $P(t + \Delta t) = P(t) + \beta P(t)C(t)\Delta t$; and the number of passive citizens is expressed by: $C(t + \Delta t) = C(t) - \beta P(t)C(t)\Delta t$. It should also be noted that the participation rate increases as $C(t)$ increases, since the participation rate is expected to rise as the number of potential protesters increases. Similarly, the participation rate is expected to increase as $P(t)$ increases, i.e., based on the bandwagon mechanism, it is assumed that people who do not protest will do so when the number of participants reaches a certain subjective threshold.

However, in collective action dynamics people may not only join protest activities but also withdraw from them. This parameter is added to the model as follows: For every time interval, the number of people who leave the protest activity and become passive citizens is represented by $rP(t)\Delta t$, where the parameter r represents the rate of withdrawal from the collective action. This rate is significantly influenced by the government resources invested to counteract social protest. At the end of this time interval, the number of protesters is expressed by: $P(t + \Delta t) = P(t) - rP(t)\Delta t$; and, the number of citizens who do not participate is expressed by $C(t + \Delta t) = C(t) + rP(t)\Delta t$.

Combining the movement from and to each group, the dynamic can be expressed as follows:

$$P(t + \Delta t) = P(t) + \beta P(t)C(t)\Delta t - rP(t)\Delta t \tag{1}$$

$$C(t + \Delta t) = C(t) - \beta P(t)C(t)\Delta t + rP(t)\Delta t \tag{2}$$

Note that the sum of Eqs. (1) and (2) yields $P(t + \Delta t) + C(t + \Delta t) = P(t) + C(t) = N$, which is consistent with the assumption that the size of the population remains constant over time. To find the instantaneous changing rate in $P(t)$ and $C(t)$, $P(t)$ is subtracted from (1) and $C(t)$ is subtracted from (2). To understand the trend of the system at a certain point in time, let $\Delta t \to 0$, which yields the following population dynamic equations:

$$\frac{dP}{dt} = \beta P(t)C(t) - rP(t) \tag{3}$$

$$\frac{dC}{dt} = -\beta P(t)C(t) + rP(t) \tag{4}$$

In order to determine whether this dynamic converges and what the limit is, we find the equilibrium in this system. In this type of dynamic equation, equilibrium is a point (P_e, C_e) in the P-C plane such that after the functions $P(t)$ and $C(t)$ have reached this point for some t_0, they stay there for $t \geq t_0$. It follows that if (P_e, C_e) is an equilibrium, then $\frac{dP}{dt} = \frac{dC}{dt} = 0$ at this point. Thus, to find the equilibrium, we should solve for P and C satisfying

$$\beta PC - rP = 0 \tag{5}$$

$$-\beta PC + rP = 0 \tag{6}$$

Equations (5) and (6) produce two equilibria. When $(P, C) = (0, N)$, i.e., when the number of protesters is zero, the system stays at this point forever. This conforms with the assumption underlying the bandwagon effect, according to which the rate of joining the protest depends on the number of people already protesting. This actually means that no player has a threshold of zero, such that no one initiates collective action. If $\frac{r}{\beta} < N$, then the second equilibrium is reached at $C = \frac{r}{\beta}$ and $P = N - \frac{r}{\beta}$. It follows that the number of passive citizens will stabilize at the ratio between the rate of leaving the protest and the rate of joining the protest. For $\frac{r}{\beta} \geq N$, the protest asymptotically approaches zero where $C = N$, resulting in a unique equilibrium. The dynamic leading to this equilibrium is demonstrated in Fig. 1, where it is assumed that $P_0 = 1$, $C_0 = 99$, $\beta = 0.02$, $r = 1.5$. As can be seen in Fig. 1, $C(t)$ starts from 99 and decreases to $\frac{r}{\beta} = 75$, while $P(t)$ starts from 1 and increases to 25.

The stability of equilibria are determined as follows: In a dynamic system, an equilibrium is asymptotically stable if there is a minor deviation from the equilibrium point and the functions $P(t)$ and $C(t)$ converge back to that point.

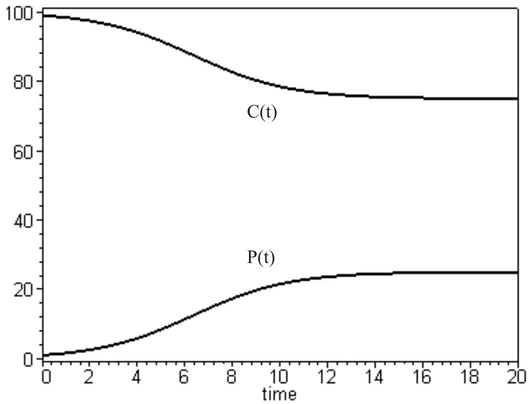

Fig. 1. The development of $P(t)$ and $C(t)$ over time

Proposition 1.

1. *If* $\frac{r}{\beta} < N$*, then the equilibrium* $[C = N, P = 0]$ *is not asymptotically stable, whereas the equilibrium* $\left[C = \frac{r}{\beta}, P = N - \frac{r}{\beta}\right]$ *is asymptotically stable.*
2. *If* $\frac{r}{\beta} \geq N$*, then the unique equilibrium* $[C = N, P = 0]$ *is asymptotically stable.*

Proof. see Appendix.

This equilibrium analysis identifies two parameters that can explain the bandwagon dynamic and the number of participants, i.e. the efforts of social activists to mobilize collective action, and the resources spent by the government to counteract this activity. Since these parameters strongly influence the participation and leaving rate respectively, they are also central parameters in determining the scope of the bandwagon effect. In other words, based on certain behavioral assumptions, we suggest a model that does not require specification of individual preferences and beliefs in order to explain the bandwagon effect. In this way, the methodological weakness of threshold models, as proposed by Granovetter (1978) and Kuran (1995), is partially solved.

More specifically, Proposition 1 shows that as the mobilization efforts of social activists increase, while the amount of resources invested by the government to counteract protest decreases and the target population increases, then there are greater chances that in equilibrium mass mobilization will succeed. However, as the amount of resources invested by the government increases, while the mobilization efforts of social activists and the size of the target population decrease, the chances are greater of reaching the second stable equilibrium in which there are no participants. Proposition 1 proves this intuitive rationale.

Furthermore, the continuous time model makes it possible to determine how long it takes the dynamic to converge to equilibrium. The system of equations (3) and (4) can be explicitly solved to obtain the solution presented in Proposition 2.

Proposition 2.

1. *If* $\frac{r}{\beta} \neq N$, *then*

$$C(t) = \frac{r}{\beta} + \left(N - \frac{r}{\beta}\right) \times \frac{1}{1 + \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}} \tag{7}$$

2. *If* $\frac{r}{\beta} = N$ *then*

$$C(t) = \frac{C_0 + \beta N P_0 t}{1 + \beta P_0 t}. \tag{8}$$

where $P(t) = N - C(t)$.

Proof. See Appendix.

The solution for $\frac{r}{\beta} \neq N$, includes two components. The first one, $\frac{r}{\beta}$, is the equilibrium at which the dynamic converges if $\frac{r}{\beta} < N$, as presented in Proposition 1. The second component, $\left(N - \frac{r}{\beta}\right) \times \frac{1}{1 + \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}}$, declines to zero if $\frac{r}{\beta} < N$ and converges to $N - \frac{r}{\beta}$ if $\frac{r}{\beta} > N$. This second component dictates the converging rate to the equilibrium. As we can see by developing the term $\frac{1}{1 + \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}}$ in Taylor's series, we have the first two leading terms $\frac{1}{1 + \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}} \sim 1 - \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}$ when t is large enough and $\frac{r}{\beta} > N$. From that time, the converging rate is exponential. When $\frac{r}{\beta} < N$ the first leading term is $\frac{1}{1 + \frac{P_0}{C_0 - r/\beta} e^{(\beta N - r)t}} \sim \frac{C_0 - r/\beta}{P_0} e^{-(\beta N - r)t}$, and again, for a large enough t, the converging rate is exponential. When $\frac{r}{\beta} = N$, we find that when t is large then $\frac{C_0 + \beta N P_0 t}{1 + \beta P_0 t} \sim N - \frac{1}{t\beta}$. Thus, the converging rate is $\frac{1}{t}$.

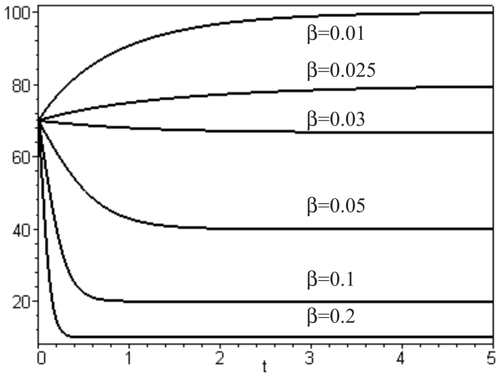

Fig. 2. The development of $C(t)$ over time for constant r and different β

Figure 2 shows the development of $C(t)$ over time for a constant r and different β. Starting from $N = 100$, $C_0 = 70$, $P_0 = 30$ and $r = 2$, the upper curve represents the dynamic when $\beta = 0.01$, meaning that $\frac{r}{\beta} > N$. Then $C(t)$ converges to N, which means that after five time units collective action stops. When $\frac{r}{\beta} = N$, $C(t)$ converges to N as well. The other five curves represent the dynamics for $\beta = 0.025, 0.03, 0.05, 0.1, 0.2$ (from top to bottom) where $\frac{r}{\beta} < N$. As the participation rate increases, the ratio between the leaving rate and the participation rate decreases. This causes the group of potential participants represented by $C(t)$, to shrink. Furthermore, as the participation rate increases, the mobilization graph converges more rapidly. This intuitive rationale is mathematically proven by Proposition 2, that also offers a practical tool for explaining and planning a specific bandwagon effect.

To complete this framework, let us find the point in time t_α such that for every $t > t_\alpha$,

$$\left| \frac{C(t) - \frac{r}{\beta}}{\frac{r}{\beta}} \right| < \alpha \tag{9}$$

where t_α is the point after which the proportional distance between the number of potential participants and the number of passive citizens in equilibrium is less than a certain value, α. Substituting $C(t)$ into (9) and rearranging yields that the distance between $C(t)$ and the limit $\frac{r}{\beta}$ is less than α after a period of time expressed in Proposition 3.

Proposition 3.

1. *If* $\frac{r}{\beta} \neq N$, *then*

$$t_\alpha = \frac{1}{\beta N - r} \ln \frac{\frac{N}{r/\beta} - 1 - \alpha}{\alpha \frac{\beta P_0}{\beta C_0 - r}} \tag{10}$$

2. *If* $\frac{r}{\beta} = N$, *then*

$$t_\alpha = \frac{N(1-\alpha) - C_0}{\alpha \beta P_0} \tag{11}$$

where time is measured by time units.

Proof. See Appendix.

As can be seen in (10), the period of time until the dynamic converges depends on all of the parameters in the system, including the size of the entire population, N, and the starting points P_0 and C_0. Returning to the numerical example presented in Fig. 1, if $\alpha = 0.01$ let us find how long does it take the level of protest to converge to the limit 25. If the parameters of the situation are substituted into (9), it is found that after 13.26 time units, the distance between the protest level and the equilibrium is less than 1%.

Thus far, the model only considers two groups that are influenced by moves from and to them. The following considers a third population group.

3 Two types of bandwagon effect

The model introduced in the previous section shows that the dynamic of protest converges to a certain level of activity. However, as social history has shown, a protest movement may have a life cycle that ends with the decline of the movement. To explain this dynamic and make the model more realistic, a third group is added to the two discussed above. This group consists of people who leave the protest cycle permanently or begin supporting the government. The number of people in this group at a given time is represented by $S(t)$. In this new model, each person who quits the protest moves to this group and never returns to participate in the protest. As in the previous model, it is assumed that the number of protestors who leave the movement is expressed by $rP(t)$, but here they switch to the group $S(t)$ rather than $C(t)$. The system of equations is as follows:

$$\frac{dP}{dt} = \beta P(t)C(t) - rP(t) \tag{12}$$

$$\frac{dC}{dt} = -\beta P(t)C(t) \tag{13}$$

$$\frac{dS}{dt} = rP(t) \tag{14}$$

In line with the previous assumption, the system should preserve the total size of the population, such that $P(t) + C(t) + S(t) = N$. This system, developed by Kermach and McKendrick (1927), is well known in epidemic theory where there are three groups (Bailey 1976). The first consists of people with a contagious disease, which may spread to the second group, consisting of people who have not yet been infected with the disease. The third group consists of people who have either died of the disease or recovered from it, and can no longer be infected. The fundamental threshold theorem of epidemic can be applied to the problem at hand, because it is based on the same system. This theorem states that if $C_0 < \frac{r}{\beta}$, then the number of protesters $P(t)$ monotonically declines to zero from $t > 0$. This means that if the size of the potential protest population is less than the ratio between the rate of leaving the protest and the rate of participation, then the level of protest begins to decline right from the beginning of the activity. If $C_0 > \frac{r}{\beta}$, then $P(t)$ increases during the first period until it reaches a certain maximal level and then declines to zero. The threshold theorem reveals that $C(t)$ converges to the solution, x, by the equation

$$C_0 e^{-(\beta/r)(N-x)} = x \tag{15}$$

(For proof of this limit, see Eisen 1988).

Figure 3 demonstrates the dynamic of this model for $C_0 < \frac{r}{\beta}$, where

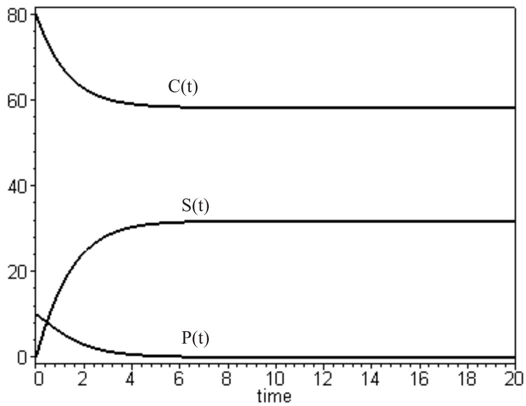

Fig. 3. The development of $P(t)$, $C(t)$ and $S(t)$ over time when $C_0 < \frac{r}{\beta}$

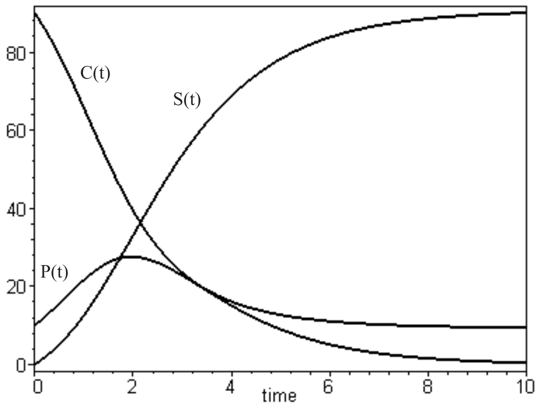

Fig. 4. The development of $P(t)$, $C(t)$ and $S(t)$ over time when $C_0 > \frac{r}{\beta}$

$P_0 = 10$, $C_0 = 90$, $S_0 = 0$, $\beta = 0.02$, $r = 2$. The upper line in Fig. 3 represents the number of people who have not yet joined the protest. This number decreases with time and converges to a certain limit. The middle line represents the number of protesters, which begins at a certain level and declines from the very beginning. The third line represents the number of citizens who began participating and withdrew or started supporting the government. Figure 3 shows that $C(t)$ converges to a certain limit while $S(t)$ increases.

Figure 4 portrays the dynamic of this model for $C_0 > \frac{r}{\beta}$, where $P_0 = 10$, $C_0 = 90$, $S_0 = 0$, $\beta = 0.02$, $r = 0.8$. It differs from the dynamic in Fig. 3 in that the middle line, which represents the number of protesters, increases during the first period and then declines to zero.

Thus, according to the fundamental threshold theorem of epidemic, for any starting conditions where $P_0 > 0$, the $P(t)$ converges to zero, such that this is the unique stable equilibrium for this system. There is another equilibrium – when $P_0 = 0$ – but it is not stable. As explained in the previous section, in this case no one initiates the protest dynamic and the system remains at the initial state. However, for any deviation that increases P_0, the dynamic moves to the second equilibrium where the number of protesters declines to zero while the number of citizens who do not take part in the protest is indicated in (15).

It follows that when there are people who withdraw from the mass movement and do not return under any circumstances, two types of a bandwagon effect are evident. One is more likely to develop as the group of potential protesters diminishes while the rate of leaving the movement increases and the rate of joining decreases. Then the movement can be expected to decline right from the beginning. If the protest starts with very few participants, it will seem as if no real protest activity took place. The second type of a bandwagon effect is more likely to develop as the group of potential protestors increases while the rate of leaving the movement declines and the rate of joining increases. In this case, the movement can be expected to grow gradually until it reaches a certain maximal level from which it will begin to decline reaching a participation level of zero at the end of the cycle. It is also easy to verify that if some of the people who discontinue their participation in the collective action effort return to the group of potential participants, i.e., moving from $S(t)$ to $C(t)$, the system will behave according to the same rationale. Although the rate of convergence may differ numerically from the situation analyzed above, the dynamic develops in the same way.

It should be emphasized again that the rates of joining and leaving the protest activity are strongly influenced, respectively, by the efforts of social activists and by the resources invested by the government to counteract the protest activity. In comparison, Chong (1991: 191–220) proposes a discrete model which portrays the rise and decline of mass movements but does not explicitly consider those who leave the movement. The model explains the possible decline of collective action, and attributes it mainly to the fact that participants' demands are responded to by the government, so that government opposition to collective action is not too strong. Although this rationale is similar to ours, Chong's model includes numerous parameters which are difficult to estimate. The model proposed here only requires an estimate of the ratio between the resources invested by the government to counteract protest activity and the efforts of social activists to mobilize collective action. This is a simple model of a complex dynamic.

4 Conclusion

The bandwagon effect is central in most dynamics of collective action – especially in explaining the rise and decline of mass movements. Any such dynamic is influenced by the efforts and resources invested by social activists and the government for and against collective action, respectively. This basic rationale guides many analyses of collective action, yet none of them have attempted to systematically explain these parameters by continuous time models.

The model developed in this paper highlights the central parameters in determining the rate and nature of the bandwagon effect, and suggests mathematical tools for calculating the resources required to achieve a certain level of collective action at a certain point in time. The model has characteristics similar to those of epidemic models, which counteract the process of exposure to and spread of infectious diseases. According to the same rationale, the bandwagon effect develops when people are exposed to others who are willing to take action and risk the consequences of their participation.

The main contribution of the model lies in the tools it proposes for explaining or actually planning a certain bandwagon effect without direct knowledge of the players' beliefs and preferences. All that has to be known is the size of the population, the number of participants at the starting point, and the approximate ratio between the resources invested by the government and those invested by social activists. The estimation process involves varying interpretations, but it is easier than precise identification of individual preferences and beliefs in society at large.

Furthermore, the theoretical framework enables to draw several testable implications. First, both in the simple model, presented in Sect. 2, and in the complex one, presented in Sect. 3, the final size of the movement, i.e., the equilibrium, $P(\infty)$, does not depend on the number of participants at the starting point, P_0. Proposition 2 and the analysis that follows it show that in the simple model the final size of the movement depends on the parameters r, β and N, but not on the initial size of the movement. Figures 3 and 4 show that in the complex model the final size of the movement converges to zero so it does not depend on the initial number of participants. The time it takes the dynamic to converge to zero depends on the parameters r, β and C, but not on the initial size of the movement. This conclusion means that the number of activists who initiate a mass movement does not influence the final results of their activity. The efforts invested by that group and the resources invested by the government to counteract this activity are the key factors in predicting the outcome of this activity.

Second, Proposition 3 shows that in the simple model the time it takes for the movement to reach approximately its equilibrium size, t_α, depends on all of the parameters in the system, including the size of the entire population, N, and the initial starting points, P_0 and C_0. Thus, although a very small group of activists can successfully initiate a mass movement, it may take much more time to reach the maximal size as compared to initiation by a large group of activists. This conclusion can explain the strategy of creating informal networks of social activists before making a protest movement visible.

Third, Proposition 2 shows that in the simple model the number of participants in the movement, $P(t)$, is maximized either at the starting point, when $t = 0$, or at infinity depending on whether the expression $\beta N - r$ is negative or positive, respectively. Regarding the complex model, Eq. (12) shows that given $C_0 < \frac{r}{\beta}$, $P(t)$ is maximized when $C = \frac{r}{\beta}$, while the fundamental theorem of epidemic shows that given $C_0 > \frac{r}{\beta}$, $P(t)$ is maximized at $t = 0$. Yet, in the complex model the size of the movement at this point depends on the initial number of participants, P_0, while in the simple model it does not. Again, this conclusion has practical implications for planning a certain bandwagon effect.

Fourth, the complex model shows that the ratio between the initial number of potential protesters, C_0, the leaving and joining rate, r and β respectively, determines whether the movement will decline from the very beginning or will grow up to a certain maximal level and then will decline to zero.

Finally, we should emphasize that 'real-world' applications of this theoretical framework do not require empirical measurement of individual preferences and beliefs. Therefore, such applications may improve our understanding of the shared mental models guiding individual behavior discussed by Denzau and North (1994).

Appendix

Proof of Proposition 1

It is simple to verify by a conventional linearization method that if $\frac{r}{\beta} < N$, the equilibrium $[C = N, P = 0]$ is not stable, since for any deviation that increases P to more than zero, the dynamic moves away from this point. However, the system of differential Eqs. (3) and (4) is highly dependent, based on the expression $N = P + C$. Substitution of this expression into (3) yields the differential equation $\frac{dP}{dt} = P(\beta N - r - \beta P)$. For a positive P, we find that when $\frac{r}{\beta} < N$, the derivative of P is positive. Thus, P increases with time and $P = 0$ is not asymptotically stable. When $\frac{r}{\beta} \geq N$, $\frac{dP}{dt} < 0$ for any positive P, and $P = 0$ is the unique equilibrium and, in this case, it is asymptotically stable. When $\frac{r}{\beta} < N$, the second equilibrium $\left[P = N - \frac{r}{\beta}, C = \frac{r}{\beta}\right]$ is asymptotically stable, since if $P > N - \frac{r}{\beta}$, then $\frac{dP}{dt} < 0$ and if $P < N - \frac{r}{\beta}$, then $\frac{dP}{dt} > 0$. We conclude that P converges with the equilibrium and is thus asymptotically stable.

Proof of Proposition 2

1. Refer to the system of differential equations (3) and (4). The variable P can be eliminated from the first equation by substituting $P = N - C$ in (4). This yields, for $\frac{r}{\beta} \neq N$, the differential equation

$$\frac{dC}{dt} = -\beta(N - C(t))C(t) + r(N - C(t)). \tag{A1}$$

Separating variables, yields

$$\frac{1}{(N - C)(r - \beta C)}\, dC = dt. \tag{A2}$$

We can rewrite the fraction and have

$$\frac{1}{r - \beta N}\left(\frac{1}{N - C} - \frac{\beta}{r - \beta C}\right) dC = dt \tag{A3}$$

Integrating (A3) gives

$$\frac{-1}{r-\beta N}\ln(N-C)+\frac{1}{r-\beta N}\ln(r-\beta C)=t+\text{constant} \tag{A4}$$

or

$$\ln\left(\frac{r-\beta C}{N-C}\right)^{1/(r-\beta N)}=t+\text{constant}. \tag{A5}$$

From (A5) we have

$$e^{t+\text{constant}}=Ae^{t}=\left(\frac{r-\beta C}{N-C}\right)^{1/(r-\beta N)} \tag{A6}$$

where A is a constant. Rearranging (A6) and extracting for $C(t)$ yields, after substituting the initial condition $C(0)=C_0$, the result.

2. For $N=\frac{r}{\beta}$ the differential equation is

$$\frac{dC}{dt}=\beta(N-C)^2 \tag{A7}$$

Again, separating variables yields

$$\frac{1}{(N-C)^2}\,dC=\beta\,dt. \tag{A8}$$

Integrating (A8) and substituting the initial condition $C(0)=C_0$, yields the result.

Proof of Proposition 3

When $N\neq\frac{r}{\beta}$, the result can be obtained by substituting the function $C(t)$ from (7) into (9) and rearranging. When $N=\frac{r}{\beta}$, part 2 can be obtained by substituting $C(t)$ from (8) into $\left|\frac{C(t)-N}{N}\right|<\alpha$ and rearranging.

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