On the motion of small heavy particles in an unsteady flow

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(Received 3 March 2004; accepted 27 April 2004; published online 7 July 2004)

Motion of a small spherical heavy particle subjected to the Stokes drag force and the force of gravity in a nonstationary fluid flow is studied. An analytical solution of the equation of the particle motion at a short-time interval and conditions for validity of this solution are obtained. For the case of a long-time motion the velocity and the acceleration of the particle are found. The velocity is obtained up to the second-order terms with respect to the Stokes response time and the acceleration is obtained up to the first-order terms.
[DOI: 10.1063/1.1763946]

Transport of small particles in unsteady fluid flows frequently occurs in both natural phenomena and technological processes. Droplets in atmospheric clouds and aerosol particles in the atmosphere represent examples of naturally occurring turbulent flows with heavy particles. Examples of such flows in technology are combustion of pulverized coal, pneumatic transport, turbulent mixing, etc. Hereafter the particle is considered as heavy if the material density of the particle is much higher than the density of the fluid. Investigation of the particle motion in an unsteady flow is of fundamental importance because of different phenomena which are determined by this motion. Examples of these phenomena include collision between particles,[1,2] turbulent particle dispersion,[3] fluctuations of particle concentration,[4] and others.

Due to inertia velocity of the particle in an unsteady flow depends, in general, upon the history of its motion (Basset force). However, when a particle is small and it moves in the fluid for a long time, velocity of the particle is determined entirely by the fluid velocity field at the particle's location.[5] For this case Saffman and Turner[1] expressed the variance of the relative velocity of the particle through the acceleration of the fluid element at the same location. Maxey[5] obtained a formula which expresses the particle velocity through the velocity and the acceleration of the fluid element at the particle's location. Higher-order terms for this formula in the special case when gravity is neglected were determined by Druzhinin.[6]

Behavior of a particle in a turbulent flow at a small time interval has special features since in this case the particle moves in the viscous spatial range and does not feel the vortical structure of the turbulence. Friedlander[7] considered motion of a particle at an initial time interval and obtained the variance of the particle displacement for the case when the force of gravity is neglected and a particle is initially at rest.

In this study we analyzed motion of a small heavy particle in a turbulent flow with gravity. At first the particle motion at the initial time interval is considered. The time interval is determined by the condition that the particle moves all the time inside the same eddy with the Kolmogorov length scale. In this case the equation of the particle motion can be linearized and it has an analytical solution. Thereafter we find velocity and acceleration of the particle when it moves in the fluid for a long time.

Let a small heavy spherical particle moves in a turbulent flow. It was found[8–10] that in this case the forces of buoyancy, added mass, pressure, and Basset are all negligible. The same conclusion was obtained from calculations for the case when the fluid velocity varies impulsively.[11,12] Thus one can neglect all these forces also when initial particle velocity differs essentially from the fluid velocity. Then equation of the particle motion reads

$$\dot{\mathbf{v}}+\frac{1}{\tau_p}\mathbf{v}=\frac{1}{\tau_p}(\mathbf{u}+\mathbf{g}\tau_p), \tag{1}$$

where $\mathbf{v}=\mathbf{v}(t)$ is the particle's velocity at time t, $\mathbf{u}=\mathbf{u}(t,\mathbf{r})$ is the fluid velocity at time t at particle's location $\mathbf{r}$, $\mathbf{g}$ is the acceleration of gravity, the dot denotes the time derivative, and $\tau_p=(2/9)\rho_p r_p^2/\mu$ is the Stokes response time. Here ρ_p is the material density of the particle, $\mathbf{r}_p$ is radius of the particle, and μ is the dynamic viscosity of the fluid.

We consider small particles so that if a particle moves in the surrounding fluid for a sufficiently long time then its velocity is close to the fluid velocity at the same location, that is

$$|\mathbf{v}-\mathbf{u}|\ll u_m, \tag{2}$$

where u_m is the root mean square velocity of the turbulent flow, $u_m^2=\overline{\mathbf{u}^2}$. Suppose that a fluid flow is homogeneous, isotropic, and statistically stationary. Let at the initial time $t=0$ any particle is located at $\mathbf{r}=0$ and its velocity is $\mathbf{v}_0$, that is, $\mathbf{v}(0)=\mathbf{v}_0$. In parallel with the particle motion let us consider a motion of the fluid element which at the initial time is located at $\mathbf{r}=0$. This fluid element moves along the pathline $\mathbf{r}_f(t)$. Let $\mathbf{u}_0$ be the velocity of the marked fluid element at time $t=0$ and $\varphi(t)$ is its velocity at time t, that is, $\boldsymbol{\varphi}(0)=\mathbf{u}_0$. Clearly, if the particle is small and the initial particle velocity $\mathbf{v}_0$ is close to the fluid velocity $\mathbf{u}_0$ then the particle trajectory $\mathbf{r}(t)$ is close to the pathline $\mathbf{r}_f(t)$. If the

1070-6631/2004/16(8)/3231/4/$22.00

particle is large or the initial particle velocity $\mathbf{v}_0$ differs essentially from $\mathbf{u}_0$ then the particle moves along the trajectory which differs from the trajectory of the fluid element. However, even in this case at some time interval the fluid velocity $\mathbf{u}(t,\mathbf{r})$, which affects the particle at the time t, is close to the velocity $\varphi(t)$ of the fluid element that moves along the pathline $\mathbf{r}_f(t)$. At this time interval one can expand the function $\mathbf{u}(t,\mathbf{r})$ in the power series of coordinates in the vicinity of $\mathbf{r}_f(t)$ and keep only the linear terms. This expansion is valid when the distance between the particle and the considered fluid element is much less then the Kolmogorov length scale l_K, that is

$$|\mathbf{r}(t)-\mathbf{r}_f(t)|\ll l_K. \tag{3}$$

Consider now the motion of the marked fluid element. There exists some time interval where the velocity $\varphi(t)$ is close to the initial velocity $\mathbf{u}_0$. For such values of time one can expand the function $\varphi(t)$ in to the power series in time in the vicinity of $t=0$ and keep only the linear terms. This expansion is valid when the velocity of the fluid element varies in magnitude much less then the Kolmogorov velocity scale

$$|\boldsymbol{\varphi}(t)-\mathbf{u}_0|\ll u_K. \tag{4}$$

Our goal is to determine a solution of Eq. (1) at a short-time interval where the inequalities (3) and (4) are valid.

Let $\dot{\mathbf{u}}(t,\mathbf{r})$ be an acceleration of the fluid element at the time t at the location $\mathbf{r}$,

$$\dot{\mathbf{u}}(t,\mathbf{r})=D\mathbf{u}/Dt=\partial\mathbf{u}/\partial t+(\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}=\mathbf{b}+B\mathbf{u},$$

where

$$\mathbf{b}=\begin{pmatrix}\partial u_x/\partial t\\ \partial u_y/\partial t\\ \partial u_z/\partial t\end{pmatrix},\quad B=\begin{pmatrix}\partial u_x/\partial x & \partial u_x/\partial y & \partial u_x/\partial z\\ \partial u_y/\partial x & \partial u_y/\partial y & \partial u_y/\partial z\\ \partial u_z/\partial x & \partial u_z/\partial y & \partial u_z/\partial z\end{pmatrix}.$$

Then the acceleration of the fluid element at $t=0$ and $\mathbf{r}=0$ is $\dot{\mathbf{u}}_0=\mathbf{b}_0+B_0\mathbf{u}_0$, where index "0" denotes that the associated variable is calculated at $t=0$, $\mathbf{r}=0$. Consider the particle motion at the time interval defined by the inequalities (3) and (4). Let us expand $\varphi(t)$ in the vicinity of $t=0$ in Taylor's series upto the linear term $\boldsymbol{\varphi}(t)=\mathbf{u}_0+\dot{\mathbf{u}}_0t$, end the velocity $\mathbf{u}(t,\mathbf{r})$ in Taylor's series of coordinates in the vicinity of the location $\mathbf{r}_f(t)$ and keep only the linear term, $\mathbf{u}(t,\mathbf{r})=\boldsymbol{\varphi}(t)+B(\mathbf{r}-\mathbf{r}_f)$, where all elements of the matrix B are calculated at the time t at the location $\mathbf{r}_f(t)$. According to the assumption (4) the velocity of the fluid at the vicinity of the marked fluid element varies slightly when the element moves from the origin to the location $\mathbf{r}_f(t)$. Therefore the matrix B is close to the matrix B_0 and with the accuracy of the first-order terms with respect to coordinates the expression for $\mathbf{u}(t,\mathbf{r})$ can be rewritten as $\mathbf{u}(t,\mathbf{r})=\boldsymbol{\varphi}(t)+B_0(\mathbf{r}-\mathbf{r}_f)$. This equation together with expressions for $\dot{\mathbf{u}}_0$ and $\boldsymbol{\varphi}(t)$ yield $\mathbf{u}(t,\mathbf{r})=\mathbf{u}_0+(\mathbf{b}_0+B_0\mathbf{u}_0)t+B_0(\mathbf{r}-\mathbf{r}_f)$. Note that vector $\mathbf{r}_f(t)$ differs from vector $\mathbf{u}_0t$ by a small value. Hence, the term $B_0\mathbf{u}_0t$ is close to $B_0\mathbf{r}_f$ and with the accuracy of the first-order terms

$$\mathbf{u}(t,\mathbf{r})=\mathbf{u}_0+\mathbf{b}_0t+B_0\mathbf{r}. \tag{5}$$

Thus the obtained equation is merely an expansion of the function $\mathbf{u}(t,\mathbf{r})$ in Taylor's series in t and $\mathbf{r}$ in the vicinity of $t=0$ and $\mathbf{r}=0$. We obtained this equation alternatively by expanding the function $\mathbf{u}(t,\mathbf{r})$ in the vicinity of $\mathbf{r}_f(t)$. This enables us to find the conditions (3) and (4) for the validity of the expansion (5). Substituting expansion (5) into the right-hand side of Eq. (1) we obtain an approximate equation of the particle motion

$$\dot{\mathbf{v}}+\frac{1}{\tau_p}\mathbf{v}=\frac{1}{\tau_p}(\mathbf{u}_0+\Delta\mathbf{u}_1+\mathbf{g}\tau_p), \tag{6}$$

where $\Delta\mathbf{u}_1$ represents the first-order terms of the expansion, $\Delta\mathbf{u}_1(t,\mathbf{r})=\mathbf{b}_0t+B_0\mathbf{r}$. Equation (6) is a linear inhomogeneous ordinary differential equation with constant coefficients. Stochastic character of the fluid motion in this equation is reflected in the fluid velocity $\mathbf{u}_0$ and the random variables $\mathbf{b}_0$ and B_0. Thus at the small time interval which is determined by the inequalities (3) and (4) the nonlinear differential equation (1) is reduced to a linear one. According to the assumptions (2)–(4) the term $\mathbf{u}_0$ in the right-hand side of Eq. (6) is the leading term. Keeping only this term one can obtain particle trajectory up to the terms of the first order. The first-order terms $\Delta\mathbf{u}_1+\mathbf{g}\tau_p$ contribute to the trajectory by the terms of the second order. Equation (6) can be solved by the method of successive approximations. Keeping only the leading term in the right hand side of the equation one has

$$\dot{\mathbf{v}}+\frac{1}{\tau_p}\mathbf{v}=\frac{1}{\tau_p}\mathbf{u}_0. \tag{7}$$

Unlike Eq. (6) the right-hand side of Eq. (7) does not depend on the particle coordinates. Let $\mathbf{q}$ be the relative velocity of the particle with respect to the fluid velocity at the same location, $\mathbf{q}(t)=\mathbf{v}(t)-\mathbf{u}[t,\mathbf{r}(t)]$. Then solution of Eq. (7) with the initial condition $\mathbf{v}(0)=\mathbf{v}_0$ reads

$$\mathbf{v}^{(0)}(t)=\mathbf{u}_0+\mathbf{q}_0e^{-t/\tau_p}, \tag{8}$$

where $\mathbf{q}_0=\mathbf{q}(0)$. Integrating Eq. (8) with the initial condition $\mathbf{r}(0)=0$ yields

$$\mathbf{r}^{(1)}(t)=\mathbf{u}_0t+\mathbf{q}_0\tau_p(1-e^{-t/\tau_p}). \tag{9}$$

Note that expression (8) for the velocity comprises zero-order terms with respect to t while both terms of the right-hand side of Eq. (9) are of the first order accuracy.

Let us determine now a more accurate solution of Eq. (6). Substituting the first-order accurate trajectory $\mathbf{r}^{(1)}(t)$ instead of $\mathbf{r}$ in the right-hand side of Eq. (6) we find

$$\dot{\mathbf{v}}+\frac{1}{\tau_p}\mathbf{v}=\frac{1}{\tau_p}[\mathbf{u}_0+\Delta\mathbf{u}_1(t,\mathbf{r}^{(1)})+\mathbf{g}\tau_p], \tag{10}$$

where $\Delta\mathbf{u}_1(t,\mathbf{r}^{(1)})=\dot{\mathbf{u}}_0t+B_0\mathbf{q}_0\tau_p(1-e^{-t/\tau_p})$. The right-hand side of Eq. (10) is a known function of time. Solution of Eq. (10) with the initial conditions $\mathbf{v}(0)=\mathbf{v}_0$ reads

$$\mathbf{v}^{(1)}(t)=\mathbf{u}_0+\mathbf{q}_0e^{-t/\tau_p}+\dot{\mathbf{u}}_0[t-f(t)] + B_0\mathbf{q}_0[f(t)-te^{-t/\tau_p}]+\mathbf{g}f(t), \tag{11}$$

where $f(t)=\tau_p(1-e^{-t/\tau_p})$. Integration of this equation with the condition $\mathbf{r}(0)=0$ yields

$$\mathbf{r}^{(2)}(t)=\mathbf{u}_0 t+\mathbf{q}_0 f(t)+\dot{\mathbf{u}}_0(0.5t^2-\tau_p t+\tau_p f(t)) + B_0\mathbf{q}_0[2\tau_p t - t f(t)+2\tau_p f(t)] + \mathbf{g}\tau_p[t-f(t)]. \tag{12}$$

In the obtained approximation the velocity is correct up to the first-order terms including and the position comprises the second-order terms. Expressions (11) and (12) coincide partially with those derived by Graham.[13] The difference is that Eqs. (11) and (12) are more exact since they contain two additional terms with $\dot{\mathbf{u}}$ and with B_0. Let us determine the first-order approximation for the relative velocity. Using Eq. (5) we obtain $\mathbf{q}^{(1)}(t)=\mathbf{v}^{(1)}(t)-(\mathbf{u}_0+\mathbf{b}_0 t+B_0\mathbf{r}^{(1)}(t))$. Substituting $\mathbf{r}^{(1)}(t)$ and $\mathbf{v}^{(1)}(t)$ in this equation yields

$$\mathbf{q}^{(1)}(t)=\mathbf{q}_0 e^{-t/\tau_p}-\dot{\mathbf{u}}_0 f(t)-B_0\mathbf{q}_0 t e^{-t/\tau_p}+\mathbf{g}f(t). \tag{13}$$

Expression (12) provides a displacement of the particle during time t which is valid for arbitrary initial conditions. As an example, let us consider the case when a particle is initially at rest, that is $\mathbf{v}_0=0$, $\mathbf{q}_0=-\mathbf{u}_0$, and assume that $t \ll \tau_p$. Expanding the exponents in the right hand side of Eq. (12) into Taylor's series in t/τ_p and keeping the leading terms only we obtain $\mathbf{r}(t)=\mathbf{u}_0 0.5t^2/\tau_p+\mathbf{g}0.5t^2$. According to the adopted assumption the first term of this expression is the leading one. It describes a motion in the direction of the velocity $\mathbf{u}_0$. The variance of this term, $l^2=u_m^2 0.25t^4/\tau_p^2$, coincides with that obtained by Friedlander[7] who considered the case without gravity.

The obtained above expressions are valid on the time interval which is determined by the inequalities (3) and (4). If a particle is sufficiently small this interval includes such values of t that $t \gg \tau_p$. In this case $e^{-t/\tau_p} \ll 1$ and Eq. (13) reduces to $\mathbf{q}=-\dot{\mathbf{u}}_0\tau_p+\mathbf{g}\tau_p$. Since $\dot{\mathbf{u}}_0$ differs from $\dot{\mathbf{u}}(t,\mathbf{r})$ by a small value the relative particle velocity with the accuracy of the first-order small terms including can be rewritten as $\mathbf{q}=-\dot{\mathbf{u}}\tau_p+\mathbf{g}\tau_p$. Using equation $\mathbf{v}=\mathbf{u}+\mathbf{q}$ we find the particle velocity

$$\mathbf{v}=\mathbf{u}-\dot{\mathbf{u}}\tau_p+\mathbf{g}\tau_p. \tag{14}$$

Expression (14) was obtained first by Maxey[5] whose formula contains also an additional term $(\mathbf{g}\cdot\boldsymbol{\nabla})\mathbf{u}\tau_p^2$ which is of the second order with respect to τ_p. The reason that this term does not appear in Eq. (14) is that this formula represents the velocity of the particle upto the terms of the first order. Expression for the particle velocity upto the terms of the second order contains several additional terms, as it will be shown below.

With the goal to elucidate the significance of each term in formula (14) let us estimate the magnitudes of these terms for the case of water droplet motion in atmospheric clouds. The magnitude of the first term is determined by the turbulent velocity fluctuations which can be estimated by the root mean square velocity u_m. The influence of the inertia on the particle velocity is determined by the second term in Eq. (14). We can evaluate fluid acceleration $\dot{\mathbf{u}}$ as acceleration at

TABLE I. Values of α for particles of different sizes.

r_p (μm)	1	5	10	20
α	0.000 39	0.010	0.039	0.16

the Kolmogorov length scale a_K and therefore, the inertial term is of the order of $a_K\tau_p$. Note that according to experimental data[2,14] a turbulent energy dissipation rate ε in clouds is in the range 1–2000 cm^2/s^3. For this range of values $a_K < g$ and the inertial term can be neglected with respect to the gravitational one. Therefore, the difference of the particle velocity from the fluid one is determined mainly by the gravitational term $\mathbf{g}\tau_p$. Consider the ratio $\alpha=g\tau_p/u_m$ which characterizes the angle between the particle velocity and the fluid velocity at the same location. Let us adopt the following values of parameters: $\mu=1.8\times10^{-4}$ g/cm s, $u_m=30$ cm/s. The values of α calculated for the droplets of different sizes are presented in Table I. One can see from the table that in the considered range of droplet sizes the droplet velocity is very close to the fluid velocity.

All the expressions derived above are valid when conditions (2)–(4) are satisfied. Let us examine these conditions for the case when the initial velocity of the particle, $\mathbf{v}_0$, is close to the fluid velocity $\mathbf{u}_0$. The range of particles sizes is determined by the inequality (2). Equation (14) shows that the inertial component of the particle velocity is $\dot{\mathbf{u}}\tau_p$ and the gravitational component is $g\tau_p$. One can estimate acceleration $\mathbf{u}$ as the acceleration of the Kolmogorov scale, a_K. Then the inertial contribution in the difference $\mathbf{v}-\mathbf{u}$ can be estimated as $a_K\tau_p$ and, hence, the inequality (2) implies that $a_K\tau_p \ll u_m$ and $g\tau_p \ll u_m$. One can write these inequalities in the following form:

$$St \ll \min\left\{\frac{u_m}{u_K}, \frac{u_m}{g\tau_K}\right\}, \tag{15}$$

where the Stokes number $St=\tau_p/\tau_K$.

The inequality (3) imposes a limitation on the displacement of the particle from the fluid element. The inertial contribution in the particle displacement on the time interval t is $a_K\tau_p t$ and, hence, inequality (3) yields $a_K\tau_p t \ll l_K$. Contribution of gravity into the particle displacement at the time interval t can be evaluated as $g\tau_p t$ and the inequality (3) yields $g\tau_p t \ll l_K$. The inequality (4) imposes a limitation on the velocity variation of an arbitrary fluid element. It is known that during a time τ_K velocity of a fluid element varies by the value of the order of v_K. Therefore inequality (4) implies that $t \ll \tau_K$. The obtained time limitations can be combined in the following form:

$$t \ll \tau_0=\min\left\{\tau_K, \frac{1}{St}\tau_K, \frac{1}{St}\frac{a_K}{g}\tau_K\right\}. \tag{16}$$

Let us examine the obtained conditions for the considered above case of the water droplet motion in atmospheric clouds. As it was indicated $a_K < g$, therefore the first term in the brackets in Eq. (15) and the last term in the brackets in Eq. (16) can be omitted. Suppose that $\varepsilon=50$ cm^2/s^3, then Kolmogorov's time, length, velocity, and acceleration are

$\tau_K=0.055$ s, $l_K=0.091$ cm, $u_K=1.7$ cm/s, $a_K=30$ cm/s^2, correspondingly. The inequality (18) yields $St\ll 0.93$ and the inequality (16) yields $t\ll\tau_0=\min\{\tau_K,0.031\tau_K/St\}$. Note that for small particles with $St<0.03$, $\tau_0=\tau_K$ and, consequently, τ_0 does not depend on the particle size.

Consider now the validity of the formula (14) for a long time motion. The formula is derived from Eq. (13) by using the inequality $\tau_p\ll t$. Combining this condition with inequality (16) we obtain

$$St\ll\min(1,\sqrt{a_K/g}). \tag{17}$$

Thus the particle size for the formula (14) is determined by the conditions (15) and (17). In the case of the atmospheric turbulence these inequalities reduce to $St\ll\min\{u_m/(g\tau_K),\sqrt{a_K/g}\}$. For the above values of parameters this yields $\tau_p\ll 0.0096$ s, and, hence, droplets radius must be less than 20 μm.

Formula (14) provides the expression for a particle velocity for a long-time motion up to the first-order terms. Druzhinin[6] used the method of successive approximations and found the higher-order terms in the case when gravity is neglected. Let us find the particle velocity using the same method when gravity is involved. For this end write Eq. (1) of particle motion in the following form $\dot{\mathbf{v}}=\mathbf{u}-\mathbf{v}\tau_p+\mathbf{g}\tau_p$. For a small particle two last terms in this equation are small with respect to $\mathbf{u}$. Therefore $\mathbf{u}$ is the leading term in the particle velocity and we can set the zero approximation as $\mathbf{v}^{(0)}=\mathbf{u}$. Substitute this expression into the expression for $\mathbf{v}$ and find the first approximation for the particle velocity $\mathbf{v}^{(1)}=\mathbf{u}-\dot{\mathbf{u}}\tau_p+\mathbf{g}\tau_p$. The second approximation can be found by similar substitution, $\mathbf{v}^{(2)}=\mathbf{u}-\dot{\mathbf{v}}^{(1)}\tau_p+\mathbf{g}\tau_p$. Here $\dot{\mathbf{v}}^{(1)}$ is the acceleration of the particle which moves with the velocity $\mathbf{v}^{(1)}$. One can find this acceleration up to the first-order terms $\dot{\mathbf{v}}^{(1)}=\dot{\mathbf{u}}-\ddot{\mathbf{u}}\tau_p-(\dot{\mathbf{u}}\cdot\nabla)\mathbf{u}\tau_p+(\mathbf{g}\cdot\nabla)\mathbf{u}\tau_p$. Substituting this expression into the expression for $\mathbf{v}^{(2)}$ we obtain a particle velocity for a long-time motion with the accuracy of the second-order terms

$$\mathbf{v}=\mathbf{u}-\dot{\mathbf{u}}\tau_p+\mathbf{g}\tau_p+\ddot{\mathbf{u}}\tau_p^2+(\dot{\mathbf{u}}\cdot\nabla)\mathbf{u}\tau_p^2-(\mathbf{g}\cdot\nabla)\mathbf{u}\tau_p^2. \tag{18}$$

Note that Eq. (18) can be obtained from Eqs. (15)–(17) in Druzhinin[15] by changing St to τ_p and e_z/Fr to $-g$ and keeping only terms up to the second order in τ_p. The formula (18) comprises three terms of the second order with respect to τ_p. Two first of them coincide with those of Druzhinin[6] and the last term with gravity coincides with that of Maxey.[5] Using $\mathbf{q}=\mathbf{v}-\mathbf{u}$ we find

$$\mathbf{q}=-\dot{\mathbf{u}}\tau_p+\mathbf{g}\tau_p+\ddot{\mathbf{u}}\tau_p^2+(\dot{\mathbf{u}}\cdot\nabla)\mathbf{u}\tau_p^2-(\mathbf{g}\cdot\nabla)\mathbf{u}\tau_p^2.$$

Substituting expression (18) in Eq. (1) we find the acceleration of the particle

$$\dot{\mathbf{v}}=\dot{\mathbf{u}}-\ddot{\mathbf{u}}\tau_p-(\dot{\mathbf{u}}\cdot\nabla)\mathbf{u}\tau_p+(\mathbf{g}\cdot\nabla)\mathbf{u}\tau_p.$$

We have benefited from useful discussions with Dr. N. Kleeorin. This study was partially supported by the German–Israeli Project Cooperation (Grant No. DIP B2.1) administrated by the Federal Ministry of Education and Research, and by the Israel Science Foundation governed by the Israeli Academy of Science.

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