



Hysteresis processes in the regular reflection $\leftrightarrow$ Mach reflection transition in steady flows

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Abstract

Ernst Mach recorded experimentally, in the late 1870s, two different shock-wave reflection configurations and laid the foundations for one of the most exciting and active research field in an area that is generally known as *Shock Wave Reflection Phenomena*. The first wave reflection, a two-shock wave configuration, is known nowadays as *regular reflection*, RR, and the second wave reflection, a three-shock wave configuration, was named after Ernst Mach and is called nowadays *Mach reflection*, MR.

A monograph entitled *Shock Wave Reflection Phenomena*, which was published by Ben-Dor in 1990, summarized the state-of-the-art of the reflection phenomena of shock waves in steady, pseudo-steady and unsteady flows.

Intensive analytical, experimental and numerical investigations in the last decade, which were led mainly by Ben-Dor's research group and his collaboration with Chpoun's, Zeitoun's and Ivanov's research groups, shattered the state-of-the-knowledge, as it was presented in Ben-Dor (*Shock Wave Reflection Phenomena*, Springer, New York, 1991), for the case of steady flows. Skews's and Hornung's research groups joined in later and also contributed to the establishment of the new state-of-the-knowledge of the reflection of shock waves in steady flows.

The new state-of-the-knowledge will be presented in this review. Specifically, the hysteresis phenomenon in the RR $\leftrightarrow$ MR transition process, which until the early 1990s was believed not to exist, will be presented and described in detail, in a variety of experimental set-ups and geometries.

Analytical, experimental and numerical investigations of the various hysteresis processes will be presented.

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Nomenclature		Δ	difference
h_m	Mach stem height	θ_i	flow deflection angle while crossing a shock wave into region (<i>i</i>)
H_{st}	Mach stem height	θ_w	reflecting wedge angle
H_m	non-dimensional Mach stem height	θ_w^D	reflecting wedge angle at the detachment condition
ℓ	Mach stem height	θ_w^E	reflecting wedge angle at the condition analogous to detachment condition in the reflection of asymmetric shock waves
L	length	θ_w^N	reflecting wedge angle at the von Neumann condition
M_0	oncoming flow Mach number	θ_w^T	reflecting wedge angle at the condition analogous to von Neumann condition in the reflection of asymmetric shock waves
M_{0C}	oncoming flow Mach number for which the von Neumann and the detachment RR ↔ MR transition criteria coincide	ρ_i	density in region (<i>i</i>)
M_f	flight Mach number	τ	non-dimensional time
M_{tr}	flight Mach number at which a transition takes place	<i>Waves and points</i>	
p_i	the pressure in region (<i>i</i>)	i	incident shock wave
p_w	the wake pressure behind the tail of the reflecting wedge	m	Mach stem
t	time	r	reflected shock wave
U_0	oncoming free-stream velocity	s	slipstream
x	coordinate, horizontal distance	T	triple point of an MR
X	non-dimensional horizontal distance	<i>Wave configurations</i>	
Y	coordinate, vertical distance	MR	Mach reflection
<i>Greek letters</i>		RR	regular reflection
β	incident shock wave angle	<i>Overall wave configurations</i>	
β_i	incident shock wave angle	oMR	overall Mach reflection
β_r	reflected shock wave angle	oRR	overall regular reflection
β^D	incident shock wave angle at the detachment condition		
β^N	incident shock wave angle at the von Neumann condition		
γ	specific heats ratio		

Types of Mach reflection

DiMR direct-Mach reflection

InMR inverse-Mach reflection

StMR stationary-Mach reflection

Types of regular reflection

sRR strong regular reflection (an RR with a strong reflected shock wave)

wRR weak regular reflection (an RR with a weak reflected shock wave)

Types of overall Mach reflection

oMR[DiMR + DiMR]

an oMR that consists of two DiMRs

oMR[DiMR + StMR]

an oMR that consists of one DiMR and one StMR

oMR[DiMR + InMR]

an oMR that consists of one DiMR and one InMR

Types of overall regular reflection

oRR[wRR + sRR]

an oRR that consists of one wRR and one sRR

oRR[wRR + wRR]

an oRR that consists of two wRRs

1. Introduction

When a supersonic flow, $M_0 > 1$, encounters a straight compressive wedge the generated shock wave is straight and attached to the leading edge of the reflecting wedge provided the reflecting wedge angle is smaller than the maximum flow deflection angle appropriate to the flow-Mach number, M_0 .

If the reflecting wedge is positioned over a straight surface the oblique shock wave will be reflected from the surface resulting in either a regular reflection (RR) or a Mach reflection (MR) wave configuration. Schematic illustrations of the wave configurations of an RR and an MR are shown in Figs. 1a and b, respectively. The interaction between the supersonic flow and the reflecting wedge generates a straight oblique incident shock

wave. It is attached to the leading edge of the reflecting wedge. The oncoming flow is deflected by an angle of $\theta_1 = \theta_w$, while passing through the oblique shock wave, to become parallel to the reflecting wedge surface. The flow behind the incident shock wave remains supersonic. The deflected flow obliquely approaches the bottom surface (with an incident angle equal to θ_w if the bottom surface is parallel to the direction of the oncoming flow ahead of the incident shock wave). The supersonic flow can negotiate this obstacle only with the aids of either an RR or an MR as shown in Figs. 1a and b, respectively.

Two out of a variety of conditions, which were proposed by various investigators, for the $RR \leftrightarrow MR$ transition, in the past 125 years, are extreme. They are the detachment condition beyond which an RR wave configuration is theoretically impossible and the von Neumann condition beyond which an MR wave configuration is theoretically impossible. Von Neumann [2,3] was the first to introduce these two conditions as possible $RR \leftrightarrow MR$ transition criteria. For more details regarding these and other proposed $RR \leftrightarrow MR$ transition criteria see Section 1.5.4 in Ben-Dor [1] and Chapter 8.1.2.4 in Ben-Dor [4].

Hornung and Robinson [5] showed that the $RR \leftrightarrow MR$ transition criterion in steady flows depends upon whether the incident flow-Mach number, M_0 , is smaller or larger than M_{0C} , which is the value appropriate to the point at which the transition lines arising from the above-mentioned von Neumann and detachment criteria intersect. Molder [6] calculated the exact value of M_{0C} and found that it was 2.202 for a perfect diatomic gas ($\gamma = 7/5$) and 2.470 for a perfect monatomic gas ($\gamma = 5/3$). Based on their experimental results Hornung and Robinson [5] concluded that the $RR \leftrightarrow MR$ transition (i.e., both the $RR \rightarrow MR$ and the $MR \rightarrow RR$ transitions) occurs at the von Neumann criterion for $M_0 \geq M_{0C}$, and at the sonic condition, which is very close to the detachment criterion, and

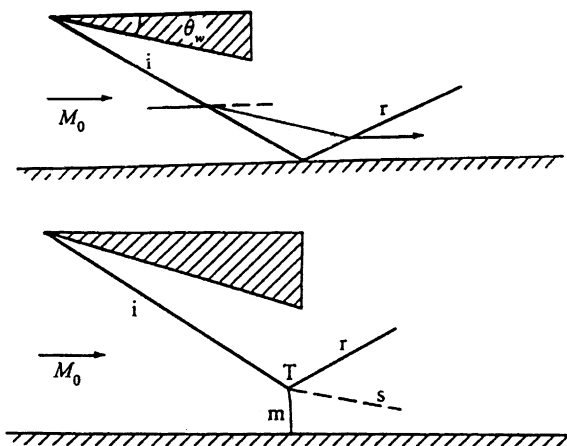


Fig. 1. Schematic illustrations of the wave configurations of a regular reflection—RR (a); and a Mach reflection—MR (b) in the reflection of straight oblique shock waves from straight surfaces in steady flows.

hence cannot be practically distinguished from it, for $M_0 \leq M_{0C}$.

By defining the angles of incidence of the incident shock wave that are appropriate to the von Neumann and the detachment conditions as β^N and β^D , respectively, one obtains that only RR wave configurations are theoretically possible in the range $\beta < \beta^N$ and only MR wave configurations are theoretically possible in the range $\beta > \beta^D$. In the intermediate range $\beta^N \leq \beta \leq \beta^D$ both RR and MR wave configurations are theoretically possible. For this reason the intermediate domain, which is bounded by the von Neumann condition, β^N , and the detachment condition, β^D , is known as the *dual-solution-domain*.

The dual-solution-domain in the (M_0, θ_w) -plane is shown in Fig. 2. As just mentioned it is divided into three domains:

- A domain inside which only RR-configurations are theoretically possible.
- A domain inside which only MR-configurations are theoretically possible.
- A domain inside which both RR and MR-configurations are theoretically possible.

The existence of conditions beyond which only RR or only MR wave configurations are theoretically impossible and the existence of a domain inside which both RR and MR wave configurations are theoretically possible led Hornung et al. [7] to hypothesize that a hysteresis could exist in the $RR \leftrightarrow MR$ transition process.

An inspection Fig. 2 indicates that two general hysteresis processes are theoretically possible:

- *A wedge-angle-variation-induced hysteresis.*
In this hysteresis process the flow-Mach number is kept constant and the wedge angle is changed.
- *A flow-Mach-number-variation-induced hysteresis.*
In this hysteresis process the wedge angle is kept constant and the flow-Mach number is changed.

It is noted that since $\beta = \beta(M_0, \theta_w)$, the above two hysteresis processes are, in fact, *angle-of-incidence-variation-induced hysteresis* processes.

Henderson and Lozzi [8,9] failed in their experimental attempt to record the wedge-angle-variation-induced hysteresis process and concluded that the RR wave configuration is unstable inside the dual-solution-domain and that as a consequence both the $MR \rightarrow RR$ and $RR \rightarrow MR$ transitions occur at the von Neumann condition.

In a following experimental attempt, Hornung and Robinson [5] also were unable to confirm the hypothesized hysteresis. As a consequence they adopted Henderson and Lozzi's [9] conclusion that the RR wave

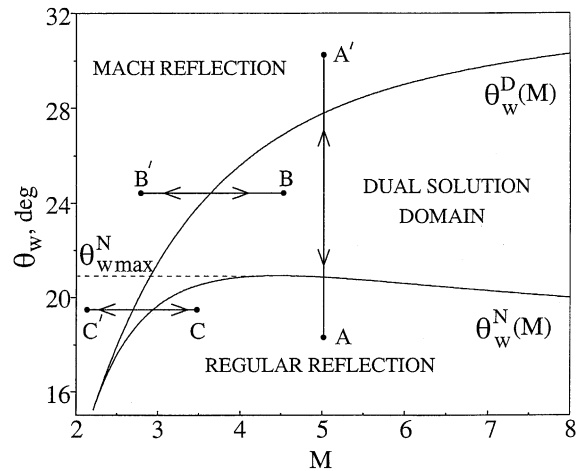


Fig. 2. Domains of possible shock wave reflection wave configurations in the (M_0, θ_w) -plane.

configuration is unstable in the dual-solution-domain and that both the $MR \rightarrow RR$ and $RR \rightarrow MR$ transitions occur at the von Neumann condition.

Teshukov [10] analytically proved, by using a linear stability technique, that the RR wave configuration is stable inside the dual-solution-domain. Li and Ben-Dor [11] analytically proved, by applying the principle of minimum entropy production, that the RR wave configuration is stable in most of the dual-solution-domain.

Chpoun et al. [12] were the first to experimentally record both stable RR wave configurations inside the dual-solution-domain, and a wedge-angle-variation-induced hysteresis in the $RR \leftrightarrow MR$ transition. As will be presented and discussed subsequently it was shown later, by some investigators, that Chpoun et al.'s [12] experimental results were not purely two-dimensional. As a result these investigators claimed and showed that the hysteresis process that was recorded by Chpoun et al. [12] was influenced and promoted by three-dimensional effects.

Vuillon et al. [13] were the first to numerically obtain stable RR and MR wave configurations for the same flow-Mach numbers and reflecting wedge angles but different aspect ratios inside the dual-solution-domain.

The above-mentioned experimental and numerical findings that the RR wave configuration is stable inside the dual-solution-domain and the experimental finding that a hysteresis in the $RR \leftrightarrow MR$ transition indeed exists, re-initiated the interest of the scientific community in the reflection process in steady flows, in general, and the hysteresis process in the $RR \leftrightarrow MR$ transition, in particular. The revived interest led to the publication of tens of papers, directly related to both the hysteresis process and the stability of the RR wave configuration

inside the dual-solution-domain, that eventually shattered the then existing state-of-knowledge and consequently led to a new state-of-knowledge, which is outlined in this review.

It is important to note here that in spite of the fact that the early reasons for the interest in studying the hysteresis process in the $RR \leftrightarrow MR$ transition were purely academic, it turned out recently that the existence of the hysteresis process has an important impact on flight performance at supersonic and hypersonic speeds. Consequently, there is a clear aeronautical and aerospace engineering interest in better understanding this phenomenon. As will be shown subsequently, some of the geometries that were investigated, in recent years, resemble geometries of supersonic/hypersonic intakes. The findings regarding the existence of hysteresis loops, in general, and overlapping hysteresis loops, in particular, which will be reported in the following, can be relevant to flight performances of vehicles flying at supersonic and hypersonic speeds. The possible dependence of the flow pattern that is established inside an intake, in general, and the accompanied pressure distribution, in particular, on the preceding variations in the speed of flight of a supersonic/hypersonic aircraft should be accounted for in designing intakes and flight conditions for supersonic and hypersonic vehicles. Especially due to the fact that different flow fields would result in different flow conditions that can significantly affect the combustion process and the entire performance of the vehicle.

2. The hysteresis process in the reflection of symmetric shock waves

2.1. Wedge-angle-variation-induced hysteresis

As mentioned earlier, Chpoun et al. [12] were the first to experimentally record the wedge-angle-variation-induced hysteresis in the $RR \leftrightarrow MR$ transition and thereby verify Hornung et al.'s [7] hypothesis. (As noted earlier it will be presented subsequently that some investigators claimed and showed that Chpoun et al.'s [12] experimental results were not purely two-dimensional. However, when Chpoun et al.'s [12] experimental results were published, their claims and the results that showed that the experiments were influenced by three-dimensional effects were unavailable.)

An example of Chpoun et al.'s [12] experimental results is shown in Fig. 3 in the (β_i, β_r) -plane, where β_i and β_r are the wave angles of the incident and reflected shock waves, respectively. Experimentally recorded MR and RR wave configurations are marked with open triangles and closed circles, respectively. The theoretical von Neumann and the detachment transition angles for the flow-Mach number of the experiment, $M_0 = 4.96$,

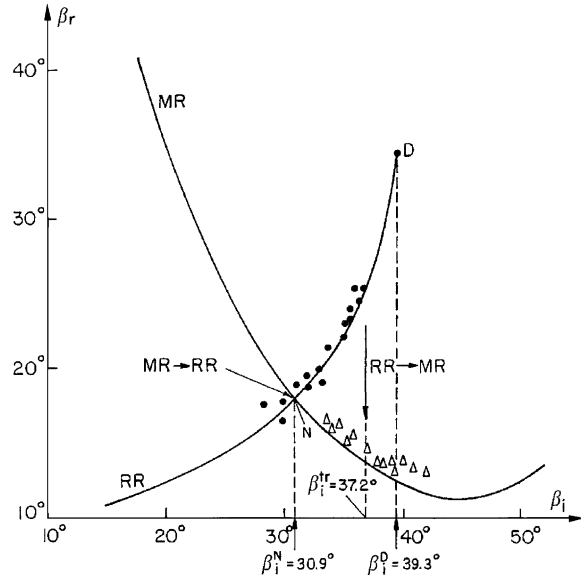


Fig. 3. Chpoun et al.'s [12] experimental results illustrating the hysteresis process in the $RR \leftrightarrow MR$ transition in the (β_i, β_r) -plane.

are $\beta_i^N = 30.9^\circ$ and $\beta_i^D = 39.3^\circ$, respectively. The experimental results reveal that the $MR \rightarrow RR$ transition occurred very close to the appropriate theoretical von Neumann angle, i.e., $\beta_i^{tr}(MR \rightarrow RR) \cong \beta_i^N = 30.9^\circ$, while the reversed $RR \rightarrow MR$ transition, took place at about $\beta_i^{tr}(RR \rightarrow MR) = 37.2^\circ$, which is about 2.1° smaller than the appropriate theoretical detachment angle. Consequently, a hysteresis phenomenon in the $RR \leftrightarrow MR$ transition is clearly evident in the experimental results that are presented in Fig. 3. The very good agreement between the experimental results and the von Neumann criterion and the poor agreement with the detachment criterion might suggest that the above-mentioned three dimensional effects (to be discussed in more details subsequently) have a larger influence on the $RR \rightarrow MR$ transition than on the $MR \rightarrow RR$ transition and that the RR wave configuration is less stable than the MR wave configuration.

Schlieren photographs of RR and MR wave configurations, taken from the above-mentioned experimental investigation, that illustrate the hysteresis phenomenon are shown in Fig. 4. Fig. 4a shows an MR wave configuration at $\beta_i \cong 42^\circ > \beta_i^D$. When β_i was decreased the MR wave configuration was maintained as shown in Fig. 4b where $\beta_i^N < \beta_i \cong 34.5^\circ < \beta_i^D$. When β_i was decreased below β_i^N the MR wave configuration was terminated and an RR wave configuration was obtained as shown in Fig. 4c for $\beta_i \cong 29.5^\circ < \beta_i^N$. When the process was reversed and β_i was increased beyond β_i^N back to a value of $\beta_i^N < \beta_i \cong 34.5^\circ < \beta_i^D$, the wave configuration remained an RR as shown in Fig. 4d. Note that the MR

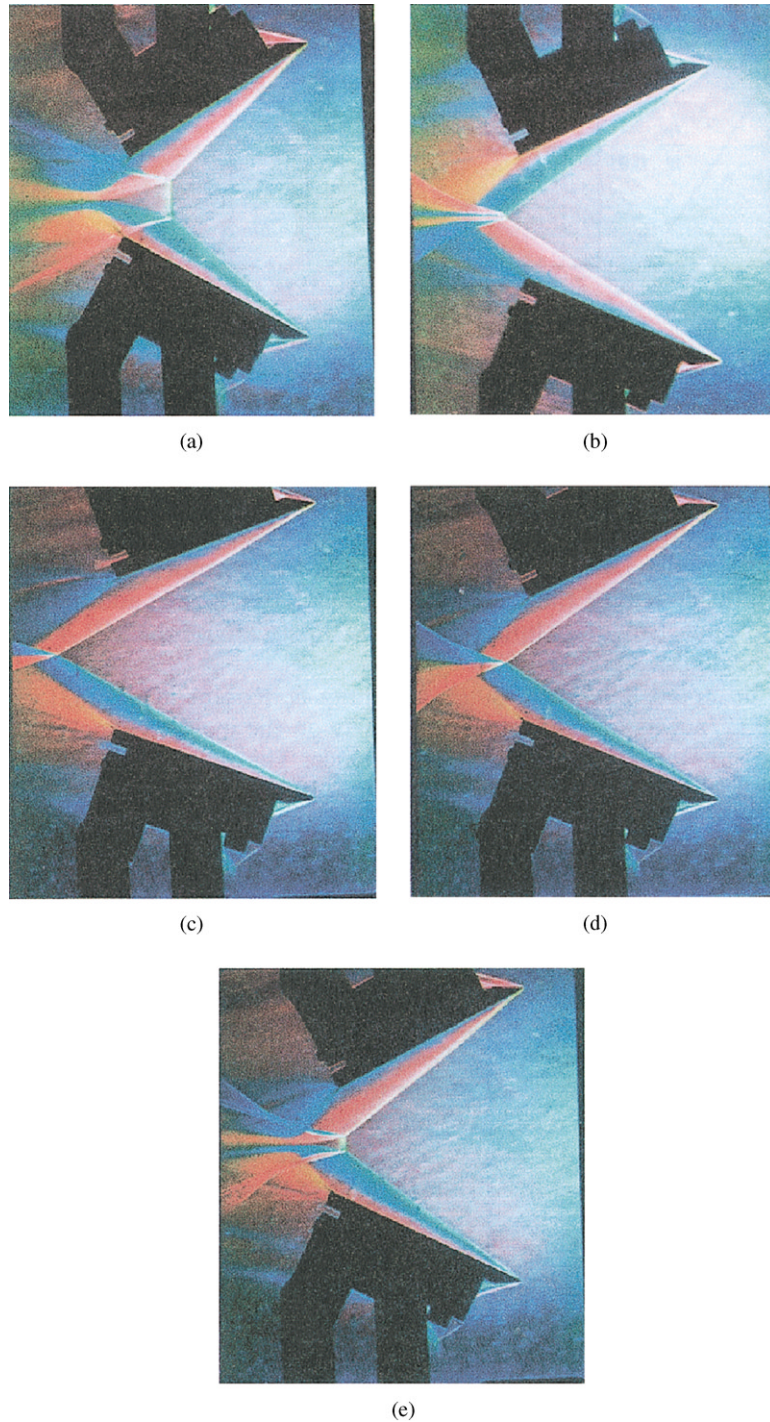


Fig. 4. Schlieren photographs from Chpoun et al.'s [12] experimental investigation illustrating the hysteresis phenomenon in the $RR \leftrightarrow MR$ transition process. (a) An MR at $\beta_i \cong 42^\circ > \beta_i^D$, (b) an MR at $\beta_i^N < \beta_i \cong 34.5^\circ < \beta_i^D$, (c) an RR at $\beta_i \cong 29.5^\circ < \beta_i^N$, (d) an RR at $\beta_i^N < \beta_i \cong 34.5^\circ < \beta_i^D$, (e) an MR at $\beta_i \cong 37.5^\circ < \beta_i^D$. Note that the MR wave configuration shown in (b), and the RR wave configuration shown in (d), have the same initial conditions, i.e., $M_0 = 4.96$ and $\beta_i \cong 34.5^\circ$.

wave configuration, shown in Fig. 4b, and the RR wave configuration, shown in Fig. 4d, had practically the same initial conditions, i.e., $M_0 = 4.96$ and $\beta_i \cong 34.5^\circ$. The fact that both the RR and the MR wave configurations were stable, clearly verifies the fact that a hysteresis exists in the $RR \leftrightarrow MR$ transition process. When β_i was further increased, the RR wave configuration was suddenly terminated and an MR wave configuration was formed as shown in Fig. 4e where $\beta_i \cong 37.5^\circ < \beta_i^D$. It is noted here that Ivanov et al. [14] experimentally recorded a similar hysteresis process a few years later.

Chpoun et al. [15], using a Navier–Stokes solver, were the first to numerically simulate and thereby verify the existence of a wedge-angle-variation-induced hysteresis in the $RR \leftrightarrow MR$ transition. Unfortunately, since their study was published in a French journal it has not caught the attention of the relevant scientific community. Ivanov et al. [16] conducted independently a direct simulation Monte Carlo (DSMC) based study of the phenomena and confirmed the existence of the hysteresis process. Following their numerical study, which was published in an English journal, many investigators using a variety of numerical codes numerically simulated the hysteresis process. Additional direct simulation Monte Carlo (DSMC) calculations were published by Ivanov et al. [17], Ben-Dor et al. [18] and Ivanov et al. [19]; Ivanov et al. [17] also presented high resolution FCT scheme calculations; calculations based on total variation diminishing (TVD) scheme were presented by Shirozu and Nishida [20]; calculations based on Godunov and van Leer scheme were conducted by Chpoun and Ben-Dor [21], calculations based on HLLE MUSCL TVD scheme were conducted by Ivanov et al. [19]; and calculations based on Steger and Warming flux splitting scheme were presented by Ben-Dor et al. [22]. It should be noted here that unlike the experiments the above-mentioned numerical simulations were purely two-dimensional.

A numerical illustration of the wedge-angle-variation-induced hysteresis, for $M_0 = 5$, is shown in Fig. 5. These Euler equations based simulations were carried out using high-order finite volume MUSCL TVD scheme with HLLE approximate Riemann solver. The details of the numerical code can be found in Ivanov et al. [19]. The simulation starts with a wedge angle $\theta_w = 20^\circ$ that corresponds to the incident shock angle $\beta = 29.8^\circ$ for which, as can be seen in frame (1), an RR wave configuration is obtained. Upon an increase in the wedge angle (or the angle of incidence) the RR wave configuration is maintained [follow frames (2), (3) and (4)] until it is suddenly changed to an MR wave configuration between the calculations appropriate to $\theta_w = 27.9^\circ$ [frame (5)] and $\theta_w = 28^\circ$ [frame (6)]. If at this point the direction of changing the wedge angle is reversed the MR wave configuration is seen to persist

[follow frames (7), (8) and (9)] until it is changed back to an RR wave configuration between the calculations appropriate to $\theta_w = 22.15^\circ$ [frame 9] and $\theta_w = 22.1^\circ$ (not shown in the figure). Finally, when the wedge angle is decreased to its initial value, $\theta_w = 20^\circ$, a steady RR wave configuration, identical to that shown in frame (1), is again obtained.

The numerically obtained transition angles do not agree exactly with the appropriate theoretical von Neumann and detachment wedge angles (the theoretical values of the angles for $M_0 = 5$ are $\theta_w^N = 20.9^\circ$ and $\theta_w^D = 27.8^\circ$, respectively). The numerical $MR \rightarrow RR$ transition angle is 1° higher than the theoretical von Neumann angle $\theta_w^N = 20.9^\circ$. This is probably due to the fact that the very small Mach stem, in the vicinity of the von Neumann transition angle, is not resolved in the computations. Grid refinement studies confirmed that the numerically obtained $MR \rightarrow RR$ transition angle approached the theoretical value of $\theta_w^N = 20.9^\circ$ as the grid was refined (see Fig. 6 where the normalized Mach stem height versus the wedge angle is given). The minimum normalized Mach stem height $H_{st}/w \approx 1\%$ was obtained on a very fine grid (the grid cell size in the vertical direction was $\Delta y/w = 0.001$).

It should be noted that the $RR \rightarrow MR$ transition angle does not depend on the grid resolution for fine enough grids but strongly depends on the numerical dissipation inherent in any shock-capturing solver. Large numerical dissipation or low order reconstruction can result in significant differences between the numerical and the theoretical values of the transition angles. For example, the $RR \rightarrow MR$ transition angle, for $M_0 = 4.96$, in the computations of Chpoun and Ben-Dor [21] who used an INCA code, was 33° instead of $\theta_w^D = 27.7^\circ$, i.e. more than 5° higher. The use of a high-order shock-capturing scheme gave a transition wedge angle $\theta_w^D \approx 27.95^\circ$, which is much closer to the theoretical value (see frames 5 and 6 in Fig. 5). The numerical transition angle, in this computation, is only 0.2° higher than the theoretical value of $\theta_w^D = 27.8^\circ$.

2.1.1. Three-dimensional effects

The foregoing presentation led the investigators to the unavoidable question: why had the hysteresis phenomenon been recorded in the course of some experimental investigations and not in others? Although the complete answer to this question is not determined yet, two possible reasons were suggested and forwarded in the past years:

1. The extent of the hysteresis depends on the type of the wind tunnel used for the experiment.

Fomin et al. [23], Ivanov et al. [14,24] showed, experimentally, that while in a closed wind tunnel the hysteresis was hardly detected, a clear hysteresis was obtained in an open wind tunnel. Not surprisingly

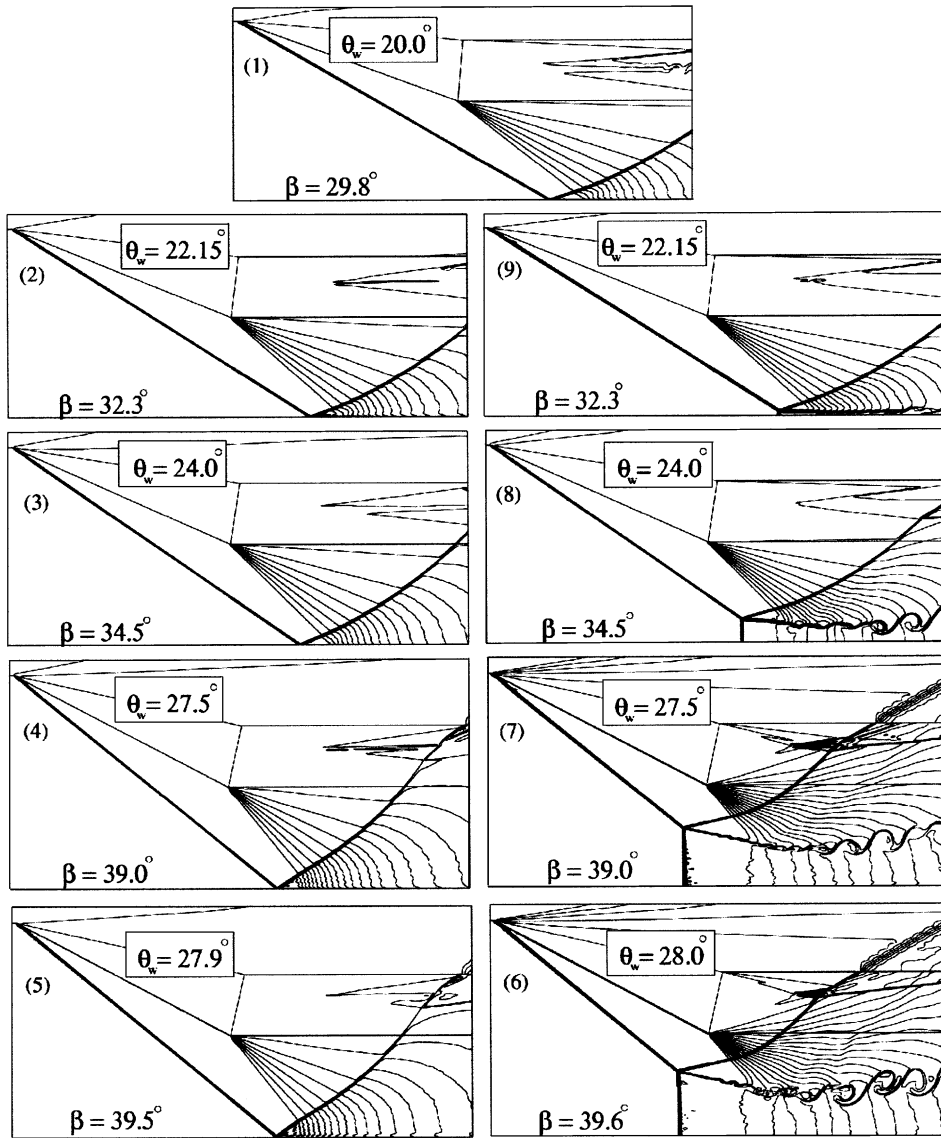


Fig. 5. Wedge-angle-variation-induced hysteresis for $M_0 = 5$.

Henderson and Lozzi [8,9], Hornung et al. [7] and Hornung and Robinson [5] who did not detect the hysteresis, in their experimental investigations, used closed wind tunnels, while Chpoun et al. [12] and Fomin et al. [23] who did detect it used open jet type wind tunnels.

2. Three-dimensional edge effects affect the experiment and promote the hysteresis.

Skews et al. [25], Skews [26,27] and Ivanov et al. [14,24,28], claimed and showed that the experimental investigations, in which hysteresis in the $RR \leftrightarrow MR$ transition were recorded, were all contaminated by three-dimensional edge effects and hence could not

be considered as purely two-dimensional. Skews [29] showed that three-dimensional edge effects are evident in actual wave configurations associated with the reflection of plane shock waves over plane wedges.

It should be noted here that using the same reflecting wedge (i.e., the same aspect ratios) a hysteresis was observed in an open wind tunnel by Chpoun et al. [12] and was not observed in a closed wind tunnel by Ivanov et al. [14,24], in spite of the fact that very similar to identical three-dimensional effects, that mainly depend on the aspect ratios of the reflecting wedge, were present

in both cases. These results clearly indicate that three-dimensional effects by themselves are not sufficient to promote the hysteresis and that the type of the wind

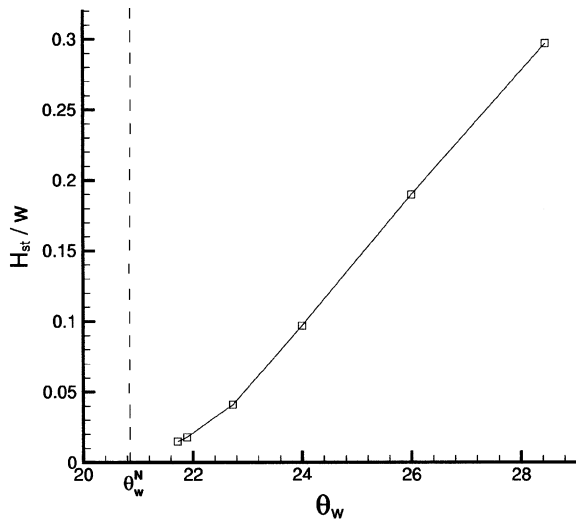


Fig. 6. The calculated normalized Mach stem height versus reflecting wedge angle. Note the problem in the vicinity of the von Neumann condition.

tunnel, i.e., open or closed, has a significant, and unfortunately not yet understood, role in the occurrence of hysteresis in the RR $\leftrightarrow$ MR transition in steady flows.

Fig. 7 shows numerically generated schlieren-like photos of RR and MR wave configurations in a purely two-dimensional flow, on the left side, and a supposedly two-dimensional flow, which is affected by three-dimensionally edge effects, on the right side. Beside the clear differences between the purely two-dimensional and the three-dimensionally affected wave configurations, the dramatic shortening of the Mach stem height, owing to the three-dimensional edge effects, should be noted. Consequently, the numerically calculated height of the Mach stem of a purely two-dimensional MR wave configuration, for a given geometry and flow conditions, could be used as a measure of the two-dimensionality of the considered actual MR wave configuration.

Kudryavtsev et al. [30] demonstrated both numerically and experimentally that increasing the aspect ratio could reduce the influence of the three-dimensional edge effects. Consider Fig. 8 and note that by increasing the aspect ratio, the actual Mach stem height approaches the numerically calculated height, which is appropriate to a purely two-dimensional MR wave configuration. Hence, an actual MR wave configuration cannot be

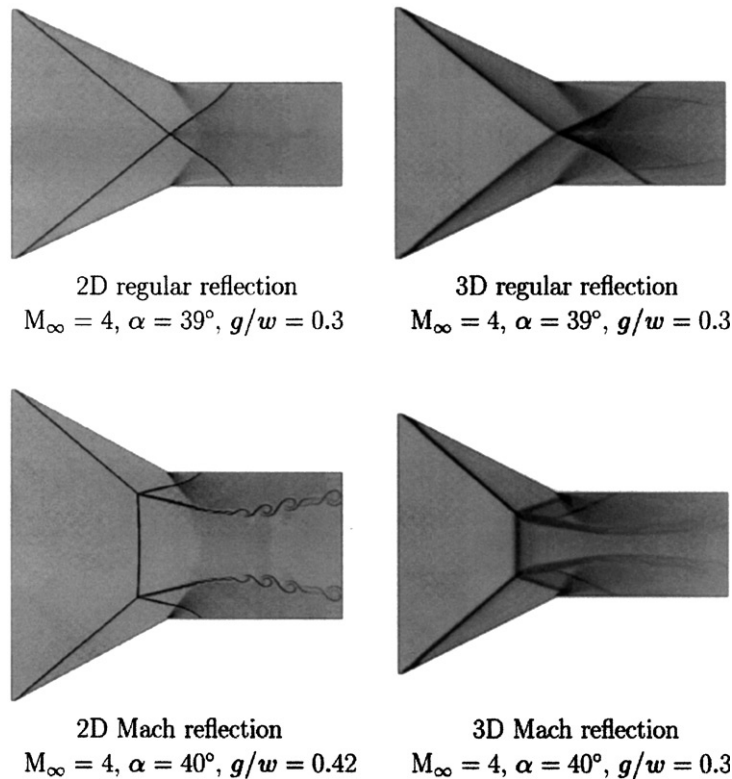


Fig. 7. Numerically generated schlieren-like photos of RR and MR wave configurations in a purely two-dimensional flow, on the left, and a supposedly two-dimensional flow, which is affected by three-dimensionally edge effect, on the right.

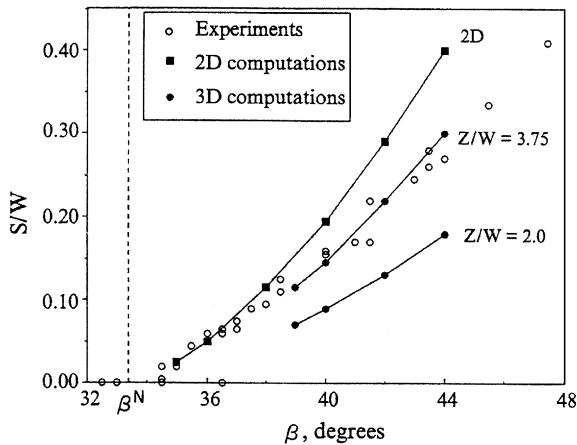


Fig. 8. Dependence of the Mach stem height on the aspect ratio. Note that equality between the Mach stem height of an actual Mach reflection and the one numerically calculated, using a 2D code, is a necessary but not a sufficient condition for the actual Mach reflection to be two-dimensional.

considered as free of three-dimensional edge effects as long as its Mach stem height is smaller than that appropriate to a calculated purely two-dimensional MR wave configuration. Note that this condition is a necessary but not a sufficient one.

2.2. Flow-Mach-number-variation-induced hysteresis

As mentioned earlier keeping the wedge angle constant and changing the oncoming flow-Mach number can also lead to a hysteresis process in the $RR \leftrightarrow MR$ transition. It is evident from Fig. 2 that there are two possible hysteresis processes in this case:

- If $\theta_w > \theta_{w,max}^N$ the Mach number can be changed along the path $BB'B$ from a value inside the dual-solution-domain where both RR and MR wave configurations are theoretically possible, to a value outside the dual-solution-domain for which only an MR wave configuration is theoretically possible and then back to the initial value. If one starts inside the dual-solution-domain with an RR wave configuration¹ then after transition to an MR wave configuration, the wave configuration never returns to be an

¹ Depending on the way the flow is numerically initiated either an RR or an MR wave configuration can be established inside the dual-solution-domain where both of them are theoretically possible. If the computation is initiated with an oncoming supersonic flow having a Mach number, M_0 , a stable RR will be established. However, if the computation is initiated with an oncoming normal shock wave behind which the induced flow has a Mach number equal to M_0 , a stable MR will be established.

RR wave configuration because the $MR \rightarrow RR$ transition is not compulsory on the return path. Note that the above-described loop does not represent a full hysteresis loop, though both RR and MR wave configurations can be observed for the same values of the reflecting wedge angle and the flow-Mach number.

- If $\theta_w < \theta_{w,max}^N$ the Mach number can be changed from a value for which only an RR wave configuration is theoretically possible to a value for which only an MR wave configuration is theoretically possible and then back to the initial value crossing both $\theta_w^N(M)$ and $\theta_w^D(M)$ curves (see the path $CC'C$ in Fig. 2). Consequently, in similar to the wedge-angle-variation-induced hysteresis, a full hysteresis loop is obtained in this case, with a return to the initial wave configuration.

The above two cases were numerically investigated by Ivanov et al. [31]. Typical results for the former case ($\theta_w > \theta_{w,max}^N = 20.92^\circ$) are shown in Fig. 9a. The reflecting wedge angle was kept constant at $\theta_w = 27^\circ$ while the oncoming flow-Mach number, M_0 , was first decreased from 5 to 4.45 and then increased back to its initial value of 5. The first frame, with $M_0 = 5$, shows an RR wave configuration inside the dual-solution-domain. As M_0 was decreased, the detachment transition line, $\theta_w^D(M)$, beyond which an RR wave configuration is theoretically impossible, was reached at $M_0 = 4.57$. The $RR \rightarrow MR$ transition took place between frames (3) and (4) when the flow-Mach number was changed from 4.5 to 4.45, i.e., at $M_0 = 4.475 \pm 0.025$. This numerical value is in reasonable agreement with the theoretical value of 4.57. The existence of an RR wave configuration slightly beyond the theoretical limit has been also observed in other numerical simulations of the wedge-angle-variation-induced hysteresis and can be explained by the influence of numerical viscosity that is inherent in shock-capturing codes. Once an MR wave configuration was established, the flow-Mach number, M_0 , was increased back to its initial value of $M_0 = 5$. Since theoretically an MR wave configuration can exist for values of M_0 , which is inside the dual-solution-domain, the reversed $MR \rightarrow RR$ transition did not take place at the detachment transition line. As a result, two different wave configurations, an RR and an MR, were obtained for identical flow conditions (i.e., same values of M_0 and θ_w). Compare the pairs of the frames (1) and (7), (2) and (6) and (3) and (5) in which the first frame shows an RR wave configuration and the second one shows an MR wave configuration.

Typical results for the latter case ($\theta_w < \theta_{w,max}^N = 20.92^\circ$) are shown in Fig. 9b. The reflecting wedge angle was kept constant at $\theta_w = 20.5^\circ$. For this value of θ_w , the Mach number values that correspond to the von Neumann criterion are 3.47 and 6.31 while that

corresponding to the detachment criterion is 2.84. The oncoming flow Mach number was decreased from $M_0 = 3.5$ to $M_0 = 2.8$ and then increased back to the initial value. Some frames showing the sequence of events that were encountered are shown in Fig. 9b. The RR→MR transition occurs between the frames appropriate to $M_0 = 2.9$ (not shown in the figure) and $M_0 = 2.8$ in close agreement with the theoretical value while the reverse MR→RR transition is observed between the frames appropriate to $M_0 = 3.2$ and $M_0 = 3.3$ (not

shown in the figure), i.e., slightly earlier than the theoretical value. This disagreement can be attributed to the very small height of the Mach stem near the von Neumann criterion, which makes its numerical resolution very difficult (see Fig. 6 and relevant discussion).

It should be noted here that an independent investigation of the flow-Mach-number-induced hysteresis was also conducted and reported by Onofri and Natusi [32]. The results obtained by them were in close agreement with those obtained by Ivanov et al. [31].

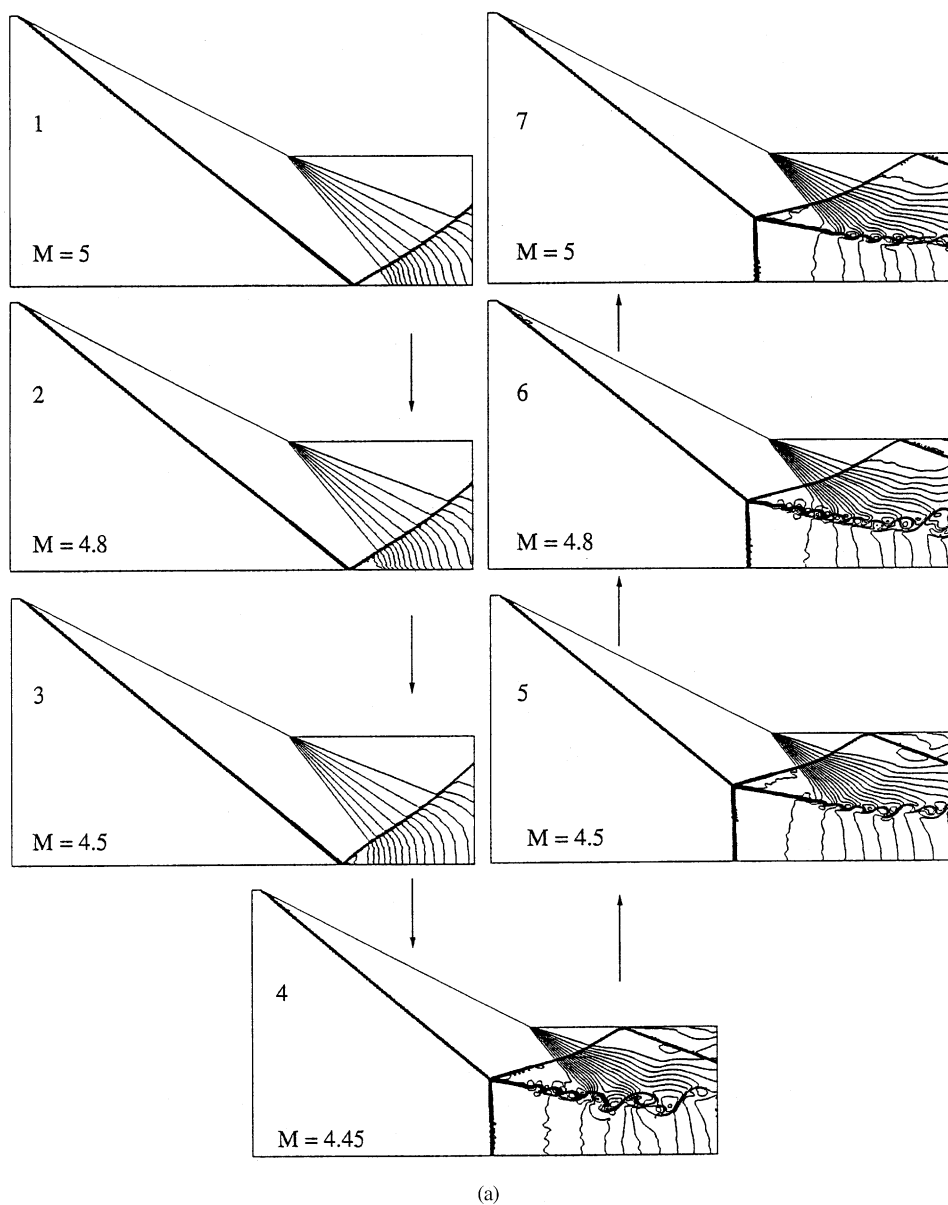


Fig. 9. (a) Flow-Mach-number-variation-induced hysteresis for $\theta_w = 27^\circ$. (b) Flow-Mach-number-variation-induced hysteresis for $\theta_w = 20.5^\circ$.

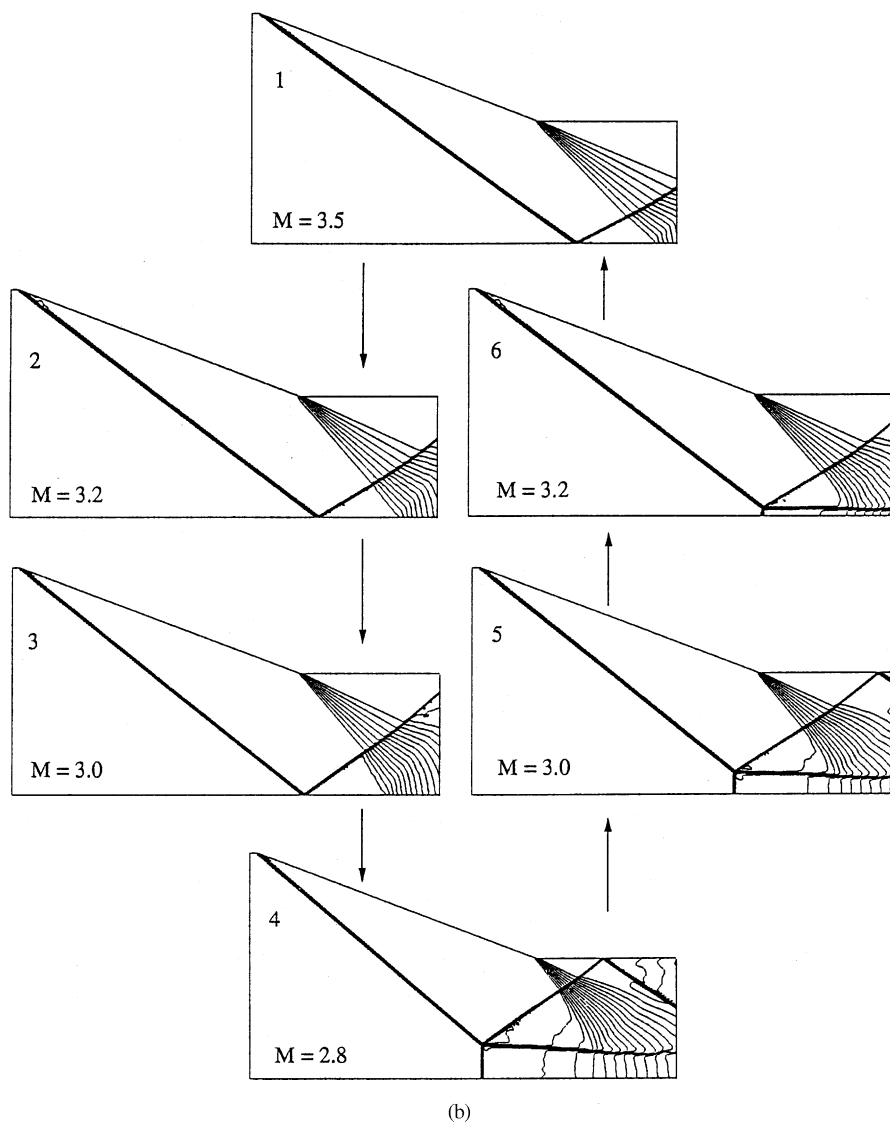


Fig. 9 (continued).

3. The hysteresis process in the reflection of asymmetric shock waves

The reflection process of asymmetric shock waves was investigated experimentally by Chpoun and Lengrand [33], both analytically and experimentally by Li et al. [34] and numerically by Ivanov et al. [35].

3.1. The overall wave configurations

Li et al. [34] conducted a detailed analysis of the two-dimensional reflection of asymmetric shock waves in steady flows. Similarly to the interaction of symmetric shock waves in steady flows, the interaction of asym-

metric shock waves leads to two types of overall wave configurations, namely; an overall regular reflection (oRR) and an overall Mach reflection (oMR). Schematic drawings of these two overall wave configurations are shown in Figs. 10a and b, respectively.

An oRR wave configuration, shown in Fig. 10a, consists of two incident shock waves (i_1 and i_2), two reflected shock waves (r_1 and r_2), and one slipstream (s). These five discontinuities meet at a single point (R). The slipstream results from the fact that the streamlines of the oncoming flow pass through two unequal shock wave sequences, i.e., the (i_1, r_1)- and (i_2, r_2)-sequences. If the flow deflection angles are $\theta_1, \theta_2, \theta_3$ and θ_4 through the i_1, i_2, r_1 and r_2 shock waves, respectively, then the

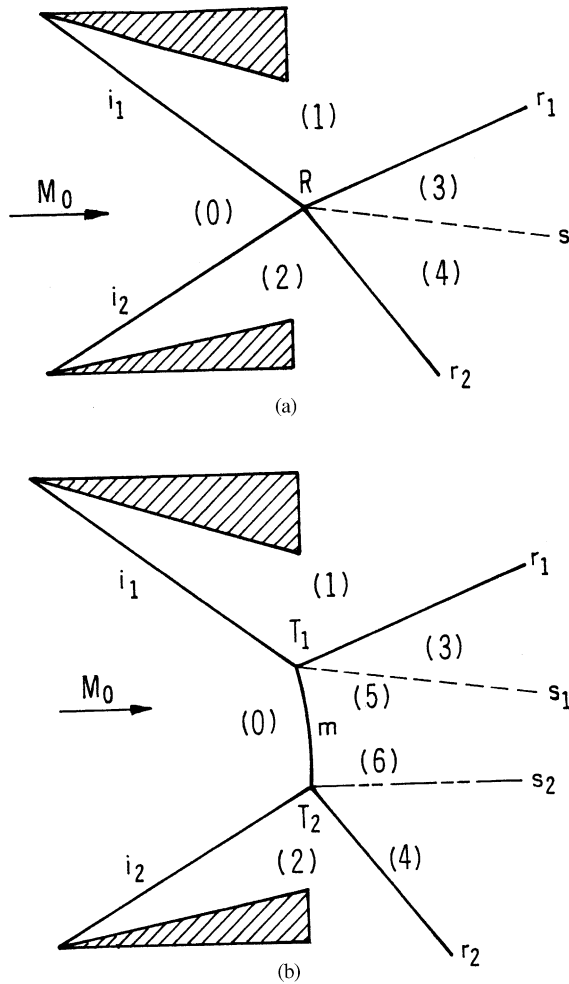


Fig. 10. Schematic illustration of two general overall wave configurations which can be obtained from the reflection of asymmetric shock waves in steady flows: (a) an overall regular reflection (oRR), and (b) an overall Mach reflection (oMR).

boundary condition for an oRR is $\theta_1 - \theta_3 = \theta_2 - \theta_4 = \delta$. (Note that for case of the reflection of symmetric shock waves $\theta_1 = \theta_2$ and $\delta = 0$).

Li et al. [34] showed that two different oRR wave configurations are theoretically possible:

- An oRR wave configuration that consists of two regular reflections that are both weak, wRR, and hence will be labelled as oRR[wRR + wRR]; and
- an oRR wave configuration that also consists of two regular reflections of which one is weak, wRR, and the other is strong, sRR, and hence will be labelled as oRR[wRR + sRR].

Fig. 10b indicates that in addition to the incident and reflected shock waves (i_1, i_2, r_1, r_2) a Mach stem (m)

appears in an oMR wave configuration. The Mach stem bridges two triple points (T_1 and T_2) from which two slipstreams (s_1 and s_2) emanate. If the flow deflection angles are again $\theta_1, \theta_2, \theta_3$ and θ_4 through the i_1, i_2, r_1 and r_2 shock waves, respectively, then the boundary conditions for an oMR wave configuration are $\theta_1 - \theta_3 = \delta_1$ and $\theta_2 - \theta_4 = \delta_2$. (Note that for the case of the reflection of symmetric shock waves $\theta_1 = \theta_2$ and $\delta_1 = \delta_2$.)

Li et al. [34] also showed that three different oMR wave configurations are theoretically possible. They are:

- An oMR wave configuration that consists of two direct-Mach reflections, DiMR, and hence will be labelled as oMR[DiMR + DiMR];
- an oMR wave configuration that consists of one direct-Mach reflection, DiMR, and one stationary-Mach reflection, StMR, and hence will be labelled as oMR[DiMR + StMR]; and
- an oMR wave configuration that consists of one direct-Mach reflection, DiMR, and one inverse-Mach reflection, InMR, and hence will be labelled as oMR[DiMR + InMR].

Details regarding the direct-, stationary- and inverse-Mach reflections can be found in Courant and Friedrichs [36] and Ben-Dor [1].

3.2. The dual-solution-domain

Li et al. [34] identified, in the course of their analytical study, two extreme transition criteria, which as recognized by them were analogous to the above-mentioned two extreme transition criteria in the case of the reflection of symmetric shock waves, namely the detachment and the von Neumann criteria. Similarly to the case of the reflection of symmetric shock waves, the two extreme transition criteria also resulted in a dual-solution-domain.

The dual-solution-domain, in the $(\theta_{w1}, \theta_{w2})$ -plane for $M_0 = 4.96$ is shown in Fig. 11. (Note that the wedge angles θ_{w1} and θ_{w2} play a symmetric role.) The transition lines analogous to the “detachment”, θ_{w2}^E , and the “von Neumann”, θ_{w2}^T , criteria are drawn as solid lines. The dual-solution-domain, inside which the overall wave configuration can be either an oRR or an oMR, extends between these two transition lines. The two dashed lines, marked θ_{w1}^N and θ_{w2}^N , indicate the von Neumann condition for the shock wave reflection over a single wedge (i.e., a symmetric reflection). On one of its sides the Mach reflection wave configuration is direct, DiMR, and on its other side the Mach reflection wave configuration is inverse, InMR, on the line itself the Mach reflection wave configuration is stationary, StMR.

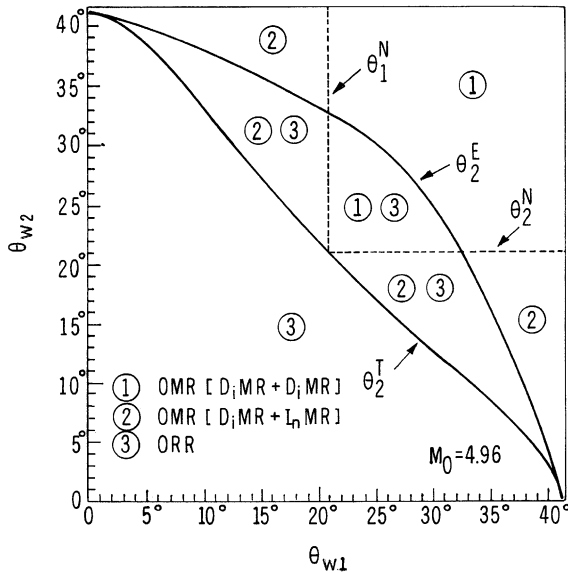


Fig. 11. The dual-solution-domain in the reflection of asymmetric shock waves.

Based on Fig. 11 the dual-solution-domain can be divided into two parts:

- In one part, labelled oMR[DiMR + DiMR], the overall wave configuration can be either an oRR or an oMR, which consists of two DiMR wave configurations.
- In the other part, labelled oMR[DiMR + InMR], the overall wave configuration can be either an oRR or an oMR, which consists of one DiMR wave configuration and one InMR wave configuration.

3.3. Wedge-angle-variation-induced hysteresis

Fig. 11 suggests that one can start with an oRR wave configuration having a value of $\theta_{w2} < \theta_{w2}^T$ and then increase θ_{w2} until the “detachment” transition line, θ_{w2}^E , above which an oRR wave configuration is theoretically impossible, is crossed. At the “detachment” transition line the oRR wave configuration must change to an oMR wave configuration. If, after the transition, the direction of changing θ_{w2} is reversed and θ_{w2} is decreased, the oMR wave configuration can continue to exist until the “von Neumann” transition line, θ_{w2}^T , below which an oMR wave configuration is theoretically impossible, will be reached. At the “von Neumann” transition line the oMR wave configuration must change to an oRR wave configuration.

Based on Fig. 11, which corresponds to $M_0 = 4.96$, it is evident that depending on whether θ_{w1} is smaller or larger than θ_{w1}^N , two different sequences of transition of the overall reflection wave configuration are theoret-

cally possible during the process of first increasing θ_{w2} and then decreasing it back to its initial value:

1. For $\theta_{w1} < \theta_{w1}^N$ the process starts with an oRR wave configuration that is maintained until the “detachment” transition line, θ_{w2}^E , is reached. At this point a transition takes place and the oRR wave configuration changes to an oMR wave configuration that consist of one direct- and one inverse-Mach reflection, i.e., an oMR[DiMR + InMR]. On the reverse path the oMR[DiMR + InMR] wave configuration is maintained until the “von Neumann” transition line, θ_{w2}^T , is reached. At this point the reversed transition takes place and the oMR[DiMR + InMR] wave configuration changes back to an oRR wave configuration.
2. For $\theta_{w1} > \theta_{w1}^N$ the process again starts with an oRR wave configuration that is maintained until the “detachment” transition line, θ_{w2}^E , is reached. At this point a transition takes place and the oRR wave configuration changes to an oMR wave configuration that consists of two direct-Mach reflection, i.e., oMR[DiMR + DiMR]. On the reverse path the oMR[DiMR + DiMR] wave configuration is maintained until the line θ_{w2}^N is reached. Exactly on this line the reflection wave configuration becomes an oMR[DiMR + StMR], i.e., an oMR wave configuration that consists of one direct- and one stationary-Mach reflections. Then it changes to an oMR[DiMR + InMR] wave configuration, that consists of one direct- and one inverse-Mach reflections. The oMR[DiMR + InMR] is maintained until the “von Neumann” transition line, θ_{w2}^T , is reached. At this point the reversed transition takes place and the oMR[DiMR + InMR] wave configuration changes back to an oRR wave configuration.

Chpoun and Lengrand [33] and Li et al. [34] experimentally verified both the existence of an oMR[DiMR + InMR] wave configuration and the existence of a wedge-angle-variation-induced hysteresis in the oRR $\leftrightarrow$ oMR transition. Owing to resolution limitations of their experimental capabilities, the above-described two theoretically possible sequences of events in the oRR $\leftrightarrow$ oMR transition were not recorded. The experimental results of Li et al. [34] very well agreed with the above-presented analytical transition lines.

It is important to note here that the experimental and geometrical set-ups of the reflection experiments over asymmetric wedges were very similar to those over symmetric wedges, which were described in the previous section. Hence, the three-dimensional effects in both cases were probably also similar. The fact that very good agreements between the analytical predictions and the experimental results were obtained regarding both the transition and the wave angles might suggest that the

influence of the three-dimensional effects was not too significant.

Typical schlieren photographs showing an overall MR wave configuration, oMR, and an overall RR wave configuration, oRR, for identical values of $M_0 = 4.96$, $\theta_{w1} = 28^\circ$ and $\theta_{w2} = 24^\circ$ are shown in Figs. 12a and b, respectively. These two schlieren photographs are clear evidence that different overall wave configurations can be obtained for identical flow conditions when two asymmetric shock waves interact.

Ivanov et al. [35] numerically investigated the just-mentioned wedge-angle-variation-induced hysteresis. Numerical simulations of the above-mentioned two different sequences of events, which were obtained by Ivanov et al. [35], are shown in Figs. 13a and b for $M_0 = 4.96$ and for $\theta_{w1} = 18^\circ$ and $\theta_{w1} = 28^\circ$, respectively. Constant density contours (isopycnics) are shown in each of these figures. The calculation of each case starts at the top frame and then goes around in a counter clockwise direction. The flow conditions for each

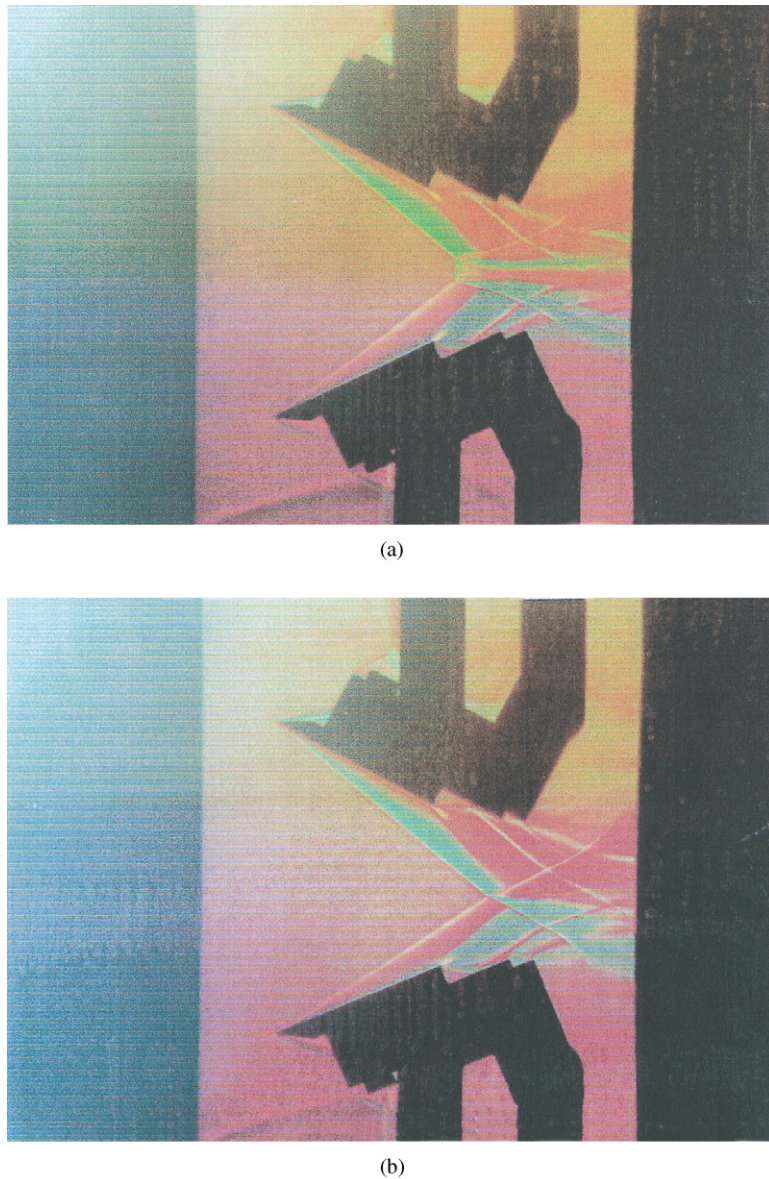


Fig. 12. Typical schlieren photographs, taken from the experimental study of Li et al. [34], showing (a) an overall MR wave configuration, and (b) an overall RR wave configuration for identical values of $M_0 = 4.96$, $\theta_{w1} = 28^\circ$ and $\theta_{w2} = 24^\circ$.

horizontal pair of the frames are the same. Hence, the wedge-angle-variation-induced hysteresis is clearly seen in these figures.

The sequence that is shown in Fig. 13a starts with an oRR wave configuration at $\theta_{w2} = 22^\circ$. When θ_{w2} is

increased the oRR wave configuration still exists at $\theta_{w2} = 28^\circ$. At $\theta_{w2} = 36^\circ$, that is above the corresponding value of θ_{w2}^E , an oRR wave configuration can no longer exist and the overall reflection wave configuration is an oMR[DiMR + InMR] in which the upper MR wave

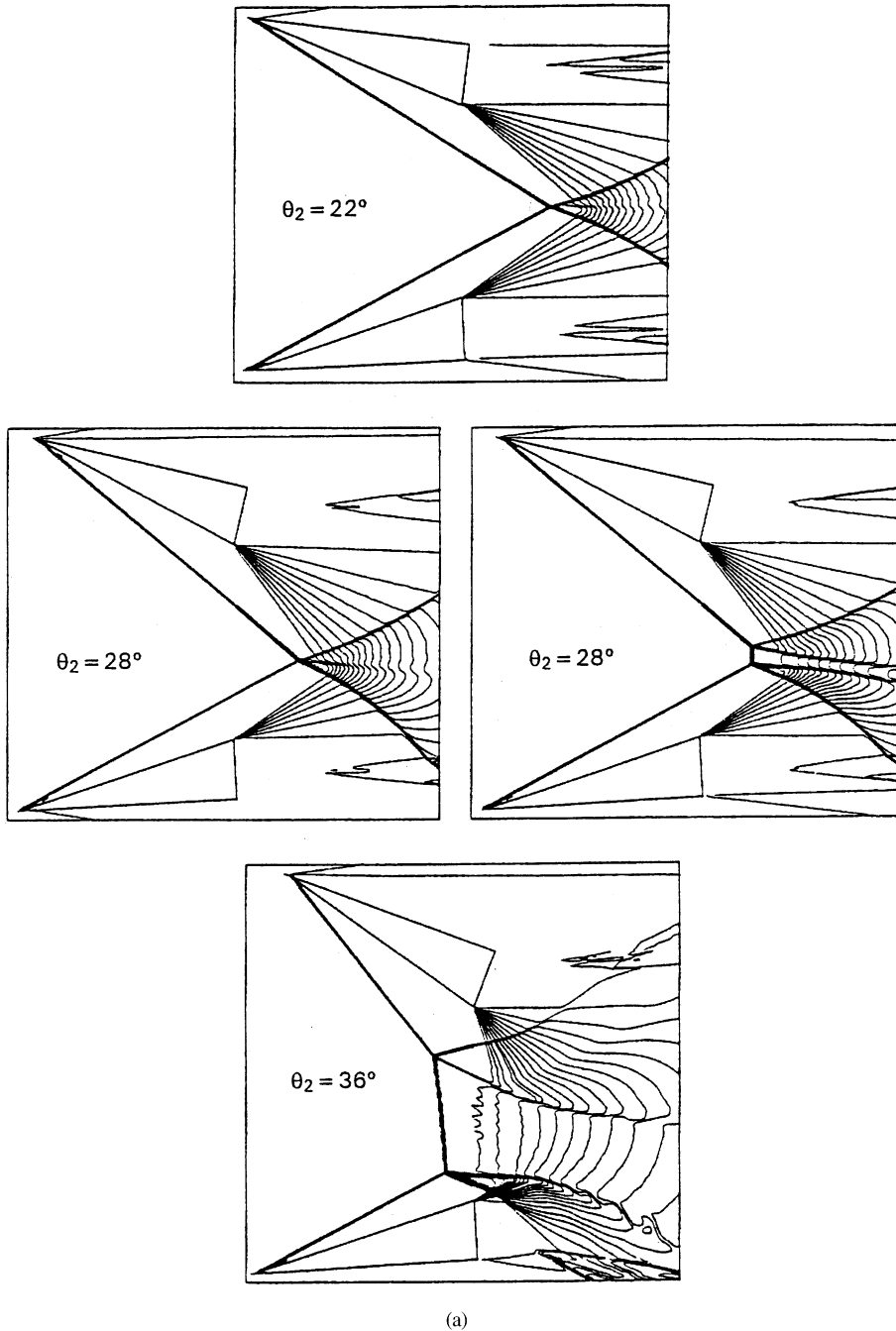


Fig. 13. Numerical frames, taken from Ivanov et al.'s [35] numerical investigation, illustrating the hysteresis loop for (a) $M_0 = 4.96$ and $\theta_{w1} = 18^\circ$ and (b) $M_0 = 4.96$ and $\theta_{w1} = 28^\circ$. Constant density contours are displayed.

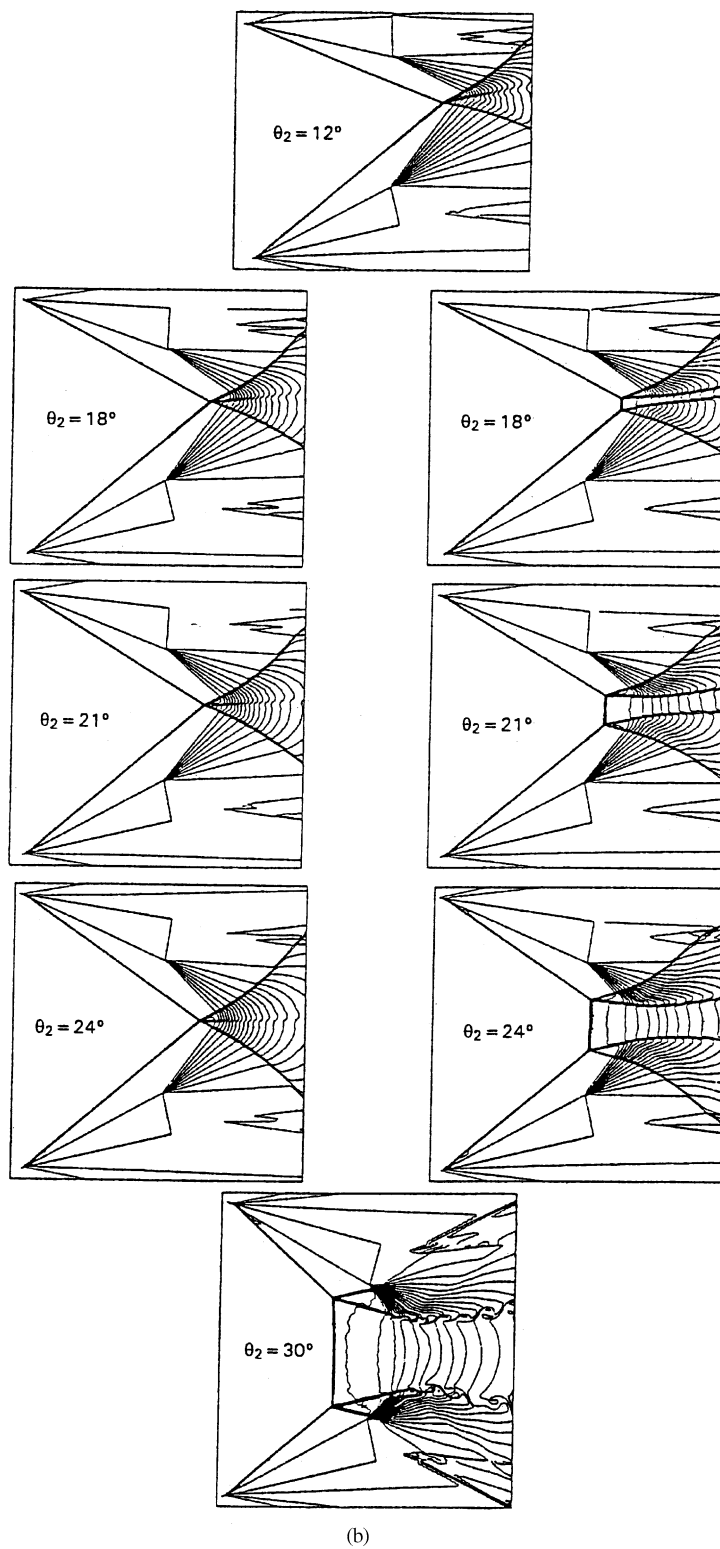


Fig. 13 (continued).

configuration is a DiMR wave configuration and the lower MR wave configuration is an InMR wave configuration. When θ_{w2} is decreased from $\theta_{w2} = 36^\circ$ back to its initial value, i.e., $\theta_{w2} = 22^\circ$, the oMR[DiMR + InMR] wave configuration is maintained in the dual-solution-domain. Thus, for example, at $\theta_{w2} = 28^\circ$ (see Fig. 13a) two different shock wave configurations, one oRR and one oMR, are actually observed, in perfect agreement with the analysis. This sequence can be labelled

$$\text{oRR} \xrightarrow{\text{at } \theta_{w2}^E} \text{oMR [DiMR + InMR]} \xrightarrow{\text{at } \theta_{w2}^T} \text{oRR}.$$

The sequence that is shown in Fig. 13b starts with an oRR wave configuration at $\theta_{w2} = 12^\circ$. When θ_{w2} is increased the oRR wave configuration still exists at $\theta_{w2} = 24^\circ$. At $\theta_{w2} = 30^\circ > \theta_{w2}^E$ the overall reflection wave configuration is an oMR[DiMR + DiMR] in which both the upper and the lower MR wave configurations are DiMR wave configurations. When θ_{w2} is decreased from $\theta_{w2} = 30^\circ$ back to its initial value, i.e., $\theta_{w2} = 12^\circ$, the orientation of the slipstream of the lower MR wave configuration changes continuously. As a result at $\theta_{w2} = 21^\circ$, which is very close to the analytical value $\theta_{w2}^N = 20.87^\circ$, the upper wave configuration is close to being an StMR (i.e., its slipstream is almost parallel to the oncoming flow at the triple point). Upon a further decrease in θ_{w2} the lower wave configuration changes to an InMR, and the oMR wave configuration consists now of a DiMR and an InMR, i.e., it is an oMR[DiMR + InMR] wave configuration, as shown for $\theta_{w2} = 18^\circ$. This sequence can be labelled

$$\begin{aligned} \text{oRR} &\xrightarrow{\text{at } \theta_{w2}^E} \text{oMR[DiMR + DiMR]} \\ &\xrightarrow{\text{on } \theta_{w2}^N} \text{oMR[DiMR + StMR]} \\ &\xrightarrow{\text{at } \theta_{w2}^N} \text{oMR[DiMR + InMR]} \\ &\xrightarrow{\text{at } \theta_{w2}^T} \text{oRR}. \end{aligned}$$

3.4. Flow-Mach-number-variation-induced hysteresis

Similarly to the flow-Mach-number-variation-induced hysteresis process, in the reflection of symmetric shock waves, which was numerically illustrated and verified both by Ivanov et al. [31] and Onofri and Nasuti [32], it is hypothesized here that a similar flow-Mach-number-variation-induced hysteresis process also exists in the reflection of asymmetric shock waves.

Owing to the fact that conducting experiments in a wind tunnel in which the flow-Mach number is continuously changed is extremely complicated the existence of a flow-Mach-number-variation-induced

hysteresis process in the reflection of asymmetric shock waves will have to be proven numerically.

4. The hysteresis process in the reflection of axisymmetric (conical) shock waves

Regarding the reflection of both symmetric and asymmetric shock waves, as mentioned earlier, in spite of the good to excellent agreement that was evident when the experimentally recorded hysteresis processes in the RR $\leftrightarrow$ MR transition were compared with predictions based on two-dimensional analytical models (see e.g., Chpoun et al. [12] and Li et al. [34]), it was claimed and shown by some investigators (see e.g., Skews et al. [25], Skews [26,27], Fomin et al. [23], Ivanov et al. [28], and Kudryavtsev et al. [30]) that these hysteresis processes were influenced and triggered and thereby promoted by three-dimensional edge effects, and are, therefore, not purely two-dimensional. This in turn, led to the questions whether an actual RR wave configuration is indeed stable inside the dual-solution-domain, and whether the hysteresis process indeed exists in a purely two-dimensional flow. For this reason Chpoun et al. [37] and Ben-Dor et al. [38] designed an axisymmetric geometrical set-up, which was free of three-dimensional effect.

A schematic illustration of the experimental set-up fulfilling this requirement is shown in Fig. 14. A 70-mm in diameter and 28-mm wide conical ring was placed in the center of a 127-mm supersonic jet, which emanated from the wind tunnel. The head angle of the conical ring was $\theta = 8.5^\circ$. A curvilinear cone was placed downstream of the conical ring. Their axes of symmetry coincided. The shape of the curvilinear cone was: $y(x) = 0.000115x^3 + 0.002717x^2 + 0.08749x$ (x and y are in mm). The base diameter and the length (height) of the curvilinear cone were 30.4 and 40 mm, respectively. The conical ring generated an incident converging straight conical shock wave, i_1 . This incident converging straight conical shock wave interacted with the incident diverging curvilinear conical shock wave, i_2 , which was generated by the curvilinear cone. Depending on the angle of interaction between these two incident shock waves three different types, A, B and C, of overall wave configurations were recorded in the course of Chpoun et al.'s [37] experimental investigation. They are shown schematically in Fig. 15.

The Type A wave pattern, shown in Fig. 15a, consists of a straight diverging conical shock wave attached to the nose of the curvilinear cone that interacts with the converging straight conical shock wave, which is generated by the conical ring. As a result, the boundary layer separates, and a large slow re-circulation zone is induced near the surface of the cone. The slow re-circulation zone is the mechanism supporting the

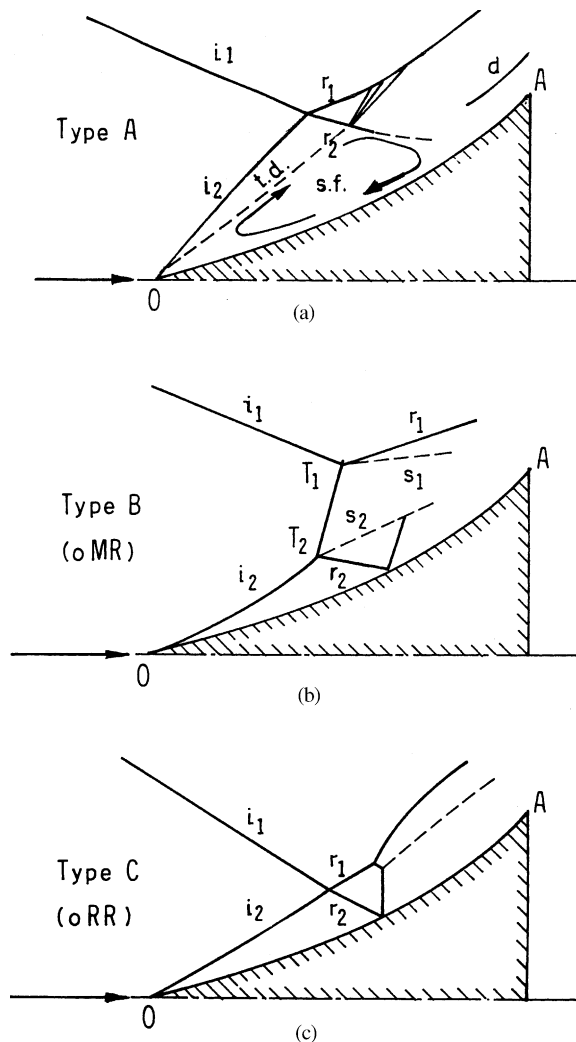


Fig. 15. Schematic illustration of the three different types of wave configurations that were obtained in the course of Chpoun et al.'s [37] experimental investigation of the reflection of conical shock waves, in the set-up shown in Fig. 14: (a) Type A, (b) Type B (oMR), and (c) Type C (oRR).

From the initial position [frame (a)] with coordinate $X = 1.56$ the curvilinear cone was shifted slowly with the velocity 0.22 mm/s along the axis of symmetry towards the conical ring. Whenever a drastic change, in the wave pattern, took place, the curvilinear cone was stopped and the flow was allowed to stabilize. Type A wave patterns are seen in frames (a)–(d). When the curvilinear cone was shifted from $X = 0.86$ [frame (d)] to $X = 0.70$ [frame (e)] the wave configuration changed suddenly from Type A to Type B. This transition involved a reattachment of the boundary layer and, a disappearance of the separated zone. As a result the incident shock wave that was attached to the nose of the

curvilinear cone, changed suddenly from a strong straight diverging conical shock wave to a weak curvilinear diverging conical shock wave. This transition, the Type A $\rightarrow$ Type B transition, occurred at $X \approx 0.78$. The subsequent frames (f)–(j) correspond to the shift of the curvilinear cone in the reverse direction. Two transitions in the wave pattern were encountered, in the course of shifting the curvilinear cone from $X = 0.70$ [frame (e)] to $X = 1.68$ [frame (j)]. While shifting the curvilinear cone from $X = 0.70$ [frame (e)] to $X = 1.29$ [frame (h)] the height of the Mach stem continuously decreased until it disappeared between $X = 1.29$ [frame (h)] and $X = 1.56$ [frame (i)], and the wave pattern changed from Type B to Type C. This transition, the Type B $\rightarrow$ Type C transition, occurred at $X \approx 1.38$. While shifting the curvilinear cone from $X = 1.56$ [frame (i)] to $X = 1.68$ [frame (j)] the boundary layer was separated from the surface of the curvilinear cone. As a result, the incident shock wave that was attached to the nose of the curvilinear cone, suddenly changed from a weak diverging curvilinear conical shock wave to a strong diverging straight conical shock wave, and the Type A wave pattern was formed again. This transition, the Type C $\rightarrow$ Type A transition, occurred at $X \approx 1.61$.

Note that the pairs of frames (a) and (i), (b) and (h), (c) and (g) and (d) and (f), which correspond to the same X -position of the curvilinear cone, clearly demonstrate the existence of two different wave patterns for the same flow conditions. The entire set of frames in Fig. 16 provides a clear experimental evidence for the existence of a hysteresis process, which consists of the following three transitions:

- Type A $\rightarrow$ Type B at $X \approx 0.78$ in the course of decreasing X ,
- Type B $\rightarrow$ Type C at $X \approx 1.38$ in the course of increasing X , and
- Type C $\rightarrow$ Type A at $X \approx 1.61$ in the course of increasing X .

The hysteresis loop can be summarized as:

$$\begin{aligned} \text{Type A} &\xrightarrow{X \approx 0.78} \text{Type B} \\ &\xrightarrow{X \approx 1.38} \text{Type C} \xrightarrow{X \approx 1.61} \text{Type A.} \end{aligned}$$

Ben-Dor et al. [38] also found that the just mentioned Type B $\rightarrow$ Type C transition at $X \approx 1.38$ (between frames (h) and (i) in Fig. 16) was the origin of an additional internal hysteresis loop. In order to detect this loop in some of the experiments, immediately after encountering the Type B $\rightarrow$ Type C transition, at $X \approx 1.38$, the curvilinear cone was stopped and its shifting direction was reversed.

Fig. 17 shows a sequence of 8 experimental frames corresponding to this case. The coordinate of the

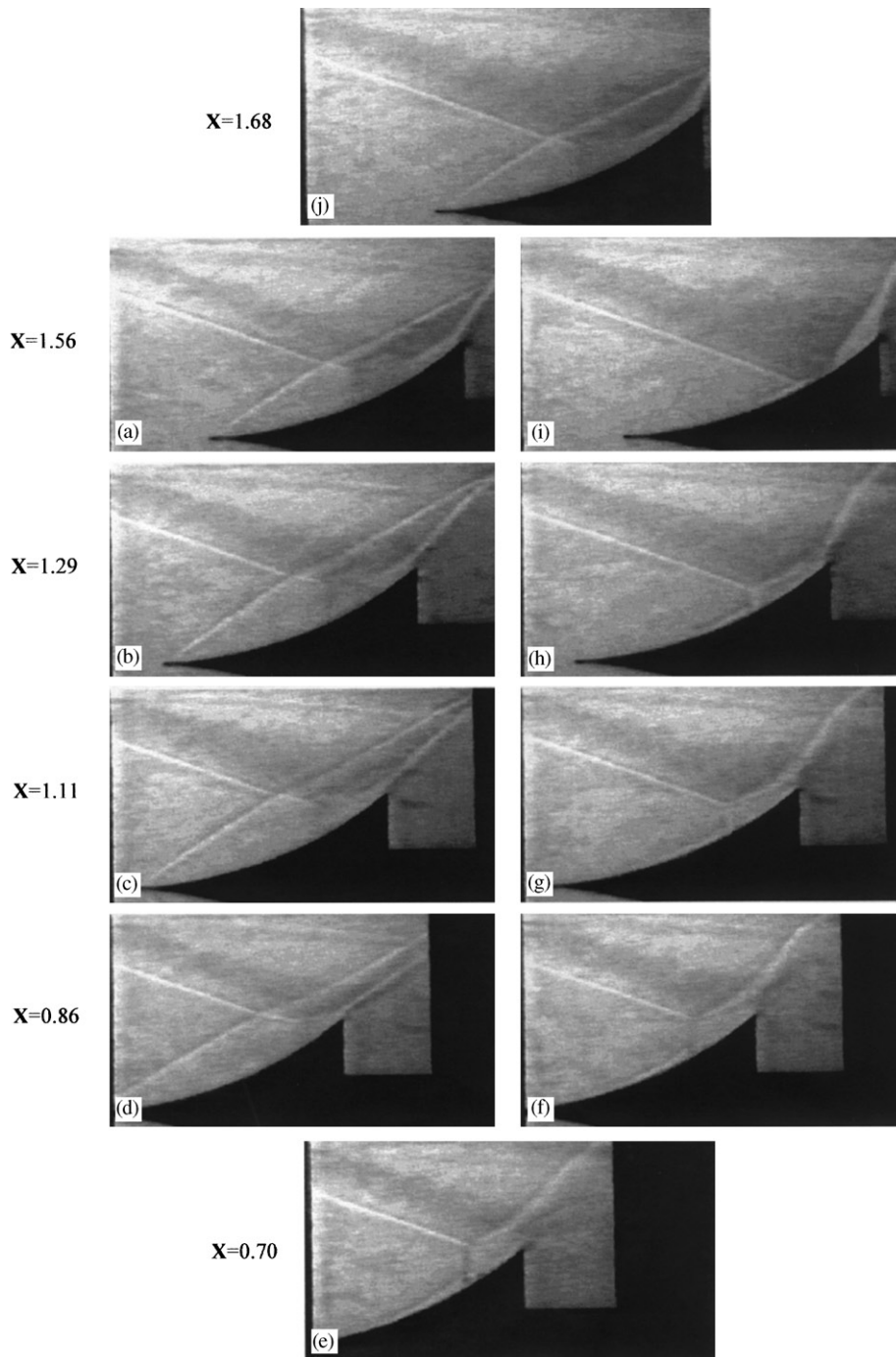


Fig. 16. A sequence of 10 frames taken from one of Ben-Dor et al.'s [38] experimentally recorded videotapes. The sequence shows the evolution of the wave pattern during the shift of the cone in the course of X : $1.56 \rightarrow 0.70 \rightarrow 1.68$. The three different wave patterns, Types A, B and C, which are shown in Fig. 15, are clearly seen.

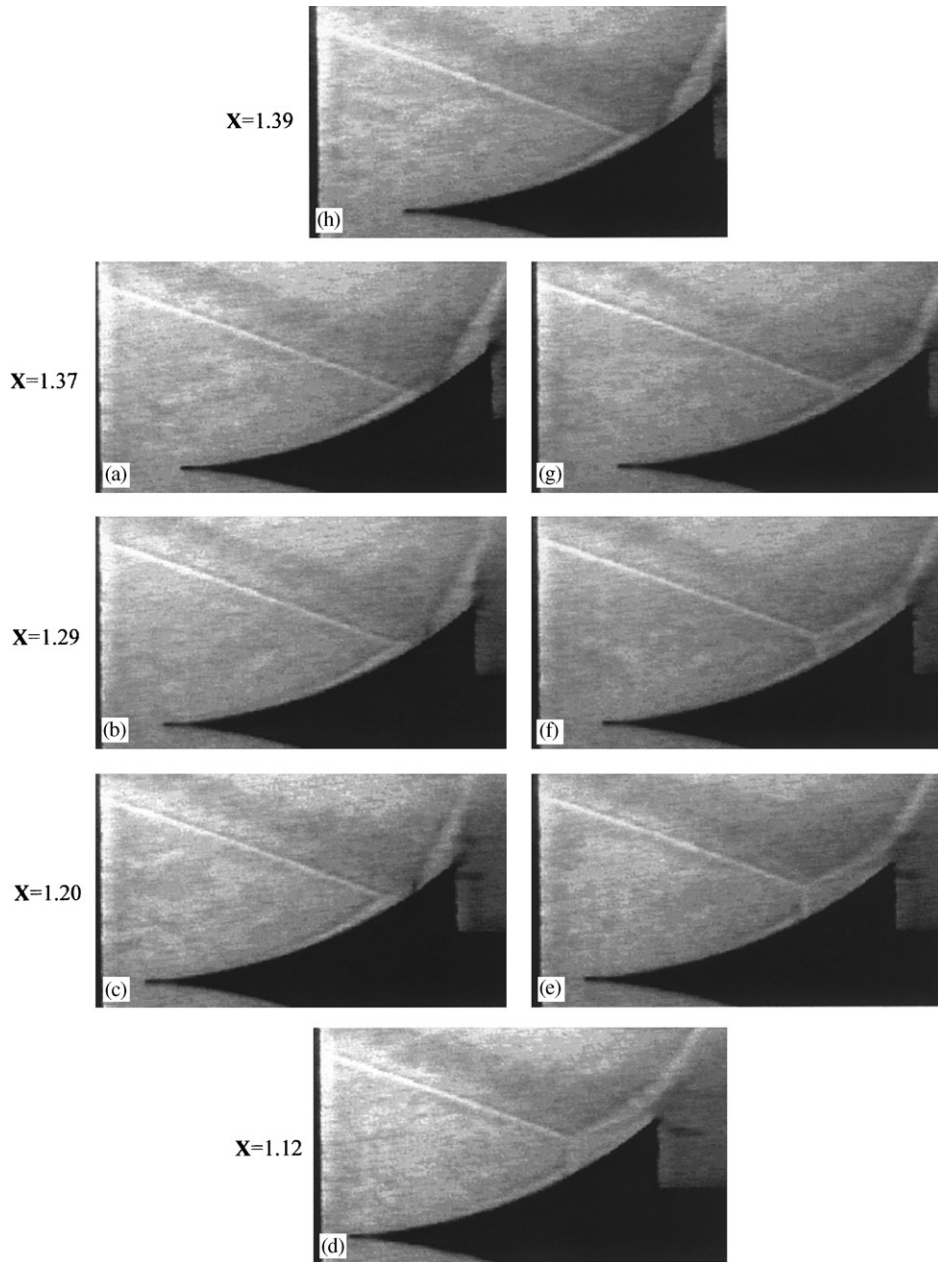


Fig. 17. A sequence of 8 frames taken from one of Ben-Dor et al.'s [38] experimentally recorded videotapes. The sequence shows the evolution of the wave pattern during the shift of the cone in the course of X : $1.39 \rightarrow 1.12 \rightarrow 1.39$. Two transitions of the wave pattern are evident.

curvilinear cone was changed in the course of X : $1.39 \rightarrow 1.12 \rightarrow 1.39$. Two transitions of the wave pattern were evident:

- A Type C $\rightarrow$ Type B transition took place at $X \approx 1.16$, when X was changed from 1.20 [frame (c)] to 1.12 [frame (d)], and an opposite
- Type B $\rightarrow$ Type C transition occurred at $X \approx 1.38$, when X was changed from 1.37 [frame (g)] to 1.39 [frame (h)].

It should be noted here that, based on the experiments, the Type B $\rightarrow$ Type C transition occurred when the Mach stem had a finite height. This was not the case

in the inviscid numerical simulations, where the Mach stem decreased continuously until it vanished at the Type B $\rightarrow$ Type C transition. Probably viscous effects behind the Mach stem are the cause for this peculiar behavior. It is noted again, that the Type C $\leftrightarrow$ Type B hysteresis loop, shown in Fig. 17, is an internal hysteresis loop of the major hysteresis loop shown in Fig. 16. This internal hysteresis loop can be summarized as: Type C $\rightarrow$ $X \approx 1.16$ Type B $\rightarrow$ $X \approx 1.38$ Type C.

Finally, it should be noted that frames (b) and (h) in Fig. 16 and frames (b) and (f) in Fig. 17, which correspond to the same X -position of the curvilinear cone, clearly demonstrate that three different shock wave patterns, Types A, B and C, were obtained experimentally, for the same flow conditions.

The above-described experimental double-loop hysteresis is shown in Fig. 18. The frames shown in Fig. 16 describe a viscous-dependent major hysteresis loop, which is accompanied by three transitions:

- Type A $\rightarrow$ Type B at $X \approx 0.78$;
- Type B $\rightarrow$ Type C at $X \approx 1.38$; and
- Type C $\rightarrow$ Type A at $X \approx 1.61$.

Similarly, the frames shown in Fig. 17 describe an internal (non-viscous-dependent) hysteresis loop with two transitions.

- Type B $\rightarrow$ Type C (or oMR $\rightarrow$ oRR) at $X \approx 1.38$; and
- Type C $\rightarrow$ Type B (or oRR $\rightarrow$ oMR) at $X \approx 1.16$.

Note that the above values of X are directly related to the dimensions and shapes of the conical ring and the curvilinear cone. It is believed that different dimensions and shapes would result in a similar hysteresis phenomenon with transitions at different values of X .

The frames shown in Figs. 16 and 17 were added to Fig. 18. Each one of them is labeled as (Nn) where N is the number of the Figs. 16 or 17 and n is the designation of the frame in the appropriate figure [i.e., (a), (b), (c), etc.].

In summary, the experimental results provide a clear proof of the existence of a hysteresis phenomenon in the oRR $\leftrightarrow$ oMR transition in a flow that is free of three-dimensional effects. They also reveal a complex double-loop hysteresis.

As will be shown subsequently, the Euler solver, which was applied by Ben-Dor et al. [38], failed, as expected, to reproduce the viscous dependent Type A wave configuration and the hysteresis loop that is associated with it. As a result only the non-viscous-dependent internal hysteresis loop, i.e., the Type B $\rightarrow$ Type C hysteresis loop, was simulated. However, in addition, a few secondary hysteresis loops, associated with the interaction between the overall shock wave configuration and the rear edge of the curvilinear cone were obtained. Burstschell et al. [39], who simulated the process using a Navier–Stokes code, did succeed in obtaining the Type A wave configuration.

The Euler solver based overall multi-loop hysteresis is shown in Fig. 19 in the (X, H_m)-plane. Here $H_m = h_m/L$

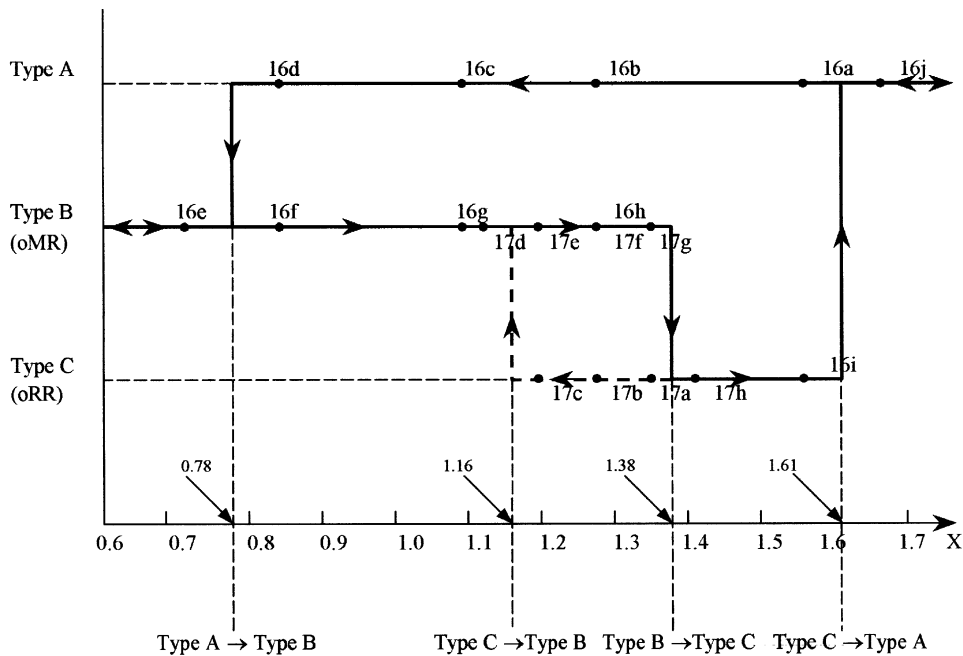


Fig. 18. The experimental double-loop hysteresis.

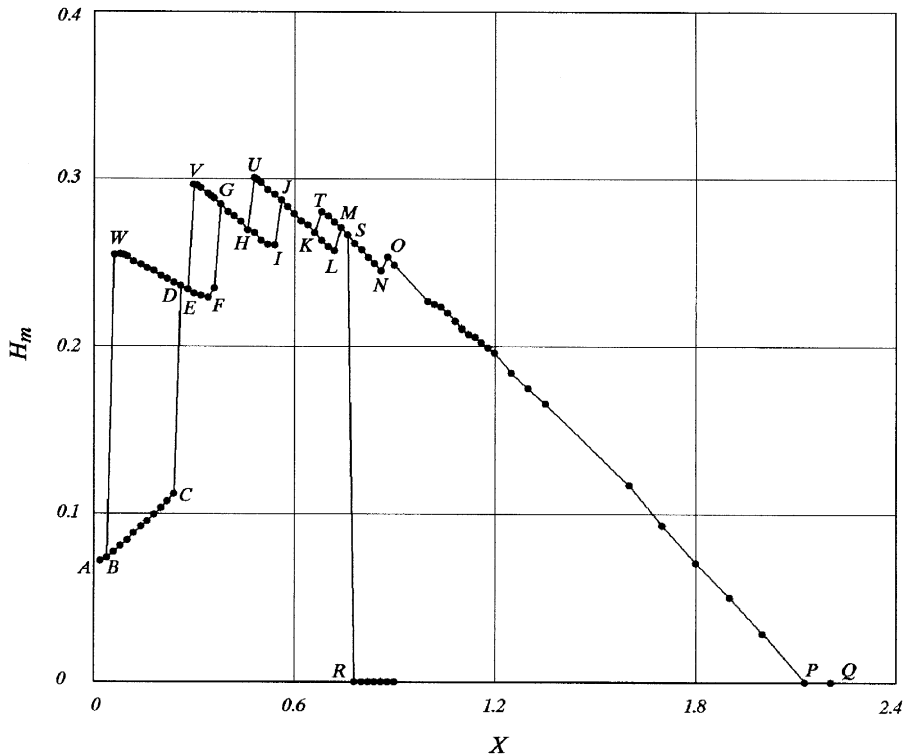


Fig. 19. The numerical (Euler solver based) overall multi-loop hysteresis.

is the non-dimensional length of the Mach stem, where h_m , the length of the Mach stem, i.e., the distance between the two triple points when the resulted overall wave configuration is an oMR. Evidently, for an oRR wave configuration $h_m = 0$, i.e., $H_m = 0$. The stable flow states, which were encountered when the curvilinear cone was numerically shifted along the $X = 0 \rightarrow 2.2$ path and along the reversed $X = 2.2 \rightarrow 0$ path were A–B–C–D–E–F–G–H–I–J–K–L–M–N–O–P–Q and Q–P–R–S–M–T–K–J–U–H–G–V–E–D–W–B–A, respectively. These two different paths of stable flow states consisted of two different types of hysteresis loops. The first one, the major one, which is similar to the one obtained experimentally (see Fig. 18), consists of both oMR and oRR wave configurations for the same flow conditions. It is described in Fig. 19 by the S–N–O–P–R–S loop. The second one, the minor one, has not been recorded experimentally, probably because of the poor experimental resolution along the surface of the curvilinear cone. It consists of only oMR wave configurations, which have different flow patterns for the same flow conditions. Fig. 19 reveals four such minor hysteresis loops: B–C–D–W–B, E–F–G–V–E, H–I–J–U–H and K–L–M–T–K.

The major S–N–O–P–R–S hysteresis loop is shown in Fig. 20. This hysteresis, which is non-viscous dependent,

probably occurs due to the existence of a dual-solution-domain. The wave configuration for $X = 1.3$, shown in frame (a), is an oRR. The incident converging straight conical and the incident diverging curvilinear conical shock waves intersect each other in a regular manner. The refracted converging conical shock wave reflects from the surface of the curvilinear cone as an RR. The reflected shock wave of this RR interacts with the refracted diverging curvilinear conical shock wave. When X is decreased to 0.80 [frame (c)] the reflection of the transmitted converging conical shock wave from the surface of the curvilinear cone changes from an RR to an MR. When X is decreased to 0.70 [frame (d)] the overall wave configuration changes suddenly from an oRR to an oMR. If now the direction of shifting the curvilinear cone is reversed and X is increased back towards its initial value, the flow patterns shown in frames (e)–(g) are encountered. The length of the Mach stem, of the oMR wave configurations, gradually decreases. It is also seen in frames (d)–(g) that once an oMR wave configuration is formed the interaction of the wave reflected from the surface of the curvilinear cone and the contact surface results in a special structure of “humps” of the contact surface. The number of the humps is seen to increase as X is increased. There are three full humps for $X = 0.7$ and 0.80 [frames (d) and

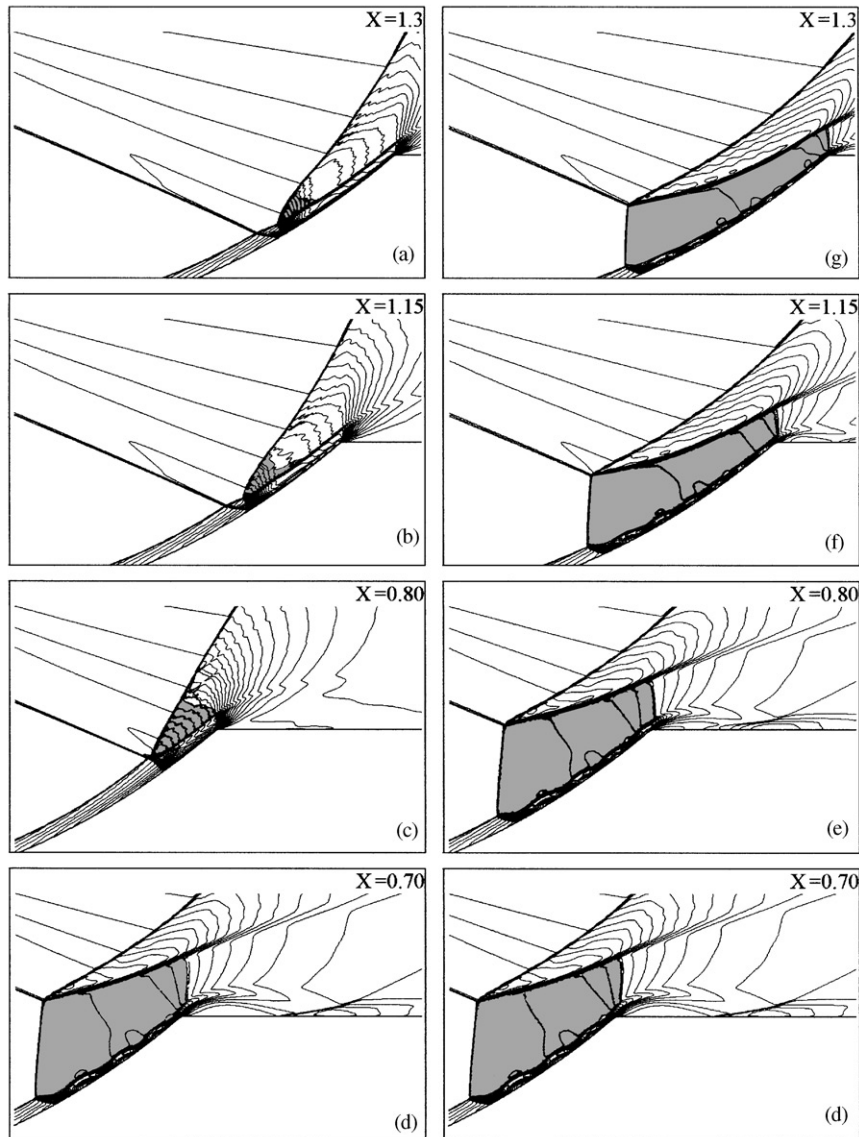


Fig. 20. Numerical frames showing the wave patterns associated with the major S–N–O–P–R–S hysteresis loop, (see Fig. 19 for details).

(e), respectively], five full humps for $X = 1.15$ [frame (f)] and seven full humps for $X = 1.3$ [frame (g)]. The three pairs c and e (for $X = 0.8$), b and f (for $X = 1.15$), and a and g (for $X = 1.3$) which show different overall shock wave configurations for identical flow conditions provide a clear evidence of the hysteresis phenomenon. Based on Fig. 20 the oRR $\rightarrow$ oMR transition takes place at $X = 0.75 \pm 0.05$.

Ben-Dor et al. [38] found that the location of the curvilinear cone when a given hump detached from the rear edge of the curvilinear cone was different from the location of the curvilinear cone when the same hump

was reattached. The attachment/detachment mechanism led to the identification of the above-mentioned additional minor hysteresis loops in which the wave configuration was always an oMR, but the flow patterns along the surface of the curvilinear cone were different. Four such hysteresis loops are shown in Fig. 19. They are B–C–D–W–B, E–F–G–V–E, H–I–J–U–H and K–L–M–T–K.

The B–C–D–W–B hysteresis loop is shown in Fig. 21. At $X = 0.02$ [frame (a)] an oMR wave configuration is obtained. It should be noted that unlike the previously discussed oMR wave configurations where the reflected

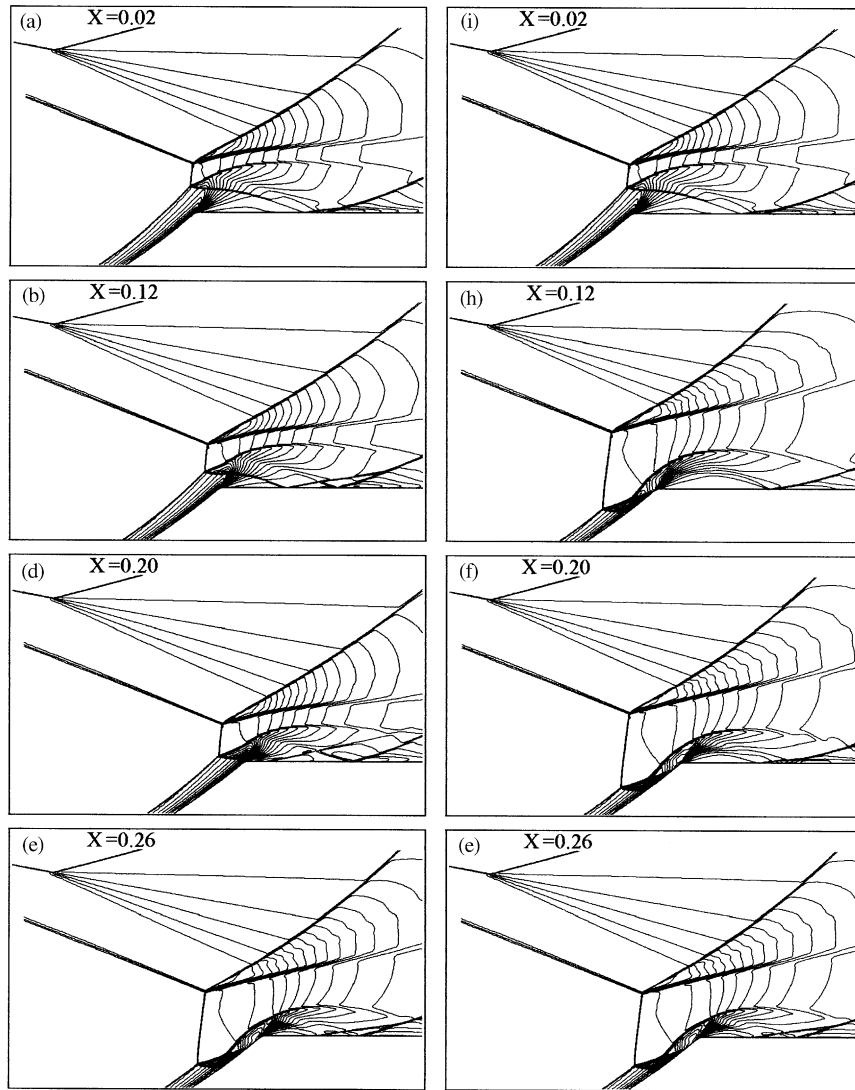


Fig. 21. Numerical frames showing the wave patterns associated with the B–C–D–W–B hysteresis loop, (see Fig. 19 for details).

shock wave of the lower MR interacted with the curvilinear cone upstream of its rear edge, here it interacts with the surface of the curvilinear cone downstream of its rear edge. When X is increased [frames (b) and (d)] the Mach stem connecting the two triple points moves upstream and its length increases. Note that a situation in which the reflected shock wave of the lower MR touches the rear edge of the curvilinear cone is reached in the vicinity of $X = 0.20$ [frame (d)]. An increase of X beyond this value results in a sudden jump of the Mach stem upstream that is associated with a sudden increase in its length, as can be seen in frame (e) for $X = 0.26$. When X is decreased back to $X = 0.02$ the stable solutions shown in frames (f) to (i) are

obtained. The two pairs d and f (for $X \approx 0.2$) and b and h (for $X \approx 0.12$), which show different overall wave configurations for identical flow conditions are clear evidences of the hysteresis. The hysteresis is typified by a sudden increase in the length of the Mach stem at $X = 0.23 \pm 0.03$ and a reverse sudden decrease in its length at $X = 0.07 \pm 0.05$. The calculations reveal that the mechanism, which triggers the sudden transition from an oMR with a short Mach stem [frame (d)] to an oMR with a much larger Mach stem [frame (e)], is the interaction of the reflected shock wave of the lower MR wave configuration (of the two MR wave configurations that consist the oMR) with the rear edge of the curvilinear cone [frame (d)]. It should be noted that

once the transition takes place [(frame (e)) the reflected shock wave of the lower MR wave configuration reflects from the surface of the curvilinear cone as an RR wave configuration. The reflected shock wave of this RR wave configuration interacts with the contact surface of the lower MR wave configuration. The result of this interaction is an expansion wave, which hits the solid surface of the curvilinear cone and reflects back towards the contact surface as an expansion wave. The reflected expansion wave interacts with the contact surface and turns it back towards the curvilinear cone. Had X been further increased, as will be shown subsequently, this turn of the contact surface would have been developed to become the first full hump of the contact surface over the surface of the curvilinear cone. Frames (h) and (i) indicate that the reverse transition is driven by the interaction of the above-mentioned expansion wave with the rear edge of the curvilinear cone.

The minor E–F–G–V–E hysteresis loop is shown in Fig. 22. At $X = 0.26$ an oMR wave configuration is obtained. When X is increased the Mach stem connecting the two triple points moves upstream and its length increases. The flow patterns in frame (a), (b) and (c) for $X = 0.26, 0.30$ and 0.34 , respectively, are similar to that shown in Fig. 21f. A further increase to $X = 0.38$, results in the flow pattern shown in frame (d), in which the above-mentioned expansion wave succeeded to develop a full hump of the contact surface of the lower MR wave configuration. When X is decreased back to $X = 0.26$ the stable flow patterns shown in frame (e), (f) and (g) are obtained. The pairs c and e and b and f that show different flow patterns for identical flow conditions are clear evidences of the hysteresis. It is evident from Fig. 22 that the hysteresis is driven by a mechanism associated with the attachment/detachment of the first hump of the contact surface to the surface of the curvilinear cone. For this reason the E–F–G–V–E hysteresis is termed: the 0 hump $\leftrightarrow$ 1 hump hysteresis. The transitions of this hysteresis loop are:

- 0 hump $\rightarrow^{X=0.36 \pm 0.02}$ 1 hump
- 1 hump $\rightarrow^{X=0.28 \pm 0.02}$ 0 hump

Ben-Dor et al. [38] showed that the minor H–I–J–U–H–E hysteresis loop is driven by a mechanism associated with the attachment/detachment of the second hump to the surface of the cone. For this hysteresis it is termed: the 1 hump $\leftrightarrow$ 2 humps hysteresis. The transitions of this hysteresis loop are:

- 1 hump $\rightarrow^{X \approx 0.55}$ 2 humps
- 2 humps $\rightarrow^{X \approx 0.47}$ 1 hump

Similarly, the minor K–L–M–T–K hysteresis loop is driven by a mechanism associated with the attachment/

detachment of the third hump to the surface of the cone. For this reason this hysteresis is termed: the 2 humps $\leftrightarrow$ 3 humps hysteresis. The transitions of this hysteresis loop are:

- 2 humps $\rightarrow^{X \approx 0.69}$ 3 humps
- 3 humps $\rightarrow^{X \approx 0.67}$ 2 humps

It is important to note here that all the above-determined transition values of X are appropriate to the specific geometrical set-up (i.e., dimensions and shapes of both the conical ring and the curvilinear cone) that was investigated by Ben-Dor et al. [38]. It is believed that a different geometrical set-up would result in similar hysteresis processes but with transitions at different values of X .

As mentioned earlier, Ben-Dor et al. [38], who used an Euler code in their numerical study, failed, as expected, to numerically reproduce the Type A wave configuration, shown in Fig. 15a, which involves a boundary layer separation, and as a result is viscous dependent. Burstchell et al. [39] also investigated the reflection process associated with the geometry shown in Fig. 14 using both an Euler and a Navier–Stokes numerical codes. While the results of their Euler calculations were similar to those of Ben-Dor et al. [38], they did succeed to reproduce the Type A wave configuration in their Navier–Stokes calculations. Furthermore, their Navier–Stokes calculations revealed a strong dependence of the number of the humps and their intensities on the viscosity. Hence, the role of viscous effects in forming the humps and determining their intensities, and as a result the existence of the above-described minor hysteresis loops is yet to be investigated and understood.

A Navier–Stokes numerical simulation of the experiment shown in Fig. 16a is shown in Fig. 23. The numerically generated Type A viscous-dependent wave configuration supports the hypothetical description of the fine details of the flow field as was forwarded by Ben-Dor et al. [38].

4.2. Flow-Mach-number-variation-induced hysteresis

Ben-Dor et al. [40] numerically investigated the flow-Mach-number-variation-induced hysteresis process in the interactions of conical shock wave over the axisymmetric geometry shown in Fig. 14. Three cases were investigated in the course of their numerical study. They differed in the location of the curvilinear cone with respect to the conical ring (i.e., in the value of X). The values of X that were used by them were: -0.3 , -0.2 and -0.1 (the minus sign means that the nose of the curvilinear cone was located upstream of the entrance cross section of the conical ring). The results

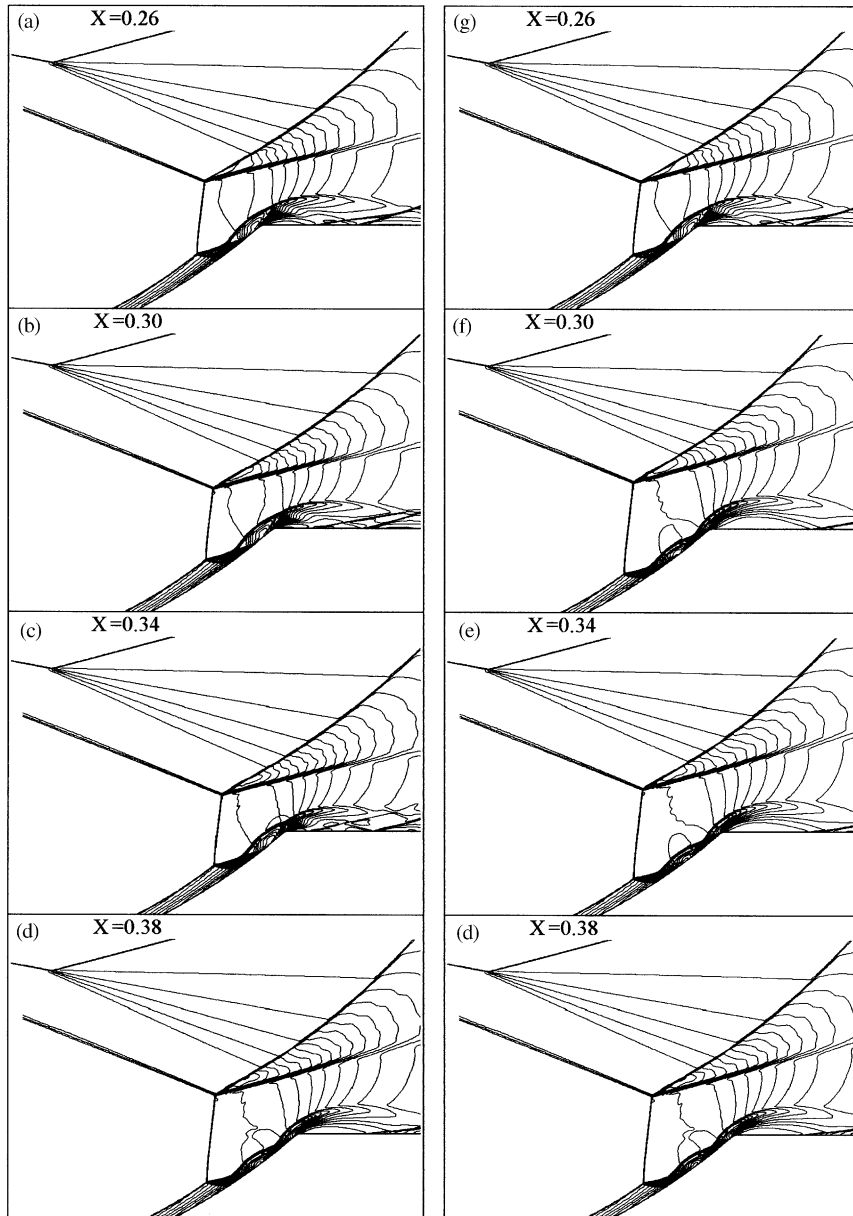


Fig. 22. Numerical frames showing the wave patterns associated with the E–F–G–V–E hysteresis loop (see Fig. 19 for details).

of their numerical simulations are presented in the followings.

Case 1: $X = -0.3$. The dependence of the non-dimensional Mach stem length (i.e., the distance between the two triple points of an oMR) on the free-stream-flow-Mach number when the latter was changed in the course $4.8 \rightarrow 2.6 \rightarrow 4.8$ is shown in Fig. 24 (recall that $H_m = 0$ for an oRR).

As is evident in Fig. 24, the wave configuration was an oRR during the variation of the flow-Mach number

from 4.8 to 3.8. A sudden transition from an oRR wave configuration to an oMR wave configuration occurred at $M_0 \cong 3.65$. Further reductions of the flow-Mach number were associated with a gradual increase in the length of the Mach stem. This trend continued until $M_0 \cong 3.15$ where a sharp increase of the Mach stem length was observed. As it turned out, this transition was caused by the intersection of the reflected shock wave of the lower MR wave configuration with the rear edge of the curvilinear cone. When the direction of changing the

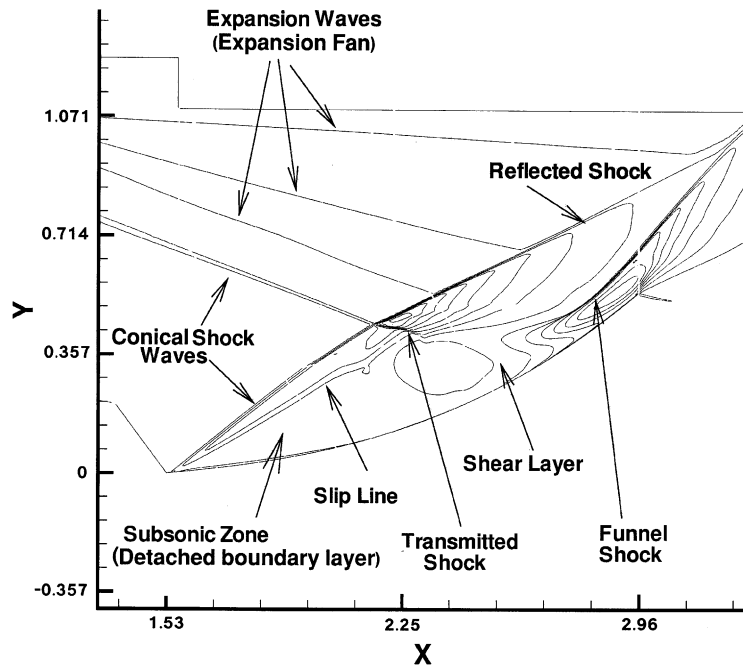


Fig. 23. A Navier–Stokes numerical simulation of the experiment shown in Fig. 16. The Type A viscous-dependent wave configuration is seen.

flow-Mach-number was reversed and the flow-Mach-number was increased, the reversed transitions occurred at different values of M_0 . As a consequence two hysteresis loops, A and B, were observed. While loop A involved both oRR and oMR wave configurations loop B involved only oMR wave configurations that were associated with different flow patterns. More details regarding the wave configurations associated with hysteresis loops A and B can be found in Ben-Dor et al. [40].

Case 2: $X = -0.2$. The dependence of the Mach stem length on the free-stream-flow-Mach number when the latter was changed in the course $4.8 \rightarrow 2.6 \rightarrow 4.8$ is shown in Fig. 25. It is evident from Fig. 25 that the change in the location of the curvilinear cone resulted in a significant increase of the range of the hysteresis loop A. As a result the hysteresis loops A and B overlap. This in turn results in a situation in which there is a flow-Mach-number range for which three stationary wave configurations, one oRR and two oMRs, are possible. More details regarding the wave configurations associated with the hysteresis loops A and B can be found in Ben-Dor et al. [40].

Case 3: $X = -0.1$. The dependence of the Mach stem length on the free-stream-flow-Mach number when the latter was changed in the course $5.0 \rightarrow 2.6 \rightarrow 5.0$ is shown in Fig. 26. It is evident from Fig. 26 that the change in the location of the curvilinear cone resulted in a further

significant increase of the range of the hysteresis loop A. Furthermore, unlike the previous two cases, now there are three hysteresis loops: the previously observed loops A and B and a small additional hysteresis loop, loop C. Similar to the previous case here again the hysteresis loops overlap. There is an overlap of loops A and C, and an overlap of loops A and B. Consequently, once again based on Fig. 26 three different wave configurations for identical flow conditions (flow-Mach number and geometry) are possible. The wave configurations associated with the hysteresis loops A, B and C are shown in Figs. 27–29, respectively. The important role of the rear edge of the curvilinear cone in forming the hysteresis loops A, B and C is clearly seen in these figures. More details regarding the wave configurations associated with hysteresis loops A, B and C can be found in Ben-Dor et al. [40].

Ben-Dor et al. [40] showed that the different wave configurations for identical flow-Mach numbers were associated with significantly different pressure distributions along the cone surface.

Fig. 30 represents the pressure distributions for $M_0 = 3.45$ (for which loops A and C overlap). Three different wave configurations; an oRR, an oMR having a short Mach stem and an oMR having a longer Mach stem are possible for this flow Mach number (see Fig. 26). The pressure distribution associated with the oMR having the longer Mach stem (solid line) has two pressure

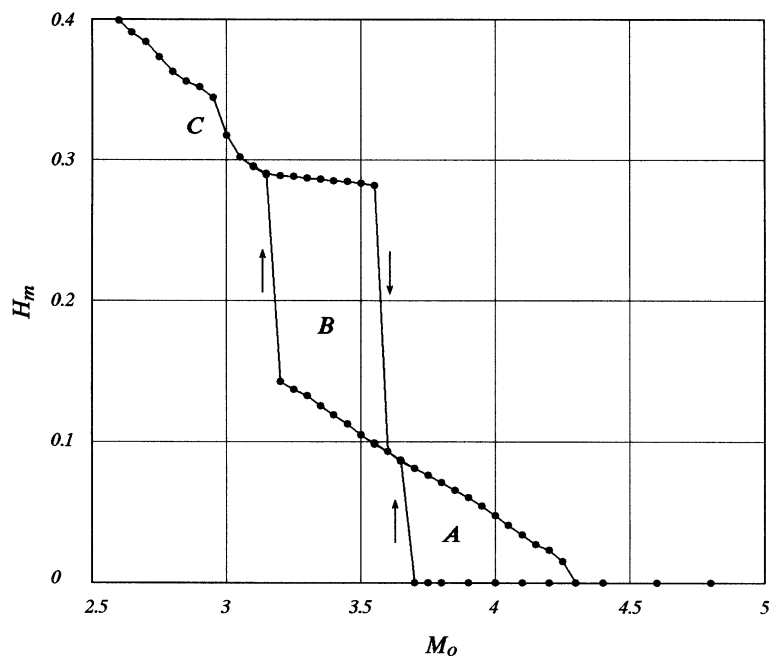


Fig. 24. The flow-Mach-number-variation-induced hysteresis loop for $X = -0.3$ when the flow-Mach number was changed in the course $4.8 \rightarrow 2.6 \rightarrow 4.8$.

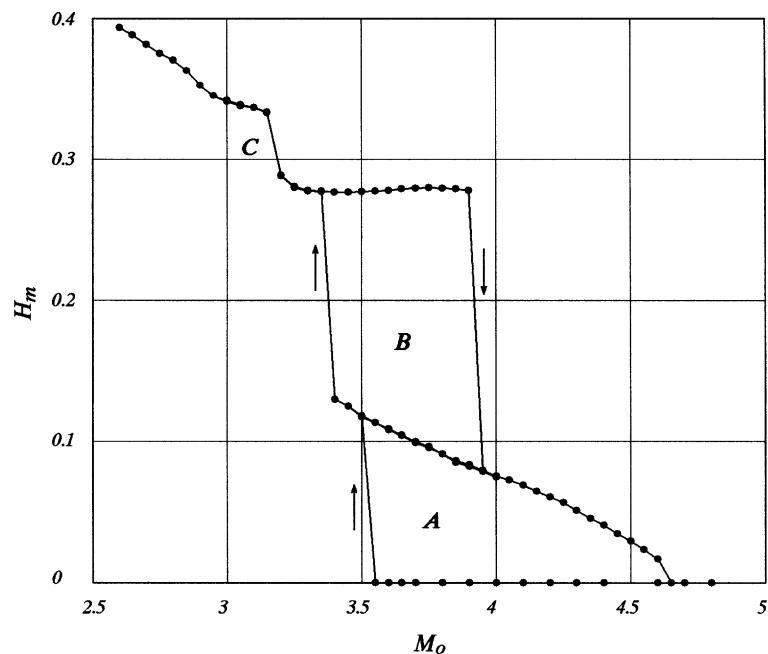


Fig. 25. The flow-Mach-number-variation-induced hysteresis loop for $X = -0.2$ when the flow-Mach number was changed in the course $4.8 \rightarrow 2.6 \rightarrow 4.8$.

peaks. While the first peak reaches a value of about 37 times higher than the ambient pressure the second peak reaches a value of about 24 times higher than the

ambient pressure. The reason for the double peak is clearly understood when the actual wave configuration that is shown in Fig. 29 ($M_0 = 3.45$, left frame) is

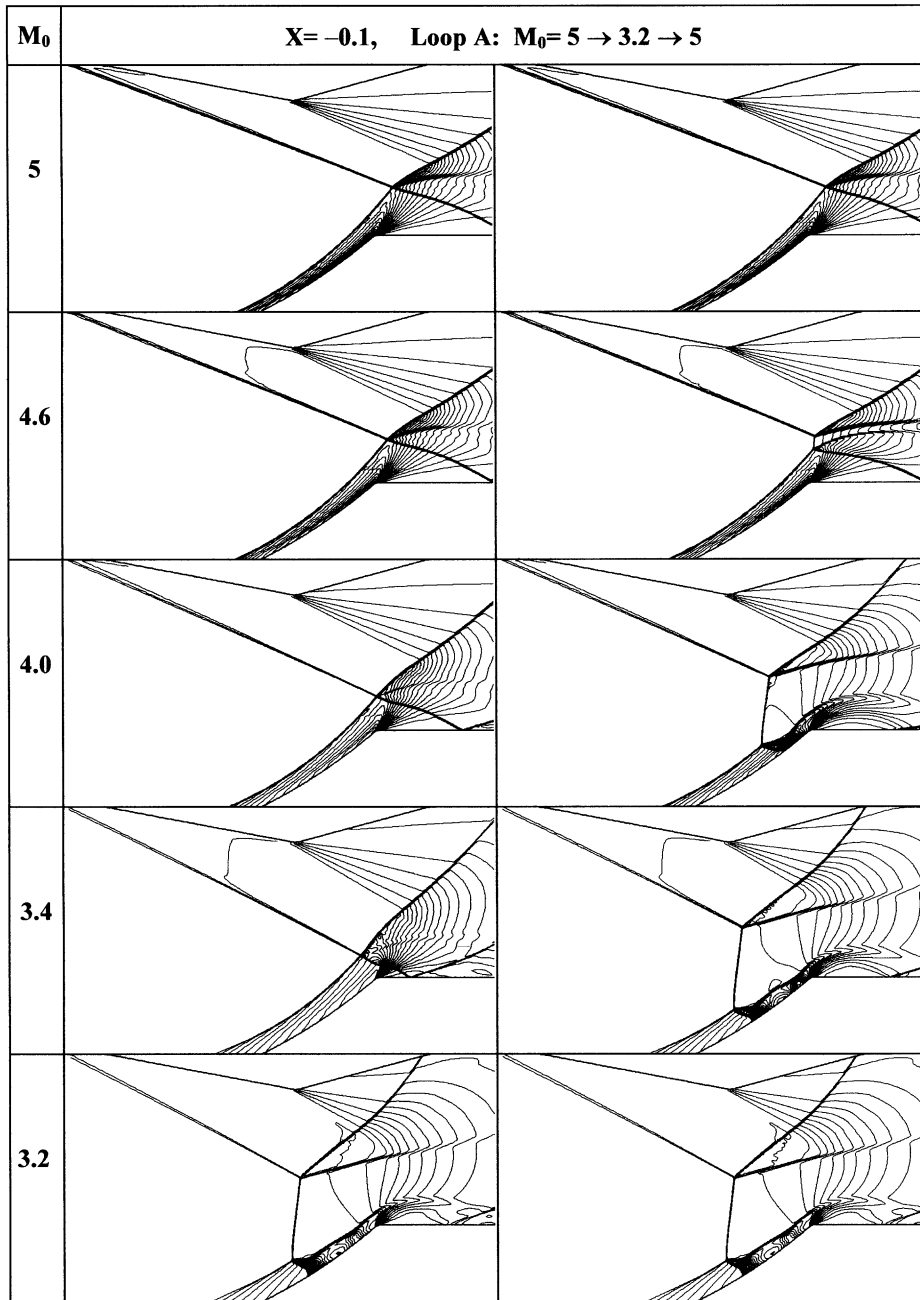


Fig. 27. The wave configurations associated with hysteresis loop A of Fig. 26.

results for $p_w/p_0 = 10$ were used as the initial conditions for $p_w/p_0 = 12$. This procedure was repeated until $p_w/p_0 = 22$ was reached. Then p_w/p_0 was decreased, again by using the final results of the previous case as the initial conditions for the next case until $p_w/p_0 = 10$ was reached again.

As can be seen from the results shown in Fig. 31 a hysteresis exists in the reflection process. While the

RR→MR transition took place between frames (5) and (6), the reversed MR→RR transition occurred between frames (12) and (13).

The downstream-pressure-variation-induced hysteresis loop is shown in Fig. 32 in the $(\ell_m/L, p_w/p_0)$ -plane (ℓ_m is the height of the Mach stem and L is the length of the reflecting wedge surface). As can be seen the RR→MR transition occurs at $p_w/p_0 = 19.63$ and

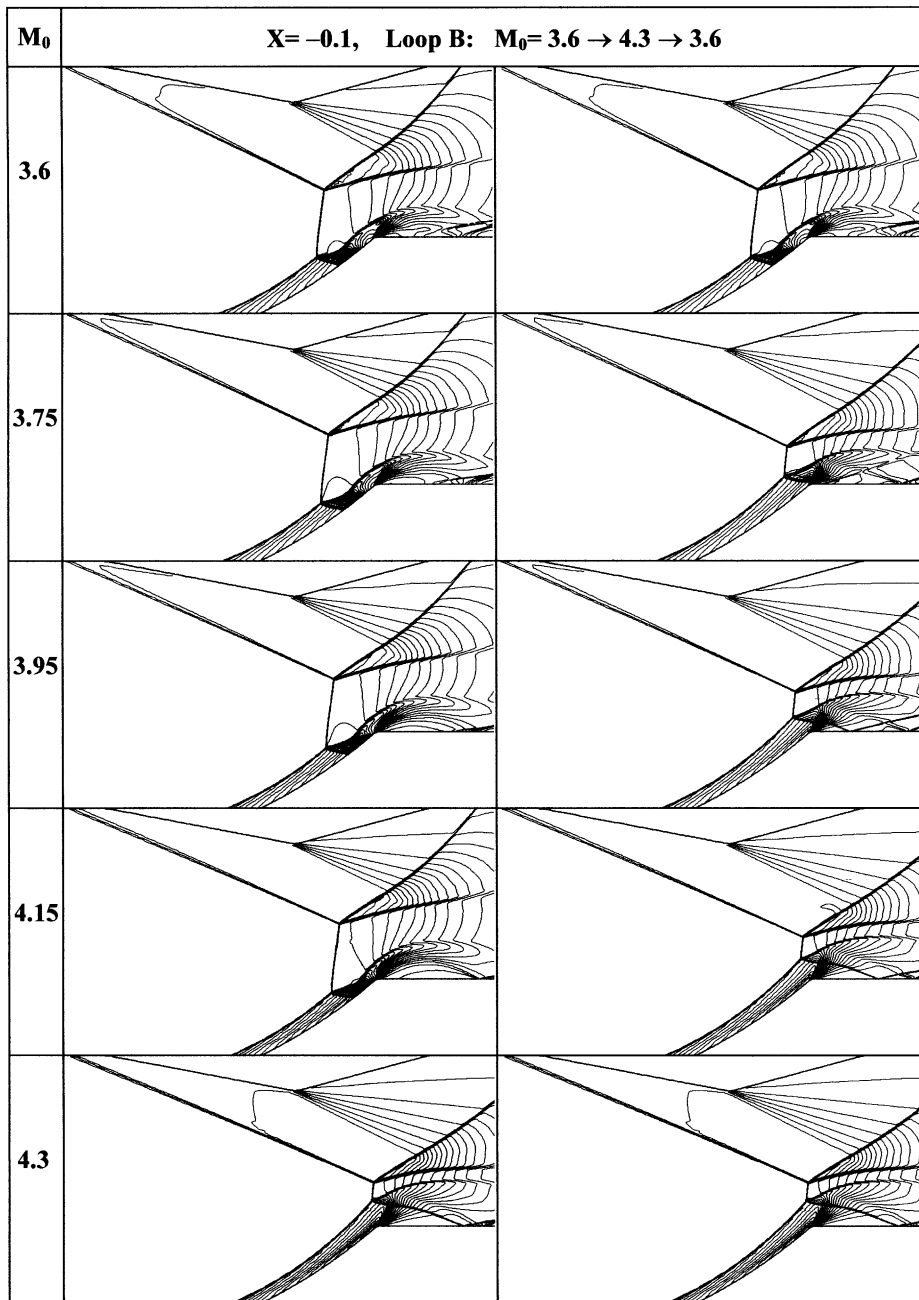


Fig. 28. The wave configurations associated with hysteresis loop B of Fig. 26.

the reversed MR $\rightarrow$ RR transition takes place at $p_w/p_0 = 10$. Both the MR $\rightarrow$ RR and the RR $\rightarrow$ MR transitions were associated with a sudden disappearance and appearance of a finite size Mach stem. These observations contradict Henderson and Lozzi's [8] postulation that a mechanical equilibrium should exist at these transitions.

6. Free-stream perturbations effect on the transition between regular and Mach reflections

As both RR and MR wave configurations are observed in the numerical simulations and experiments inside the dual-solution-domain, these wave configurations are stable to infinitesimal perturbations. However,

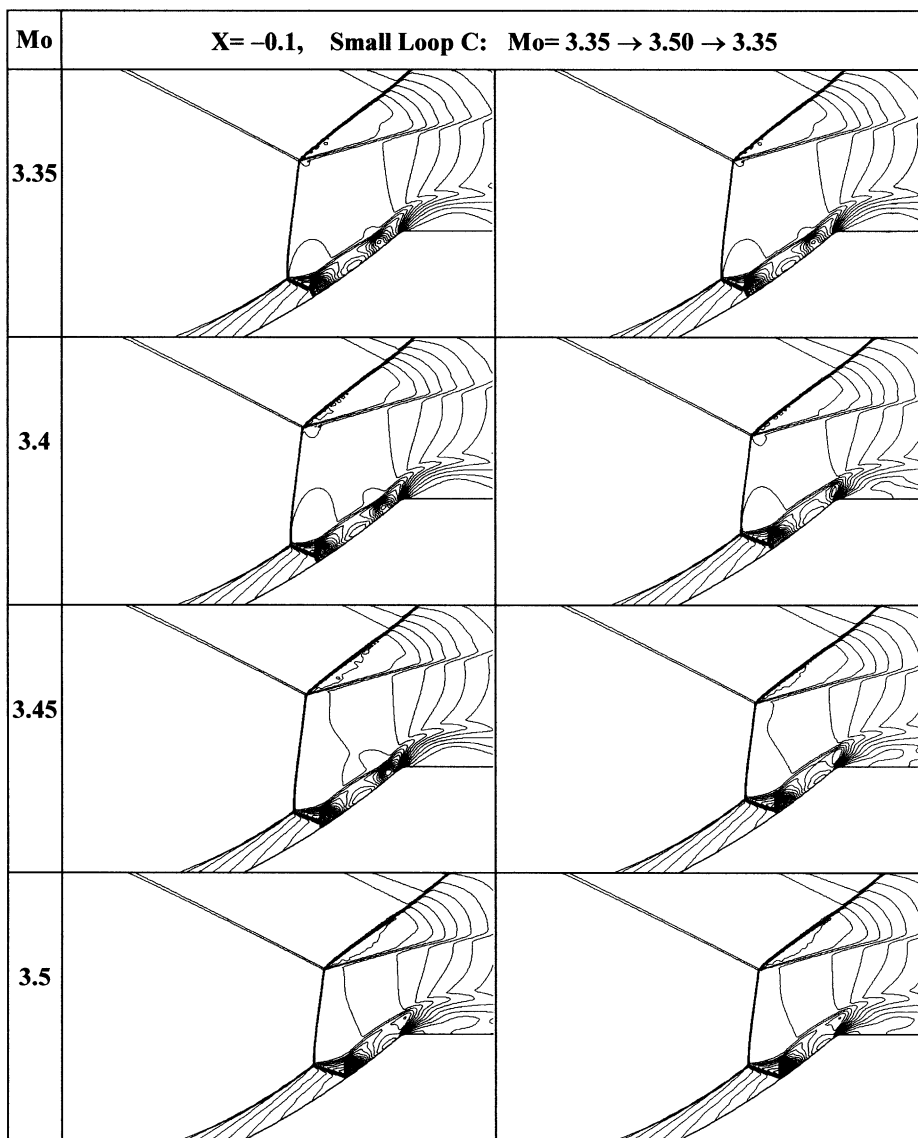


Fig. 29. The wave configurations associated with hysteresis loop C of Fig. 26.

large-amplitude perturbations may cause a transition (flip) between these two wave configurations. Since experimental facilities have certain levels of perturbations that differ from each other in their nature and magnitude, a question arises whether the discrepancies between the data on the transition angles that was obtained at different experimental facilities and numerical results could indeed be explained by the presence of disturbances of some kind in the wind tunnels on the one hand and by their absence in the numerical computations, on the other hand. Furthermore, if the ability of such disturbances to promote or induce the transition

could be realized, could this be modeled in the computations?

Vuillon et al. [13] investigated numerically the stability of an RR wave configuration by introducing perturbations behind the reflection point (the velocity was set to zero in several rows of the grid cells). Ivanov et al. [41] investigated, using a similar method, the influence of the velocity perturbations near the reflection point using a DSMC method. It was revealed, in these studies, that velocity perturbations could indeed promote the transition from RR to MR.

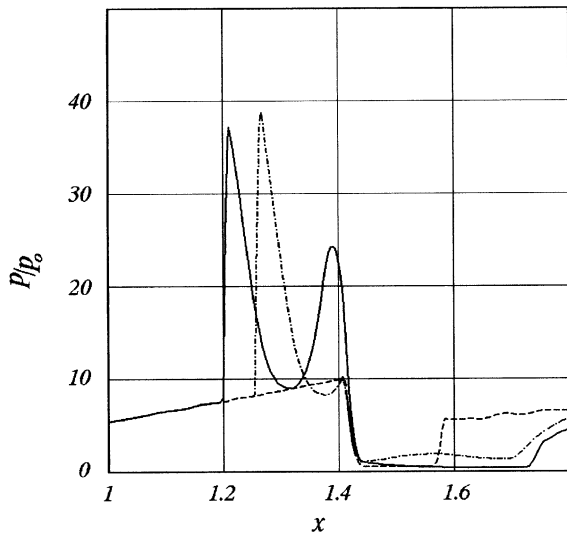


Fig. 30. The pressure distribution along the surface of the curvilinear cone for $M_0 = 3.45$ (Loops A and C of Fig. 26). The distance is measured from the tip of the curvilinear cone. Solid line—oMR wave configuration having a long Mach shock; dash-dotted line—oMR wave configuration having a short Mach shock; dashed line—oRR wave configuration.

Ivanov et al. [41,42] using a DSMC method and Ivanov et al. [19] using an Eulerian code also investigated the influence of strong, but short-time perturbations of the free stream that are more physical. The perturbation, in their investigation, consisted of a short duration change in the free-stream velocity at the upstream boundary. The computations showed that such perturbations indeed affected the RR→MR transitions.

The RR→MR transition process is shown in Fig. 33. The perturbation consisted of a short duration 40% increase in the free stream velocity at the upstream boundary. The early stages of the interaction of the perturbation with the RR wave configuration are seen in Fig. 33a. Following the interaction of the perturbations with the RR wave configuration the RR (Fig. 33a) is changed to an MR wave configuration (Fig. 33d). Obviously, the reason for the formation of an MR wave configuration is a temporary existence of the shock wave with a large angle of incidence during this unsteady interaction.

The reverse MR→RR transition by means of impulsive free stream perturbations (a short duration decrease in the free stream velocity) was also observed in DSMC computations of Ivanov et al. [41,42]. The computations revealed that it was easier to promote the RR→MR transition in the larger part of the dual-solution-domain than the reverse MR→RR transition, where stronger perturbations had to be applied. It is

important to note that the Mach stem height was found to be independent of the way the MR wave configuration was obtained and to solely depend on the flow geometry.

Khotyanovsky et al. [43] investigated numerically the effect of free-stream density perturbations. Moderate amplitude density perturbations were introduced in a thin layer near the reflection point. Unlike the previously discussed perturbation in the free stream velocity, this form of perturbing the flow has an advantage that it does not affect the entire flow field and is simply convected downstream, while being separated from the undisturbed flow by a contact surface. It was shown that by perturbing the flow in this way both the RR→MR and the MR→RR transitions could be promoted inside the dual-solution-domain. Such a “forced” transition between RR and MR wave configurations is illustrated in Fig. 34 for the following conditions inside the dual-solution-domain: $M_0 = 4$ and $\beta = 36^\circ$. The steady RR wave configuration shown in Fig. 34a, was exposed to a short duration density perturbation, i.e., the density was decreased at the inflow in the 10 lower cells by 25%, i.e., $\Delta\rho/\rho_0 = -0.25$. As a result, the RR→MR transition was promoted and an MR wave configuration started to be formed (see Fig. 34b). The flow field in Fig. 34b corresponds to a non-dimensional time $\tau = 2.4w/U_0$ after the flow perturbation began (here w is the length of the reflecting wedge along the windward plate and U_0 is the free-stream velocity). The perturbation was “switched off” after that moment, and the Mach stem persisted and continued to grow. Finally, a steady-state MR wave configuration, shown in Fig. 34c was obtained.

The forced reverse MR→RR transition is also shown in Fig. 34. The density was increased in the 20 lower cells by 50%, i.e., $\Delta\rho/\rho_0 = 0.5$, so that the triple point of the MR wave configuration was inside the perturbed layer. As a result of the interaction of the perturbation with the triple shock configuration, the Mach stem height started to decrease until it finally disappeared. This is shown in Fig. 34d, which corresponds to a non-dimensional time $\tau = 10w/U_0$ from the beginning of flow perturbation. At this time, the perturbation was already “switched off” and its “tail” was upstream from the reflection point. Once the perturbation disappeared an RR wave configuration was formed. It should be noted that the time necessary for the MR→RR transition to be completed is much larger than that required for the completion of the RR→MR transition.

The reason for the “switch” between the two shock wave configurations is the refraction of the incident shock wave in the perturbed region (see Fig. 34d). As a result, the shock wave angle near the reflection point is changed. If the level of the perturbation is high enough then the refracted shock angle in the perturbed layer could become larger than the detachment

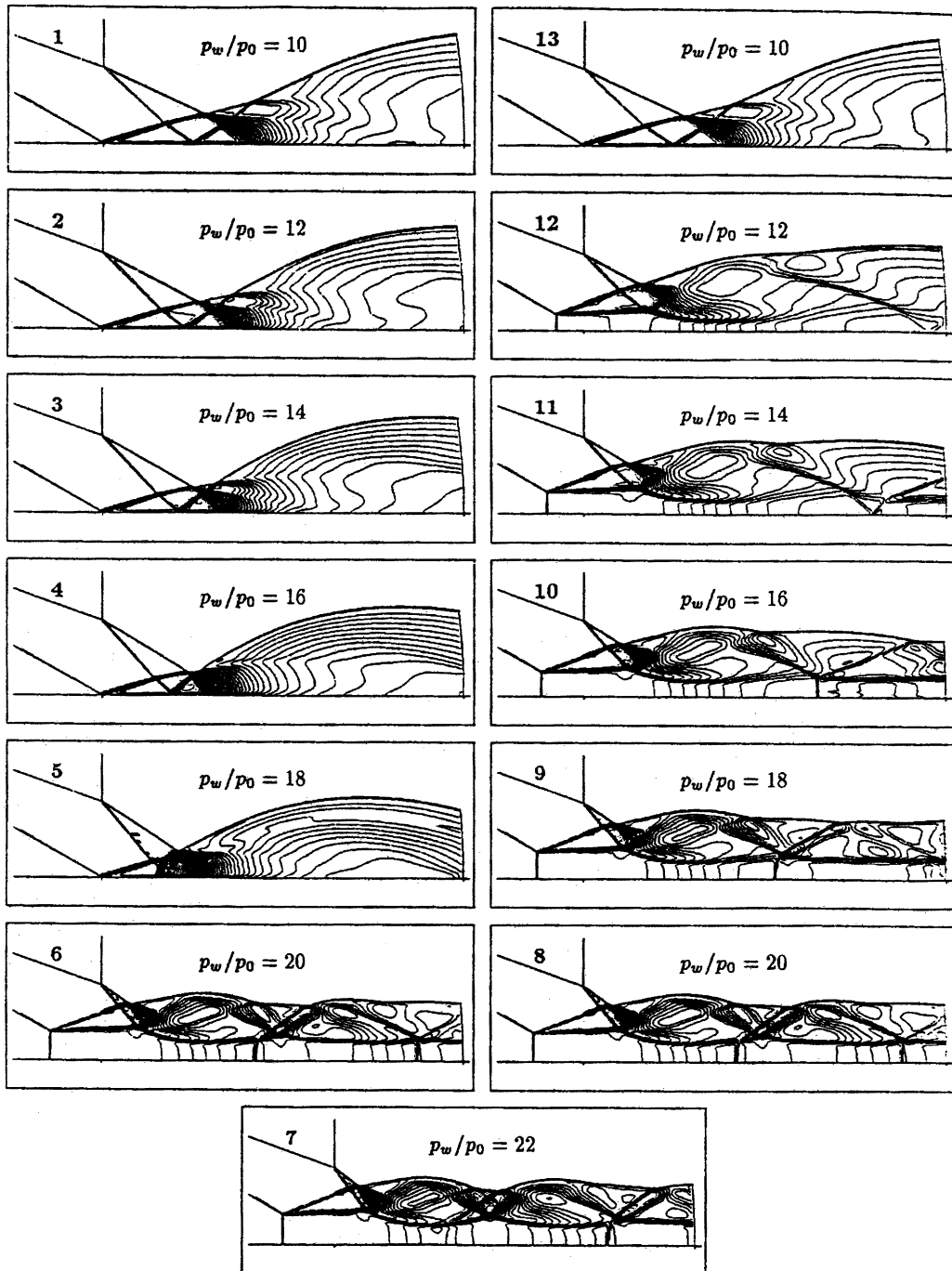


Fig. 31. The numerical downstream-pressure-variation-induced hysteresis.

criterion or smaller than the von Neumann criterion. As a result, the $RR \rightarrow MR$ or $MR \rightarrow RR$ transitions become inevitable. The resulted shock wave configuration continued to exist after the perturbation was “switched off”.

The threshold levels of the perturbations that are required to cause the transitions were evaluated theoretically, and are shown in Fig. 35. It was also found that the $RR \rightarrow MR$ transition could be promoted easier (i.e., with smaller perturbation intensities) than

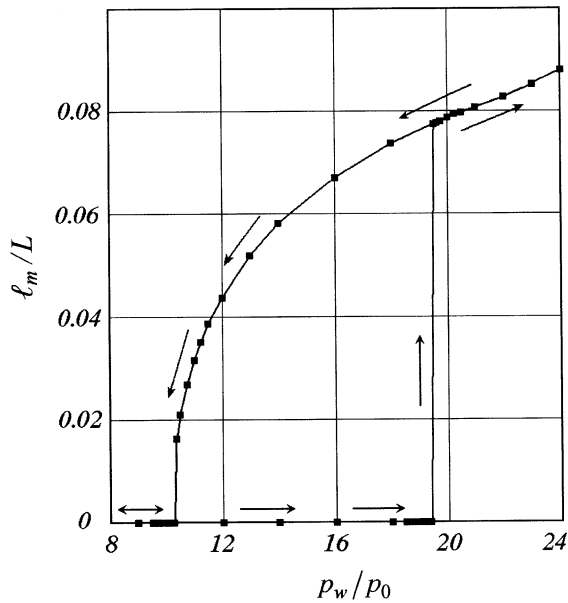


Fig. 32. The numerical downstream-pressure-variation-induced hysteresis loop in the $(\ell_m/L, p_w/p_0)$ -plane.

the reverse $MR \rightarrow RR$ transition. It is apparent from Fig. 35 that the slope of the line corresponding to the $MR \rightarrow RR$ transition is much larger than that for the $RR \rightarrow MR$ transition. Consequently, the levels of the perturbation capable of changing the MR wave configuration into an RR wave configuration are very high and comparable in magnitude, for some angles of incidence, to the free-stream density. Moreover, there is another reason making the promotion of the $RR \rightarrow MR$ transition much easier to achieve than that of the reverse $MR \rightarrow RR$ transition. For promoting the $RR \rightarrow MR$ transition it is enough to have a perturbation localized in a very thin region near the reflection point. The thickness of this layer depends on the grid resolution, and is usually a few grid cells, so that the formation of the triple shock configuration inside the perturbed region can be resolved. The perturbation that is required for promoting the $MR \rightarrow RR$ transition must have much larger scales both in space and in time. As mentioned above, its spatial extent must exceed the height of the Mach stem and its temporal duration must be long enough so that the relatively slow process of the decrease of the Mach stem height could be completed and the Mach stem can vanish before the perturbation is vanished.

7. Summary and conclusions

It has been shown that the theoretical prediction that multiple shock wave configuration could exist for

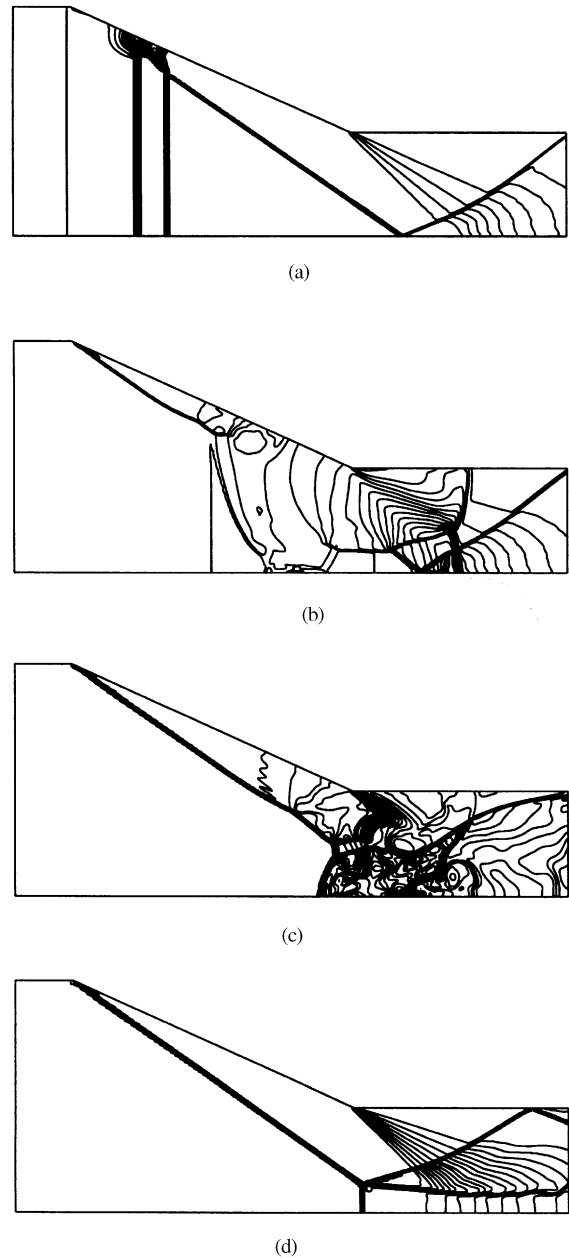


Fig. 33. The perturbation induced $RR \rightarrow MR$ transition process.

identical flow conditions in a variety of shock wave related problems in steady flows, could lead to the existence of hysteresis processes. The analytically predicted hysteresis processes were verified both numerically and experimentally for a variety of steady flows with shock waves reflections.

Since the investigated geometry resembles the geometry of supersonic intakes, the findings regarding the

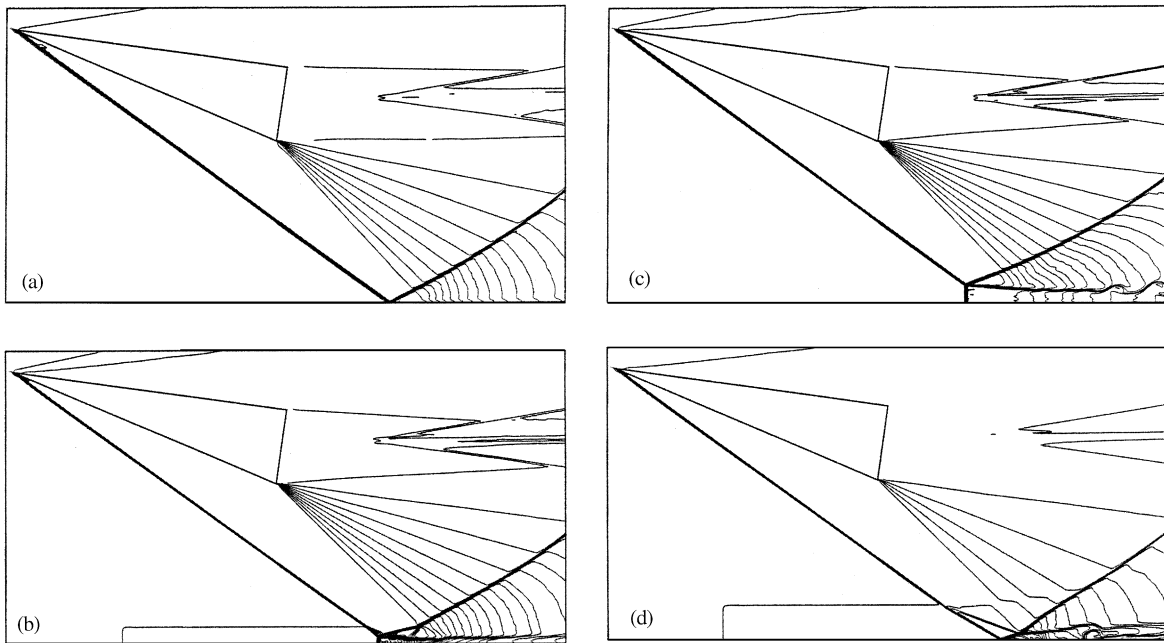


Fig. 34. The promotion of the RR $\rightarrow$ MR and the MR $\rightarrow$ RR transitions by density perturbations for $M_0 = 4$ and $\beta = 36^\circ$. (a) The RR wave configuration prior to the perturbation; (b) the beginning of the RR $\rightarrow$ MR transition as a result of a density perturbation of $\Delta\rho/\rho_0 = -0.25$; (c) the steady-state MR wave configuration that was finally obtained after the perturbation was “switched off”; (d) the beginning of the MR $\rightarrow$ RR transition as a result of a density perturbation of $\Delta\rho/\rho_0 = 0.5$. After the perturbation was “switched off” the RR wave configuration shown in (a) was established.

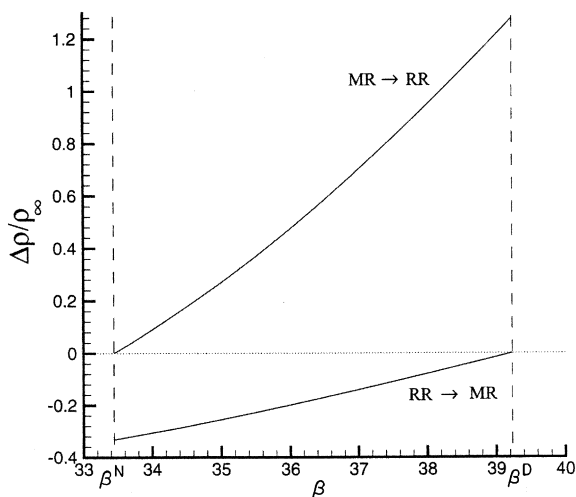


Fig. 35. The theoretically evaluated threshold levels of the perturbations, which are required to promote the RR $\rightarrow$ MR (see Fig. 34) and the MR $\rightarrow$ RR (see Fig. 35) transitions.

hysteresis loops that are reported in this study can be relevant to flight performances at high supersonic speeds. The possible dependence of the flow

pattern, in general, and the pressure distribution, in particular, on the preceding variations in the speed of flight of a supersonic aircraft should be taken into account in designing intakes and flight conditions for supersonic and hypersonic vehicles.

Consider Fig. 36 in which the upper part is a reproduction of Fig. 25. The overlap of the hysteresis loops A and B suggests, as mentioned earlier, that there is a flow-Mach number range for which three different wave configurations are possible. For example for the flow-Mach number $M_f = 3.8$ one can obtain an oMR wave configuration with a long Mach stem, or an oMR wave configuration with a short Mach stem or an oRR wave configuration. These three wave configurations are marked in Fig. 36 as (1), (2) and (3), respectively. The flow-Mach number at which the transition from an oMR wave configuration with a long Mach stem to an oMR wave configuration with a short Mach stem takes place is labeled M_{tr1} and the flow-Mach number at which the transition from an oMR wave configuration with a short Mach stem to an oRR wave configuration takes place is labeled M_{tr2} . The lower part of Fig. 36 shows possible flight-Mach number histories of a supersonic vehicle whose intake is identical in its geometry to that shown in Fig. 14 and the leading edge

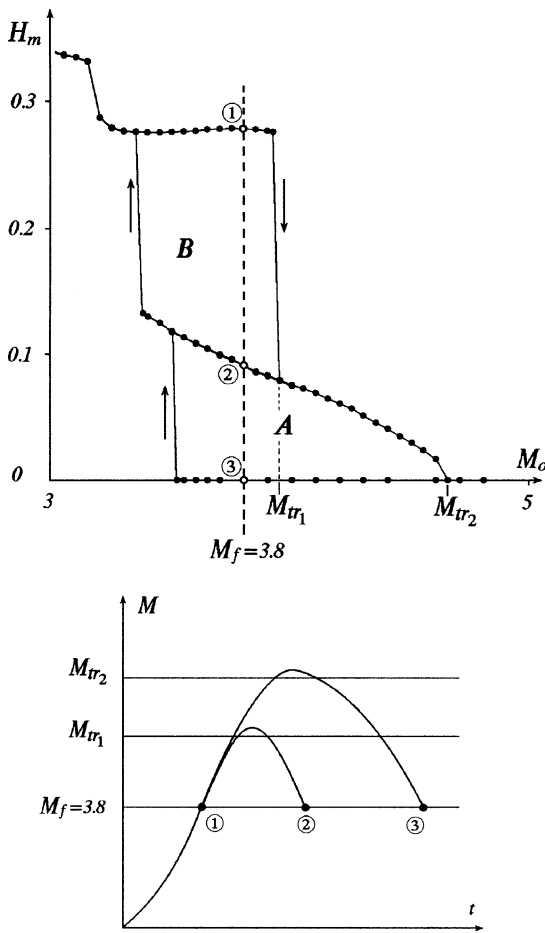


Fig. 36. Schematic illustration of the possibility of having a supersonic vehicle flying at the same speed and having different wave configurations in its supersonic intake.

of the curvilinear cone is at $X = -0.2$. At $t = 0$ the vehicle starts accelerating to reach $M_f = 3.8$. Having reached this speed the wave configuration in its supersonic intake will be an oMR wave configuration with a long Mach stem. If at this stage the vehicle accelerates to a speed in the range $M_{tr1} < M_f < M_{tr2}$ and then returns to $M_f = 3.8$ then the wave configuration in the supersonic intake will change to an oMR with a short Mach stem. If, however, the vehicle accelerates to a speed in the range $M_f > M_{tr2}$ and then returns to $M_f = 3.8$ then the wave configuration in the supersonic intake will change to an oRR. Hence, at identical supersonic flight speeds, i.e., $M_f = 3.8$, three different wave configurations might exist in the supersonic intake. As shown earlier (see Fig. 30), these different wave configurations are associated with different pressure distributions and hence different dynamic and thermodynamic properties of the gas in the intake.

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