

Thermodynamics of phase transitions in current-carrying conductors

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This work studies the thermodynamics of phase transitions of the first kind in current-carrying conductors when these transitions are accompanied by a sharp change of the electrical conductivity. It is shown that the critical current in the normal conductor, i.e., the current that generates the critical pressure, may be considerably lower than is generally believed. The reason for the lower value of the critical current is the shift of the whole curve of phase equilibrium in the presence of a strong electric current. This shift arises due to the additional work performed against ponderomotive forces, which prevents the formation of the nucleus of a phase with the lower value of electric conductivity. In case of the van der Waals model of the critical state the value of the critical current calculated taking into account the shift of the phase equilibrium curve is 2–3 times less than the critical current determined when this shift is neglected. It is shown that under these conditions there occurs a splitting of the phase-equilibrium curve into two separate curves for direct and inverse phase transitions. Depending upon the mutual location of both curves two opposite situations may occur. The first case is that of regular hysteresis when there exists a domain of stability of both phases and the realization of a particular phase is determined by the initial conditions and the direction of the process. In the second case there exists a region where both phases are unstable. This region is considered as a domain of the fragmentation of material into small particles. This work determines various thermodynamic parameters: latent heat of the phase transition, shift of the phase-equilibrium curve, and the size of the critical nucleus. It is shown that the value of the shift of the phase-equilibrium curve under the current densities employed in the experiments with exploding wires is of order 1. A mechanism for the formation of small particles is suggested and theoretical results are compared with experimental data.

I. INTRODUCTION

An efficient method for conversion of electric energy into other kinds of energy is the fast discharge of powerful electric currents through the conductors. One of the widely used methods in technology is the electric explosion of conductors.¹ In an electric explosion the conductor passes through various phase states during relatively short-time intervals and its physical parameters vary over a very wide range. Naturally such dynamics cause various physical phenomena, which were investigated in a number of works.^{2–5} Nevertheless, until recently the main efforts were directed towards the analysis of various types of thermal and hydrodynamic instabilities initiated in the vicinity of the conductor's surface and the calculations of the characteristic times of the development of these instabilities. One of the most comprehensive investigations of these problems was reported in Ref. 5.

In the present investigation we analyze the thermodynamics of phase transitions in a high-density current-carrying conductor far from its surface. The importance of this problem is associated with the significant contribution of the related phenomena into the processes accompanying the regular electric explosion of conductors and with the feasibility to create special conditions when these phenomena play the dominant role, e.g., enhanced cooling of the conductor's surface.

The results of the present investigation can be summa-

rized as follows. Denote by the index "2" the phase with higher conductivity σ_2 and by index "1" the phase with lower conductivity σ_1 . It is shown that the phase equilibrium curve for phase transition from phase 2 to phase 1 ($2 \rightarrow 1$) does not coincide with the phase equilibrium curve for the phase transition $1 \rightarrow 2$.

This effect arises due to the different value and sign of the work performed by the ponderomotive forces in the case of formation of the nucleus with the lower value of electric conductivity in the medium with the higher value of electric conductivity (i.e., $2 \rightarrow 1$ phase transition) and the work performed by the ponderomotive forces in the case of inverse phase transition. It is shown that both phase equilibrium curves for the $2 \rightarrow 1$ and $1 \rightarrow 2$ phase transitions are shifted with respect to the phase equilibrium curve in case of current free conductor towards the spinodal line of the phase with higher conductivity. Hereafter the spinodal line is defined as a boundary of thermodynamic stability of the phase and the phase "2" is assumed to be a high-pressure and a low-temperature phase.

Thus, if one neglects the Joule heating, the electric currents in the conductor stabilize the phase with a higher value of electric conductivity. It is shown that the spinodal lines of phases 1 and 2 coincide with the spinodal line of the current free system. Therefore, the phase equilibrium curve for the $2 \rightarrow 1$ phase transition intersects with the spinodal line of phase 2 at pressure $p_c(I) < p_c^0$ and temperature $T_c(I) < T_c^0$, where p_c^0 and T_c^0 are the crit-

ical pressure and temperature of a given material (see Fig. 1). In case when the temperature $T > T_c(I)$, the system can be "stretched" up to the boundary of its thermodynamic stability. Similarly when $p > p_c(I)$, the phase equilibrium curve is located beyond the spinodal line of phase 2 at all temperatures and inside this pressure range the system can be overheated up to its spinodal line.

Pressure in the conductor depends upon the value of electric current and in the magnetostatic approximation can be determined from the expression

$$p(I) = p_0 + \frac{I^2}{k^2 r_0^2} \left(1 - \frac{r^2}{r_0^2} \right), \quad (1)$$

where p_0 is the pressure at the surface of a conductor, r is the distance from the center, k is the coefficient dependent upon the system of units, and r_0 is the radius of the conductor. The above arguments were employed to renormalize the value of the critical current. Instead of the regular definition of the critical current,

$$I_c = r_0 \sqrt{k p_c^0}, \quad (2)$$

the critical current must be determined as a root of the equation

$$I = r_0 \sqrt{k p_c(I)}. \quad (3)$$

Denote by I_c^0 the electric current defined by expression (2) and the electric current defined by expression (3) by $\bar{I}_c$. Then it is shown that, in case of the van der Waals model of the critical state (see Ref. 6, Chap. 14, Sec. 152), the critical current $\bar{I}_c$ may be considerably lower than the critical current I_c^0 .

The next result is associated with splitting of the phase equilibrium curve of the current free system into the two separate curves for the $2 \rightarrow 1$ and $1 \rightarrow 2$ phase transitions. It is shown that, depending upon the mutual location of both curves, two opposite situations may occur. The first case is when the phase equilibrium curve for the $2 \rightarrow 1$ phase transition is located at a higher-temperature region than the phase equilibrium curve of the phase transition $1 \rightarrow 2$ at the same pressure. This is the case of a regular hysteresis when there exists a domain of stability of both

phases which is located between the phase equilibrium curves for the phase transitions $2 \rightarrow 1$ and $1 \rightarrow 2$. The realization of a particular phase is determined by the initial conditions and the direction of the process.

In the second case, when the phase equilibrium curve for the $2 \rightarrow 1$ phase transition is located at a lower-temperature region than the phase equilibrium curve of the phase transition $1 \rightarrow 2$ at the same pressure, there exists a region between these two curves where both phases are unstable. This region is considered as a domain of the fragmentation of the material into small particles. The peculiar feature of this domain is that within it there exist critical nuclei of finite size for the phase transitions from phase 2 to phase 1 and from phase 1 to phase 2 simultaneously. Therefore, the stable growth of the nuclei in this domain is possible only until the growing nucleus remains less than the critical nucleus for the inverse phase transition. At some temperature the sizes of the critical nuclei for the phase transitions from phase 2 to phase 1 and from phase 1 to phase 2 coincide. In this work we calculate this temperature and determine the size of the critical nucleus at this temperature.

The determined size of the critical nucleus is compared with the size of small particles formed in the experiments⁷ with exploding wires. In this work, on the basis of experiments⁷ we determine the value of the effect of the shift of the fusion temperature and the width of the domain of fragmentation of the material into small particles. It is shown that the value of the shift of phase equilibrium curve under the current densities employed in the experiments⁷ is of order 1.

The paper is organized as follows. In Sec. II, we derive the conditions for adiabatic formation of a nucleus when the analysis can be performed with the aid of a thermodynamic approach and determine the equations of the phase equilibrium curves in current-carrying conductors. In Sec. III, we calculate critical currents in normal conductors. In Sec. IV, we analyze the peculiarities of the phase transitions associated with the existence of the two phase equilibrium curves.

II. EQUILIBRIUM OF PHASES WITH DIFFERENT CONDUCTIVITY CARRYING HIGH-DENSITY CURRENTS

Consider an infinitely long cylindrical conductor of radius r_0 carrying an electrical current of density j . Assume that the conductor is sufficiently thin and that its radius satisfies the condition

$$r_0^2 < \frac{c^2 \tau_0}{2\pi\sigma}, \quad (4)$$

where c is the velocity of light, σ is the electric conductivity, and τ_0 is the duration of current impulse.

Condition (4) allows one to neglect the skin effect and to assume the homogeneous current distribution along the cross section of the conductor. Assume also that the nucleus of the new phase with conductivity σ_1 is formed far from the conductor surface and that the characteristic time τ_a of the formation of the nucleus of size a satisfies the condition

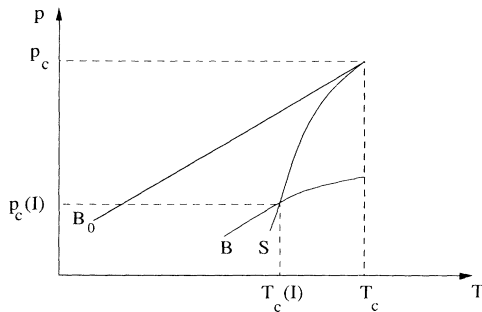


FIG. 1. B_0 is the phase equilibrium curve $\mu_1 = \mu_2$ in the absence of electric current. B is the phase equilibrium curve $\mu_2 = \mu_1 + \bar{p}v_1$ for the phase transition $2 \rightarrow 1$ in the presence of electric current and S is the spinodal line $(\partial p / \partial v)_v = v_2 = 0$.

$$\tau_a \sim \frac{a}{\dot{a}} \gg \frac{2\pi\sigma a^2}{c^2}, \quad \tau_a \ll \tau_0. \quad (5)$$

We also neglect the mass fluxes through the surface of the nucleus that allows one to neglect the effect of electrodynamic induction. The relative magnetic permeability of the media μ is assumed $\mu = 1$. Under these assumptions we may seek for the solution of the electrodynamic problem in the form

$$\mathbf{E} = \frac{\mathbf{j}_s}{\sigma} - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \nabla \times \mathbf{A}, \quad (6)$$

where $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, $\mathbf{j}_s$ is the "adiabatic" part of the total current density $\mathbf{j} = \sigma \mathbf{E}$ which satisfies the equations

$$\nabla \times \frac{\mathbf{j}_s}{\sigma} = 0, \quad \nabla \cdot \mathbf{j}_s = 0 \quad (7)$$

and does not depend upon time explicitly, i.e., $\mathbf{j}_s = \mathbf{j}_s[r, a(t)]$.

The condition for adiabatic behavior of the system is that the value of nucleus growth rate is small and it is possible to neglect all the quantities proportional to this value in the solution of the electrodynamic problem. Substituting (6) into the electrodynamic equations we arrive at the following conditions for the validity of the adiabatic regime of nucleation

$$\sigma_2 a \dot{a} \ll c^2, \quad \dot{a} \ll \sigma_2 a,$$

which are satisfied in the wide range of the parameters. When the latter conditions are satisfied the configuration of the electric field is determined from Eq. (7) and

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j}_s. \quad (8)$$

In the conditions of the adiabatic regime, which is assumed in this investigation, the work of the formation of the new phase can be determined as a difference between the free energy of the system before and after formation of the nucleus.

Consider a simple isotropic thermodynamic system with the free energy per unit volume in the absence of the electromagnetic field $f(\rho, T)$ where ρ is density and T is temperature. In the regime when the value of the electrical current is fixed, the free energy per unit volume is determined by the following expression (see Ref. 8, Chap. 14, Sec. 33):

$$\tilde{f} = f(\rho, T) - \frac{H^2}{8\pi}. \quad (9)$$

Then the work of the formation of the nucleus of the new phase W can be found from the equation

$$W = F_s + \int \left[\frac{H_0^2 - H_n^2}{8\pi} + (\rho_n \mu_n - \rho_0 \mu_0) + [p_0(\rho_0, T) - p_n(\rho_n, T)] \right] d\mathbf{r}, \quad (10)$$

where F_s is the contribution of the density gradient at the

interphase boundary to the free energy, H_0 and H_n are magnetic fields before and after formation of the nucleus, p_0, p_n and μ_0, μ_n are the thermodynamic pressure and chemical potential before and after formation of the nucleus which are related with the function $f(\rho, T)$ as follows:

$$f(\rho, T) = \rho \mu(\rho, T) - p(\rho, T).$$

To simplify the problem we adopt the "disperse" model of the formation of a new phase^{9,10} when the change of the initial state of the system occurs in the nucleus of the new phase and in the additional volume V_{ad} occupied or vacated by external phase due to the difference between the specific volumes of the external and the new phases. In the remaining volume of the system the thermodynamic parameters p and μ do not change. The variation of the thermodynamic parameters inside the nucleus is neglected, i.e., the nucleus is characterized by the volume-averaged value of the thermodynamic parameters. The contribution of the density gradient at the interphase boundary to the free energy F_s according to the "disperse" model can be represented as $F_s = \alpha S$, where α is a coefficient of surface tension and S is the surface area of the nucleus. The additional volume occupied by the new phase V_{ad} is determined by the expression

$$V_{ad} = N_n \left(\frac{1}{\bar{\rho}_n} - \frac{1}{\bar{\rho}_0} \right),$$

where N_n is the number of molecules of the nucleus of the new phase, $\bar{\rho}_n$ and $\bar{\rho}_0$ are the densities of the "new" and "old" phases averaged over the volume of the nucleus.

Using the expression for V_{ad} and substituting the expansion $\mu_n(p_n) = \mu_0(p_0) + v_0(p_0 - p_n)$ into expression (10), we arrive at the following formulas for the work of the formation of the nucleus of a new phase:

$$W = \alpha S + N_n(\mu_n - \mu_0) + W_f, \quad W_f = \int \left[\frac{H_0^2 - H_n^2}{8\pi} \right] d\mathbf{r}. \quad (11)$$

According to the derived expression there exist two mechanisms through which the electric current affects the phase transition. The first mechanism is associated with the pressure change as given by expression (1). This mechanism does not alter the phase equilibrium curve but changes only the temperature of the phase transition due to the pressure change.

The second mechanism is associated with the additional term W_f in expression (11) and causes the shift of the phase equilibrium curve. It is the contribution of this term that results in all the effects described in this work. Generally the nucleus formation work W_f depends upon its geometry and the distance between the nucleus and the conductor surface.

Consider the case when the longitudinal and transversal dimensions of the conductor are approximately equal and the nucleus is formed at a distance d from the center of the conductor. Assuming that $d \ll r_0$ and $a \ll r_0$,

where a is the characteristic size of a nucleus, it can be shown (see the Appendix) that, with an accuracy of order $(d/r_0)^2$ and $(a/r_0)^2$, the nucleus formation work W_f is given by the expression

$$W_f = 4p_m \xi V, \quad (12)$$

where V is the volume of the nucleus, $p_m = I^2/k^2 r_0^2$ is the pressure in the center of a conductor in a magnetostatic approximation when the pressure at its surface $p_0 = 0$ [see (1)]. The value ξ is determined by the expression

$$\xi = \frac{\sigma_0 - \sigma_n}{\sigma_n + 2\sigma_0}, \quad (13)$$

where σ_0 is the conductivity of the medium where the nucleus was formed, and σ_n is the conductivity of the nucleus.

If the length of the conductor $L \gg r_0$, then in expression (12) there appears the term proportional to $\ln(L/r_0)$. The expression (12) for W_f is still valid but with the logarithmic accuracy the coefficient ξ is given by the formula

$$\xi = 2 \frac{\sigma_0 - \sigma_n}{\sigma_n + 2\sigma_0} \ln \left[\frac{L}{r_0} \right]. \quad (14)$$

It is readily seen from expressions (12)–(14) that for phase transition from the phase with higher conductivity to the phase with lower conductivity the work W_f is positive. The ponderomotive forces prevent the formation of the nucleus because of the reduction of the inductance of the system. In the case of the inverse phase transition, the inductance of the system increases and the work W_f is negative.

As before, we denote by the index “2” the phase with higher conductivity and by the index “1” the phase with lower conductivity. We will consider such geometry of the nucleus of phase “1” when the absolute value of W_f is a minimum and such geometry of the nucleus of phase “2” when the absolute value of W_f is a maximum since, for such nuclei, the metastable phase is most unstable.

In long conductors the minimum of the absolute value of the nucleus formation work is attained for thin long nuclei of length $b \gg r_0$ oriented in the direction of the conductor axis. Assume that the shape of these nuclei is close to cylindrical and that they are formed at a distance $d \ll r_0$ from the axis of a conductor. Then their formation work is given by expression (12), where ξ can be found from the expression

$$\xi \equiv \xi_{21} = \frac{\sigma_2 - \sigma_1}{4\sigma_2}. \quad (15)$$

The critical dimensions of such a nucleus can be determined from the conditions $\partial W_f / \partial b = 0$ and $\partial W_f / \partial a = 0$, where a is transversal size of a nucleus. The latter conditions and formulas (11) and (12) yield the critical dimensions of the nucleus:

$$a_1 = \frac{\alpha v_1}{\mu_2 - \mu_1 - \bar{p} v_1}, \quad a_2 = \frac{2\alpha v_1}{\mu_2 - \mu_1 - \bar{p} v_1}, \quad (16)$$

where a_1 is a transversal size of a nucleus which ensures its growth in the transversal direction, a_2 is the same size of the nucleus which ensures its growth in the direction of the electric current, $\bar{p} = 4p_m \xi_{21}$ and ξ_{21} is given by expression (15).

It can be seen from expressions (16) that the phase equilibrium curve for the phase transition $2 \rightarrow 1$ is determined by the following relation:

$$\mu_2 = \mu_1 + \bar{p} v_1. \quad (17)$$

Similarly, assuming the spherical shape of the critical nucleus in order to ensure its maximum metastability, for an inverse phase transition $1 \rightarrow 2$ the critical size of this nucleus and the phase equilibrium curve are determined from the following expressions:

$$a_{12} = \frac{2\alpha v_2}{\mu_1 - \mu_2 - \bar{p}' v_2}, \quad (18)$$

$$\mu_1 = \mu_2 + \bar{p}' v_2, \quad (19)$$

where $\bar{p}' = 4p_m \xi_{12}$ and

$$\xi_{12} = 2 \frac{\sigma_1 - \sigma_2}{\sigma_2 + 2\sigma_1} \ln \frac{L}{r_0}.$$

It can be readily seen from formulas (17) and (19) that the electric current in a conductor stabilizes the phase with higher conductivity and causes shift of the phase equilibrium curve towards the spinodal line of this phase. The condition for thermodynamic stability in a current-carrying conductor reads

$$\frac{\partial^2 f}{\partial \rho^2} \geq 0$$

and, as can be seen from formula (9), the boundaries of the stability domain coincide with the spinodal line of the current-free system. Taking into account this fact determines the values of the critical currents for which the high-temperature phase can be overheated up to the boundary of its thermodynamic stability provided that all the alternative phases have significantly lower conductivity.

III. CRITICAL CURRENTS IN CONDUCTORS

The values of critical currents can be determined from expression (17) which we rewrite for this purpose as follows:

$$\int_{\rho_1}^{\rho_2} [p - P(\rho', T)] d\rho' \frac{d\rho'}{\rho'^2} = - \frac{\bar{p}}{\rho_1}. \quad (20)$$

Expression (20) determines the phase equilibrium curve in the current-carrying conductor $p = p_{21}(I, T)$ for the phase transition $2 \rightarrow 1$ provided that the equation of state $P(\rho, T)$ is known. Here ρ_1 and ρ_2 are equilibrium densities of phases 1 and 2 which are the roots of equation $p = P(\rho, T)$.

If the equation of the spinodal line of phase 2 is known,

$$p = P_s(T), \quad (21)$$

then pressure can be eliminated from Eqs. (20) and (21) and the critical temperature as a function of electric current $T_c = T_c(I)$ can be found. At temperatures $T \geq T_c(I)$ a high-temperature phase can be "stretched" up to its spinodal line. Similarly eliminating temperature from Eqs. (20) and (21) yields critical pressure $p_c = p_c(I)$.

Equating relation (1) which determines pressure in a conductor in the magnetostatic approximation to the critical pressure $p_c(I)$, we arrive at definition (3) of the critical current $\bar{I}_c$ for a conductor with a free surface ($p_0 = 0$). The critical current $\bar{I}_c$ does not depend upon temperature and is a property of a given conductor. It is shown below that the critical current $\bar{I}_c$ is considerably less than the critical current I_c^0 determined by expression (2).

Determine the parameter $\gamma = \bar{I}_c / I_c^0$ in case of the van der Waals model of the critical state (see Ref. 6, Chap. 14, Sec. 152). The equation of state is given by

$$\pi(\eta, \tau) = -A\tau - 2B\tau\eta + 4C\eta^3, \quad (22)$$

where $\pi(\eta, \tau) = p/p_c^0 - 1$, $\eta = \rho/\rho_c^0 - 1$, $\tau = 1 - T/T_c^0$ and coefficients A , B , and C are the parameters of the model. Hereafter we consider the case $\eta \ll 1$, i.e., the vicinity of the critical point. The equation of the binodal line (20) can be written as follows:

$$\int_{\eta_1}^{\eta_2} [\pi - \pi(\eta', \tau)] d\eta' = -\tilde{\pi}, \quad (23)$$

where $\tilde{\pi} = \bar{p}/p_c^0$. The equation of the spinodal line for this model reads

$$\pi = -A\tau - \frac{B^{3/2}}{C^{1/2}} \frac{4}{3 \cdot 6^{1/2}} \tau^{3/2}. \quad (24)$$

Taking into account that, at the spinodal temperature $\tau = \tau_s(\pi)$, which is determined by expression (24), $\eta_2(\tau) = (B\tau/6C)^{1/2}$ and $\eta_1 = -2\eta_2(\tau)$, and using expressions (23) and (24) we arrive at the following expression for the critical temperature as a function of the electric current:

$$\tau_c = \frac{2}{3^{1/2}} \frac{C^{1/2}}{B} \tilde{\pi}^{1/2} \quad (25)$$

or

$$T_c(I) = T_c^0 \left[1 - \frac{4}{3^{1/2}} \xi^{1/2} \frac{C^{1/2}}{B} \frac{I}{I_c^0} \right].$$

Formula (25) provides the value of temperature at which the phase equilibrium curve (23) intersects with the spinodal line at a given electric current. Substituting (25) into (24) yields the value of the corresponding pressure. Equating thus determined pressure $p_c(I)$ to the pressure in a conductor given by expression (1), we arrive at the equation which determines the parameter $\gamma = \bar{I}_c / I_c^0$:

$$\frac{4}{3^{1/2}} \frac{AC^{1/2}}{B} \xi^{1/2} \gamma + \left[\frac{4}{3} \right]^{9/4} C^{1/4} \xi^{3/4} \gamma^{3/2} = 1 - \gamma^2. \quad (26)$$

Equation (26) shows that $\gamma(\xi)$ is a monotonically decreasing function of ξ . In order to demonstrate the magnitude of the effect, determine the value γ at the following

values of the parameters of the problem: $\sigma_2 \gg \sigma_1$, $A = 4$, $B = 2$, $C = \frac{3}{8}$ (the values of parameters A , B , and C are adopted equal to those of the van der Waals gas). As discussed above, the necessary value ξ_{21} is given by expression (15) since we deal with the phase transition from the high conductivity phase into the low conductivity phase. Then $\xi \approx 0.25$ and solution of Eq. (26) in this case is $\gamma \approx 0.45$.

Therefore, the value of the critical current $\bar{I}_c$, determined by taking into account the shift of the phase equilibrium curve, is significantly less than the value of the critical current I_c^0 determined by the relation for the critical pressure (2). In this connection it is of interest to study this effect using the equation of state of real metals.

IV. PECULIARITIES OF PHASE DYNAMICS

Consider now effects which are caused by the occurrence of the two-phase equilibrium curves determined by expressions (17) and (19) depending upon their mutual location. The location of these curves can be characterized by the temperature shift at a given pressure p with respect to the phase equilibrium curve of the current free system $T_0(p)$:

$$\kappa_{21}(p) = T_{21}(p) - T_0(p).$$

In the linear approximation formula (17) yields the following expression for $\kappa_{21}(p)$:

$$\kappa_{21}(p) = T_0(p) \frac{\bar{p}v_{1,0}}{\lambda_0}, \quad (27)$$

where $v_{1,0}$ and $v_{2,0}$ are the specific volumes of phases at the phase equilibrium curve $T_0(p)$; λ_0 is the specific latent heat of the phase transition from phase 2 to phase 1; $\lambda_0 = T_0(S_1^0 - S_2^0)$; S_1^0 and S_2^0 are specific entropies of phases 1 and 2 at temperature $T_0(p)$.

Similarly, using expression (19) for the phase equilibrium curve $1 \rightarrow 2$, we find that

$$\kappa_{12}(p) = -T_0(p) \frac{\bar{p}v_{2,0}}{\lambda_0}. \quad (28)$$

Expressions (27) and (28) show that both phase equilibrium curves are located at the same side of the binodal line $\mu_1 = \mu_2$. The following relation is valid for values κ_{12} and κ_{21} :

$$\kappa_{12} = -\kappa_{21} \frac{\xi_{12}v_{2,0}}{\xi_{21}v_{1,0}}.$$

The behavior of the system depends essentially upon the value of the parameter

$$\theta = -\frac{\xi_{12}v_{2,0}}{\xi_{21}v_{1,0}}.$$

Taking into account that phase "2" is a low-temperature phase and phase "1" is a high-temperature phase, it can be readily established that the condition $\theta < 1$ implies a regular hysteresis when the phase transition from phase "2" to phase "1" occurs at a higher temperature than the

inverse phase transition. Under these circumstances in the temperature range T , which is determined by the relation

$$\theta < \frac{T - T_0(p)}{\kappa_{21}} < 1,$$

both phases (a low-temperature phase and high-temperature phase) are stable. The existence in this domain of one of the two phases depends upon the initial conditions and the direction of the process.

In the opposite case when $\theta > 1$, the system is overheated with respect to the low-temperature phase but may remain overcooled with respect to the high-temperature phase. In this case, in the temperature range T , which is determined by the relation

$$1 < \frac{T - T_0(p)}{\kappa_{21}} < \theta,$$

there occurs a peculiar situation when both phases are simultaneously unstable.

The dynamics of the system is determined by the size of the critical nucleus $a_{21} \equiv a_2$ for a transition from phase 2 to phase 1 and by the size of the critical nucleus a_{12} for the transition from phase 1 to phase 2. From relations (16) and (18) it can be established that

$$a_{12}(p) = \frac{2\alpha(v_2/v_1)}{\bar{p}(\theta - 1) - 2\alpha/a_{21}}. \quad (29)$$

As can be seen from the formula (29), the coexistence of the critical nuclei of both phases ($a_{12} > 0$ and $a_{21} > 0$ simultaneously) is possible only if $\theta > 1$.

From the general dynamic theory of nucleation (see Ref. 11, Chap. 12, Sec. 99) it follows that the stable growth of a nucleus of a new phase can occur only until its size remains smaller than the size of the critical nucleus of the alternative phase.

In order to study the dynamics of nucleation in the situation when both phases are metastable, it is necessary to take into consideration that the size of the critical nucleus formed inside a metastable phase depends upon the size of the domain occupied by this phase. However, the above analysis is valid only if the external phase occupies almost all the wire and under the fixed value of electric current. Therefore, the consistent dynamic theory of nucleation under these conditions is a challenging problem which is beyond the scope of present research which employs only general thermodynamic relations.

The distance between two phase equilibrium curves for the inverse and direct phase transition $\Delta(p) = \kappa_{12} - \kappa_{21}$ can be determined from the following formula:

$$\Delta(p) = T_0 \frac{\bar{p}v_{1,0}}{\lambda_0} (\theta - 1).$$

If Δ is sufficiently high to provide the existence of the domain with the small size of the critical nuclei a_{12} and a_{21} and to support therefore the high nucleation rates, it may be anticipated that this domain has a profound influence upon the dynamics of phase transition. Besides, the above phenomenon may constitute the separate

mechanism of the fragmentation of the system into small particles.

The temperature range where $a_{12} \approx a_{21}$ is of particular interest. Consider the fusion process when $v_1^0 \approx v_2^0$. Then expression (29) yields

$$a_0 = a_{12} = a_{21} = \frac{4\alpha}{\bar{p}(\theta - 1)}. \quad (30)$$

The temperature κ_s (reckoned from the temperature T_0), at which the sizes of the critical nuclei of both phases are equal, can be derived from the above formulas:

$$\kappa_s = \kappa_{21} \frac{1 + \theta}{2}.$$

Note that at temperature κ_{21} , $a_{21} = \infty$ and $a_{12} = 2\alpha/\bar{p}(\theta - 1)$; similarly, at temperature κ_{12} , $a_{12} = \infty$ and $a_{21} = 2\alpha/\bar{p}(\theta - 1)$.

The above-discussed mechanism changes not only the condition for phase equilibrium but also alters significantly the energy balance in the system. The latent heat of phase transition from phase "2" to phase "1" $q(2 \rightarrow 1)$ is determined by the expression

$$q(2 \rightarrow 1) = W_1 - W_2,$$

where W is a specific enthalpy $W = \mu + TS$. Taking into account expression (17), we arrive at

$$q(2 \rightarrow 1) = -\bar{p}v_1 + (T_0 + \kappa_{21}) \times [S_1(T_0 + \kappa_{21}) - S_2(T_0 + \kappa_{21})].$$

Expanding the latter expression into the series of the parameter κ , we find that

$$q(2 \rightarrow 1) = \lambda_0 + (c_{p,1} - c_{p,2})\bar{p}v_{1,0} \frac{T_0}{\lambda_0}, \quad (31)$$

where c_p is a specific isobaric heat capacity. Similarly, for the inverse phase transition we determine that

$$q(1 \rightarrow 2) = -\lambda_0 + (c_{p,2} - c_{p,1})\bar{p}v_{1,0} \frac{T_0}{\lambda_0}.$$

As can be seen from these expressions, the latent heat of the phase transition in both cases changes in the same direction depending upon the relation between the specific heat of both phases. The absolute value of the latent heat of the phase transition decreases if $c_{p,1} < c_{p,2}$ and increases if $c_{p,2} > c_{p,1}$.

In order to obtain the numerical values of the above-determined parameters we used the data presented in Ref. 7 where the size of particles obtained in exploding aluminum, copper, and tungsten wires was measured. The maximum value of the electric current employed in these experiments was $I_0 = 20$ kA, the cross-section area of the wires was $S = 0.14$ mm², and its length was $L = 4$ cm. Since the maximum value of the current is attained in the vicinity of the melting point,⁷ it is legitimate to use this value for the calculations of the parameters given by the above derived expressions. Then employing these data and assuming that $\sigma_2 \gg \sigma_1$, so that formula (17) yields $\xi_{21} = 0.25$, we obtain that $\bar{p} = 2.85$ Kbar. The

TABLE I. The characteristic values for relative shift of the fusion temperature κ_{21}/T_0 determined from Eq. (27) and the size of the critical nucleus a_0 calculated from Eq. (30).

	ρ (kg/m ³) density in solid phase	λ (KJ/mole) latent heat of phase transition	α (erg/cm ²) coefficient of surface tension	M (at. weight)	κ_{21}/T_0 relative shift of fusion temperature	a_0 (Å) radius of critical nucleus	Measured characteristic radius of formed particles (Ref. 7) (Å)
Al	2.70×10^3	10.70	914	26.980	0.265	2.74	200
Cu	8.93×10^3	13.01	1351	63.546	0.150	4.05	200
W	5.96×10^3	35.20	1750	183.85	0.250	5.25	100

characteristic values for relative shift of the fusion temperature κ_{21}/T_0 determined from expression (27) and the size of the critical nucleus a_0 calculated from expression (30) are presented in Table I. Note that the magnitudes of the parameters presented in Table I demonstrate the significance of the above-considered effects.

V. CONCLUSIONS

The above-presented analysis demonstrated the occurrence of several peculiar effects which accompany phase transitions in current-carrying conductors: significant reduction of the magnitude of critical currents, splitting of the phase equilibrium curve into two curves for the direct and inverse phase transitions, formation of the domain of instability of both phases, and re-normalization of the latent heat of phase transition. The above phenomena are essential for understanding the processes occurring in the conductors in the presence of high electric currents and, in particular, for understanding the mechanism of the formation of ultrafine particles in exploding wires. It is well known that the previously suggested mechanisms of small-particle formation in exploding wires² cannot explain the formation of ultrafine particles which was observed in the experiments. Thus, the sizes of small particles formed during condensation of an expanding metal cloud are of order 10^{-6} m (see Ref. 13, Chap. 8, Secs. 12 and 13). The characteristic lengths of various magnetohydrodynamic instabilities which occur in exploding wires and may, in principle, cause formation of small particles, are even higher.^{2,3} Hence, it is of particular interest to study the dynamics of fusion and to study this process numerically taking into account the above-considered effects.

The theoretical value of the above-presented analysis stems from the fact that the obtained results are fairly general. Even though in every specific case the developed approach requires additional modifications for its adaptation, it is conceivable to indicate the fields where there exists similarity with the above-performed analysis. Many of the aspects of the above-considered problems have direct similarity with the problem of phase transition of the first kind accompanied by change of dielectric constant in strong electric fields.¹² These problems include melting (crystallization) and evaporation (condensation) of liquids with high dielectric constant in strong electric fields, chemical reactions in the presence of elec-

tric fields, and polymerization in a weakly ionized medium.

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APPENDIX: CALCULATION OF NUCLEUS FORMATION WORK

Taking into account expression (8), the expression for nucleus formation work (11) can be represented as

$$W_f = \frac{1}{2c^2} \int \frac{\mathbf{j}_0(\mathbf{r}) \cdot \mathbf{j}_0(\mathbf{r}') - \mathbf{j}_n(\mathbf{r}) \cdot \mathbf{j}_n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}', \quad (\text{A1})$$

where $\mathbf{j}_0(\mathbf{r})$ and $\mathbf{j}_n(\mathbf{r})$ are current density distribution $\mathbf{j}_s(\mathbf{r})$ before and after formation of the nucleus, respectively, which are determined from Eq. (7).

Consider the spherical nucleus of radius $a \ll r_0$. Assume also that this nucleus is formed at distance $d \ll r_0$ from the axis of a conductor. Under these conditions, in the calculations of current density distribution, the medium can be assumed to be infinite and the solution of Eq. (7) for $\mathbf{j}_n(\mathbf{r})$ is as follows:

$$\mathbf{j}_n(\mathbf{r}) = \mathbf{j}_1(\mathbf{r})\eta(a-r) + \mathbf{j}_2(\mathbf{r})\eta(r-a), \quad (\text{A2})$$

where

$$\begin{aligned} \mathbf{j}_1(\mathbf{r}) &= \frac{3\sigma_n}{\sigma_n + 2\sigma_0} \mathbf{j}_0, \\ \mathbf{j}_2(\mathbf{r}) &= \mathbf{j}_0 \left[1 - \xi \frac{a^3}{r^3} (3 \cos^2 \theta - 1) \right] \\ &\quad - 3\xi \mathbf{e}_\perp j_0 \sin \theta \cos \theta \frac{a^3}{r^3}, \end{aligned}$$

ξ is determined by expression (13), $\cos \theta = \mathbf{r} \cdot \mathbf{j}_0 / r j_0$,

$$\eta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases},$$

r is the distance from the center of the nucleus, $\mathbf{e}_\perp$ is a vector in the plane perpendicular to the direction of electric current $\mathbf{j}_0$. Note that electric current $\mathbf{j}_0$ before formation of the nucleus remains constant due to the ab-

sence of the skin effect.

Consider a case when the longitudinal and transversal dimensions of the conductor are approximately equal. Assume for simplicity that the conductor is of a spherical shape and that a spherical nucleus is formed far from the surface of the conductor, i.e., $d \ll r_0$. Then, in the zero-order approximation in parameter d/r_0 and with the accuracy $O(a/r_0)^2$, expressions (A1) and (A2) yield formulas (12) and (13). In the case of a long conductor in the same order approximation in the parameters d/r_0 , a/r_0 , and

with the logarithmic accuracy in L/r_0 , expressions (A1) and (A2) yield formulas (12) and (14).

Formation work of a thin and long nucleus when its length $b \gg r_0$ is calculated directly from Eq. (8) and expression (11) for W_f . Assume that the nucleus is of a cylindrical shape. In the zero-order approximation in the parameters a/b and d/r_0 we can neglect edge effects and consider that the nucleus is formed at the axis of a conductor. Then with the accuracy $O(a/r_0)^2$, we arrive at expressions (12) and (15).

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