European Journal of Operational Research 215 (2011) 705–712

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R

Selection of entrepreneurs in the venture capital industry: An asymptotic analysis ☆

Ramy Elitzur [a], Arieh Gavious [b,c,*]

[a] The Rotman School of Management, University of Toronto, 105 St. George St., Toronto, Ontario, Canada M5S 3E6
[b] Faculty of Business Administration, Ono Academic College, Kiryat Ono 55000, Israel
[c] Department of Industrial Engineering and Management, Ben Gurion University, Beer Sheva 84105, Israel

ARTICLE INFO

Article history:
Received 8 September 2009
Accepted 24 June 2011
Available online 14 July 2011

Keywords:
Asymptotic methods
Auctions
Contests
Entrepreneurs
Venture capital

ABSTRACT

We study a model of entrepreneurs who compete in an auction-like setting for venture capital (VC) funding in a setting where limited capital dictates that the VC can only finance the best entrepreneurs. With asymmetric information, VCs can only assess entrepreneurs by the progress of development, which, in equilibrium, reveals the quality of the new technology. Using an asymptotic analysis, we prove that in attractive industries having a large number of entrepreneurs competing for VC funding could lead to underinvestment in technology by entrepreneurs as the effort exerted by losing entrepreneurs is wasted. The study then proceeds to characterize the conditions under which a greater number of competing entrepreneurs is better. The model also demonstrates that VCs could possibly increase their payoff by concentrating on a single industry. In addition, the study also provides some insights on the effects of multiple investments by VCs and the effects of competition among VCs on the same investments.

1. Introduction

Venture capitalists (VCs) thrive by successfully gambling on a small number of companies that they fund from all the applications that they receive. This study focuses on whether increasing traffic in the VC firm would have a positive effect on the firm or, on the contrary, be counterproductive. Our model considers entrepreneurs who compete for VC funding in an auction-like setting where the VC acts as the auctioneer that sells financing to *n* entrepreneurs who bid for financing. The surprising finding is that having a large number of entrepreneurs who vie for funding can cause underinvestment in technology by entrepreneurs. Moreover, we find that this phenomenon is likely to occur when the industry is very attractive and populated with many high quality entrepreneurs. The reason for this result is that when the number of competitors is high, and there are many entrepreneurs who are likely to have high quality technology, the probability of getting funding from a VC decreases as competition becomes fierce. In turn, unfunded entrepreneurs would lose their development investments and, thus, as a preemptive move, they will reduce their technology investments prior to participation. Another interesting result is that VCs could possibly increase their payoff if they avoid overextending themselves and focus, instead, on a small number of industries. In addition, the study also provides some insights on the effects of multiple investments by VCs and the effects of competition among VCs on the same investments.

Venture capital financing for early-stage companies has dramatically increased in importance in the last two decades, and so has the academic research on this topic. The majority of the VC literature entails descriptive field and empirical studies (see, for example, Sahlman, 1990; Lerner, 1994; Gompers, 1995; Gompers and Lerner, 1999; Hellmann and Puri, 2000; Kaplan and Stromberg, 2002). The theoretical research in this area has largely focused on the mechanism of staged investments (see, for example, Neher, 1999; Wang and Zhou, 2004; Shepherd et al., 2005). Others have investigated whether financing should be provided in the form of debt, equity, or a hybrid instrument (Bergemann and Hege, 1998; Trester, 1998; Schmidt, 2003; Elitzur and Gavious, 2003). Several theoretical studies (see for example, Amit et al., 1998; Ueda, 2004) focus on the raison d'être of VCs and argue that VCs exist because of their ability to reduce informational asymmetries. Specifically, banks and other institutional lenders, in contrast to VCs, cannot distinguish between high and low quality entrepreneurs for such early stage companies. As such, VCs act essentially as financial intermediaries who thrive because of their superior ability to screen and monitor entrepreneurs. While several studies argue that screening prospective

☆ The authors acknowledge the helpful comments made by Yan Alperovych, Thomas Astebro, Fabio Bertoni, Massimo G. Colombo, Gaetan De Rassenfosse, Mariana Mazzucato, Tereza Tykvova and other participants in the VICO final conference "*Entrepreneurial finance: the real impact*". The authors also thank two anonymous reviewers, the Associate Editor, and the Editor-in Chief of this journal. The first author gratefully acknowledges the financial support from the Edward J. Kernaghan Professorship in Financial Analysis.

* Corresponding author at: Department of Industrial Engineering and Management, Ben Gurion University, Beer Sheva 84105, Israel. Tel.: +972 8 6472212; fax: +972 8 6472958.

E-mail addresses: elitzu@rotman.utoronto.ca (R. Elitzur), ariehg@bgu.ac.il (A. Gavious).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2011.06.029

investments by VCs is crucial for the VC's success (see, for example, Zacharakis and Meyer, 2000; Shepherd et al., 2005), or that the VCs' superior ability to do so is the very reason for their existence (Amit et al., 1998; Ueda, 2004, for example), research on the screening pro cess is scarce. As such, this is the focus of this study: the screening process itself and its impact on technology development by entrepreneurs prior to their participation in the funding competition.

Our modeling method is related to the economic literature on private-value contests with incomplete information where many entrepreneurs seek venture capital financing. The venture capitalist has the power to choose the entrepreneur and boost the start-up firm. This type of modeling is different from the case of the double auction where both parties are engaged in simultaneous offers and neither of them has an advantage over the other (see Chatterjee and Samuelson's work (1983) on double auctions). The literature in this field (which includes, for example, Weber, 1985; Hillman and Riley, 1989; Krishna and Morgan, 1997) deals with an auctioneer who benefits from the bids (or efforts) made by the players while assuming a linear cost function. In this sense, our model is related to Moldovanu and Sela (2005) where a non-linear cost function is assumed. However, in contrast to the traditional literature in this field, our model assumes (in order to fit the venture capital industry) that the auctioneer (the venture capitalist in our model) benefits, in addition to the bid, also from the private value of the winner, which represents the firm's quality.

A recent line of literature that is related to our paper in the contests area includes Taylor (1995), Fullerton and McAfee (1999), Moldovanu and Sela (2005), and Fibich and Gavious (2009). However, the significant difference in the current work is that the VC benefits only from the winning bid and the highest technology (i.e., $\max(b_i + v_i)$) as opposed to the contest literature where the auctioneer receives also a payoff from the losing bids (i.e., $\sum_i b_i$).

The paper is organized as follows: Section 2 presents the model. Section 3 provides the analysis of the equilibrium bids. In Section 4 we endogenize the contracting between the VC and the entrepreneur and examine optimal contracting between the parties. Section 5 examines what would happen when VCs compete among themselves on entrepreneurs. Section 6 concludes.

2. The basic model

2.1. Brief description of the model

Consider n entrepreneurs competing for a single investment unit with size P offered by a VC at a cost of capital of d. Usually the decision made by the VC is a "go" or "no go" one. Namely, if the VC and other investors decide to support a new startup firm they will raise and invest as much money as needed. Hence, the investment amount, P, in our setting is independent of the entrepreneur's effort (and is based on the amount required by the entrepreneur to proceed) while the decision whether to invest in the entrepreneur ultimately depends on his effort. For sake of simplicity and without loss of generality we normalize P to 1. It is well accepted in practice that VCs invest a given amount of money per venture (or within a well defined range). Each entrepreneur i invests development effort e_i, $i = 1,2,\ldots,n$ where his idea has a value v_i which is private information and known only to the entrepreneur. The VC observes the efforts made by the entrepreneurs e_i, $i = 1,2,\ldots,n$ and decides on which entrepreneur he invests the investment unit in. The cost of effort for an i entrepreneur is $0.5e_i^2$. Using the investment unit, the entrepreneur starts a firm where it expected value is given by $(v + e)P$. The entrepreneur gets a fraction α of this value where the VC gets the rest. The VC chooses the entrepreneur with the highest effort as the winner. An entrepreneur i's payoff if he wins is $\alpha(v + e)P - 0.5e_i^2$ and his payoff in the case that he loses is the (negative) cost of effort $0.5e_i^2$. The VC's payoff is $(1 - \alpha)(v + e)P - (1 + d)P$.

2.2. Detailed assumptions

We model the selection of entrepreneurs by the VC as an all-pay auction. An-all pay auction is one where all bidders must pay regardless of whether they win the prize and thus, it is used to model tournaments. Araujo et al. (2008) state that, an important example of all-pay auctions is a tournament" (p. 416) since the tools used for analyzing all pay auctions are the same such as applied for tournaments. All-pay auction model makes sense here because when entrepreneurs compete for funding they have already made their investment in the technology (the payment), regardless of whether they get subsequent venture capital financing (the prize). Suppose there are n entrepreneurs competing over VC financing. We assume that the VC will finance $K \geqslant 1$ entrepreneurs, where in Sections 3 and 4 we study the case $K = 1$ and in Section 5 we let $K > 1$. Each entrepreneur i, $i = 1,\ldots,n$ knows the value of his technology v_i where $v_i \in [0,1]$ is private information of entrepreneur i. The value of each entrepreneur's technology, v_i, is drawn independently from a twice continuous distribution $F(v)$ defined over $[0,1]$. It is assumed that F has a strictly positive density $f(v)$,with bounded derivative f'. Observe that the term value of technology" is not in terms of money but in term of quality. As we will see later on, the firm's expected value in monetary units is a linear function of v.

We assume that the entrepreneur takes some actions to develop the product before approaching the VC and reaches a certain phase of development. These actions by the entrepreneurs (often referred to as effort in the game theory and principal-agent literatures, e.g., Amit et al., 1998; Moldovanu and Sela, 2005) are denoted as $e_i \geqslant 0$, $i = 1,\ldots,n$. The cost of these actions is $0.5e_i^2$, $i = 1,\ldots,n$. The specification $0.5e_i^2$ provides a simple cost function ensuring tractable analysis and incorporates costs that are increasing in development effort. Moreover, it is a strictly convex cost function with an increasing marginal cost, a standard assumption in microeconomics modeling.[1] Note that the cost function is the same across all entrepreneurs but they differentiate themselves in their technologies. We assume that e_i, is observed by the VC.

Let P be the VC's expected investment in the winning entrepreneur. We may assume that P is a random variable varying among entrepreneurs.[2] To avoid complexity we assume that all n entrepreneurs are in the same industry and in a similar stage. This assumption is reasonable as VCs normally specialize in an industry and in a stage of development (e.g., seed, first- or second-round, expansion, mezzanine and so forth). The realization of the investment is unknown to the VC and the entrepreneur and becomes known after the winning entrepreneur starts up the firm and the VC raises the money needed (probably, in several investment rounds). Note that, while the ex-post value of the investment is ex-ante unknown to the VC, its range is known. This assumption of having a range of investment amounts by the VC in each stage is consistent with the literature (as shown, for example, in Table V in Gompers (1995)) and actual practice (as evidenced, for example, in the website (n.d.) of Sequoia Capital). Since the VC and the entrepreneurs make their decisions based on their expected payoffs, we can avoid unnecessary complexity (which will not change the results) and define immediately the expected investment made by the VC. We assume that the winning firm's value increases in both the value of the technology, v, and the effort made by the entrepreneur, e. For mathematical simplicity we consider a linear relation between v, e and the firm's value. We assume that winning firm's ex-post value is

[1] Note, that, one can replace the constant 0.5 with any other constant. The advantage of using 0.5 as the coefficient (as opposed to, say, c) is that it provides a tangible and tractable function, without losing generality.

[2] We can define P_i, $i = 1,2,\ldots,n$, to be the VC's investment given that entrepreneur i wins. The investments P_i assumed to be independent and identically distributed (iid) random variables. The distribution of the investments P_i depends on the type of the industry and stage of the start up firm. However, P_i vanishes in the analysis since we consider expected payoffs and what is left is the expectation $E(P_i) = P$.

given by $(v + e)rP$, where $r > 0$ is the expected magnitude of the return on the investment in the firm. In practice, v and e are positive because otherwise the VC will not invest in an entrepreneur who is not exerting an effort, or when the value of the technology is zero. Nevertheless, this formulation suggests that the ex-ante value of the firm is positive even if one of the parameters is zero. The rationale behind having a value to the firm despite having a zero v is that acquiring knowledge, creating a team, and having a research organization is valuable in itself. This assumption is consistent with Zider (1998) who reports that "... should the venture fail, they (the VCs) are given first claim to all the company's assets and technology" (p. 134). To simplify notation we assume that expected level of investment is scaled to one unit namely, $P = 1$.[3] Observe that this setting does not assume a deterministic outcome. The firm may still fail and all investment may be lost, or generate different level of exit payoffs. The underlying assumption is that the expected ex-post value is $(v + e)rP = (v + e)r$. Note that it is possible that the VC will invest in the future additional resources, or approach some other investors, to provide these resources. Our setting does not rule out the last possibility because $(v + e)r$ is expected value and thus includes future events.

The VC observes development progress, e, and cooperates with the winner of the contest, the entrepreneur with the highest development progress. If several entrepreneurs happen to have the highest level of development the VC then chooses randomly among these entrepreneurs. The VC, however, has the option to reject all proposals if none of them are expected to generate a profit. Obviously, the VC may have other criteria, which are not modeled in the current study, to decide on the winning entrepreneur. The VC may have some intuitive criteria, or even private decision criteria, which are not observed by the entrepreneur. One possible way to address this issue is to assume that the VC's decision is randomized although it depends to some extent on the progress made by the entrepreneurs. A mechanism which offers a solution for this difficulty is the Tullock (1980) contest, where the winner is dictated by a random selection and the probability of a player to win is proportional to his progress. Namely, making more effort leads to high chance of winning. In the current study we avoid this complication which leads to much more complex and an intractable model. A different approach for dictating who is the winner is suggested by Lazear and Rosen (1981) where the VC observes the progress, e_i jammed by a noise ensuing in a random decision if the difference between the highest efforts made by the entrepreneur is too close. We avoid these complications since the analysis becomes intractable.

We assume that the sharing rule between the VC and the entrepreneur stipulates that the entrepreneur receives a percentage of the firm's value, α where $0 < \alpha < 1$, while the VC gets $(1 - \alpha)$ of the firm's value. The VC announces α before the contest and he is committed to this sharing rule. In the first part of this paper we assume that α is typical to the VC industry and thus, is an exogenous and known number. This assumption simplifies the mathematics and, thus, we can obtain a closed-form solution. Later on, we relax this assumption and determine, through numerical analysis (in contrast with a closed-form solution), the value of α endogenously.

The VC invests $P = 1$ dollars in the firm ($P = 1$ is common knowledge). We assume, consistent with the literature (see, for example, Mason and Harrison, 2002; Manigart et al., 2002), that the VC requires a certain rate of return (known as hurdle rates), d, where $d > 0$ (namely, the expected opportunity cost of resources for the VC is $d \times P$ and the VC aims for expected profits above $(1 + d) \times P$). We also assume that $(1 - \alpha)r \geqslant 1 + d$. This latter assumption ensures that the VC will be involved only in areas with strictly positive expected return.

[3] We found that assuming an investment of $P \neq 1$, instead of a single monetary unit, does not add much to our analysis.

Table 1
Summary of notations.

Symbol	Explanation
α	The share of the firm retained by the entrepreneur after the investment by the venture capitalist
d	The VC's hurdle rate
e_i	Actions taken by the entrepreneur to develop the technology
$e(v)$	Development progress (as a function of v_i)
$E(P_i)$	Expected P_i
$F(v)$	Distribution of v_i
$f(v)$	Density function of $F(v)$
$G(v)$	The probability that an entrepreneur will receive VC funding when the venture capitalist makes K investments
K	The number of entrepreneurs that the VC funds
n	Number of entrepreneurs participating in the auction
P_i	The VC's investment given that entrepreneur i wins
Pr	Probability
r	The expected magnitude of the return on the investment in the firm
$Rhr(v)$	Reverse hazard rate $\left(\text{equal to } \frac{f(v)}{F(v)}\right)$
u_i	The utility of the entrepreneur
U_i	Expected utility of the entrepreneur
v_i	Value of technology of entrepreneur i
$\underline{v}$	The minimum acceptable technology to the VC
v^*	The threshold level of technology set by the VC in the auction
VC	Venture capitalist (acronym)
W	Expected utility of the VC

The utility of entrepreneur i is given by

$$u_i = \begin{cases} -\frac{1}{2}e^2; & lose, \\ \alpha r(v + e)P - \frac{1}{2}e^2; & win. \end{cases} \tag{1}$$

Consequently, since $P = 1$, entrepreneur's i expected utility is

$$U = \alpha r \,\mathrm{Prob}(i \text{ wins}|development\ progress\ e)(v + e) - \frac{1}{2}e^2. \tag{2}$$

Table 1 summarizes the notations we use in this study.

3. One entrepreneur–exogenous contract case

In this section and in the following section we assume that $K = 1$. The case where the VC selects only one entrepreneur ($K = 1$) out of all candidates is realistic when the VC decides that she is going to work only with the industry, or a technology leader. This could be motivated by the desire to avoid conflict of interests as the entrepreneurs might not want enter a contest for VC funding with a VC that is known to be working with their competition. Having an exogenous contract (a sharing rule) between the entrepreneur and the VC is not unreasonable because such sharing rules are standard for a given industry and a given stage of development and well known to both parties.

If the VC selects a winner, his expected payoff is given by

$$V = (1 - \alpha)r(v + e)P - (1 + d)P = (1 - \alpha)r(v + e) - (1 + d). \tag{3}$$

It is clear that if the winning bid results in an ex-ante loss ($V < 0$) to the VC then, the winner would be rejected and thus, the VC has the following constraint with respect to the winner's type:

$$v + e \geqslant \frac{1 + d}{(1 - \alpha)r}. \tag{4}$$

Thus, the VC should have a minimum acceptable level of technology and development level that does not entail a loss. In equilibrium, the entrepreneur reveals his technology through the development progress $e(v)$ and thus the VC can set a threshold level of technology $\underline{v}$ such that $\underline{v} + e(\underline{v}) = (1 + d)/[(1 - \alpha)r]$ where $e(\underline{v})$ is the minimal equilibrium progress made by entrepreneur if she participates in the contest. Thus, the VC dictates the minimum level of progress, $e(\underline{v})$. To summarize, the game stages are:

1. The VC announces the minimum level of progress that he is willing to accept, $e(\underline{v})$.
2. Nature chooses the value of the technology v_i for every entrepreneur i.
3. Every entrepreneur i is informed (privately) about v_i.
4. Entrepreneurs, simultaneously and independently, decide on the level of progress that they make.
5. The VC observes the level of effort made by the entrepreneurs and grant the investment to the entrepreneur with the highest progress.

Assuming a monotonic equilibrium function $e(v)$ *for* the entrepreneurs, the VC maximizes in equilibrium her profit. Accordingly, her ex-ante expected payoff can be represented by (the equalities below follow the assumption that the investments P_i, $i = 1,2,\ldots,n$ are i.i.d random variables)

$$\begin{aligned} W &= \{(1-\alpha)rE(\max_{i=1,\ldots,n}(e(v_i)+v_i)P|\max_{i=1,\ldots,n} v_i \\ &\geqslant \underline{v}) - (1+d)P\}\Pr(\max_{i=1,\ldots,n} v_i \geqslant \underline{v}) \\ &= \{(1-\alpha)rE(\max_{i=1,\ldots,n}(e(v_i)+v_i)|\max_{i=1,\ldots,n} v_i \\ &\geqslant \underline{v}) - (1+d)\}P\cdot\Pr(\max_{i=1,\ldots,n} v_i \geqslant \underline{v}) \\ &= \{(1-\alpha)rE(\max_{i=1,\ldots,n}(e(v_i)+v_i)|\max_{i=1,\ldots,n} v_i \\ &\geqslant \underline{v}) - (1+d)\}\Pr(\max_{i=1,\ldots,n} v_i \geqslant \underline{v}). \end{aligned} \tag{5}$$

The equilibrium progress functions, $e(v)$, and the level of minimum acceptable technology to the VC, $\underline{v}$, entail a sub-game perfect Nash equilibrium. In other words, the VC cannot set a threshold level of technology $v^* > \underline{v}$, which is too high, because each entrepreneur knows that if the highest level of technology (the winner) is below v^*, but still above $\underline{v}$, the VC will not reject him because she would still end up with a positive expected payoff. Consequently, as we discuss later on, any demand from the VC for a threshold v^* that is too high will not be credible. We calculate the symmetric equilibrium progress function.

Proposition 1. *The symmetric monotonic increasing bid is given by*

$$\begin{aligned} e(v) &= \alpha r F^{n-1}(v) \\ &\quad + \sqrt{\alpha^2 r^2 F^{2(n-1)}(v) + 2\alpha r\left(vF^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(s)ds\right)}, \end{aligned} \tag{6}$$

where $\underline{v}$ *is the solution for*

$$\underline{v} + e(\underline{v}) = \frac{1+d}{(1-\alpha)r}.$$

All proofs are relegated to Appendix 1.

It is easy to verify that (6) is increasing. We denote the VC's minimum acceptable technology, which is a function of the number of entrepreneurs, as $\underline{v}(n)$.

Proposition 2. *The VC's minimum acceptable technology,* $\underline{v}(n)$*, is monotonically increasing in n.*

Note that although $\underline{v}(n)$ is monotonically increasing with n it is still bounded below 1 by the assumption that $(1-\alpha)r > 1 + d$. The intuition behind Proposition 2 is that with limited capital, the VC only finances the best project and, thus, having too many entrepreneurs causes underinvestment in technology by low-type entrepreneurs since effort by losers is wasted (when n increases development progress decreases for low levels of technology but increases for high level of technology) and the VC increases the minimum required technology level, $\underline{v}(n)$. The VC can observe e but not v, and, thus, evaluates the value of v from e.[4] Moreover, as the following result demonstrates, $\underline{v}(n)$ is bounded by the ratio of the VC's future value coefficient, $1 + d$ to the share of the VC in the total return on all investments (including development) in the firm, $(1-\alpha)r$. Let $\underline{v}^\infty = \lim_{n\to\infty}\underline{v}(n)$ then;

Corollary 1.

$$\underline{v}^\infty = \frac{1+d}{(1-\alpha)r}. \tag{7}$$

$\underline{v}(n)$ represents the worst case for the VC and, thus, $\underline{v}^\infty$ corresponds to the worst case when the number of entrepreneur is very large. Since when there are many entrepreneurs the one with the marginal value $\underline{v}$ reduces his effort to zero, Eq. (7) can be explained as the worst case for the VC if his payoff depends only on the value of technology and he is not willing to lose money. From (7) and (3) the VC's payoff is $V = (1-\alpha)r\underline{v}^\infty - (1 + d) = 0$. Observe that the assumption $(1-\alpha)r > 1 + d$ guarantees that (7) is bounded below 1. We can write the equation for $\underline{v}(n)$ (by using equations (A.3) in Appendix) as $\underline{v} = \frac{1+d}{(1-\alpha)r} - \alpha r F^{n-1}(\underline{v}) - \sqrt{\alpha^2 r^2 F^{2(n-1)}(\underline{v}) + 2\alpha r \underline{v} F^{n-1}(\underline{v})}$. Note that because $\underline{v}(n) < \underline{v}^\infty$ is bounded away from 1, $F^{n-1}(\underline{v}(n))$ rapidly converges to zero (the convergence rate is exponential). Thus, if the industry is such that the distribution over v is skewed towards high value technology, the minimum required technology level, $\underline{v}^\infty$, approaches the limit with only a few entrepreneurs. Fig. 1 depicts the value of $\underline{v}(n)$ as a function of n when the distribution is $F(v) = v^4$, $r = 2$, $d = 0$ and $\alpha = 0.25$.

As Fig. 1 indicates, $\underline{v}^\infty$ is a good approximation for the minimum technology level required by the VC with as few entrepreneurs as five or six. Moreover, the limit value $\underline{v}^\infty$ is independent of the shape of the distribution (although the convergence is faster for positively skewed distributions). From (5), the VC's expected payoff is given by[5]

$$W = (1-\alpha)rn\int_{\underline{v}}^{1}[e(v)+v]F^{n-1}(v)f(v)dv - (1+d)(1-F^n(\underline{v})). \tag{8}$$

Next, we find the optimal minimum technology level that maximizes the VC's expected payoff. Observe that this minimum technology level, although desirable by the VC, is not supported by the sub-game prefect Nash equilibrium. Denote by v^* the optimal minimum technology level that the VC would like to dictate.

Proposition 3. *The optimal threshold technology level that the VC would like to dictate,* v^**, will exceed the VC's breakeven threshold technology level,* $\underline{v}$*.*

The VC ideally will increase the minimum required level of technology in order to eliminate weak entrepreneurs (those below the minimum level that guarantees non-negative payoffs). At the same time, the VC would take the risk that she could end up with nothing if the best entrepreneur is between v^* and $\underline{v}$, the interval where it is still profitable to support the firm. However, this choice of $v^* > \underline{v}$ by the VC is not credible (it is merely 'cheap talk') because nothing would prevent her from changing her mind ex-post as she would prefer to invest in a firm with technology level v such that $v^* > v \geqslant \underline{v}$, if this happens to be the maximum she gets from the n entrepreneur. Thus, if the VC has no way to guarantee that she will not accept technology below v^*, an entrepreneur with

[4] This, in essence, states that the VC can calculate the equilibrium $e(v)$ and, in turn, reverse-engineer v by mapping e to the corresponding value of v.

[5] Observe that in equilibrium if $e(v)$ is increasing then, the entrepreneur with the highest valuation v_i also invest the highest effort $e(v_i)$ and thus, $1 = \max_{i=1..n} e(v_i) + \max_{i=1..n} v_i$. It follows that the distribution of $\max_{i=1..n}(e(v_i) + v_i)$ is the distribution of $\max_{i=1..n} v_i$ given by $nF^{n-1}(v)f(v)$.

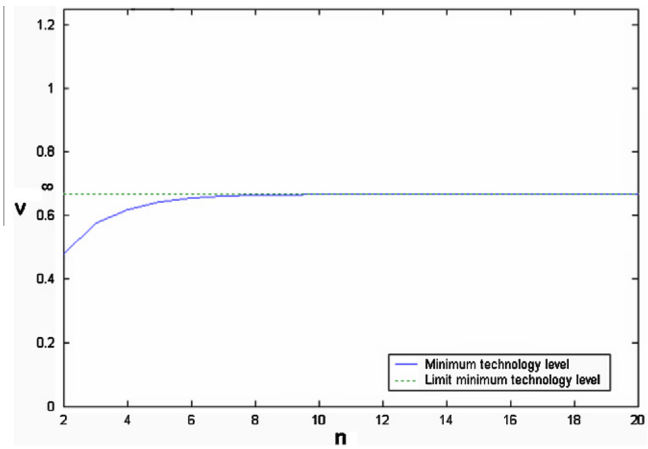

Fig. 1. The value of $\underline{v}(n)$ as a function of n.

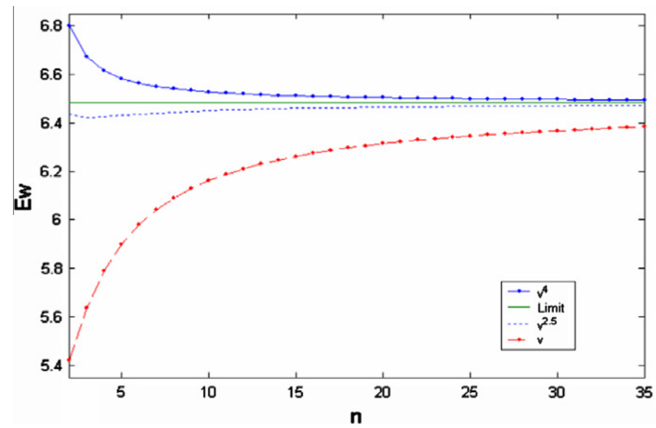

Fig. 2. The expected payoff of VC as a function of n.

technology $v^* > v \geqslant \underline{v}$ may still participate in the contest despite the limitation by the VC. The reason for this is that the entrepreneur is hoping to be the one with the highest v and receive VC funding because the development stage is above the VC's break-even threshold level, $\underline{v}$.

Next, we provide some characterization of when having more entrepreneurs is better (i.e. when the optimal number of entrepreneurs is infinite) and show how it depends on the shape of the distribution of types. First, we denote reverse hazard rate[6] as $Rhr(v) = \frac{f(v)}{F(v)}$. The reverse hazard rate in this context is the probability of observing an outcome in a neighborhood of v, conditional on the outcome being no more than v. Then, $Rhr(v)$ would be non-increasing at the maximum technology level if $Rhr'(1) = \left(\frac{f(v)}{F(v)}\right)'_{v=1} \leqslant 0$, which is equivalent to $\frac{f'(1)}{f^2(1)} \leqslant 1$. In the following proposition, we investigate the optimal number of participating entrepreneurs in a contest for VC funding.

Theorem 1. *If the density of types at the maximum technology is large and Rhr′ (1) is non positive then, the optimal number of entrepreneurs in the auction will be finite.*

The above proposition shows that if the density level of technology $f(1)$ is likely to be high then the optimal number of entrepreneurs is finite (for instance, n could be 2). Observe that since the distribution is continuous, a large $f(1)$ implies that the distribution of technology carries a high weight near $v = 1$. A distribution of the form $F(v) = v^\beta$, $\beta > 1$ also has this feature. Sometimes in auctions and contests the revenue for the seller does not monotonically increase with the number of entrepreneurs (see for example Moldovanu and Sela, 2005). This is not straightforward in the current model as the firm's value in equilibrium depends on the sum of $e(v) + v$ where the VC takes the maximum over all n entrepreneurs. Holding v fixed then, when n is increasing, the equilibrium progress function, $e(v)$, is decreasing in n for low v and increasing in n for large v. There are two different impact of increasing the number of entrepreneur on the expected revenue for the VC. On one hand, while n is increasing the entrepreneurs with high technology are increasing their effort. On the other hand, increasing n forces entrepreneur with lower technology to reduce their effort. In all contest-like models there is a tension between these two phenomena. In the current model, when there is high density at $v = 1$, every entrepreneur expects fiercer competition as n is increasing. Note that the increase in the effort for entrepreneur with higher technology does not compensate for the reduction in effort for entrepreneur with lower technology.

Fig. 2 demonstrates that for $\alpha = 0.2$, $d = 0$, $r = 4$, $F(v) = v^\beta$, the expected revenue for the VC, as a function of n for $\beta = 1$, is increasing with the number of entrepreneurs and strictly decreasing with n if $\beta = 4$. Moreover, for $\beta = 4$ the optimal number of entrepreneurs is two. Finally, when $\beta = 2.5$ the expected revenue is not sensitive to the number of entrepreneurs although it starts off by decreasing and then increasing with n.

4. One entrepreneur-endogenous contract case

In this section, we relax the previous assumption of an exogenous market-determined sharing rule between the VC and the winning entrepreneur's, α, and let the VC dictate α before the contest. We also assume that the VC commits to this α and cannot change her mind later on. Thus, we search for a sub-game prefect Nash equilibrium, assuming that in the next stage the entrepreneurs will play their equilibrium strategies, given the sharing rule α. In Corollary 2 below we characterize the optimal α.

Corollary 2. *The optimal sharing rule between the VC and entrepreneur, α, satisfies the following equation*

$$\int_{\underline{v}}^{1} \left[(1-\alpha)\frac{de(v)}{d\alpha} - (e(v)+v)\right]F^{n-1}(v)f(v)dv = 0.$$

Finding a closed-form solution for α is very complex, and so instead we use the VC's payoff, W, from (8) to numerically solve for the optimal α. Obviously, the solution depends on the distribution $F(v)$. However, since for large n the expected profit for the VC is close to the limit value we can use the limit and obtain an approximate solution. This solution is independent of the distribution and is still close to the optimal value.[7] Fig. 3 depicts the VC's expected profits as a function of the entrepreneur's share, α (represented by the dotted line), for five entrepreneurs, $r = 4$, $d = 0$, $F(v) = v^4$, where the solid line represents the expected profit of the VC at the limit when the number of entrepreneurs approaches infinity. Fig. 3 demonstrates that there is a maximum α above which there will be diminishing incremental returns for the VC and that α is close to the optimal α if we use the limit function instead.

[6] The reverse hazard rate is commonly denoted as $\sigma_F(v)$. The ratio is also known as inverse Mills' ratio.

[7] It seems from the proof of Proposition 4 that when the industry is abundant with entrepreneurs holding high quality technologies (i.e., high density near $v = 1$), the convergence is even faster, thus, the approximation is good even for a relatively small number of entrepreneurs.

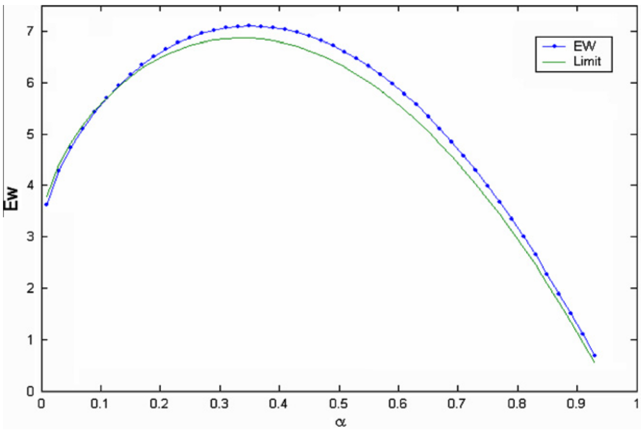

Fig. 3. The expected payoff of the VC, EW, as a function of α.

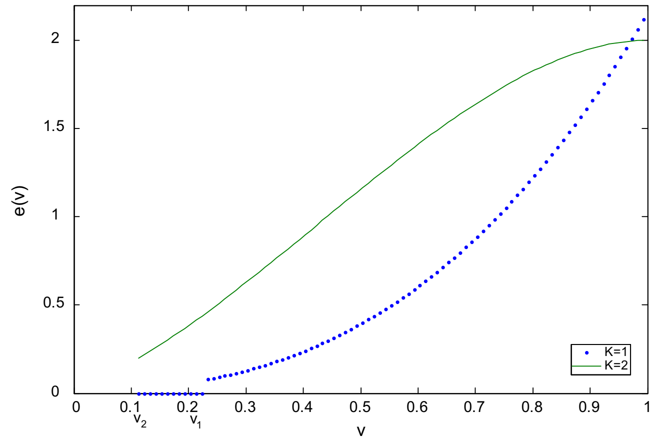

Fig. 4. Development progress for $K = 1, 2$.

5. *K* entrepreneurs–exogenous contract case

In this section, we relax our previous assumption that there is only one VC who makes an investment in a firm. Instead we now assume that the VC has the resources to invest in more than one firm and that this amount is identical for all firms. Namely, the VC intends to invest in the K firms with the highest level of progress and, as previously assumed, the expected investment by the VC is $E(P_i) = 1$, $i = 1,2,\ldots,n$. We assume as before that the VC will not invest if she expects to lose. The case when the VC invests in more than one firm within an industry, or technology, $(K > 1)$, and the entrepreneurs are still willing to participate, implies that the entrepreneurs are so eager to obtain VC financing that they are willing to take a chance and cooperate with a VC who is working with their competition and, hence, could potentially have a conflict of interests. A winning entrepreneur obtains, as previously discussed, α of the firm's value, where α is pre-announced and identical for all winners. The model is a multi-unit auction model but since the demand for each entrepreneur is only for a single unit of investment, the model is similar to the one with a single investment and the equilibrium is given by the following proposition.

Proposition 4. *In the case of K identical investments the equilibrium bid function, e(v), is given by*

$$e(v) = \alpha r G(v) + \sqrt{\alpha^2 r^2 G^2(v) + 2\alpha r\left(vG(v) - \int_{\underline{v}}^{v} G(s)ds\right)}$$

where $G(v) = \sum_{j=1}^{K}\binom{n-1}{j-1}F^{n-j}(v)(1-F(v))^{j-1}$ *is the probability that an entrepreneur will receive VC funding and* $\underline{v}$ *is given in Proposition 1.*

Because the probability of winning for each given technology level v is increasing with the number of investments, K, one might expect that the level of progress made by an entrepreneur to decrease since the competition on VC funding is less fierce. However, this conclusion is not straightforward because, on one hand, the entrepreneur with a high level of technology (i.e., v close to 1) reaches a lower development stage when the number of investments K increases by 1, and, on the other hand, an entrepreneur with a low level of technology (i.e., v close to $\underline{v}$) will make greater progress. Moreover, the minimum technology level required by the VC, $\underline{v}$, will be lower.

Proposition 5. *Increasing the number of investments, K, by the VC would increase the development progress made by low technology entrepreneurs and decrease the development made by high technology entrepreneurs. Moreover, the VC's breakeven threshold technology level* $\underline{v}$*, decreases with the number of investments, K.*

The value of the threshold technology level, $\underline{v}$, decreases with the number of investments, K. This decrease occurs because the development stage, $e(v)$, increases for low technology levels and thus, the VC can reduce the level of the minimum technology required to guarantee non-negative profits. Fig. 4 provides an example of an equilibrium function $e(v)$ for 1 and 2 investments for $r = 4$, $\alpha = 0.2$, $n = 4$, $d = 0$ and uniform distribution. In this example, the progress function $e(v)$ for the two investments is above the one relating to a single investment, except when the technology parameter, v, is very close to 1.

Using the same example for a setting where the VC has two investments we may guess that she will prefer to invest in two different industries. Assume that the two industries are independent with respect to the entrepreneurs' behavior and that the VC find $n = 4$ entrepreneurs in each industry. We compare the VC's expected profits from two investments in different industries to the profit when she invests the two units in a single industry.[8] For simplicity we assume that although there are two different industries in this example, the expected investments in a firm in both industries is the same and scaled to one as we did in the previous sections namely, $P = 1$ in both industries. This phenomenon however is confusing. On one hand, we have two investments in one industry with four entrepreneurs, which should boost the entrepreneurs' willingness to develop to a further stage since there are more investments available to them (see Fig. 4). However, investing in two industries introduces a total of eight entrepreneurs, which, in turn, increases the possibility for a promising technology. In our example, the expected revenue from one investment in one industry with four entrepreneurs is 5.786 and thus, the VC's total expected revenue from the two industries is $5.786 \times 2 = 11.572$. However, in this example, when the VC invests in one industry her expected revenue is higher. She obtains from the first winner 7.189 and from the second winner 5.28. Observe that in this example the $f(1) = 1$ is not high and thus, the result is not driven by the increases in n as we have found in Proposition 4. The practical implication of this result is that spreading into different industries not necessarily increases the VC profits, which could

[8] This setting is different from the common models in contests. Usually, in contests the focus is on dividing the n competitors into subgroups where the total number is fixed. Here, the alternative is many groups with the same size as the single group, which increases the total number of entrepreneurs.

provide some intuition for VCs' tendency to specialize in terms of the industries that they invest in.

Let us now consider a scenario with competition in the same industry among K VCs, each with a single unit of investment and a constant exogenous α. Every entrepreneur in this case would approach all VCs and thus,[9] the model is equivalent to a situation of a single VC with K investments (where K is the total number of investments available by all VCs) and the analysis above still holds. In this setting, the K entrepreneurs with the highest progress win since all the VCs observe the same level of progresses made by the entrepreneurs. The only piece still missing is matching between winning entrepreneurs and the VCs (i.e., which VC gets the entrepreneur with the highest progress made, which one gets the second highest and so forth). The mechanism of market clearing in this setting, however, is not covered in our analysis. We learned from the previous example that the total expected profits of all VCs might be higher than the setting where each VC becomes a monopolist in a different industry. However, we cannot conclude that all VCs will ends up with higher expected payoff since the allocation of winning entrepreneur to each VC is unknown and thus, some VC's may benefit from competition among VCs and some may lose.

6. Conclusions and summary

A crucial factor in the success of venture capitalists is the quality of the firms that they invest in. The approach that we take in this study models the competition for VC funding as an auction with asymmetric information favoring the entrepreneur. An important insight that this study provides is that having a large number of entrepreneurs who compete simultaneously for VC funds could be suboptimal from the VC's standpoint, especially in industries abundant with high quality entrepreneurs. The intuition behind this is that effort, which is costly, is wasted for the losing entrepreneurs and, thus, if they perceive their chances of winning the auction to be relatively slim many of the better entrepreneurs will opt out. The study also examines the optimal contracting between VC and entrepreneur and sheds some light on a setting with multiple VC investments, and a scenario with competing VCs.

Table 2 below summarizes the numerical results presented in this study. Figs. 1 and 2 depict the case where the contract is exogenous and the VC selects only one entrepreneur. Fig. 3 depicts the case where the contract is endogenous and the VC selects only one entrepreneur. Fig. 4 describes the case where the VC invests in K entrepreneurs and the contract is exogenous.

A possible extension to this paper could involve further investigation of VCs investments in different industries and examine what should be the optimal number of industries that VCs would get into and their characteristics.

A possible extension to the paper could incorporate a different decision rule for the VC. The model in our study adopts a simplified approach and suggests that the VC invests in the entrepreneur who makes the highest progress. One could argue that the VC decision rule should be fuzzy and his ability to observe progress made by the entrepreneurs would be limited. In such situations, the mechanism that dictates who is the winner is partially random. Similar to Lazear and Rosen (1981) we assume that entrepreneur i chooses a level of progress e_i and bears a cost $0.5e_i^2$ however, the VC observes progress $e_i + \varepsilon_i$ where ε_i, $i = 1, 2, \ldots, n$ are iid random variables. Another possible extension to the study could have the VC's decision may depend not just on the level of progress but rather on some private and possibly intuitive parameters. A possible direction to implement this extension would follow the model used in Tullock (1980), thus, having the entrepreneur i winning with the probability $\frac{e_i}{\sum_{j=1}^{n} e_j}$. Lastly, the study can be extended by allowing the investment by the VC to be stochastic but dependent on the entrepreneurs' efforts.

[9] We assume that the entrepreneurs submit the same proposal to all VCs.

Table 2
Summary of the numerical analysis performed.

Setting	Figure	Findings
Investment by the VC in one entrepreneur and the contract is exogenous	Fig. 1	The value of $\underline{v}(n)$ as a function of n increases until it asymptotically converges to $\underline{v}^{\infty}$ with as few entrepreneurs as five or six
Investment by the VC in one entrepreneur and the contract is exogenous	Fig. 2	The expected payoff of VC as a function of n for $\beta = 1$ is increasing with the number of entrepreneurs and strictly decreasing with n if $\beta = 4$. Moreover, for $\beta = 4$ the optimal number of entrepreneurs is two. Finally, when $\beta = 2.5$ the expected revenue is not sensitive to the number of entrepreneurs although it starts off by decreasing and then increasing with n
Investment by the VC in one entrepreneur and the contract is endogenous	Fig. 3	There is a maximum α above which there will be diminishing incremental returns for the VC and that α is close to the optimal α if we use the limit function instead
Investment by the VC in K entrepreneurs and the contract is exogenous	Fig. 4	The progress function $e(v)$ for two investments is above that relating to a single investment except when the technology parameter, v, is very close to 1

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2011.06.029.

References

Amit, R., Brander, J., Zott, C., 1998. Why do venture capital firms exist? Theory and Canadian evidence. Journal of Business Venturing 13 (6), 441–465.

Araujo, A., de Castro, L.I., Moreira, H., 2008. Non-monotoniticies and the all-pay auction tie-breaking rule. Economic Theory 35, 407–440.

Bergemann, D., Hege, U., 1998. Venture capital financing, moral hazard, and learning. Journal of Banking & Finance 22, 703–735.

Chatterjee, K., Samuelson, W., 1983. Bargaining under incomplete information. Operations Research 31, 835–851.

Elitzur, R., Gavious, A., 2003. A multi-period game theoretic model of venture capital investment. European Journal of Operational Research. 144 (2), 440–453.

Fibich, G., Gavious, A., 2009. Large auctions with risk-averse bidders. International Journal of Game Theory 39, 359–390.

Fullerton, R.L., McAfee, R.P., 1999. Auctioning entry into tournaments. Journal of Political Economy 107 (3), 573–605.

Gompers, Paul A., 1995. Optimal investment, monitoring, and the staging of venture capital. Journal of Finance 50, 1461–1489.

Gompers, P., Lerner, J., 1999. The Venture Capital Cycle. MIT Press, Cambridge, Mass.

Hellmann, T., Puri, M., 2000. The interaction between product market and financing strategy: The role of venture capital. Review of Financial Studies 13, 959–984.

Hillman, A., Riley, J., 1989. Politically contestable rents and transfers. Economics and Politics 1, 17–39.

Kaplan, S.N., Stromberg, P., 2002. Financial contracting theory meets the real world: Evidence from venture capital contracts. Review of Economic Studies 70, 281–315.

Krishna, V., Morgan, J., 1997. An analysis of the war of attrition and the all-pay auction. Journal of Economic Theory 72, 343–362.

Lazear, E.P., Rosen, S., 1981. Rank-order tournaments as optimum labor contracts. Journal of Political Economy 89 (5), 841–864.

Lerner, J., 1994. The syndication of venture capital investments. Financial Management 23, 16–27.

Mason, C.M., Harrison, R.T., 2002. Is it worth it? The rates of return from informal venture capital investments. Journal of Business Venturing 17 (3), 211–236.

Manigart, S., De Waele, K., Wright, M., Robbie, K., Desbrieres, P., Sapienza, H., Beekman, A., 2002. Determinants of required return in venture capital investments: A five-country study. Journal of Business Venturing 17 (4), 291–312.

Moldovanu, B., Sela, A., 2005. Contest architecture. Journal of Economic Theory 126 (1), 70–97.
Neher, D., 1999. Staged financing: An agency perspective. Review of Economic Studies 66, 255–274.
Sahlman, W., 1990. The structure and governance of venture capital organization. Journal of Financial Economics 27, 473–524.
Schmidt, K.M., 2003. Convertible securities and venture capital finance. The Journal of Finance 58 (3), 1139–1166.
Sequoia Capital, n.d., Sizing, Retrieved August 9, 2009 from <http://www.sequoiacap.com/sequoia-capital/>.
Shepherd, D.A., Armstrong, M.J., Levesque, L., 2005. Allocation of attention within venture capital firms. European Journal of Operational Research 163 (2), 545–564.
Trester, J.J., 1998. Venture capital contracting under asymmetric information. Journal of Banking & Finance 22 (6–8), 675–699.
Taylor, C.R., 1995. Digging for golden carrots: An analysis of research tournaments. American Economic Review 85 (4), 872–890.
Tullock, G., 1980. Efficient rent seeking. In: Buchanan, Tollison, Tullock (Eds.), Towards a Theory of Rent-Seeking Society. Texas A&M University Press, College Station, TX, pp. 97–112.
Ueda, M., 2004. Banks versus venture capital: Project evaluation, screening, and expropriation. The Journal of Finance 59 (2), 601–621.
Wang, S., Zhou, H., 2004. Staged financing in venture capital: Moral hazard and risks. Journal of Corporate Finance 10 (1), 131–155.
Weber, R., 1985. Auctions and competitive bidding. In: Young, H.P. (Ed.), Fair Allocation. American Mathematical Society, pp. 143–170.
Zacharakis, A.L., Meyer, G.D., 2000. The potential of actuarial decision models; Can they improve the venture capital investment decision? Journal of Business Venturing 15 (4), 323–346.
Zider, B., 1998. How venture capital works. Harvard Business Review, 131–139.