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Decision Aiding

Maximizing political efficiency via electoral cycles: An optimal control model

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Abstract

The exchange between an elected politician, such as a president, prime minister or a local governor and interest groups is analyzed as an optimization problem. The optimal control model shows the conditions required from regulatory policy and resource investment in order to maximize the politician's utility from the interest group's support. Given one interest group, such a policy includes two time intervals: Well in advance of the elections the politician in office should invest a constant level of resources, while for a certain period close to the elections the politician increases or decreases investment, depending on the electoral significance of that interest group. This proves that electoral cycles not only empirically exist, but also maximize the politician's utility from interest groups' support. Given several interest groups, at each point in time, the politician should invest in the group that contribute the most for his or her political interests.

Keywords: Government; Control; Interest groups; Political efficiency; Electoral cycles

1. Introduction

Political interactions often raise optimization problems, yet operational research methods are usually applied to evaluate the social and economic aspects of specific projects or policies (Nagel, 1994). Such methods are seldom used to explain the conditions for maximizing specific interests, e.g., the interests of politicians, interest groups or bureaucrats. The involvement of such diverse interests often leads to different policy outcomes than expected based on traditional policy evaluation approach. For example, a certain policy or a project may maximize social or economic efficiency, but since it does not maximize the special interests of a politician in office, it will not be implemented in the way recommended by traditional policy evaluators. Thus, policy outcomes often reflect the specific interests involved in the decision making process rather than the goal of maximizing social welfare.

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In this paper we focus on two specific interests in the process of political decision making, i.e., the interests of politicians in office and those of interest groups. We analyze the optimization problem that a politician in office faces when attempting to maximize interest groups' support. In particular, we ask whether electoral cycles, which empirically exist (Kau and Rubin, 1982; Fremdreis and Waterman, 1985; Tosini and Tower, 1987), maximize the politician's utility from interest groups' support. This interaction is analyzed by an optimal control model.

The influence of interest groups on public policy and on election outcomes, as well as on social welfare, has been widely discussed by political economists (Barro, 1973; Becker, 1985; Buchanan et al., 1980; Coate and Morris, 1995; Downs, 1957; Grossman and Helpman, 1996; Mitchell and Munger, 1990; Olson, 1965; Pelzman, 1976; Posner, 1971; Stigler and Friedland, 1962). This influence is usually attributed to the exchange between politicians in office and interest groups. The politicians are usually understood to be the suppliers of regulatory services, such as price fixing, subsidies, restriction of entry, promotion of complementary goods and suppression of substitutes. In exchange for these highly valuable services, the regulated industry, i.e., the interest group, can offer politicians campaign contributions, speaking honoraria and votes of industry employees (Austen-Smith, 1993; Mitchell and Munger, 1990).

Starting from this point, political economists try to explain the consequences of these exchanges in terms of economic efficiency and social welfare. Economists such as Barro (1973), Pelzman (1976), Posner (1971) and Stigler and Friedland (1962), theorize that open competition between interest groups that try to maximize their own benefits, also termed "rents", will, under certain conditions, maximize market efficiency and social welfare. On the other hand, other researchers argue that the rents received by interest groups damage economic efficiency as well as social welfare but may maximize political efficiency, i.e., maximize politicians' chances of being re-elected (Buchanan et al., 1980; Coate and Morris, 1995; Ekelund and Tollison, 1980). According to this view, political reality is better explained by assuming that politicians in office aim to maximize political rather than economic efficiency and therefore interest groups' activity should be analyzed in this context.

This paper adopts the second approach, namely, that reality is better explained by assuming that politicians in office try to maximize political efficiency, i.e., their chances of being re-elected, in their interaction with interest groups while economic efficiency enters their calculations only indirectly. Unlike most studies, we do not address the macroeconomic consequences of the interaction between a government and interest groups. Nor do we concentrate on strategies adopted by interest groups as many studies do (Grossman and Helpman, 1996; Lohmann, 1995; Potters and Sloof, 1996). Rather, we focus on a specific optimization problem that is often neglected – i.e., the conditions that will enable a politician in office to invest minimal resources required in order to maximize the support of an interest group, or several ones, during the period between elections and on election day. We do not explicitly model the competition between politicians over the interest group's support although it is implicitly modeled as will be explained later.

The model developed in this paper assumes that a politician in office can expect two different benefits from the interest group's support – one is in terms of financial contribution during the term in office as well as on election day and the second is the benefit in terms of votes on election day. This distinction between two types of benefits helps explaining how electoral cycles maximize the politician's utility from interest groups' support. Yet, studies about the relations between politicians and interest groups usually neglect this distinction and assume that the same electoral considerations guide politicians through the entire term in office.

The model mathematically proves that a regulatory policy that maximizes political efficiency in this interaction is based on the rationale of electoral cycles, i.e., a significant growth in policy regulations in favor of a specific interest group and in allocation of resources for public projects related to this group starting several months before elections until the election day inclusively in order to gain electoral support. [1] This rationale can also be regarded as a means to minimize economic inefficiency, because when political efficiency is maximized, the politicians only invest the necessary resources to maximize support, not more than that. The model for several interest groups shows that a politician in office should never invest simultaneously in both groups, i.e., should not distribute resources among them, but, rather, at each point in time the politician should invest in the group that contributes the most for his or her chances of re-election. We also explain why politicians often do not apply such optimal policy. Finally, the model suggests a formula to calculate the period in time before elections when a politician should start or cease investments, or should switch the investment between groups. The formula also explains the conditions for optimally setting election day by the politician when electoral laws enable such manipulation.

This paper is organized as follows. Section 2 introduces the components of the model and the optimization problem when there is only one interest group seeking for regulatory services. Section 3 analyzes the solution. Section 4 explains the conditions for optimally setting election day by the politician. Section 5 presents the optimization problem when there are several interest groups. Section 6 concludes the analysis.

2. The optimization problem in politician–interest group exchange

In the exchange between a politician in office and one interest group the politician attempts to maximize the interest group's support, which may have various forms both in terms of means and length of support. For example, an interest group may raise a financial contribution to the politicians' electoral effort either on a regular or a special occasion basis. Alternatively, an interest group may mobilize activists or guarantee voters' support on election day. Then, the support is focused on one day and measured by the number of voters. An interest group may also provide a passive support, meaning that it does not take to the streets or mobilize mass demonstrations against the politician as long as its interests are kept unharmed. In constructing the model we are aware of these variations in the form of support, but, in order to include all of them in the model and still keep it as simple as possible, the level of support that a given group provides the politician at time t is defined as $r(t)$. Later, we will explain how the variations in the form of support are expressed in the model and its results. At $t = 0$ the level of initial support is denoted by $r_0 = r(0) \geqslant 0$.

In exchange for the group's support, the politician in office who wishes to maintain or increase this support attempts to invest resources either by direct financial assistance given to that interest group or in the form of regulatory services on a regular or a special occasion basis. Although this policy has to be approved by other players and may finally fail due to institutional or political opposition, the model explains the policy that can potentially maximize the politician's chances of being re-elected and should be the politician's goal. Thus, at every point in time, t, the politician has to spend a certain amount of resources, expressed by $m(t)$, to maintain or increase the group's support.

The planning time horizon is represented by the parameter, T, i.e., the time until the next election. Later in the analysis, T will be treated as a decision variable, thus enabling the politician in office to dictate the value of T. The model is limited to one term in office which means that the time horizon is finite and interaction is not repeated.

The politician in office gains the support of the interest group at a rate which is linearly proportional to the level of support $r(t)$. This is the simplest structure for these relations, yet a more complex structure, e.g., exponential relations, does not significantly influence the core of the model. Let $a \geqslant 0$ be the coefficient for the advantage that

[1] The rationale of electoral cycles as well as the "political business cycle" approach have been widely discussed in the political-economy literature (Alesina, 1987; Beck, 1982, 1984; Hibbs, 1977, 1987; Nordhaus, 1975). These studies focus on the macroeconomic policy and outcomes as a result of electoral considerations. However, in this paper the term "electoral cycles" is used to describe the impact of electoral considerations on a regulatory policy toward interest groups.

the politician gets from the group's support during the term in office, so that for each unit of support the politician enjoys a marginal benefit of a units. The politician's gain from the group's support at every time $t \in [0, T)$ is expressed by $ar(t)$. Note that if $a = 0$ then the politician does not benefit from the group's support during the planning interval. For example, if the group can only provide electoral support through voting on election day, the politician does not benefit from the group's support during the term in office.

In addition, it is assumed that at the end of the interval, when $t = T$, the politician's benefit from the group's support is proportional to the level of support on election day. This support may be in terms of either financial contribution, votes or both. Let $c \geqslant 0$ be the coefficient for the total advantage, so that for each unit of support at $t = T$ the politician in office enjoys a marginal benefit of c units. The parameter c is actually the aggregate of two parameters – the advantage in terms of financial contribution and the advantage in terms of votes. The value of this parameter is determined according to the number of members–voters in the interest group and the financial contribution it can raise on election day. The politician's gain from the group's support when $t = T$ is expressed by $cr(T)$. If $c = 0$ then the politician does not benefit from the group's support on election day. For example, if the group can only provide financial contribution for the politician through the term in office, but very few votes and/or small financial contribution on election day, the parameter a will be large while the parameter c will be very small.

The multiplication $ar(t)$ expresses the support rate that the group supplies the politician long before the election and the multiplication $cr(T)$ expresses the level of support provided on election day when $t = T$. By differently defining $r(t)$ and $r(T)$ as well as distinguishing between the parameters a and c, the model suggests two variables that express the group's support in the politician – the support through the term in office, $r(t)$, and the support on election day, $r(T)$. As will be explained, this distinction between two different benefits that a politician in office can secure from the interest group's support, is very significant for optimizing policy.

The politician's investment, $m(t)$, generates an increasing marginal cost, expressed by shortage of resources available for other purposes. In other words, this investment in getting the interest group's support reduces the number of alternatives for investment, i.e., causes opportunity costs. To express the fact that the marginal cost increases, it is assumed that the marginal cost is proportional to the square of $m(t)$. This is a simple mathematical structure which can be further complicated, but at this stage the assumptions are as simple as possible. [2] Let $b > 0$ be the coefficient of this cost, so that the cost is given by $bm^2(t)$ and it is convex and increasing in $m(t)$.

The possibility of discounting income and expenses throughout the planning interval $[0, T]$ is avoided because it is easy to show that it does not add a significant insight to the model. From the above assumptions it follows that the politician's utility function at each point in time through the planning horizon, is given by $(ar(t) - bm^2(t))$ and the utility function on election day is given by $cr(T)$. As a whole, the politician wishes to maximize the following utility function:

$$\max_{m(t)} \int_0^T (ar(t) - bm^2(t))\,\mathrm{d}t + cr(T), \tag{1}$$

where $m(t) \geqslant 0$.

In the next stage, the model specifies the relation between the level of support, $r(t)$, and the resources spent by the politician, $m(t)$. It is assumed that the trend of the level of support with time, $\mathrm{d}r(t)/\mathrm{d}t$, linearly increases with the level of the politician's investment, $m(t)$. Let $\beta > 0$ be the coefficient for the trend of the level of support, so that for each unit of investment the politician enjoys a marginal support of β units. [3] It is also assumed that at each point in time, t, there is a certain decline in the support level at a rate which

[2] Note that the model as a whole is based on the most simple assumptions and mathematical structures.

[3] The size of the group is ignored, since it is assumed that the size of the group is very large comparing to the maximal possible support level, $r(t)$. This assumption can be released but it allows to avoid some technical problems generated by the non-linearity of the equation. Moreover, relaxing the assumption does not give any new insight.

is proportional to $r(t)$. This decline is expressed by the retreat of a certain percentage of the supporters meaning that as the support is greater the erosion is bigger. The decline expresses retreat of supporters due to either high costs of participation, lack of sufficient benefits from supporting the politician, or attractiveness of other politicians. Let $\alpha > 0$ be the coefficient for this decline, so that for each unit of support there is a marginal decline of α units. For example, if a certain interest group includes a large number of potential voters or, alternatively, can raise a large financial contribution for a politician A, it is attractive for other politicians as well. A politician B may then invest more resources in the interest group as compared to politician A, thus leading to erosion in the support given to politician A. As the support in politician A is greater, politician B will invest more resources in order to attract support meaning that the erosion of support is bigger as the level of support is greater. In that way the decline in support implicitly expresses the competition between politicians over the interest group's support. The dynamic equation representing the development of support with time is given by

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = \beta m(t) - \alpha r(t). \tag{2}$$

Based on the above assumptions, the government's optimization problem is

$$\begin{aligned} &\max_{m(t)} \int_0^T (ar(t) - bm^2(t))\,\mathrm{d}t + cr(T) \\ &\text{s.t.} \quad \frac{\mathrm{d}r(t)}{\mathrm{d}t} = \beta m(t) - \alpha r(t), \\ &\qquad\;\; m(t) \geqslant 0, \;\; r(0) = r_0. \end{aligned} \tag{3}$$

In the following section, the solution for the optimization problem is presented, followed by an analysis of the implications and insights generated by the solution.

3. Optimal resources and level of support over time

This section presents the solution for the optimization problem showing that the exchange between a politician in office and an interest group can be divided into two time intervals in which the politician must invest resources differently. It also enables us to calculate the length of each time interval and the level of support at each time interval.

3.1. The solution for the optimization problem

The solution for the optimization problem is basically implemented by a method of optimal control (Kamien and Schwartz, 1991). The solution is given in the following proposition.

Proposition 1. *The optimal resources spent by the politician are given by*

$$m(t) = \frac{\beta}{2\alpha b}\left(a + (\alpha c - a)\,\mathrm{e}^{\alpha(t-T)}\right). \tag{4}$$

The level of support generated by the optimal resource function, $m(t)$, given in (4) is

$$\begin{aligned} r(t) = {} & \frac{a\beta^2}{2\alpha^2 b}(1 - \mathrm{e}^{-\alpha t}) + r_0\,\mathrm{e}^{-\alpha t} \\ & + \frac{\beta^2(\alpha c - a)}{4\alpha^2 b}\,\mathrm{e}^{-\alpha T}(\mathrm{e}^{\alpha t} - \mathrm{e}^{-\alpha t}). \end{aligned} \tag{5}$$

Proof. See Appendix A. □

This proposition basically provides a practical tool for the politician to plan his or her investment in order to maximize the utility from the interest group's support, assuming that this is the only interest group operating. The rest of this section analyzes the implications of the solution.

3.2. The behavior of the optimal resource function, $m(t)$, and electoral cycles

The solution for the optimization problem shows that the significant parameter in the interaction is the multiplication αT, i.e., whether αT is large enough ($\alpha T \gg 1$) or not. Since the parameter α represents decline in the support that the group provides the politician, α can be considered as the proportional decline for each support unit. Thus,

αT is the average decline in support units through the planning horizon for every support unit. The solution $m(t)$ in (4) shows that if $\alpha T \gg 1$ and the time t is far enough from $T(t \ll T)$, then

$$m(t) \approx \frac{a\beta}{2\alpha b},$$

meaning that the amount of resources that should be invested by the politician does not change over time. When time proceeds and t approaches T, the other component in $m(t)$ presented in (4) becomes significant and influences the amount of resources. When $t = T$, i.e., on election day, the level of resources is expressed by

$$m(T) = \frac{\beta c}{2b}.$$

It follows that the amount of resources the politician should invest at the end of the planning horizon, depends on the value of the parameter c, i.e., the advantage realized by the politician due to the group's support on election day. If $c = 0$ then when $t = T$, $m(T) = 0$. Differentiating $m(t)$ with respect to time yields

$$\frac{\mathrm{d}m(t)}{\mathrm{d}t} = \frac{\beta}{2b}(\alpha c - a)\,\mathrm{e}^{\alpha(t-T)}. \tag{6}$$

The level of resources that should be invested by the politician, $m(t)$, changes over time depending on the parameters α, a, c, β and b. All of these parameters influence the amount of resources, but only the parameters α, a and c determine the sign of the gradient, i.e., whether $m(t)$ increases or decreases with time. The function $m(t)$ increases or decreases in the end of the planning horizon if

$$\alpha > \frac{a}{c} \quad \text{or} \quad \alpha < \frac{a}{c},$$

respectively. In other words, if decline in the support level, α, is higher than the ratio between the advantage from the support before and on election day, the politician in office should increase resources to compensate for the decline and increase the group support near the end. It follows that in planning the optimal level of resources there are two time intervals. In the first, when $t \ll T$, the level of resources, $m(t)$, does not change with time while in the second, when t approaches T, the level of resources, $m(t)$, increases or decreases depending especially on the value of c.

Yet, if the multiplication αT is relatively small then we cannot identify two time intervals because the second component in $m(t)$ presented in (4) is always significant and approximately linear. Assuming that decline in support is not very great, this means that elections are approaching. For example, if elections are set for every four years but an elected politician subjectively regards this T as small, he or she will allocate resources based on electoral considerations at the very beginning of the term in office.

The behavior of the optimal resources function, over time, is introduced in Fig. 1 for $a = 3$, $b = 1$, $\alpha = 1$, $\beta = 3$, $r_0 = 10$, $T = 10$ and for various c, $c = 0, 0.5, 1, 2, 5, 10$ from the bottom up.

The constant investment presented in the first time interval in Fig. 1 aims to compensate for the decline in the support level. In this interval the constant investment does not depend on the value of c. The increase/decrease of the level of resources in the second interval is due to the parameter c becoming significant. In principle, as the parameter c is higher, α is lower and a is higher, the gradient of the optimal resource function in the second time interval increases. For example, if c is high and α is very low and a is high, the group's support is guaranteed and the politician's investment will be low. However, in Fig. 1 the parameters a and α are kept constant and α is not very low. Therefore, as parameter c increases, the

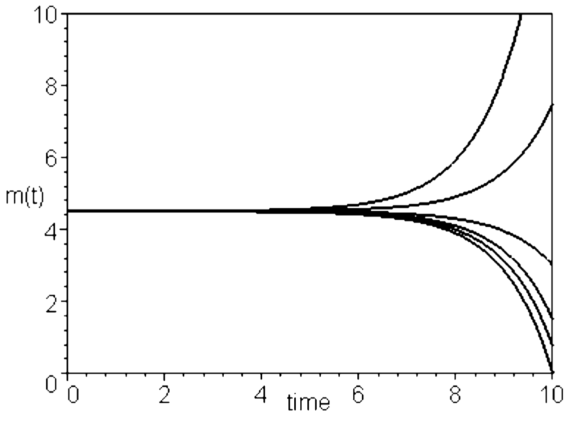

Fig. 1. The behavior of $m(t)$ for various c.

gradient of the optimal resource function in the second time interval is bigger.

Thus, the distinction between two types of benefits that the politician can secure from the interest group's support, seems to be very significant for efficiently planning policy towards the interest group over time. Studies of interest groups often disregard this distinction and, therefore, their analysis, as well as the policy implications drawn from it, may not be sensitive enough to changes in certain parameters over time.

Furthermore, the mathematical model enables calculating the length of each time interval, i.e., the period of time in which $m(t)$ does not change with time and the period in which $m(t)$ increases or decreases. Looking at the derivative $\mathrm{d}m/\mathrm{d}t$ in (6), the non-constant component in the equation is significant when the multiplication $\alpha(t-T)$ is big enough, meaning that the interval where $m(t)$ is non-constant is of order $1/\alpha$. To find the exact point in time, t^*, when $m(t)$ starts increasing or decreasing, $m(t)$ is monitored to recognize a change of, for example, 1% in the level of resources, on the assumption that $\alpha T \gg 1$. Let K be the relative distance between the constant value of $m(t)$, $a\beta/2\alpha b$, and the level where it is monitored that $m(t)$ is different from that constant. That is, when

$$\left|\frac{m(t)-(a\beta/2\alpha b)}{a\beta/2\alpha b}\right| \geqslant K$$

for every t^*, the second time interval begins.

Proposition 2.

$$T - t^* = \max\left\{0, \left(-\frac{1}{\alpha}\ln\left(K\frac{a}{(\alpha c - a)}\right)\right)\right\}. \quad (7)$$

Proof. See Appendix A. [4] $\square$

Proposition 2 suggests a formula to calculate the length of the second time interval. As mentioned, the length of that time interval is of order $1/\alpha$. It is easy to verify, as shown in (7), that the length of the last time interval is independent of β and b. This means that the cost for the politician and the level of support resulting from the politician's investment do not influence the length of time when electoral considerations guide regulatory policy toward the interest group, but, rather, only the benefits of the politician do.

This analysis explains, and mathematically proves, that regulatory policy and resource investment that maximize political efficiency are based on the rationale of electoral cycles. Respectively, self-interest politicians are expected to plan economic regulations and public investments according to electoral considerations when elections are close enough. Note, however, that the interpretations and values given to the parameters a, c and α may differ among politicians and, therefore, different politicians will start the electoral cycles at different points in time. In Section 4 the possibility of manipulating election time will also be analyzed.

3.3. The behavior of the support function, $r(t)$

The optimal resource function generates a certain level of support at each point in time. This support as a function of time is expressed by the support function, $r(t)$. The impact of the politician's investment on the level of support is expressed through the parameter β. The behavior of the support function over time is similar to that of the optimal resource function, with one exception. The dynamic support function develops in three rather than two time intervals.

Referring to the support function, $r(t)$, presented in (5), observe that when t is close to zero and $\alpha T \gg 1$, the expression $\mathrm{e}^{-\alpha T}$ is negligible and $\mathrm{e}^{-\alpha t}$ can be approximated by $\mathrm{e}^{-\alpha t} \approx 1 - \alpha t$. We substitute that into (5) and find that the last expression in (5) is negligible. From the other two expressions we obtain that the asymptotic behavior of the function is

$$r(t) \sim r_0 + \left(\frac{a\beta^2}{2\alpha b} - \alpha r_0\right)t,$$

meaning that $r(t)$ increases near $t = 0$ if

$$\left(\frac{a\beta^2}{2\alpha b} - \alpha r_0\right) > 0$$

[4] The expression "max" is used for technical reasons to avoid a negative sign for the difference $T - t^*$ when αT is not large enough.

and decreases otherwise. Since the parameter β expresses the coefficient for the increase in the support level due to the resources invested, the initial support level, r_0, changes after a short time according to the importance of the group to the politician as expressed by the investment. In other words, starting with a certain level of support, r_0, the level of support increases or decreases in the first time interval and becomes constant in the second time interval for $\alpha T \gg 1$. This constant level is determined according to the politician's investment, which expresses how much the politician in office values the group's support in the period well before the elections. When $t = T$, we find that $e^{-\alpha T}(e^{\alpha T} - e^{-\alpha T}) = 1 - e^{-2\alpha T} \approx 1$, $r_0 e^{-\alpha T}$ is negligible and $1 - e^{-\alpha T} \approx 1$. Substituting that in (5) yields that the level of support provided by the group to the politician when $t = T$ is approximately

$$r(T) \approx \frac{\beta^2(\alpha c + a)}{4\alpha^2 b}.$$

Note that $r(t)$ increases or decreases in the last time interval depending on the value of c but not necessarily depending on the level of resources, $m(t)$. It is possible that there will be a low investment of resources but the interest group will still support the politician. This happens when support is guaranteed and there is little benefit from investing resources as compared to the expected gain.

4. Setting election time, T

The model constructed so far also provides a mathematical formula for the politician to plan the optimal election time given his or her relations with one interest group and the conditions presented so far. If the politician can dictate the election time, T, given that the law specifies the maximal term in office, $\tilde{T}$, such that $T \leqslant \tilde{T}$, T is a decision variable. In this section we analyze some of the properties of the optimal T with respect to the other parameters of the problem. Although the optimal T can be explicitly found, it yields quite complex formula which may confuse rather than clarify. Therefore, the focus is on the significant relations that dictate the optimal election time, T.

Proposition 3. *The optimal interval length, T, satisfies*

$$r(T) = \frac{3\beta^2 c^2}{4b(2\alpha c - a)}, \tag{8}$$

$$-2c \frac{\mathrm{d}r(t)}{\mathrm{d}t}\bigg|_{t=T} = ar(T) - bm^2(T). \tag{9}$$

Proof. See Appendix A. □

The first part (Eq. (8)) of Proposition 3 shows the condition for optimal stopping time. If $c = 0$, then the optimal T is when $r(T) = 0$, as calculated by $r(t)$ in (5), given that $T \leqslant \tilde{T}$ (else $T = \tilde{T}$). If $2\alpha c - a < 0$, then $r(T)$ in (8) is never binding and the optimal T is set when $T = \tilde{T}$. It follows that if in the third time interval drawn from $r(t)$ in (4) the function increases, then T increases with c. Yet, if $r(t)$ decreases then T decreases with c because the politician does not gain, but probably loses, from delaying the elections.

The second part (Eq. (9)) of Proposition 3 shows that the marginal utility for the politician at the end of the planning horizon, $ar(T) - bm^2(T)$, is proportional to the marginal change in support

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t}\bigg|_{t=T}$$

and has an opposite sign. Thus, the optimal stopping time, T, is such that the marginal utility from support, $ar(T) - bm^2(T)$, at the end of the planning horizon is negative or the marginal change in the support level is negative. Thus, the optimal election time expresses a decline either in the utility from support, because the group is not important for the politician during $[0, T]$, or a decline in the support level itself.

5. Optimal resource allocation when there are several interest groups

A politician in office may also face the problem of optimally allocating resources when there are several interest groups looking for regulatory services in exchange for their support. In this section we solve this optimization problem with many groups however, for simplicity, we assume that the politician faces only two interest groups. The generalization of the analysis to interaction with many groups is straightforward.

Basically, the optimization problem when there are two interest groups is composed of similar parameters to those of the model elaborated so far. The politician in office wants to minimize the resources invested in getting maximum public support during the term in office and on election day – either from one or two interest groups. However, he or she has to decide whether to distribute the limited resources between the two interest groups or to invest in one of them. If so, the politician in office also has to decide which of the groups should enjoy regulatory services. We assume that each group is decisive, i.e., can provide the politician with the support necessary for re-election. Still, however, the politician can divide the resources among them.

For this more complex optimization problem we use the same signs and notations as in the original optimization problem. The politician's gain from a group's support at every time $t \in [0, T]$ is expressed by $a_i r_i(t)$, where $i = 1, 2$ is for Groups 1 and 2, respectively. The politician's gain from a group's support when $t = T$ is expressed by $c_i r_i(T)$ for $i = 1, 2$. Also, as before, the politician's investment in getting an interest group's support, $m_i(t)$ for $i = 1, 2$, generates an increasing marginal cost, expressed by shortage of resources available for other purposes. To express the fact that the marginal cost increases, it is assumed that the marginal cost is proportional to the square of $m(t)$. However, since the politician calculates the total amount of resources that he or she has to invest in order to maximize public support – whether it comes from one or two interest groups – then the expression $(m_1(t) + m_2(t))^2$ constitutes the marginal cost. [5] In other words, when planning a policy toward interest groups, a politician in office should see the whole picture both in terms of resources available and support, rather than considering the policy toward each interest group separately. Let $b > 0$ be the coefficient of the marginal cost, so that the cost is given by $b(m_1(t) + m_2(t))^2$ and it is convex and increasing in $m(t)$. This formulation expresses the competition between interest groups over governmental resources as well as the fact that a politician can invert the investment from one interest group to another. It is also based on the assumption that the two interest groups demand the same resources or regulatory services or such that are mutually dependent from the politician's point of view.

Using the assumptions and elaboration of the original optimization problem, the dynamic equation representing the development of support of each interest group with time is given by

$$\frac{\mathrm{d}r_i(t)}{\mathrm{d}t} = \beta_i m_i(t) - \alpha_i r_i(t), \quad i = 1, 2. \tag{10}$$

Based on these assumptions, a politician who faces two interest groups has to solve the following optimization problem:

$$\begin{aligned} \max_{m_1, m_2} \quad & \int_0^T \Big(a_1 r_1(t) + a_2 r_2(t) - b(m_1(t) \\ & \quad + m_2(t))^2\Big)\,\mathrm{d}t + c_1 r_1(T) + c_2 r_2(T) \\ \text{s.t.} \quad & \frac{\mathrm{d}r_1(t)}{\mathrm{d}t} = \beta_1 m_1(t) - \alpha_1 r_1(t), \\ & \frac{\mathrm{d}r_2(t)}{\mathrm{d}t} = \beta_2 m_2(t) - \alpha_2 r_2(t), \\ & m_1(t) \geqslant 0, \ m_2(t) \geqslant 0, \ r_1(0) = r_{01}, \\ & r_2(0) = r_{02}. \end{aligned} \tag{11}$$

The solution for the optimization problem is presented in the following proposition.

Proposition 4.

1. *The total investment of the politician, $m_1(t) + m_2(t)$ for every t, is calculated as the maximum of investment given the investment in each of the two interest groups as calculated by the solution for the original problem with one interest group. Namely,*

[5] It can be assumed that each interest group is considered separately, i.e., $bm_1^2 + bm_2^2(t)$, but then the solution for each group is similar to that of the original problem. Also, such a formulation does not express the competition between interest groups or the fact that politicians have limited resources. However, it may be used by a politician when two interest groups demand completely different resources that do not depend on each other. Then, the investment in each group is considered separately.

$$m_1(t) + m_2(t) = \max\left(\frac{\beta_1}{2\alpha_1 b}\left(a_1 + (\alpha_1 c_1 - a_1)\,\mathrm{e}^{\alpha_1(t-T)}\right), \frac{\beta_2}{2\alpha_2 b}\left(a_2 + (\alpha_2 c_2 - a_2)\,\mathrm{e}^{\alpha_2(t-T)}\right)\right).$$

2. *The politician's optimal investment for every t is either*

$$m_1(t) > 0,\ m_2(t) = 0 \quad or \quad m_1(t) = 0,\ m_2(t) > 0.$$

3. *If*

$$\frac{a_1\beta_1}{\alpha_1} > \frac{a_2\beta_2}{\alpha_2} \quad and \quad \beta_1 c_1 > \beta_2 c_2,$$

then for every t, $m_1(t) > 0$, $m_2(t) = 0$. If

$$\frac{a_1\beta_1}{\alpha_1} < \frac{a_2\beta_2}{\alpha_2} \quad and \quad \beta_1 c_1 < \beta_2 c_2,$$

then for every t, $m_1(t) = 0, m_2(t) > 0$.

4. *For $\alpha_1\ T$, $\alpha_2 T \gg 1$. If*

$$\frac{a_1\beta_1}{\alpha_1} > \frac{a_2\beta_2}{\alpha_2} \quad and \quad \beta_1 c_1 < \beta_2 c_2,$$

then there is a t^* such that for $t \leqslant t^*$, $m_1(t) > 0$, $m_2(t) = 0$ and for $t \geqslant t^*$, $m_1(t) = 0, m_2(t) > 0$. If

$$\frac{a_1\beta_1}{\alpha_1} < \frac{a_2\beta_2}{\alpha_2} \quad and \quad \beta_1 c_1 > \beta_2 c_2,$$

then there is a t^* such that for $t \leqslant t^*$, $m_1(t) = 0$, $m_2(t) > 0$ and for $t \geqslant t^*$, $m_1(t) > 0, m_2(t) = 0$.

Proof. See Appendix A. □

The first part of Proposition 4 provides a formula to calculate the optimal investment when there are several interest groups. Yet, the main insight of the proposition appears in the second part. It shows that a politician in office who seeks to optimize the investment of his or her limited resources should never simultaneously invest in both interest groups given that each group is decisive. Rather, at each point in time the politician should invest in the group that contributes the most for his or her political interests. If one of the interest groups is worth more than the other, both during the planning horizon and on election day, the politician should invest only in this group and abound the other as shown in part 3 of Proposition 4. If one of the interest group is more valuable than the other for the politician during the term in office while the other group is more valuable for the politician on election day, then we find a switching solution where the politician should switch the investment to the other group close to the election day. Also in this case, part 4 of Proposition 4 shows that the politician should never simultaneously invest in both of the groups.

Yet, in reality politicians often prefer to distribute their resources among several interest groups. This contradiction between the theoretical reasoning and reality can be explained by the fact that politicians often consider each interest group separately as proposed in Note 5. But, since they do not try to optimize their investment given the total amount of resources they have, the distribution of resources among several interest groups is necessarily not the optimal allocation of the total resources. Alternatively, if a politician in office faces two groups that demand completely different resources which are independent of each other then he or she may consider the two groups separately. Another possibility is that a politician achieves the total support of one interest group and still has resources to invest. Then, he or she may invest in another interest group in order to assure re-election.

The model also provides a formula to calculate the point in time where the politician should switch the investment from one group to another if the conditions of part 4 of Proposition 4 are fulfilled. This is presented in Proposition 5.

Proposition 5. *Given that the politician switches the investment from one group to another, the point in time when the switching should occur, t^*, is the root of the following equation*:

$$\frac{\beta_1}{\alpha_1}\left(a_1 + (\alpha_1 c_1 - a_1)\,\mathrm{e}^{\alpha_1(t-T)}\right) = \frac{\beta_2}{\alpha_2}\left(a_2 + (\alpha_2 c_2 - a_2)\,\mathrm{e}^{\alpha_2(t-T)}\right).$$

Proof. See Appendix A. □

The behavior of the optimal resource function, over time, given two interest groups and the conditions of part 4 of Proposition 4 is introduced in Figs. 2 and 3 for $a_1 = 2$, $a_2 = 3$, $b = 1$, $T = 10$, $c_1 = 3$, $c_2 = 2$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\beta_1 = 3$, $\beta_2 = 3$. Calculating the relations between the parameters,

$$\frac{a_1\beta_1}{\alpha_1} = 6 < \frac{a_2\beta_2}{\alpha_2} = 9 \quad \text{and} \quad \beta_1 c_1 = 9 > \beta_2 c_2 = 6.$$

Therefore, Fig. 2 shows that at the first time interval there is a constant investment in Group 2 while a certain time before the elections, $t^* = 9.307$, the politician switches the investment to Group 1 as shown in Fig. 3. Fig. 4 shows the behavior of the total resource function, i.e., a constant investment in the first time interval, then a decline in the switching stage and an increase of the investment close to the elections including on election day itself.

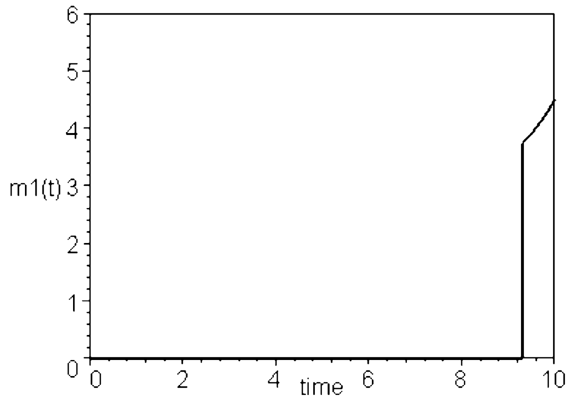

Fig. 2. The investment in Group 1 in the switching case.

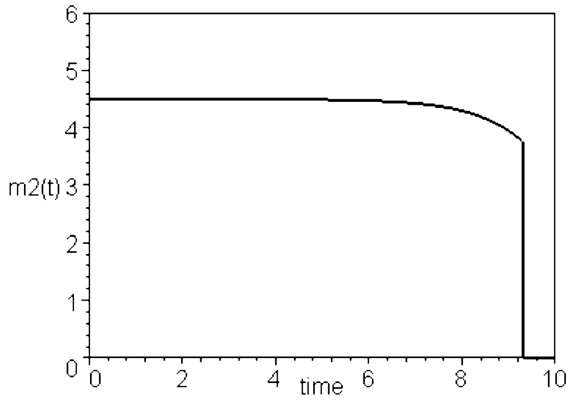

Fig. 3. The investment in Group 2 in the switching case.

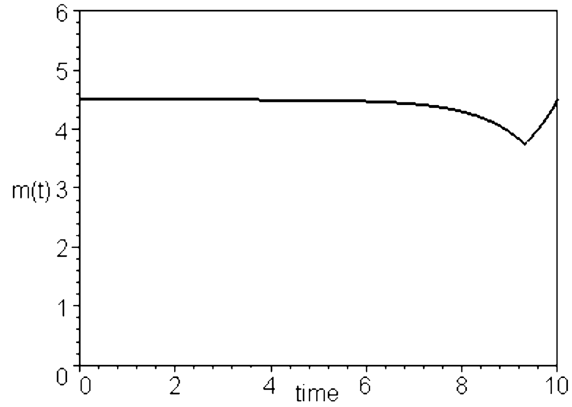

Fig. 4. The optimal investment given two interest groups in the switching case.

6. Conclusion

The continuous time model developed in this paper addresses the various aspects of a specific optimization problem that characterizes many interactions in the overlapping space between politics and economics. The need of a politician in office to gain the support of a specific interest group in order to be re-elected and the benefits that this group receives in exchange, can explain public policy that seems economically inefficient.

Assuming that politicians will always try to maximize their own political interests, the model provides analytical and practical tools to maximize political efficiency so that the self-interest motivation will only result in minimal damage to economic efficiency and social welfare. Without such planning tools politicians will act by their intuition, which may result in both political and economic inefficiency.

Under simple and straightforward assumptions, the model proves that a regulatory policy maximizing political efficiency is based on the rationale of electoral cycles. Regarding the optimization problem when several interest groups are involved, it was shown that a politician should not invest simultaneously in both groups by dividing resources among them but, rather, invest in the group that contributes the most for his or her chances to be re-elected at each point in time. This may create competition between the groups and finally reduce the resources required to guarantee their support. The effect of interest group competition will be studied in further research.

Appendix A

Proof of Proposition 1. To solve the optimization problem let us write the Hamiltonian of the problem

$$H(t, r(t), m(t)) = ar(t) - bm^2(t) + \lambda(t)(\beta m(t) - \alpha r(t)), \quad (A.1)$$

where $\lambda(t)$ is the Lagrange multiplier. The necessary conditions for optimality are as follows (see also Kamien and Schwartz, 1991):

$$\frac{\partial H}{\partial m} = -2bm(t) + \beta\lambda(t) = 0, \quad (A.2)$$

$$\frac{d\lambda(t)}{dt} = -\frac{\partial H}{\partial r} = -a + \alpha\lambda(t), \quad (A.3)$$

$$\frac{dr(t)}{dt} = \beta m(t) - \alpha r(t), \quad (A.4)$$

$$\lambda(T) = c. \quad (A.5)$$

Solving the differential equation (A.3) and substituting the boundary condition (A.5) give

$$\lambda(t) = \frac{a}{\alpha} + \frac{\alpha c - a}{\alpha} e^{\alpha(t-T)}. \quad (A.6)$$

From (A.2) and (A.6) the optimal resources function, $m(t)$, is calculated as follows:

$$m(t) = \frac{\beta\lambda(t)}{2b} = \frac{\beta}{2\alpha b}\left(a + (\alpha c - a)e^{\alpha(t-T)}\right). \quad (A.7)$$

Substituting $m(t)$ from (A.7) into (A.4) yields the differential equation for $r(t)$:

$$\frac{dr(t)}{dt} = \frac{\beta^2}{2\alpha b}\left(a + (\alpha c - a)e^{\alpha(t-T)}\right) - \alpha r(t). \quad (A.8)$$

Solving (A.8) and substituting the initial condition $r(0) = r_0$ yields

$$r(t) = \frac{a\beta^2}{2\alpha^2 b}(1 - e^{-\alpha t}) + r_0 e^{-\alpha t} + \frac{\beta^2(\alpha c - a)}{4\alpha^2 b} e^{-\alpha T}(e^{\alpha t} - e^{-\alpha t}). \quad (A.9)$$

To complete the proof, it is shown that this solution to the set of equations (A.2)–(A.5) is indeed the solution to the optimization problem. By a well-known theorem (see, for example, Kamien and Schwartz, 1991), a sufficient condition for optimality is if $(ar - bm^2)$ and $(\beta m - \alpha r)$ are concave with respect to m and r. It is easy to verify that both of them are indeed concave and thus (A.7) is the maximizer to the optimization problem presented in (3). Note that we ignore the constraint $m(t)0$ since the solution is always non-negative. □

Proof of Proposition 2. Substituting $m(t)$ from (4) into

$$\left|\frac{m(t) - (a\beta/2\alpha b)}{(a\beta/2\alpha b)}\right| > K$$

and rearranging yields (7). □

Proof of Proposition 3. Recall the optimization problem presented in (3) and replace the condition for a fixed T with a variable T, so there is a new transversality condition in addition to $\lambda(T) = c$ that exists in (A.5). This new condition is the following:

$$ar(T) - bm^2(T) + \lambda(T)(\beta m(T) - \alpha r(T)) + c\left.\frac{dr(t)}{dt}\right|_{t=T} = 0. \quad (A.10)$$

Note that the constraint $T \leqslant \tilde{T}$ is ignored at this point. Next, it is found from the function (4) that at $t = T$ the function $m(T)$ is independent of T and thus $m(T) = (\beta c/2b)$. Similarly, by the constraint in (3),

$$\frac{dr(t)}{dt} = \beta m(t) - \alpha r(t) \quad \text{and} \quad \lambda(T) = c.$$

Substituting this into (A.10) and rearranging yield the result for the first part (Eq. (8)) of Proposition 3. If at $T \leqslant \tilde{T}$ the condition presented in (A.10) is not binding, then there is inequality in (A.10) and thus the optimal T is $T = \tilde{T}$. If

$$\frac{3\beta^2c^2}{4b(2\alpha c - a)} < 0,$$

then again (A.10) has inequality for $T \leqslant \tilde{T}$ and thus $T = \tilde{T}$. Similarly, substituting $\mathrm{d}r(t)/\mathrm{d}t$ instead of $\beta m(t) - \alpha r(t)$ in (A.10) and rearranging yields the result for the second part (Eq. (9)) of Proposition 3. □

Proof of Proposition 4. Given the inequality constraints $m_1(t) \geqslant 0$ and $m_2(t) \geqslant 0$, we have the Lagrangian (see Chiang, 1992; Kamien and Schwartz, 1991):

$$L = a_1r_1(t) + a_2r_2 - b(m_1 + m_2)^2 + \lambda_1(\beta_1m_1 - \alpha_1r_1) + \lambda_2(\beta_2m_2 - \alpha_2r_2) + w_1m_1 + w_2m_2,$$

where w_1 and w_2 are Lagrange multipliers. First-order conditions for optimality are:

$$\frac{\partial L}{\partial m_i} = -2b(m_1(t) + m_2(t)) + \lambda_i(t)\beta_i + w_i(t) = 0, \quad i = 1,2, \tag{A.11}$$

$$\frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t} = -\frac{\partial L}{\partial r_i} = -a_i + \alpha_i\lambda_i(t), \quad i = 1,2, \tag{A.12}$$

$$w_i(t)m_i(t) = 0, \quad i = 1,2,$$

$$w_i(t) \geqslant 0, \ m_i(t) \geqslant 0, \quad i = 1,2,$$

$$\lambda_i(T) = c_i, \quad i = 1,2, \tag{A.13}$$

$$\frac{\mathrm{d}r_i(t)}{\mathrm{d}t} = \beta_im_i - \alpha_ir_i(t), \quad i = 1,2,$$

If both $m_1(t)$ and $m_2(t)$ are positive, we find from (A.13) that $w_1(t) = w_2(t) = 0$ and, thus, from (A.11) we have $\lambda_1(t)\beta_1 = \lambda_2(t)\beta_2$ for every t. Since the equations for the variables $\lambda_i(t)$ are the same as in the single group problem meaning that $\lambda_1(t)\beta_1 \neq \lambda_2(t)\beta_2$, we have a contradiction which proves part two of the proposition. Thus, if $m_1 > 0$ and $m_2 = 0$, then $w_1 = 0$ and $w_2 0$ and $\lambda_2(t)\beta_2 \leqslant 2bm_1(t) = \lambda_1(t)\beta_1$. If

$$\frac{a_1\beta_1}{\alpha_1} > \frac{a_2\beta_2}{\alpha_2} \quad \text{and} \quad \beta_1c_1 > \beta_2c_2,$$

we find that this condition holds for every t and thus $m_1(t) > 0$, $m_2(t) = 0$ for every t. The other case is similar and this completes the proof of part 3 of the proposition. If

$$\frac{a_1\beta_1}{\alpha_1} > \frac{a_2\beta_2}{\alpha_2} \quad \text{and} \quad \beta_1c_1 > \beta_2c_2,$$

we have a t^* where $\lambda_2\beta_2 \leqslant \lambda_1\beta_1$ for $t \leqslant t^*$ and $\lambda_2\beta_2 > \lambda_1\beta_1$ for $t > t^*$. In this case we switch from $m_1(t) > 0$, $m_2(t) = 0$ to $m_1(t) = 0$, $m_2(t) > 0$ at t^* and the proof of part 4 of the proposition is completed. By the first two equations for optimality, if $m_1(t) > 0$, $m_2(t) = 0$, then $m_1 = (\lambda_1\beta_1/2b)$ and in the second case $m_2 = (\lambda_2\beta_2/2b)$. Thus, the sum of $m_1(t) + m_2(t)$ is the maximum over the two solutions for the single group model and this proves part 1 of Proposition 4. □

Proof of Proposition 5. From the proof of part 4 of Proposition 4, we find that the switching is when $\lambda_2(t)\beta_2 = \lambda_1(t)\beta_1$. Substituting yields the result. □

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