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Innovative Applications of O.R.

Optimal response against bioterror attack on airport terminal

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ABSTRACT

We consider a potential bioterror attack on an airport. After the attack is identified, the government is faced with the problem of how to allocate limited emergency resources (human resources, vaccines, etc.) efficiently. The government is assumed to make a one-time resource allocation decision. The optimal allocation problem is discussed and it is shown how available information on the number of infected passengers can be incorporated into the model. Estimation for parameters of the cost function (number of deaths after the epidemic is over) is provided based on known epidemic models. The models proposed in the paper are demonstrated with a case study using real airport data.

1. Introduction

Since the fatal attack of anthrax on the US postal office in the fall of 2001, bioterror has become a realistic threat. The US population is more vulnerable now to the smallpox virus than three decades ago as a result of the discontinuation of smallpox vaccination in 1972. Henderson (1999) presents a discussion on the threats of bioterrorism with special attention to smallpox which is regarded, together with anthrax, as one of the two greatest potential bio-weapon threats.

Bozzette et al. (2003) call attention for the relevance of the subject and point out the magnified effect of bioterrorism if done in an airport. The web page of the Federation of American Scientists displays the Congressional Research Service Report on the bioterrorism detection program known as BioWatch. The severity of an airport attack discussed in Bozzette et al. (2003) and the fact that the US government has a program for bioterrorism detection suggests that the matter is not only an academic concern but it is a *de facto* concern. In addition, Jane's Homeland Security & Resilience Monitor (2004) cites Mark Cetron, president of a risk assessment company working for both the FBI and the Department of Defense, as evaluating the chance of terrorists obtaining smallpox to be about 80%.

There are two streams of research that discuss the new threat of smallpox bioterror attacks. One stream focuses on studying the transmission of smallpox and examining the impact of pre- or post-event vaccination strategies and control policies. Bozzette et al. (2003) develop scenarios of smallpox attacks and present a stochastic model of outcomes under various control policies (vaccination of contacts of infected persons and isolation of patients, pre- or post-attack vaccination of either 60% of the population, or 90% of the health care workers, or both). Bauch et al. (2003) present a synthesis of game theory and epidemic modeling that formalizes the conflict between self-interests of individuals and group interest for the population. They show that voluntary vaccination is unlikely to reach the group-optimal level.

The other stream of research attempts to address the process of post-event vaccination in the case of smallpox. Kaplan et al. (2002) propose a continuous-time model with 17 ordinary differential equations. The details of their model are reported in Kaplan et al. (2003), in which they model the 'race to trace' (i.e., attempting to trace and vaccinate an infected person when (s)he is still vaccine-sensitive). Meltzer et al. (2001) construct a Markov chain model to describe the epidemic progression of smallpox through a susceptible population. They examine the impact of quarantine and vaccination, separately and together on the spread of smallpox and find that only a combination of vaccination with an effective quarantine may eradicate the epidemic.

Kaplan et al. (2003) propose a model (hereafter KCW) based on the **SEIR** Model. The basic **SEIR** model (see for example Bauch et al., 2003) considers four basic stages of the disease: First stage is when the individual is susceptible (**S**), he/she is asymptomatic, non-infectious and vaccine-sensitive; in the second stage the individual

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is exposed (**E**), is asymptomatic, non-infectious and vaccine-insensitive; the third stage is when the individual is asymptomatic, infectious (**I**) and vaccine-insensitive; and finally, the individual is symptomatic and either removed (death) or recovered (**R**). People who get a vaccine in the first stage will shift from the Susceptible group to the Recovered group. KCW offer more complex analysis when they incorporate queueing issues. Similarly to the basic **SEIR** model, KCW offer an approximation to the exact solution under some mild assumptions.

In this paper, we focus on the post-event response. We discuss how a government should react and allocate limited resources (human resources, vaccines, antibiotics, etc.) to minimize the expected cost (number of deaths) in the aftermath of a bioterror attack using smallpox or a similar agent on a major airport. Once the bioterror attack is identified, the government can assign additional centralized emergency resources to each destination city[1] to deal with the disaster along with existing initial resources. We discuss two cases regarding whether or not the initial resources in each city could be re-allocated to other cities. We construct a cost (number of deaths after the epidemic is eradicated) function based on results from KCW.

This paper considers a bioterror attack on an airport using some biological weapon (i.e., smallpox, etc.).[2] The occurrence of the attack remains unknown (except to the terrorist) until the time when some infected passengers start showing symptoms. After the attack is identified, the government is faced with the problem of how to allocate limited emergency resources efficiently to the different cities. The location of the attack can be identified after a few cases of infections are observed. The health care authorities can trace backward and cross reference the time and location where infectious passengers are identified to the time of flights and the airport terminal where the attack occurred.

The government is assumed to make a one-time resource allocation decision and possible subsequent reallocations are ignored. However the Center for Disease Control (CDC) calls attention to the need for vaccinating not later than 4 days after contact with an infectious person in order to prevent or modify the disease (CDC Interim Smallpox Response Plan and Guidelines, Draft 2.0, November 21, 2001, Atlanta). Since reallocation is time consuming and the time window for applying the allocation decision is small, reallocation of resources may not be even a feasible option. Therefore, a good single (or first) allocation decision is very important for the success of fighting the epidemic.

The number of passengers who get infected is not known when the resource allocation decision is made. However, the cumulative number of passengers who have been showing symptoms by a specific time is a useful information. The optimal post-attack allocation problem is analyzed, and available information on the number of initially infected passengers can be incorporated into the model.

The cost functions used are based on known epidemic models and estimation of parameters of KCW. We prove convexity of the objective function with respect to the decision variables (subject to two reasonable conditions) which makes the solution tractable. We also solve the problem of minimizing the maximum number of deaths when little is known about the number of infected passengers arriving at each destination. In fact, our results suggest that in this case the airport is not the best place to perpetrate the attack. We provide an approach to solve the allocation problem taking into account information on infected passengers (i.e., *a posterior analysis*) and we demonstrate our models with a small case study using data from the Long Beach Airport.

We show the advantage of our approach by comparing it to two myopic approaches that may be used by the government. There is limited research about a potential bioterror attack on airports. We have identified one as mentioned above, but, to the best of our knowledge, none is addressing the problem of allocating resources for mitigating its effect.

The remainder of the paper is organized as follows. In Section 2, we present a model in which the government has to make a one-time resource allocation decision after identifying a bioterror attack. In Section 3 we discuss how to solve the allocation problem. In Section 4, we analyze the problem of allocating resources to minimize the maximum number of deaths. In Section 5, we study the problem using information on the number of passengers infected so far. In Section 6, we discuss an application of our models in a case study with real data of a small airport. In the last section, we provide concluding remarks.

2. The model

Let $\mathbf{N} = \{1,\ldots,n\}$ be the set of cities with flights arriving from the airport where the attack occurs; K_i denotes the total number of passengers flying to destination city i,[3,4] $i \in \mathbf{N}$, of which I_i^0 is the number of initially infected passengers flying to city i. Note that I_i^0's are unknown and thus treated as random variables. The government has a total of R emergency resources available excluding initial resources that may already exist in each city.[5] Let p be the probability for a passenger[6] to get infected when an attack occurs. Suppose that the government can make a one time decision on where to allocate resources after the attack is recognized and later on they cannot change that allocation (e.g., moving vaccinators between cities is not efficient). We consider two cases regarding p: known and unknown. When p is unknown which we term "stochastic", we mean that based on experts' opinion a distribution of p can be assessed. We note that Bozzette et al. (2003) consider high or low impact of airport outbreak disease (namely, high or low p).

Let $f_i\left(\mu_i, I_i^0\right)$, where μ_i is the total number of resources allocated to destination city i, be the cost (number of deaths after the epidemic is over) at city i given that the number of initially infected passengers to city i is I_i^0. Note that if there are already μ_i^0 existing resources prior to allocation, then μ_i^0 is included in μ_i. Thus, the authorities minimize the total expected number of deaths:

$$
\begin{aligned}
& f\left(R,\{K_i\}_{i=1}^n\right) = \min_{\mu_i, i=1,\ldots,n} \sum_{i=1}^{n} E_{p,I_i^0} f_i\left(\mu_i, I_i^0\right) \\
& \text{s.t.} \\
& \sum_{i=1}^{n}\left(\mu_i - \mu_i^0\right) \leqslant R \\
& \mu_i \geqslant \mu_i^0 \quad i \in N.
\end{aligned}
\tag{1}
$$

Note that when p is a known constant, the expectation in $f\left(R,\{K_i\}_{i=1}^n\right)$ is only with respect to I_i^0. The problem (1) is a resource allocation problem (see Ibaraki and Katoh, 1988).

To find the value of $f_i(\mu_i, I_i^0)$ one can solve numerically a **SEIR** model with vaccination. As our model will lead to an extremely complex problem with high computation time using the exact approach, an approximation is needed. The approximation suggested by KCW which we simplify is included in the on-line supplement.

The one-time decision is made at time $t \geqslant t_{\min}$ (the detection delay). For the time being, we ignore the information on the number of infected passengers that are already identified by time t and

[1] The destination city of the passenger from the airport where the attack occurs.

[2] Different diseases affect in different ways the parameters of the cost function which are introduced later.

[3] The airport city where the attack occurred is a dummy destination.

[4] If the attack occurs simultaneously in several terminals in different cities we do not need to change the model but we should aggregate the number of passengers going to each destination city.

[5] A resource unit might be interpreted as a vaccinator.

[6] Passenger in the airport where and when the bioterror occurs. We note that the value of p might be different for different individuals.

Table 1
Notation used and accepted parameters values for smallpox (KCW).

Parameter	Description	Accepted values
M_i	Population size of city i	
r_1	Disease stage 1 rate	$(3\text{ days})^{-1}$
r_2	Disease stage 2 rate	$(8\text{ days})^{-1}$
r_3	Disease stage 3 rate	$(3\text{ days})^{-1}$
r_4	Disease stage 4 rate	$(12\text{ days})^{-1}$
δ	Smallpox death rate	0.30
η	Vaccination fatality rate	10^{-6}
v_r	Vaccinator vaccination rate	200 person/day
$I_j^{(i)}$	Number of infected at city i, stage j when disease is discovered	
I_i^0	Initial number infected in city i	
R_0	Basic reproductive rate	[3,6]
$t_{\min}$	Detection delay	5 days

therefore we do not update the distributions of I_i^0 according to this knowledge. Assume that p is a random variable and $I_i^0|p \sim B(K_i,p)$ (a Binomial distribution with parameters K_i and p) implying that for each city i, $E\left(I_i^0\right) = K_i\bar{p}$, where $\bar{p} = E(p)$ if p is stochastic and $E(I_i^0) = K_i p$ if p is a constant.

When using the Binomial distribution we make three assumptions: (i) each passenger can be treated as a trial of an experiment; (ii) the trials are independent; and (iii) the probability p remains identical in all trials. The first assumption obviously holds. The second assumption also holds since during the attack there is no person-to-person contamination and it takes a few days of incubation until an infected individual becomes infectious. The third assumption is the strongest since the probability of getting infected may be indeed different for different passengers but it is impossible to get such information and therefore the assumption is quite reasonable.

Proposition 1.

$$E[f_i(\mu_i, I_i^0)] = f_i\left(\mu_i, E\left(I_i^0\right)\right) = f_i(\mu_i, K_i\bar{p}).$$

Proof. The cost function $f_i\left(\mu_i, I_i^0\right)$ given in (A.1) (see the on-line supplement) is linear in $I_j^{(i)}, j = 1,2,3,4$. From (A.2) we find that $I_j^{(i)}$'s are linear in the initial number of infected passengers I_i^0 and thus $f_i\left(\mu_i, I_i^0\right)$ is linear in I_i^0. □

In order to avoid the use of additional parameters, we will not use the notation K_i and p in the model we introduce shortly. Notice that when existing (prior to an attack) resources at city i can be transferred to another city, μ_i may be less than μ_i^0. Therefore, we make the following classification in terms of the value of μ_i's accordingly: (i) $\mu_i \geqslant \mu_i^0, \forall i \in \mathbf{N}$; (ii) $\mu_i \geqslant 0$ (namely, the authorities can shift some or all the initial resources available in city i). We first consider the case where $\mu_i \geqslant \mu_i^0,\ i \in N$. Let $R' = R + \sum_{i\in N}\mu_i^0$. The problem is to allocate the resources available R' (I^0 is a vector of I_i^0 values),

$$f(R, I^0) = \min_{\mu_i, i=1,\dots,n}\left\{\sum_i f_i(\mu_i, E(I_i^0)) | \sum_i \mu_i \leqslant R', \mu_i \geqslant \mu_i^0\right\}. \tag{2}$$

The disease stage rates are considered deterministic parameters. The same is implicitly assumed in KCW. Table 1 below summarizes the main parameters used herein.

3. Solving the model

From the on-line supplement, $f_i\left(\mu_i, I_i^0\right)$ can be expressed as

$$f_i\left(\mu_i, I_i^0\right) = \frac{a_0}{\mu_i^2} + \frac{a_1}{\mu_i} + a_2 + a_3\mu_i + a_4\mu_i\,e^{-\frac{a_5}{\mu_i}} \tag{3}$$

where a_j, $j = 0,\dots,5$ are city dependent and

$$\begin{aligned}
a_0 &= \frac{1}{6}\frac{I_3^{(i)}}{t_{\min}}R_0\delta r_3\left(\frac{M_i}{v_r}\right)^2,\\
a_1 &= \frac{R_0\delta r_3\left(r_1 I_3^{(i)} - \frac{I_3^{(i)}}{t_{\min}}\right)}{2r_1}\left(\frac{M_i}{v_r}\right),\\
a_2 &= \eta M_i + \delta\left(\sum_{j=2}^{4} I_j^{(i)} + I_3^{(i)} r_3 R_0\left(\frac{1}{t_{\min}r_1^2} - \frac{1}{r_1}\right) + I_1^{(i)}\right),\\
a_3 &= -a_4,\\
a_4 &= \frac{\delta\left(r_1^2 I_1^{(i)} + R_0 r_3\left(\frac{I_3^{(i)}}{t_{\min}} - r_1 I_3^{(i)}\right)\right)}{r_1^3}\left(\frac{M_i}{v_r}\right)^{-1},\\
a_5 &= r_1\left(\frac{M_i}{v_r}\right).
\end{aligned} \tag{4}$$

The use of (3) is shown by the following proposition.

Proposition 2. *If the following two conditions are met then $f_i\left(\mu_i, I_i^0\right)$ and thus $f\left(\mu, E(I_i^0)\right)$ are convex in vector $\mu = (\mu_1,\mu_2,\dots,\mu_n)$:*

(a) *The time until detection, t_{min}, is larger than the time an infected person spends in the first stage of the disease, i.e. $t_{\min} > \frac{1}{r_1}$;*
(b) *$R_0 < \frac{r_1^2 I_1^{(i)}}{r_3 I_3^{(i)}[r_1 - 1/t_{\min}]}$; that is the basic reproductive rate, measured by people infected per day, is not too large.*

Moreover, the accepted smallpox epidemic values from Table 1 support items (a) and (b).

Proof. The proof follows the second derivative of (3) with respect to μ_i:

$$\frac{6a_0}{\mu_i^4} + \frac{2a_1}{\mu_i^3} + \frac{a_4 a_5^2\,e^{-\frac{a_5}{\mu_i}}}{\mu_i^3} \tag{5}$$

which is positive, for positive values of a_0,a_1 and a_4. Clearly, a_0 is non-negative by the non-negativity of each individual parameter. Constant $a_1 \geqslant 0$ if $r_1 > 1/t_{\min}$ as stated in Condition (a). Note that $a_5 > 0$.

Constant a_4 is non-negative if

$$R_0 < \frac{r_1^2 I_1^{(i)}}{r_3 I_3^{(i)}[r_1 - 1/t_{\min}]} \tag{6}$$

as stated in condition (b). Since $f\left(\mu, I_i^0\right)$ is the sum of convex functions, the proof is complete. □

Note that the two conditions above are sufficient but not necessary conditions for convexity. If we consider the smallpox case, the parameters given in Table 1 show that both conditions are satisfied. From Table 1, $r_1 = 1/3 > 1/t_{\min} = 1/5$. Also the RHS of (6) (taking into account that numerical experiments show that $I_1^i \geqslant 3I_3^i$) is greater than 7.5 which is greater than $R_0 \in [3,6]$. However, when $I_1^i = I_3^i$ the RHS of (6) is 2.5 and therefore the second condition does not hold ($R_0 \in [3,6]$). Still, expression (5) is non-negative for all positive values of μ_i and thus convexity still holds.

As mentioned earlier our problem is a resource allocation problem which is an easy problem (that can be solved polynomially) even when μ_i's are integers, for any function f_i when R is polynomial in n, and for any R if f_i's are convex. Therefore Proposition 2 implies that solving the problem is very easy for any size of R. In this case we can do a marginal cost analysis. Therefore a greedy algorithm that allocates the next available resource to the city with the largest marginal contribution to the objective function is optimal.

We now present the algorithm to obtain the optimal solution. Let $\mu = \{\mu_1, \ldots, \mu_n\}$ and $\Delta_i(\mu, E(I^0))$ be the improvement in the objective function value obtained when one extra unit of resource is allocated to city i. That is $\Delta_i(\mu, E(I^0)) = f(\mu', E(I^0)) - f(\mu, E(I^0))$, where $\mu'_j = \mu_j$, for all $j \neq i$, and $\mu'_i = \mu_i + 1$. We will drop the explicit reference to I^0 whenever that causes no confusion. Also, whenever it is clear enough we will just write I^0 without the expectation operator. In addition, we assume here that the decision variables μ_i's are integers. If this is not the case we can either use standard convex optimization algorithms or, alternatively, we could employ a standard marginal cost analysis, that is computing the improvement in the objective function value by adding extra resources through using the derivative of each city contribution. We do not elaborate more on that since in practice we believe that resources are non-negative integers (such as number of vaccinators). The following discrete version of the marginal analysis algorithm is for the $\mu_i \geqslant \mu_i^0$ case.

Algorithm. *MA*

```
Input (EI^0, R', f)
Initialize
    μ_i = μ_i^0  ∀i = 1, ..., n
Do While Σ_i μ_i ⩽ R'
    x ← 0, best ← 0
    While j ⩽ n
      If Δ_j(μ') > x then
        x ← Δ_j(μ'), best ← j
    μ_best = μ_best + 1
Return (μ)
```

In Algorithm *MA*, we only assign a resource to city j if the additional resource would save no less lives than if assigned to city $q \quad q \neq j, q \in \{1, \ldots, n\}$. Later we will provide a simpler approximation where, through using Lagrangian relaxation, we can derive a closed form solution when either $\mu_i^0 = 0$ without the option of transferring existing resources or $\mu_i^0 \geqslant 0$ with that option.

We point out several observations related to Algorithm *MA*:

1. At the optimal solution the values of μ_i are such that the marginal cost of adding another resource to city i is *approximately* the same for all $\mu_i > \mu_i^0, i \in N$.
2. For all $\mu_i > \mu_i^0, i \in N$, the derivative of $f_i\left(\mu_i, I_i^0\right)$ with respect to μ_i is the same (the continuous counterpart of Obs.1). Although this information is useful to find numerically the optimal solution (without using the algorithm presented above), it does not imply that a closed form solution can be achieved.
3. The optimal solution does not imply that the chance of dying at city i is the same for all cities with $\mu_i > \mu_i^0, i \in N$.
4. Although all the discussion up to this point refers to the case of an attack using smallpox, the general mathematical framework could be used to address several related problems of other infectious diseases, such as the Norwalk virus or the Avian influenza.
5. KCW notes that the expected number of deaths is not heavily affected by the cities' total population. In fact, the impact caused by the number of people initially infected outweighs by far that of the total population. Hence, the total cost is less dependent on the size of the cities (destinations) connected to the original airport and more on the size of the initial population infected which is related to the size of the airport (measured by the traffic of passengers) where the attack occurred.[7]

Observation 3 brings up a variation of the problem where one could set the problem as a multi-objective optimization trying to address, in addition to efficiency, also the equity issue of not having one city with higher fraction of fatalities than others. We will not elaborate more on this variation but we suggest it for future research.

Now we assume that the existing resources prior to the attack can be transferred. Suppose that some cities have μ_i^0 existing resources available prior to the attack. Recall that $R' = R + \sum_{i \in N} \mu_i^0$ is the total resource availability in Algorithm *MA*. The problem is identical to the one when transferring resources between cities is not allowed and there are no existing resources, i.e. in the initialization stage of Algorithm *MA*, $\mu_i = 0 \, \forall i \in N$. The cities virtually shift their initial resources to the main pool and the authorities allocate them as when there are no initial resources in the cities. A city which receives less resources than their initial amount (i.e., $\mu_i < \mu_i^0$), actually gives up $\mu_i^0 - \mu_i$ of its initial resources which are transferred to another city.

3.1. A simple approximation

Here we show that in fact a simpler approximation than (3) can be obtained with the function

$$g(\mu_i) = b_1^i + \frac{b_2^i}{\mu_i^2}. \tag{7}$$

In the case of using expression (7), we assume continuity of the decision variables. In (A.2) (see on-line supplement) it is easy to verify that $I_j^i, j \in \{1, \ldots, 4\}$, are also linear in I_i^0. Thus, a_i's in (4) are also linear in I_i^0.

First we will show why (7) is a reasonable approximation to the original cost function (3). Then, we will find the parameters for (7) that will give an excellent fit with the original cost function (3). The quality of the fit depends on the possible range for the value of parameters. We have tried to use realistic values as much as possible. For example, according to the Registered Nurse Population Health Resources and Services (2001) and considering a large city (about 10 million people as considered in KCW's analysis), the city's internal resources should be around 5000. Even if the authorities allocate additional resources, it is still not expected to be too large.[8] Namely, the process of vaccination of the whole city should take a few days if we consider a unit of resource as a single vaccinator (medical staff). Assuming that each medical staff vaccinates $v_r = 200$ people every day (see Table 1), μ_i is in the range between 5000, which is equivalent to vaccinating the entire city in 10 days, and 20,000, which is equivalent to vaccinating the entire city in 2.5 days. Following KCW and (A.2), the number of people in stages [1, 2, 3, 4] is equal to [415, 662, 156, 103], respectively, and therefore the values of the a_i's in (4) are $[a_0, a_1, a_2, a_3, a_4, a_5] = [3.9 \times 10^9, 4.6 \times 10^5, 354.64, -0.0041, 0.0041, 16666]$. When μ_i is not too large (i.e., $\mu_i \sim 1000$), the term $a_2 + a_0/\mu_i^2$ is dominant and thus, the approximation (7) should be good.

However, this naïve approximation might be less accurate when μ_i is larger and other terms are needed for good approximation. We may divide the range of μ_i into segments and produce a reasonable approximation for every segment. However, to have an efficient

[7] Obviously, we may expect that there will be more passengers flying to a large city than to a small city.

[8] The Washington Post, on 12/01/08 (Page A01), reported that the U.S. Northern Command has plans to station, by 2011, around 20,000 troops inside U.S. trained for dealing with an attack using weapons of mass destruction.

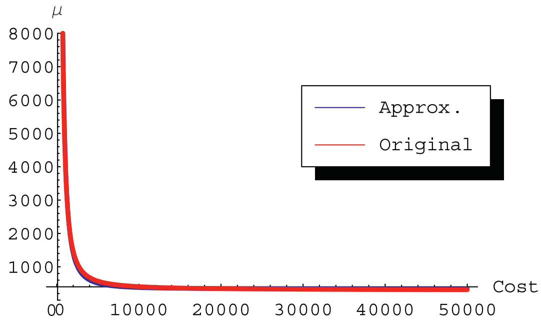

Fig. 1. Eq. (A.1) and (7).

tool we have found a good approximation which is based on running a fit between expression (7) and (A.1) (see on-line supplement). The cost function obtained is:

$$g(\mu_i) = 336.38 + \frac{4.25 \times 10^9}{\mu_i^2}.$$

The fit, using the case study data that will be presented later in the paper, was found to be very good with $R^2 = 0.993$.

Fig. 1 shows the cost functions when using equation (A.1), labeled "Original", and expression (7), labeled "Approx".The visual fit is very good. In order to improve the visualization we plot in Fig. 2 the LogLog of the two functions.

It is clear from the figures that there is almost a perfect fit. The two curves have similar asymptotic cost, when $\mu \to \infty$, of 336.38 and 290, respectively for (7) and (A.1). Although the difference between the two results is not small, we believe that in the range of interest it is not a problem. To illustrate that, note that when $\mu = 50{,}000$ the values of the two functions are 338 and 311. In order to perform the fit, we used 490 points in the range $\mu \in [1000, 50000]$.

As mentioned in Observation (5) above, the total number of deaths is less dependent on cities' size than on the number of infected people going to that particular city. Consequently, the parameters b_i are affected mostly by the number of infected people and to less extent by the population size. Thus, in a practical situation one could estimate values of b_i for small, medium and large cities. Categorization would reduce the number of estimations necessary to obtain a solution. The parameters we used are based on KCW calculations which consider a city of 10 million. However, the structure of rapid decline of the number of death/infected people remain the same for smaller cities and thus, we can use (7) for different city sizes.

Using approximation (7) in (2), we assume now that either $\mu_i^0 = 0\ \forall i \in N$ or that some μ_i^0 might be positive and we allow transfer of resources. We have the following result:

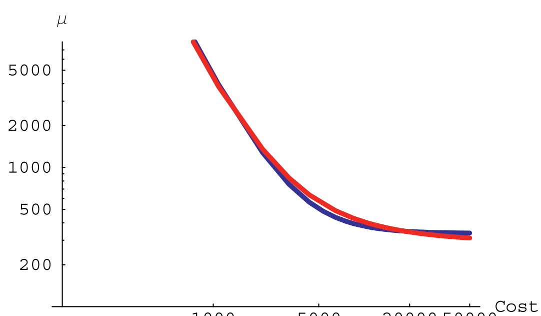

Fig. 2. LogLog plot: Eq. (A.1) and (7).

Corollary 1. *The optimal solution is*

$$\mu_i^* = \frac{\sqrt[3]{b_2^i}}{\sum_j \sqrt[3]{b_2^j}} R', \quad i \in N. \tag{8}$$

The result is obtained by using Lagrange multipliers to solve the problem:

$$\min_{\mu_i, i \in N} \sum_{i \in N} \frac{b_2^i}{\mu_i^2} \tag{9}$$

$$\text{s.t.} \quad \sum_{i \in N} \mu_i = R' \tag{10}$$

$$\mu_i \geqslant 0, \ \ i \in N. \tag{11}$$

4. The allocation problem with limited information

Until now we assumed that we can derive estimations of the number of initial infected passengers arriving to each city $i\left(E\left(I_i^0\right)\right)$. Here we assume that we only have an estimate of the total number of initially infected passengers arriving to all n cities. We define a new problem that can be used also to allocate resources in advance of an attack. In this problem, assume that the total number of initially infected passengers is, W (out of K). The decision maker then allocates resources in order to either minimize the maximum total number of deaths in case of an attack (*Problem MiniMaxSum (MMS)*) or minimize the maximum number of deaths among all cities (*Problem MiniMax (MM)*).

Note that, in contrary to the previous section, W is not a random variable. It is possible to consider a random variable W but it does not bring any value to the discussion that will follow. In the first 3 sections we focus on the authorities' optimal allocation of resources strategy. Here we discuss a hypothetical scenario where we take into consideration the best attackers' target if the maximization of the number of deaths is their objective. The key result is the understanding that attacking an airport is not necessarily the best target for maximizing the damage. This is the reason for fixing W.

We assume, in both problems, that (a) when $\mu_i^0 = 0$ for some $i \in N$, the decision maker must assign at least one resource to city i. Considering that the number of resources is always much larger than the number of destinations this assumption is not very restrictive. We also assume that (b) each destination-city will receive at least one passenger infected. This assumption is also not restrictive. Assumption (a) is needed because $\mu_i = 0$ implies that f_i is undefined due to a zero in its denominator. Assumption (b) implies that if no infected passenger travels to some destination i then we should remove that particular destination from our list of possible destinations.

First, we address the Problem *MiniMaxSum (MMS)*. The objective function is given by

$$\text{Problem } MMS: \ \text{minimize}_\mu \ \text{maximum}_{I^0} \left\{ \sum_i f_i\left(\mu_i, I_i^0\right); \ \text{for all } I_i^0 | \sum_i I_i^0 = W \right\} \tag{12}$$

where vector $\mu \in \Omega = \{\mu | \sum_i \mu_i = R', \mu_i \geqslant \max\{1, \mu_i^0\}\}$.

In order to solve the problem we must know the form of the optimal solution. Define μ^* to be the optimal allocation vector. Thus, we have the following result.

Proposition 3. *In the optimal solution of Problem MMS the maximum number of deaths will occur for a vector I^0 with the following profile: Let i,j be the indices of destination cities. $I_j^0 = W - n^0 + \mathbb{I}_j$ (n^0 is the number of cities without existing resources) for some city j and $I_i^0 = 1$ for $i \neq j$, $\mu_i^0 = 0$.*[9]

[9] $\mathbb{I}_j$ is a binary indication that assumes the value *0* when $\mu_j^0 > 0$ and 1 otherwise.

Proof. We use a logical argument in this proof. To make the proof easier to understand, assume that there is an evil agent that can assign the W infected passengers to cities at his own will. Consider an optimal solution where μ^* is the allocation vector that minimizes the maximum number of deaths. The agent, knowing that the resource allocation has been made and how the resources are allocated, will assign the first n passengers one to each destination city (Assumption (*b*) above). The agent will then compute the number of deaths caused by an extra infected passenger to each city. However, it is easy to see that expression (A.1) is linear in $I_k^{(i)}$ and $I_k^{(i)}$ (where k is the stage of the disease) is linear in I_i^0 given in (A.2). Hence, the best the agent can do is to allocate all remaining passengers to city i with the highest coefficient of I_i^0 . Since the same reasoning would hold for any vector $\mu \in \Omega$, the proof is complete. □

Corollary 2. *Causing maximum damage can be also achieved by attacking a particular city directly (rather than through an indirect mechanism; i.e. attacking an airport).*

This result is surprising since there is a belief that attacking an airport would be a very efficient way to maximize damage (see Bozzette et al., 2003). However, it is important to note that attacking an airport may cause more fear than attacking a particular city directly.

Corollary 3. *Problem MMS can be solved by standard convex minimization procedures.*

Proof. According to Proposition 3, there are only n different possible vectors I^0. Let $I^0_{(i,k)}$ denote the value corresponding to the ith city in the kth vector. Therefore we have to solve

$$\begin{aligned}&\min_\mu \max_{I^0}\left\{\sum_i f_i\left(\mu_i, I_i^0\right) \text{for all } I_i^0 \,|\, \sum_i I_i^0 = W\right\}\\ &\quad = \min_\mu \max\left\{\sum_i f_i\left(\mu_i|I^0_{(i,k)}\right)|k=1,\ldots,n\right\}\end{aligned} \tag{13}$$

Note that the term between the curly brackets, $\left(\mu_i|I^0_{(i,k)}\right)$, is a function of μ_i for each of the $k = 1,\ldots,n$ possible vectors I^0. Thus, we have a maximum of n different convex functions of μ_i which is indeed convex in μ_i. Hence, Problem *MMS* can be solved by standard convex algorithms. □

Problem *MM* is to minimize the maximum number of deaths among all cities. The problem is then defined as

$$\text{Problem } MM: \quad Min_\mu Max_{I^0}\left\{f_i\left(\mu_i, I_i^0\right) \text{for all } I_i^0 \,|\, \sum_i I_i^0 = W\right\}$$

A similar reasoning to our last two proofs, which is omitted, leads us to the following corollary.

Corollary 4. *A standard convex minimization algorithm solves Problem MM.*

5. A learning process

One of the main problems in models addressing bioterror attacks is the missing information about some parameters' values. Most of the parameters can be reasonably estimated from historical data (such as data in the medical literature), as has been done by KCW and others. However, the values of p and I_i^0 cannot be found in the literature and thus, the usual approach is to assume some values for the unknown parameters and apply sensitivity analysis. In this section, we consider the learning process of I_i^0, and p, by using the available information.[10] The tools we offer can help decision makers to estimate these parameters based on the available information on the total number of identified infected passengers.

Define $t_{\max}$ to be the maximum elapsed time for all infected passengers to be identified, $0 \leqslant t_{\min} \leqslant t_{\max}$. Assume the government decides to allocate emergency resources at t with $0 \leqslant t_{\min} \leqslant t \leqslant t_{\max}$. Let $I_i^1(t)$ be the cumulative number of infected passengers identified until time t in city i from the K_i passengers flying to this city and let $I_1(t) = \sum_{i=1}^n I_i^1(t)$. Given p and $I_i^1(t)$, we can derive the conditional probability of I_i^0 given $I_i^1(t)$ for each $i \in N$ as follows:

$$\Pr\left(I_i^0 = k_i|I_i^1(t) = m_i\right) = \begin{cases}\frac{\Pr\left(I_i^0=k_i, I_i^1(t)=m_i\right)}{\Pr\left(I_i^1(t)=m_i\right)}, & k_i \geqslant m_i,\\ 0, & k_i < m_i.\end{cases}$$

Given that a passenger is infected, let the random variable S be the length of time it takes her to first show symptoms, and be identified as a disease carrier. Define $s_j, j = 1,2$ to be the time that an infected passenger spends in stages E and I and let $S = s_1 + s_2$, obviously $t_{\min} \leqslant S \leqslant t_{\max}$. Assume that S has a known cumulative distribution H, i.e., $P(S \leqslant t) = H(t)$.[11]

Therefore given p, when $k_i \geqslant m_i$,

$$\begin{aligned}\Pr\left(I_i^0 = k_i|I_i^1(t) = m_i\right) &= \frac{\Pr\left(I_i^1(t)=m_i|I_i^0=k_i\right)\Pr\left(I_i^0=k_i\right)}{\sum_{j=m_i}^{K_i}\Pr\left(I_i^1(t)=m_i|I_i^0=j\right)\Pr\left(I_i^0=j\right)}\\ &= \frac{\binom{k_i}{m_i}H(t)^{m_i}(1-H(t))^{k_i-m_i}\binom{K_i}{k_i}p^{k_i}(1-p)^{K_i-k_i}}{\sum_{j=m_i}^{K_i}\binom{j}{m_i}H(t)^{m_i}(1-H(t))^{j-m_i}\binom{K_i}{j}p^j(1-p)^{K_i-j}}\\ &= \binom{K_i-m_i}{k_i-m_i}\frac{[(1-H(t))p]^{k_i-m_i}(1-p)^{K_i-k_i}}{(1-H(t)p)^{K_i-m_i}}.\end{aligned} \tag{14}$$

We distinguish between the two cases: (i) p is stochastic and (ii) p is a known constant.

5.1. Stochastic p

Assume that p has a discrete probability distribution $\Pr(p = r)$ where $r \in \{d_1, d_2, \ldots,\}$. Let q_t be the probability that a random passenger will first show symptoms t days after the attack occurs. The parameter p is assumed to be independent of S. We note that it is possible that an individual resistance to the disease depends on some other individual parameters and that the disease duration may depend also on the same parameters. However, there is no indication in the literature and no model (analytical, empirical, simulation, etc.) that captures this dependency. All research and data gathered in the field of epidemic is based on aggregates and not on individuals. Thus, $q_t = pH(t)$. Since $I_i^1(t)|p \sim B(K_i, q_t)$, the posterior conditional probability of p at time t given $I_1(t)$ is

$$\begin{aligned}\Pr(p = r|I_1(t) = m) &= \frac{\Pr(p=r, I_1(t)=m)}{\Pr(I_1(t)=m)} = \frac{\Pr(I_1(t)=m|p=r)\Pr(p=r)}{\sum_p \Pr(I_1(t)=m|p)\Pr(p)}\\ &= \frac{\binom{K}{m}q_t^m(1-q_t)^{K-m}\Pr(p=r)}{\sum_p \Pr(I_1(t)=m|p)\Pr(p)}\\ &= \frac{\binom{K}{m}(rH(t))^m(1-rH(t))^{K-m}\Pr(p=r)}{\sum_p\binom{K}{m}(pH(t))^m(1-pH(t))^{K-m}\Pr(p)}\\ &= \frac{\Pr(p=r)r^m(1-rH(t))^{K-m}}{\sum_p \Pr(p)p^m(1-pH(t))^{K-m}}.\end{aligned} \tag{15}$$

[10] Walden and Kaplan (2004) apply a similar approach when they estimate the time elapsed from the beginning of the disease given the current number of disease carriers.

[11] In the case of smallpox, $H(t)$ can be found in Bozzette et al. (2003).

Observe that there is a minimal time for the appearance of symptoms. Thus, for $t < t_{\min}$ the probability of showing symptoms is $H(t) = 0$. It follows that for $t < t_{\min}$ the number of passengers with symptoms is identically zero, i.e., $I_1(t) = 0$. We then assume that $t \geqslant t_{\min}$. The modified objective function is

$$\min_{\mu_i, i \in N} \sum_{i \in N} E\left[f_i\left(\mu_i, I_i^0\right) | I_i^1(t) = m_i\right] = E_p\left(\sum_{i \in N} E_{I_i^0}\left[f_i\left(\mu_i, I_i^0\right) \middle| I_i^1(t) = m_i, p\right] \middle| I_1(t) = m\right),$$

where $m = \sum_{i \in N} m_i$.

Notice that with updated information at different points in time, the resources allocated correspondingly will have different efficacy denoted by $\gamma_i(t)$ (discussed below), which is captured in the modification of (7) given by (16). To capture the linear dependency in I_i^0 we rewrite the cost function as (16):

$$f_i\left(\mu_i, I_i^0, t\right) = \left[\tilde{b}_1^i + \frac{\tilde{b}_2^i}{(\mu_i \gamma_i(t))^2}\right] I_i^0 \tag{16}$$

where $\tilde{b}_1^i$ and $\tilde{b}_2^i$ are independent of I_i^0.[12] As mentioned earlier, this function reflects the situation when resources allocated at different times have different efficacy, i.e., $\gamma_i(t)$ is no longer a constant equal to 1. Instead, it is a non-increasing function of time t, $0 \leqslant \gamma_i(t) \leqslant 1 (\gamma(t_{\min}) = 1)$. Actually, the impact of μ_i units of resources at time $t \geqslant t_{\min}$ is $\mu_i \gamma(t) \leqslant \mu_i$. $\gamma_i(t)$ reflects the deterioration of mass vaccination efficiency due to delay in the beginning of the vaccination process. In the analysis hereafter, function $f_i(\mu_i, I_i^0, t)$ will be used instead of $f_i(\mu_i, I_i^0)$ to accommodate the time-dependent resource efficacy.

We let $I^1(t) = \left(I_1^1(t), I_2^1(t), \ldots, I_n^1(t)\right)$ and assume that the time to make a decision is discrete (e.g., number of days).

Let $J(t, I^1(t))$ be the total expected cost if we allocate emergency resources $\mu_i, i = 1, \ldots, n$, at time t, i.e.,

$$J(t, I^1(t)) = \sum_{i \in \mathbf{N}} E_p\left[E_{I_i^0}\left(f_i\left(\mu_i, I_i^0, t\right) \middle| I_i^1(t) = m_i, p\right) \middle| I_1(t) = m\right]. \tag{17}$$

From (14) and (16),

$$\begin{aligned} &E_p\left[E_{I_i^0}\left(f_i\left(\mu_i, I_i^0, t\right) \middle| I_i^1(t) = m_i, p\right) \middle| I_1(t) = m\right] \\ &= E_p\left\{\left(\tilde{b}_1^i + \tilde{b}_2^i(\mu_i \gamma_i(t))^{-2}\right) \right. \\ &\quad \left. \times \sum_{k_i = m_i}^{K_i} \binom{K_i - m_i}{k_i - m_i} \frac{[(1-H(t))p]^{k_i - m_i}(1-p)^{K_i - k_i}}{(1-H(t)p)^{K_i - m_i}} k_i \middle| I_1(t) = m\right\} \\ &= \left(\tilde{b}_1^i + \tilde{b}_2^i(\mu_i \gamma_i(t))^{-2}\right) E_p\left(\frac{m_i(1-p) + K_i p(1-H(t))}{1-pH(t)} \middle| I_1(t) = m\right). \end{aligned}$$

Notice that $\frac{(1-H(t))p}{1-H(t)p}$ is the conditional probability of a random passenger showing symptoms after t, given that they haven't been symptomatic by time t.

Let

$$D_i(t) = \frac{m_i(1-p) + K_i p(1-H(t))}{1-pH(t)} \tag{18}$$

and

$$B_i(t) = E_p(D_i(t) | I_1(t) = m)$$

where $B_i(t)$ is independent of μ_i and the expectation is calculated according to (15). Note that $D_i(t)$ is the expected number of infected passengers given the information at t when p is a known constant. In addition, from expression (18), $D_i(t)$ is the weighted average of m_i and K_i. Now the allocation problem discussed in Section 3.1 is:

$$J(t, I^1(t)) = \min_{\mu}\left\{\sum_{i \in \mathbf{N}} \frac{\tilde{b}_2^i B_i(t)}{(\mu_i \gamma_i(t))^2} | \sum_{i \in \mathbf{N}} \mu_i = R; \mu_i \geqslant 0 \quad \forall i \in \mathbf{N}\right\}$$

Similar to Section 3.1, where $\tilde{b}_2^i \frac{B_i(t)}{\gamma_i(t)^2}$ replaces b_2^i, the optimal solution when either $\mu_i^0 = 0\ \forall i \in N$ without transfer of existing resources, or when some $\mu_i^0 > 0$ with the possibility of transferring existing resources, is:

$$\mu_i^* = \frac{\sqrt[3]{\tilde{b}_2^i \frac{B_i(t)}{\gamma_i^2(t)}}}{\sum_j \sqrt[3]{\tilde{b}_2^i \frac{B_j(t)}{\gamma_j^2(t)}}} R'. \tag{19}$$

The optimal allocation given by (19) is similar to the optimal allocation (8). The weights are affected now by the information at time t and by the different efficacy at every city. Observe that if the efficacy functions are identical for all cities, then the solution is independent of the γ_i's. The difference between the solution (8) and the solution with learning and delay (19) is due to the change in the beliefs about the initial disease parameters given by $B_i(t)$ and the lost of efficiency because of the delay given by $\gamma_i(t)$. A city where the updated beliefs that the initial number of infected passengers is increasing at time t will receives higher amount of resources while a city with a lower efficacy of resources at time t will also receive more resources since the weights are divided by $\gamma_i(t)$.

5.2. Constant p

If p is a known constant, then the objective function is

$$\min_{\mu_i, i \in N} J(t, I^1(t)) = \sum_{i \in N} E_{I_i^0}\left(f_i\left(\mu_i, I_i^0, t\right) \middle| I_i^1(t)\right) = \sum_{i \in \mathbf{N}} \frac{\tilde{b}_2^i D_i(t)}{\mu_i^2 \gamma_i^2(t)},$$

where $D_i(t)$, defined in (18), is independent of μ_i, $\forall i$. $D_i(t)$ is the equivalent of $B_i(t)$ in the case where p is a constant. It is the expected number of infected passengers given the information at t, i.e., $D_i(t) = E\left[I_i^0 | I_i^1(t)\right]$. Note that $D_i(t_{\max}) = I_i^0$. Then, the optimal solution to the allocation problem discussed now is:

$$\mu_i^* = \frac{\sqrt[3]{\tilde{b}_2^i D_i(t)/\gamma_i^2(t)}}{\sum_{j \in \mathbf{N}} \sqrt[3]{\tilde{b}_2^j D_j(t)/\gamma_j^2(t)}} R'. \tag{20}$$

Observe that the optimal allocation (20) is a modification of the solution (8) of the problem without learning. The original weights $\sqrt[3]{b_2^i}$ in (8) are now $\sqrt[3]{\tilde{b}_2^i D_i(t)/\gamma_i^2(t)}$

5.3. Estimation of $\gamma(t)$

The estimation is for a given I^0 and based on the equation

$$f(\mu, t) = [\tilde{b}_1 + \tilde{b}_2/(\mu\gamma(t))^2] I^0 \tag{21}$$

where t is the time when vaccination starts and the index i is omitted. Assume that $\gamma(t) = \mathrm{e}^{-\alpha(t - t_{\min})}$. This assumption suggests that when $t = t_{\min}$ the resources have their maximum efficacy ($\gamma(t) = 1$) and when t is increasing, the efficacy is declining. Obviously, this model is valid for $t = t_{\min}, \ldots, t_{\max}$ since else, f is not bounded. We can use any epidemic dynamic modes such as that of Bauch et al. (2003) to calculate the value of $f(\mu, t)$ for pairs of μ and time delay t. The value of $\tilde{b}_1 I^0$ might be interpreted as the number of deaths if μ_i^s so large that the entire population is vaccinated immediately. Thus, $\tilde{b}_1 I^0$ should be equal to the number of people that get infected until $t_{\min}$ and die. Thus, $\tilde{b}_1 = \delta$ where δ is the death rate and it is assumed that vaccination is not efficient for the initial disease carrier. We can rewrite (21) as

[12] Observe that substituting $b_j^i = \tilde{b}_j^i I_i^0, j = 1, 2$ in (7) gives the Eq. (16) and it is justified since (7) is an approximation to (3) and the parameters in (16) have a linear dependency in I_i^0.

$$\ln(f(\mu,t)/I^0-\delta)=\ln\tilde{b}_2-2\alpha t_{\min}-2\ln\mu+2\alpha t. \quad (22)$$

Now we can estimate $\tilde{b}_2$ and α by regression to minimize LSE (Least Square Errors) of (22) with respect to the variables $\ln\mu$ and t. Note that the estimation is not based on data since there is no existing data. Instead we run the KCW model and then find the best $\tilde{b}_2$ and α such that our approximated cost function is as close as possible (best fit in minimum square error) to KCW.

When allocating resources too early, possibly better information on the number of infected people in each destination city may be missed; when allocation is too late, many people may get infected. Therefore, an interesting line of investigation to pursue further would be to optimize the timing for allocating resources. There are several issues that make this line of investigation difficult, the most important being the identification of a correct epidemic model that captures the effect of delayed response beyond the threshold of 5–6 days. Nonetheless, this is an important issue to be considered.

6. Case study

In this section, we demonstrate our models with a hypothetical case using some real airport data.[13] For simplicity, we consider a small airport (we ignore the fact that terrorists will probably choose a much larger airport to attack), Long Beach Municipal Airport in Long Beach, California (airport code LGB). It is the most accessible, centrally located alternative for air travel in and out the Los Angeles, South Bay and North Orange County areas.

6.1. Data and parameter description

On April 15, 2002, there were 13 flights arriving at Long Beach Airport and 12 flights taking off with destination cities (airport codes): Colorado Springs, CA (COS), Dallas/Fort Worth, TX (DFW), Minneapolis, MN (MSP), New Orleans, LA (MSY), New York, NY (JFK), Newark, NJ (EWR), Philadelphia, PA (PHL), and Phoenix, AZ (PHX). Among all the flights departing LGB on April 15, 2002, there were 5 direct flights and 7 flights connecting to some other airports and then going to their final destinations. All the non-direct flights departing LGB had only one stop before their final destination. For simplicity, we assume that all flights are direct flights from LGB and there is only one flight for each destination. Also, 3000 passengers arrived at LGB which is their final destination. We assume that each flight has capacity for 300 passengers and that each flight is full; because of the lack of specific information we assumed that the airplanes are Boeing 777. Airplanes with smaller capacity may be more realistic for a small airport but appropriate for many bigger airports. Moreover, we believe that the conclusion about the impact of the attack will remain similar. The population size[14] in each of the cities included is shown in Tables 2.

We assume that an attack on the airport has the potential of infecting passengers in all flights discussed above. Note that both EWR and JFK have the same population size because both serve the same metropolitan areas. Thus, we add their infected passengers when calculating the impact of the attack. The calculation below uses values found in Table 1. We also assume:

- the total number of emergency resources analyzed is $R = 5000$, 20,000;[15]
- the probability, p, that a passenger gets infected is 0.4;
- there are no existing resources prior to the attack in all the cities involved.

Therefore, the expected number of infected passengers arriving at each airport of destination $(E(I_i^0))$ is $(300)(0.4) = 120$ and LGB sees $(3000)(0.4) = 1200$ infected arriving passengers. Tables 3 shows the expected number of infected passengers at stage 0 of the disease arriving at each metropolitan area.

6.2. Model illustration

Since the problem with 9 cities is relatively small, we use the cost function (5). Table 4 presents the calculation of the number of people infected by the disease in each disease stage at time $t_{\min} = 5$. The results are direct application of expression (A.2). Note that $I_j^{(i)}$ are functions of the number of initially infected passengers and not functions of the population associated to passengers. Applying algorithm *MA* when the total amount of resources available is $R = 5000$, 20,000, gives us the results shown on respectively Tables 5 and 6 (the columns headings are self explanatory).

It seems that the number of deaths per initial number of infected patients and per hundred thousand people decreases faster when R increases for large cities than for smaller ones. Although, as stated in KCW, the objective function values is more sensitive to the number of initially infected people than to the population of each city, the results show that, when jockeying for the same pool of resources, the optimization algorithm takes into account the population size and larger cities see the reduction in deaths falling to less than half, when doubling the amount of resources, while smaller cities do not feel the same effect. This effect can be easily noted by comparing the changes, as function of the amount of resources available, between COS and PHL.

We solved Problem (MMS) for the same example considering that all initially infected passengers could in fact go to any of the eight possible destinations. When minimizing the maximum number of deaths (the worst case) we find the following results shown on Tables 7 and 8. The solution in Table 7 is to assign the resources according to the first column (for example, 1483 resources to New York City) and the expected number of total deaths is 15,480, which is also the maximum number of deaths in all cities.

Tables 5 and 6 show the interesting result that Algorithm *MA* allocates resources heavily influenced by the number of initially infected passengers that flew to each city. However, when one wants to minimize the maximum number of deaths (the worst case using solution of problem *MM*), then as seen in Tables 7 and 8 the population size plays a major role in the number of resources allocated.

The number of deaths also follows a similar pattern when Algorithm *MA* is utilized. As seen in Tables 5 and 6, the number of initially infected passengers has a strong influence on the total number of deaths. However, the result when facing Problem *MMS*, as seen in Tables 7 and 8, shows a leveling trend. By trying to minimize the maximum number of deaths, the cities see similar potential losses in absolute values—there is no strong relationship with the initial number of infected passengers. City size does play a role obviously.

Considering the case when $R = 20{,}000$, the minimization of the maximum number of deaths results in the worst case solution of 1875 deaths. This number of deaths assumes that the worst case solution is applied but if the decision makers knew with certainty the number of infected passengers were indeed equals to the expectations $(E(I_i^0))$, by allocating according to the solution proposed by Algorithm *MA*, the total number of deaths would have been 1622. Thus, we can say that the price of information in this case is measured in number of lives lost and it is 253

[13] We are grateful to Professor Mara Lederman at Rotman School of Management, University of Toronto for providing the airline data.

[14] According to Census of 2000.

[15] The parameters we use for the example are in the same scale as the parameters used by KCW. Note that, as long as the size of the attack is much smaller than the population size, it seems that KCW model is not sensitive to the scale of the attack.

Table 2
Population of metropolitan areas associated with each airport.

City airport code (index i)	LGB (1)	EWR (2)	MSP (3)	PHX (4)	MSY (5)
Population	14,531,529	19,549,649	2,538,834	2,238,480	1,285,270
City airport code (index i)	DFW (6)	COS (7)	PHL (8)	JFK (9)	
Population	4,037,282	397,014	5,892,937	19,549,649	

Table 3
Expected number of initially infected people at time zero in each metropolitan area.

City airport code (index i)	LGB (1)	JFK + EWR (2)	MSP (3)	PHX (4)	MSY (5)
Expected number of infected passengers	1200	240	120	120	120
City airport code (index i)	DFW (6)	COS (7)	PHL (8)		
Expected number of infected passengers	120	120	120		

Table 4
Number of infected people in each stage of the disease at the moment of detection ($t_{\min}$ = 5 days).

Metropolitan area	Airport code	$I_1^{(i)}$	$I_2^{(i)}$	$I_3^{(i)}$	$I_4^{(i)}$
Los Angeles	LGB (1)	492.12	806.10	172.39	1436.5
New York	JFK + EWR (2)	98.42	161.22	34.48	287.30
Minneapolis	MSP (3)	49.21	80.61	17.24	143.65
Phoenix	PHX (4)	49.21	80.61	17.24	143.65
New Orleans	MSY (5)	49.21	80.61	17.24	143.65
Dallas/Fort Worth	DFW (6)	49.21	80.61	17.24	143.65
Colorado Spring	COS (7)	49.21	80.61	17.24	143.65
Philadelphia	PHL (8)	49.21	80.61	17.24	143.65

(1875 – 1622) which is an increase of 15% above the initial expectation.

Conversely, it could happen that decision makers, assuming particular $E(I_i^0)$ allocate resources according to Algorithm *MA*. For example, suppose R = 20,000 as above and 2160 infected passengers were believed to be distributed according to Table 3 (total) when, in fact, they were all flying to a single metropolitan area. In this case, we calculated the total number of deaths, which is equal to 2135; fostered by a wrong assumption. However, if the mini-max was used then the number of deaths would had been 1875. Thus, the wrong assumption had a cost of approximately 260 lives or close to 16%.

Table 5
Results for R = 5000 (units of resources).

	Resources allocated	Total number of deaths	Deaths per $E(I_i^{(0)})$	Deaths per 10^5 people
Los Angeles	1950	3208	2.67	22.08
New York	1373	2032	8.47	10.40
Minneapolis	282	479	4.00	18.86
Phoenix	259	449	3.74	20.07
New Orleans	180	338	2.82	26.32
Dallas/Fort Worth	382	621	5.17	15.37
Colorado Springs	83	204	1.70	51.30
Philadelphia	491	769	6.41	13.05
Totals	5000	8100		

Table 6
Results for R = 20,000 (units of resources).

	Resources allocated	Total number of deaths	Deaths per $E\left(I_i^{(0)}\right)$	Deaths per 10^5 people
Los Angeles	7953	648	0.54	4.46
New York	5339	289	1.20	1.48
Minneapolis	1127	115	0.96	4.52
Phoenix	1041	112	0.93	5.00
New Orleans	737	102	0.85	7.90
Dallas/Fort Worth	1513	127	1.06	3.16
Colorado Spring	363	88	0.74	22.23
Philadelphia	1927	141	1.18	2.40
Totals	20,000	1622		

Table 7
Results for R = 5000 – minimizing the maximum number of deaths.

	Resources allocated	Total number of deaths	Deaths per $E\left(I_i^{(0)}\right)$	Deaths per 10^5 people
Los Angeles	1219	12900	10.75	88.77
New York	1483	15480	64.50	79.18
Minneapolis	386	4716	39.30	185.78
Phoenix	355	4418	36.82	197.40
New Orleans	247	3328	27.74	258.96
Dallas/Fort Worth	523	6093	50.77	150.92
Colorado Spring	115	1980	16.50	498.63
Philadelphia	672	7537	62.80	127.90
Max. Deaths		15,480		

Table 8
Results for R = 20,000 – minimizing the maximum number of deaths.

	Resources allocated	Total number of deaths	Deaths per $E\left(I_i^{(0)}\right)$	Deaths per 10^5 people
Los Angeles	4093	1463	0.68	10.06
New York	6010	1875	0.87	9.59
Minneapolis	1665	1029	0.47	40.53
Phoenix	1543	1002	0.46	44.79
New Orleans	1107	906	0.42	70.48
Dallas/Fort Worth	2214	1143	0.53	28.32
Colorado Spring	565	773	0.34	194.79
Philadelphia	2803	1261	0.58	21.41
Max. Deaths		1875		

A similar discussion could be done for the case of using Algorithm *MM*. Comparing the results with those obtained by using Algorithm *MA* results in similar analysis and insights which we will omit from this paper.

6.3. Alternative policies: myopic approaches

Here we show the advantage of our approach for allocating resources over a myopic policy that may be used by the government. In this policy, each city i receives the proportion of the total resources corresponding to the proportion of the total number of infected passengers reported. When $R = 20{,}000$, the number of deaths induced by applying the myopic policy is 2224 in comparison to 1622 deaths when MA is applied (see Table 6); an increase of 37% in number of fatalities. Analogous results hold when $R = 5000$. The myopic policy discussed above ignores the cities' population size. A second myopic approach could be to allocate resources according to the fraction of the city's population to the total population. When the number of resources is 20,000 the gap is close to 10%. When the number of resources $R = 10{,}000$ (details are not shown), applying this policy yields an increase of 16% in fatalities; a significant increase but not as high as the one induced by the first myopic policy. Obviously, both myopic policies would be very inefficient for the worst case scenarios but comparing them to that case is meaningless.

7. Conclusions and future research

In this paper, we considered a potential bioterror attack on an airport. Once the attack is identified the government has to decide how to allocate limited resources. The cost function which represents the number of death is derived either by well known epidemic models or through regression analysis.

The allocation problem is discussed assuming information or lack of information on the number of infected passengers identified prior to the decision time. We showed that simple marginal analysis algorithms can solve the problem. We also solved the problem of allocating resources given estimates of the total number of initially infected passengers (scenario analysis could be developed for estimating the number of infected passengers and allocate resources prior to an attack as proactive measure). In this case, we showed how to allocate resources given information only on the total number of infected passengers. With this information the problem solved is to find optimal allocation of resources so as to minimize the maximum number of deaths. We also provide an analysis to solve the allocation problem taking into account information on infected passengers.

Finally, we presented a small case study using data from the Long Beach Airport and twelve flights to different airports. The case study illustrates the approach discussed in Sections 3 and 4. This approach was compared to alternative myopic policies for the case study. We showed that the use of myopic policies could be very inefficient. It is important to note that even though our simple model can be used to understand the main underlaying problem of resource allocation and to clarity side effects, decision makers should be careful when using it for policy implications. At the same time we believe that our simple approximation can be handy for performing first cut evaluations. Additionally, our approach will be as precise as the KCW analysis.

We can add another level to the model by considering a game where the terrorist may choose the airport to attack assuming that the authorities will react optimally against any attack. In this case, the terrorist will choose to attack the airport which, after the optimal defender's reaction, will maximize the damage occurred. This model generate a dynamic game between the terrorist and the defender. A similar approach was used by Berman and Gavious (2007).

Future research might include: (i) for a known constant and stochastic p, develop an efficient approach to find the optimal time to allocate the resources; and, (ii) solve the resource allocation problem when the government can dynamically re-allocate the resources over time.

We note that the difficulty with these two problems is that the ordinary differential equations used to model the dynamics of the disease only hold for quick reaction time to vaccinate.

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Supplementary data

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