

Peculiarities of coexistence of phases with different electric conductivities under the influence of electric current

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Abstract

In this study using Volmer–Zeldovich model and Ginzburg–Landau models we showed that in the domain of coexistence of phases under the influence of the electric current the phase equilibrium curve splits into two different curves for the direct and inverse phase transitions. It is demonstrated that there exist thermodynamic regions where both phases are metastable simultaneously. The latter effect occurs due to the splitting of the phase equilibrium curves into two curves, i.e. the equilibrium curves for the direct and for the inverse phase transitions. We investigated the dynamics of surface phase transition in current-carrying conductors. It is showed that inductance change has a strong effect on the dynamics of phase transitions. At the initial stage of phase transition ponderomotive forces caused by the inductance change prevent surface crystallization and promote surface melting. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction.

Electric current effects the dynamics of phase transition due to the following three mechanisms. The first mechanism is associated with a Joule heating of the system. Physical consequences of this phenomenon are quite transparent, and in the framework of the Stephan problem for surface melting of current-carrying conductors they were investigated in [1]. The second mechanism is associated with a change of thermodynamic pressure in the conductor due to the magnetic field. This mechanism causes a shift in the temperature of phase transition and can be accounted for independently from the other mechanisms [2]. The third, and the most interesting, mechanism that results in a number of effects and which will be described further, is associated with a change in the configuration of an electric current due to the difference in electric conductivities of the nucleus of a new phase and conductivity of the host medium.

The change of the configuration of an electric current depends upon the conductivity of the host medium σ_0 and conductivity of the nucleus of the new phase σ_1 . Schematically this change is showed in Fig. 1. Since depending upon the sign of the difference $\sigma_0 - \sigma_1$ the configuration of an electric current changes differently, and the magnitude of the work performed during formation of a nucleus is also different. If this additional work $\Delta W > 0$, ponderomotive forces prevent from the formation of a nucleus. In the opposite case when $\Delta W < 0$, a part of the work which is required for the formation of a new phase is performed by an external source of electric current.

In the following we present our recent results on some effects occurring during phase transitions in current-carrying conductors caused by the ponderomotive forces which arise due to stratification of the conductor into the domains with different conductivities.

The paper is organized as follows. In the second section we present results which were obtained using a zero-dimensional model. In the third section we present results which were derived using the Ginzburg–Landau model, and in the last section we describe the results obtained for the surface phase transition.

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2. Peculiarities of the volumetric phase transition in the zero -dimensional model

Consider the effect of the electric current on phase transition using the Volmer–Zeldovich model. The principal problem in the generalization of the Volmer–Zeldovich model to phase transitions in current-carrying conductors is to take into account the energy change ΔW which arises due to additional work performed during formation of a nucleus of a new phase. In order to derive this energy change one must determine the configuration of an electric current. When a configuration of the electric current $\vec{j}(\vec{r})$ is known the contribution of the magnetic field to the free energy of the system $F_m = \Delta W$ can be found from the following relationship:

$$\Delta W = \frac{1}{2c^2} \int \frac{\vec{j}(\vec{r}_1) \cdot \vec{j}(\vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} d^3\vec{r}_1 \cdot d^3\vec{r}_2. \quad (1)$$

The expression for the total free energy of the system F reads as:

$$F = F_0 + F_s - F_m. \quad (2)$$

Here F_0 is a volumetric part of a free energy without electromagnetic field and F_s is a surface part of the free energy caused by the formation of the interphase boundary. The sign of the last term in the right-hand side of Eq. (2) corresponds to the attraction of the electric currents flowing in one direction. Hereafter we assume that the following adiabatic condition is satisfied:

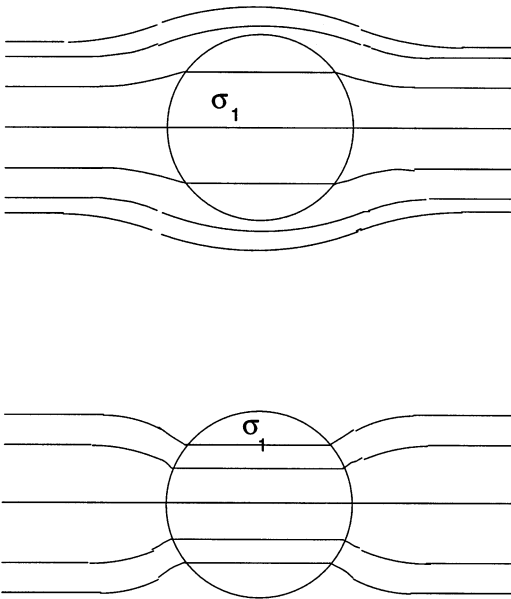


Fig. 1. Deformation of the current lines in a heterogeneous inclusion imbedded into a host medium with electric conductivity σ_0 , (a) $\sigma_0 > \sigma_1$, and (b) $\sigma_0 < \sigma_1$.

$$\frac{\sigma_m \cdot a \cdot \dot{a} c^2}{\ll 1},$$

where a and $\dot{a}$ are the nucleus size and the velocity of the interphase front, σ_m is the maximum electric conductivity.

Configuration of electric current is determined using a magnetostatic approximation:

$$\nabla \times \frac{\vec{j}}{\sigma} = 0, \quad \nabla \cdot \vec{j} = 0. \quad (3)$$

Cases of nucleus formation with various geometries were studied in Ref. [3].

In the magnetostatic approximation ΔW can be presented as a function of the volumes V_0 and V_1 occupied by the external and internal phases, respectively. The geometry of the system, namely, a shape of the conductor and the geometry of the boundary separating the phases, can be accounted for by a geometry dependent coefficient $\Phi(\vec{r})$.

Condition of thermodynamic equilibrium between phases reduces to the condition of the mechanical equilibrium in the given volume $V = V_0 + V_1$, and to the condition of a chemical equilibrium for a fixed number of particles $N = N_0 + N_1$. One of the methods to derive these condition is to introduce a functional (e.g. Refs. [4,5]):

$$\tilde{\Phi} = F + pV - \mu N, \quad (4)$$

where the external pressure p and a chemical potential of the system μ can be considered as Lagrange multipliers.

Condition of the mechanical equilibrium is determined by the following relationship:

$$\left(\frac{\partial \tilde{\Phi}}{\partial V_1} \right)_{N_1, N_0, T} = \left(\frac{\partial \tilde{\Phi}}{\partial V_0} \right)_{N_1, N_0, T} = 0 \quad (5)$$

and a condition of the chemical equilibrium reads:

$$\left(\frac{\partial \tilde{\Phi}}{\partial N_1} \right)_{v_1, v_0, T} = \left(\frac{\partial \tilde{\Phi}}{\partial N_0} \right)_{v_1, v_0, T}, \quad (6)$$

where v_0, v_1 are specific volumes of phases $v_i = V_i/N_i$.

Using Eqs. (1), (2), (5) and (6) and the expressions for F_0 and F_1 (for details see Ref. [5]) we find that:

$$\mu_1(p + \tilde{p}_m + p_s, T) = \mu_0(p, T), \quad (7)$$

where μ_1, μ_0 are chemical potentials of the internal and external phase, correspondingly.

In the case of a spherical nucleus the surface tension pressure p_s and ponderomotive pressure $\tilde{p}_m$ are determined by the following formulas:

$$p_s = \frac{2\alpha}{a}, \quad \tilde{p}_m = -2\zeta p_m \Phi(\vec{r}_1). \quad (8)$$

Here:

$$\zeta = \frac{\sigma_0 - \sigma_1}{\sigma_1 + 2\sigma_0}, \quad p_m = \frac{I^2}{\pi \rho_0^2 c^2},$$

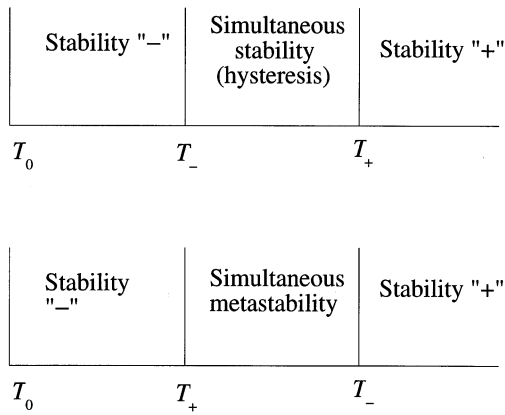


Fig. 2. Domain distribution for different magnitudes of the parameter γ , (a) $\gamma < 1$, and (b) $\gamma > 1$.

ρ_0 is a radius of the conductor, p is thermodynamic pressure, I is the total electric current, α is a coefficient of surface tension. For a cylindrical conductor p is determined by the following expression:

$$p = p_0 + \frac{I^2}{\pi \rho_0^2 c^2} \left(1 - \frac{\rho^2}{\rho_0^2} \right), \quad (9)$$

where p_0 is the pressure at the surface of a conductor.

Using Eq. (7) we can analyze some features of a phase transition. Consider a volume phase transition.

Introduce symbols ‘+’ and ‘−’ where the symbol ‘+’ denotes a high temperature phase while ‘−’ denotes a low temperature phase. Consider the most common case when the electric conductivity of a high temperature phase is less than the electric conductivity of a low temperature phase, i.e. $\sigma_+ < \sigma_-$ and $\kappa = \sigma_- / \sigma_+ > 1$. Using linear approximation in Eq. (7) in the vicinity of the temperature $T = T_0$ we arrive at the following expression for the temperature $T = T_-$ of the phase transition ‘+’ → ‘−’:

$$T_- = T_0 \left(1 - \frac{\tilde{p}_- v_-}{\lambda_0} \right), \quad (10)$$

where λ_0 is the latent heat of a phase transition, v_- is a specific volumes of phase at a temperature T_0 , $\lambda_0 = T_0(s_+^0 - s_-^0) > 0$, s_+^0 and s_-^0 are specific entropies of phases,

$$\tilde{p}_- = \frac{2(1 - \kappa)}{\kappa + 2} p_m \Phi(\tilde{r}_a). \quad (11)$$

Geometric factor $\Phi(\tilde{r}_a)$ depends upon the initial (prior to the nucleus formation) configuration of an electric current and on the location of a center of mass of a spherical nucleus $\tilde{r}_a$ [3]. A case with a non spherical nucleus requires a special analysis which is not presented here. The analysis of the latter case can be found in our studies [3,6,7].

For a temperature of the inverse phase transition we obtain:

$$T_+ = T_0 \left(1 + \frac{\tilde{p}_+ v_+}{\lambda_0} \right), \quad \tilde{p}_+ = \frac{2(\kappa - 1)}{1 + 2\kappa} p_m \Phi(\tilde{r}_a). \quad (12)$$

Define a parameter:

$$\gamma = \frac{T_- - T_0}{T_+ - T_0} = \frac{v_-(1 + 2\kappa)}{v_+(\kappa + 2)}. \quad (13)$$

Eqs. (10)–(13) show that phase equilibrium curves for the direct and inverse phase transitions T_- and T_+ are shifted towards the higher temperatures, i.e. $T_- > T_0$ and $T_+ > T_0$. Two possible situations are showed in Fig. 2.

The first case is that when $\gamma < 1$ and $T_- < T_+$. In this case in a temperature range $T_- < T < T_+$ both phases are stable and the existence of a particular phase is determined by the direction of the process. During cooling the high temperature phase remains stable while during heating the low temperature phase remains stable. In this domain a phase transition hysteresis must be observed. Nuclei of a new phase can be formed in the domain $T > T_+$ where nuclei of a high temperature phase are formed, and in the domain $T < T_-$ where nuclei of a low temperature phase are formed. In the domain $T_- < T < T_+$ nuclei of the new phase cannot be formed.

Consider now the second case with $\gamma > 1$. In this case $T_- > T_+$, and in the range $T_+ < T < T_-$ both phases are metastable simultaneously. The latter implies that in this domain when a conductor is transformed into a high temperature phase, nuclei of a low temperature phase can form inside a conductor and vice versa. The sizes of the critical nucleus for ‘+’ → ‘−’ phase transition, a_c^- , and for the inverse phase transition ‘−’ → ‘+’, a_c^+ , are determined by the following expressions:

$$a_c^- = -\frac{2\alpha v_-}{\lambda_0 \frac{\Delta T}{T_0} + v_- \tilde{p}_-}, \quad a_c^+ = \frac{2\alpha v_+}{\lambda_0 \frac{\Delta T}{T_0} - v_+ \tilde{p}_+}, \quad (14)$$

where α is a coefficient of a surface tension.

Since $\tilde{p}_- < 0$ and $\tilde{p}_+ > 0$, a critical nucleus of a low temperature phase can be formed in the domain $\Delta T > 0$ in the undercooled state while formation of a high temperature phase requires an additional overheating.

Eq. (14) yields:

$$a_c^+ = \frac{2\alpha \frac{v_+}{v_-}}{|\tilde{p}_-|^{\frac{\gamma-1}{\gamma}} - \frac{2\alpha}{a_c^-}}. \quad (15)$$

For $a_c^- = (2\alpha\gamma)/(|\tilde{p}_-|(\gamma-1)) = \min$, $a_c^+ \rightarrow \infty$, while for $a_c^+ = (2\alpha(v_+/v_-)\gamma)/(|\tilde{p}_-|(\gamma-1)) = \min$, $a_c^- \rightarrow \infty$.

Consider now a volumetric phase transition in the framework of the Ginzburg–Landau model.

3. Peculiarities of the volumetric phase transition in the framework of the Ginzburg–Landau model.

In order to construct a continual model of phase transition accompanied by a change of electric conductivity in the presence of electric current $\vec{j}$ we will use a Ginzburg–Landau model for Gibbs potential Ω . In the regime with a prescribed magnitude of electric current Ω can be written as:

$$\Omega = \Omega_0 - \Omega_m, \quad (16)$$

where Ω_m is determined as previously by Eq. (1), $\Omega_m = \Delta W$.

$$\Omega_0 = \int [f_0(\eta) - \mu\eta] d^3\vec{r} + \Omega_s, \quad (17)$$

where $\Omega_s = \chi/2 \int (\vec{\nabla}\eta)^2 d^3\vec{r}$ is a surface energy ($\chi > 0$), $f_0(\eta)$ is a specific free energy, μ is a chemical potential and η is an order parameter. It was showed in Ref. [8] that equations:

$$\dot{\eta} = -\frac{1}{\Gamma_n} \frac{\partial \Omega}{\partial \eta}, \quad \dot{\eta} = \frac{1}{\Gamma_s} \Delta \left(\frac{\partial \Omega}{\partial \eta} \right), \quad (18)$$

where Γ_n and Γ_s are kinetic coefficients, allow us to describe relaxation of the order parameter from a metastable phase to a stable phase. The first equation in Eq. (18) describes a case when the order parameter is not conserved while the second equation describes a case when the order parameter is conserved. In the latter case it has the form of a continuity equation.

Thus the problem is reduced to the problem of determining the explicit dependence of the potential Ω_m upon the order parameter η . In order to solve the latter problem we use the following approach.

According to Eq. (1) the energy of the magnetic field can be represented as follows:

$$\Omega_m = \Omega_m^0 + \Omega_m^1, \quad (19)$$

where Ω_m^0 is a part of a magnetic energy which does not depend upon a nucleus size,

$$\Omega_m^0 = \frac{\vec{j}_0^2}{2c^2} \int \frac{1}{|\vec{r}_2 - \vec{r}_1|} d^3\vec{r}_1 d^3\vec{r}_2,$$

and j_0 is electric current density far away from the nucleus.

$$\Omega_m^1 = -\frac{\vec{p}_m}{3(\sigma_0 - \sigma_1)} \int ((\vec{r} - \vec{r}_a) \vec{\nabla} \sigma) d^3\vec{r}, \quad \vec{p}_m = 2\xi \Phi p_m. \quad (20)$$

In the following we will assume that the spatial profile of electric conductivity coincides with the gradient of the order parameter, i.e.:

$$\vec{\nabla} \sigma = \frac{\sigma_0 - \sigma_1}{\eta_0 - \eta_1} \vec{\nabla} \eta$$

where η_0 and η_1 are the magnitudes of the order parameter far from the boundary between phases. Since under variation of the order parameter its values far from the interface are considered constants, after integration by parts in Eq. (20) and using Eqs. (16) and (17) we find that:

$$\frac{\partial \Omega}{\partial \eta} = -\left(\chi \Delta \eta + \mu - \frac{\partial f_0}{\partial \eta} + \frac{\vec{p}_m}{\eta_0 - \eta_1} \right). \quad (21)$$

For determining the values of order parameter $\eta(\vec{r})$ for coexisting phases, η_0 and η_1 , one can use the condition of equilibrium:

$$\frac{\partial \Omega}{\partial \eta} = 0. \quad (22)$$

Far from the boundary separating two phases the term $\Delta \eta$ in Eq. (21) can be neglected. Thus Eqs. (17), (21) and (22) yield the following equations for values of the order parameter η_0, η_1 in coexisting phases:

$$\mu + \frac{\vec{p}_m}{\eta_0 - \eta_1} = \alpha \eta_0 + \beta \eta_0^3, \quad \mu + \frac{\vec{p}_m}{\eta_0 - \eta_1} = \alpha \eta_1 + \beta \eta_1^3, \quad (23)$$

where $\alpha(T)$, $\beta(T)$ are the coefficients of the Ginzburg–Landau model:

$$f_0(\eta) = \frac{\alpha \eta^2}{2} + \frac{\beta \eta^4}{4}$$

(for details see Refs. [5,8,9]).

As it is generally accepted, assume that $\beta > 0$, and then the domain of coexisting of the phases is determined by a condition $\alpha < 0$. Introducing a dimensionless variable Ψ , $\eta = (|\alpha|/\beta)^{1/2} \Psi$ we find from Eq. (23) that for a given value of Ψ_0 there are two possible values for Ψ_1 :

$$\Psi_1 = -\frac{\Psi_0}{2} \pm \sqrt{1 - \frac{3}{4} \Psi_0^2}. \quad (24)$$

According to Eqs. (18) and (21) a condition for thermodynamic stability of a phase is $\partial^2 f / \partial \eta^2 \geq 0$ or $|\Psi_{0,1}| > 1/\sqrt{3}$. From Eq. (24) it can be easily seen that there is a unique thermodynamically stable solution Ψ_1 which corresponds to the thermodynamically stable solution Ψ_0 and vice versa. In the range $1/\sqrt{3} < \Psi_0 < 2/\sqrt{3}$, the thermodynamically stable solution reads:

$$\Psi_1 = -\frac{\Psi_0}{2} - \sqrt{1 - \frac{3}{4} \Psi_0^2}$$

and in the range $-(2/\sqrt{3}) < \Psi_0 < -(1/\sqrt{3})$ the stable solution is:

$$\Psi_1 = -\frac{\Psi_0}{2} + \sqrt{1 - \frac{3}{4} \Psi_0^2}.$$

Using these relationships and Eq. (23) we arrive at the equations for parameter Ψ_0 :

$$\begin{aligned} \frac{\tilde{\pi}_m}{E_+(\Psi_0)} &= \Psi_0^3 - \Psi_0 - h, & \frac{1}{\sqrt{3}} < \Psi_0 < \frac{2}{\sqrt{3}} \\ \frac{\tilde{\pi}_m}{E_-(\Psi_0)} &= \Psi_0^3 - \Psi_0 - h, & -\frac{2}{\sqrt{3}} < \Psi_0 < -\frac{1}{\sqrt{3}} \end{aligned} \quad (25)$$

$$E_{\pm}(\Psi_0) = \frac{3}{2}\Psi_0 \pm \sqrt{1 - \frac{3}{4}\Psi_0^2},$$

where

$$\tilde{\pi}_m = \frac{\beta}{|\alpha|^2} \tilde{p}_m, \quad h = \frac{\beta^{1/2}}{|\alpha|^{3/2}} \mu.$$

Similarly, excluding Ψ_0 from Eq. (23) we arrive at the equations for determining the thermodynamically stable value of the internal phase Ψ_1 :

$$\begin{aligned} \frac{\tilde{\pi}_m}{\tilde{E}_+(\Psi_1)} &= \Psi_1^3 - \Psi_1 - h, & \frac{1}{\sqrt{3}} < \Psi_1 < \frac{2}{\sqrt{3}} \\ \frac{\tilde{\pi}_m}{\tilde{E}_-(\Psi_1)} &= \Psi_1^3 - \Psi_1 - h, & -\frac{2}{\sqrt{3}} < \Psi_1 < -\frac{1}{\sqrt{3}} \\ \tilde{E}_{\pm}(\Psi_1) &= -\left[\frac{3}{2}\Psi_1 \pm \sqrt{1 - \frac{3}{4}\Psi_1^2} \right]. \end{aligned} \quad (26)$$

In order to determine the critical values of the chemical potential h which determine the domain of coexistence of phases we take into account that the thermodynamically unstable phase must be considered as an external phase. Then the critical value of h , at which the phase becomes thermodynamically unstable under a given magnitude of electric current, can be determined from Eq. (25). Hereafter the phases with negative and positive values of the order parameter are called the negative and the positive phases, respectively. Then for a positive phase setting in Eq. (25) $\Psi_0 = 1/\sqrt{3}$, $\sigma_0 = \sigma_+$, $h = h_c^+$ we find that:

$$h_c^+ = -h_c^0 + \frac{2}{\sqrt{3}} \frac{\kappa - 1}{\kappa + 2} A, \quad h_c^0 = \frac{2}{3\sqrt{3}}, \quad \kappa = \frac{\sigma_-}{\sigma_+}, \quad (27)$$

where $A = p_m \Phi(\beta/|\alpha|^2) \geq 0$, σ_+ and σ_- , are conductivities of the positive and negative phases, respectively. The domain of the thermodynamic stability of the positive phase is determined by relation $h > h_c^+$.

Similarly, for the negative phase $\Psi_0 = -1/\sqrt{3}$, $\sigma_0 = \sigma_-$, $h = h_c^-$ from Eq. (25) we find:

$$h_c^- = h_c^0 + \frac{2}{\sqrt{3}} \frac{\kappa - 1}{1 + 2\kappa} A. \quad (28)$$

Domain of stability of the negative phase is determined by the condition $h < h_c^-$, and the domain of coexistence of phases is determined by the condition $h_c^+ < h < h_c^-$. The critical value of electric current A_c whereby at $A > A_c$ phases cannot coexist at any value of h can be found from a condition $h_c^- = h_c^+$ or:

$$A_c = \frac{2}{\sqrt{3}} \frac{(\kappa + 2)(1 + 2\kappa)}{(\kappa - 1)^2} \geq \frac{4}{\sqrt{3}}. \quad (29)$$

For the electric current $A > A_c$ the phase transition cannot be realized as a phase transition of the first kind.

It is showed in Ref. [9] that the equilibrium value of a chemical potential is determined by the following condition:

$$\frac{\tilde{\pi}_m}{\Psi_0 - \Psi_1} + h = 0. \quad (30)$$

According to Eqs. (25) and (26) the latter condition can be satisfied only at the points $|\Psi_0| = |\Psi_1| = 1$.

In order to determine the value of the chemical potential $h = h^-$ corresponding to the equilibrium with respect to the transition from a positive phase to a negative phase, one must take into account that in this case the negative phase is the internal phase, i.e. $\Psi_0 = 1$, $\Psi_1 = -1$. Substituting $\sigma_0 = \sigma_+$, $\sigma_1 = \sigma_-$ in Eq. (30) we find that:

$$h^- = \frac{\kappa - 1}{\kappa + 2} A. \quad (31)$$

Similarly, one can determine the potential $h = h^+$ corresponding for the equilibrium with respect to the inverse phase transition from the negative phase to the positive phase by setting $\Psi_0 = -1$, $\Psi_1 = 1$ and $\sigma_0 = \sigma_-$, $\sigma_1 = \sigma_+$:

$$h^+ = \frac{\kappa - 1}{1 + 2\kappa} A. \quad (32)$$

It follows from Eqs. (31) and (32) that phase equilibrium curves for the direct and inverse phase transition are shifted in the same direction, i.e. $\text{sign}(h^+) = \text{sign}(h^-)$. Here one of the phases becomes more stable than in a current-free case while the other phase destabilizes. Naturally, it destabilizes the phase towards which shifts the phase equilibrium curve since part of the domain where this phase is stable in a current-free case becomes metastable. Eqs. (31) and (32) show that the phase with a higher electric conductivity stabilizes. The physical meaning of the domain $h^+ < h < h^-$ is elucidated by the following general considerations. In a current-free system $h > 0$ is a region of stability of the positive phase while $h < 0$ is a domain of the stability of the negative phase. Therefore, also in a current-carrying system for sufficiently large but negative h , the negative phase is stable. When h increases and attains the point h^+ where nuclei of a positive phase can form, the negative phase loses stability and transfers into a metastable state. However a positive phase is still metastable since for $h < h^-$ nuclei of a negative phase can form inside a positive phase, and only at $h > h^-$ the positive phase becomes stable. Thus in the domain $h^+ < h < h^-$ both phases are metastable simultaneously.

The points h_c^+ , h^+ , h^- , h_c^- partition the domain of h into the regions of varying thermodynamic stability of phases. Mutual location of these regions depends upon the parameter $\kappa = \sigma_-/\sigma_+$ and the magnitude of ‘electric current’ A . According to Eqs. (27) and (28) in the domain of coexistence of phases $h_c^- > h_c^+$, and according to Eqs. (31) and (32)) $h^- > h^+$. Taking into account these conditions it can be easily seen that there exist six different sequences of the domains:

- (a) $h_c^+ < h^+ < h^- < h_c^-$, (b) $h^+ < h_c^+ < h^- < h_c^-$,
 (c) $h_c^+ < h^+ < h_c^- < h^-$, (d) $h^+ < h_c^+ < h_c^- < h^-$,
 (e) $h_c^+ < h_c^- < h^+ < h^-$, (f) $h^+ < h^- < h_c^+ < h_c^-$.

The detailed analysis of the stability of phases in all these cases is performed in Ref. [9]. In Fig. 3 we showed the stability regions corresponding to the case (a). In

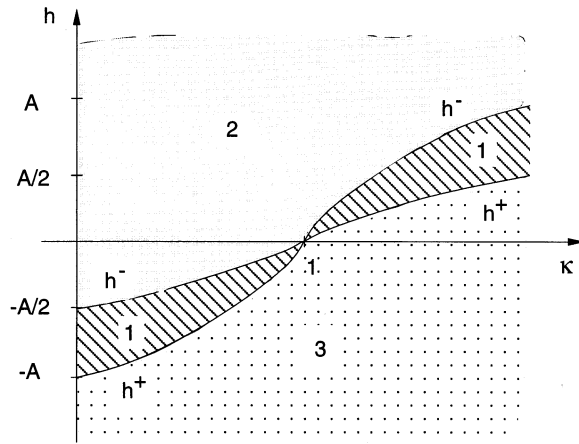


Fig. 3. Diagram of coexistence of phases at small electric currents: 1. DSM (domain of simultaneous metastability of both phases); 2 stable positive phase; 3 stable negative phase; h^+ is a binodal line for $+\Rightarrow-$ phase transition; h^- is a binodal line for $-\Rightarrow+$ phase transition.

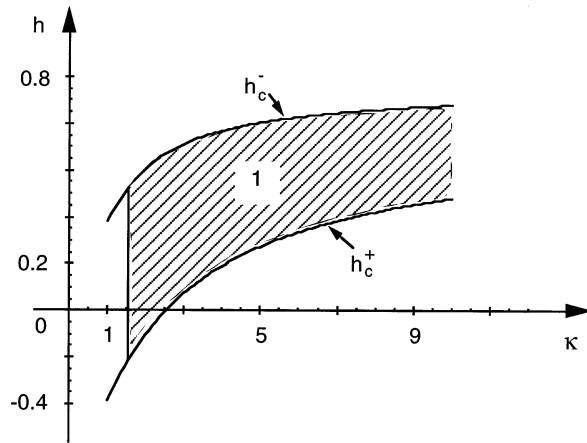


Fig. 4. Diagram of coexistence of phases, case (d): 1. DSM; h_c^- is a spinodal line of the negative phase; h_c^+ is a spinodal line of the positive phase.

the latter case the domain of simultaneous metastability of both phases borders with domains where both phases are stable. The opposite situation corresponds to the case (d), and is showed in Fig. 4. In this case the domain of the simultaneous metastability of both phases borders with the domains where the phases are unstable.

Thus the most significant effect of the electric current is the occurrence of the domain of the simultaneous metastability of two phases. Inside this domain the nuclei of a phase with a higher electric conductivity are formed due to the work of ponderomotive forces, i.e. due to an external source of electric current, while the nuclei of a phase with a smaller electric conductivity are formed due to the surplus of chemical energy.

4. Peculiarities of surface phase transition in the current-carrying conductors

Ponderomotive forces which occur during nucleation of a new phase also affect the dynamics of the surface phase transition. Without accounting for the effects of the ponderomotive forces the dynamics of a surface phase transition was considered in Ref. [1]. This process is described by a heat conduction equation with a distributed source:

$$\frac{\partial T_i}{\partial t} = \chi_i \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial T_i}{\partial \rho} \right) + \frac{Q_i}{c_i}, \quad (34)$$

where χ_i and c_i are temperature conductivity and specific heat per 1 mol of phase i , respectively, Q_i is a rate of release of Joule heat per 1 mol in phase i , $Q_i = (j_i^2 v / \sigma_i)$, j_i is electric current density, T_i is temperature of phase i , ρ is a radial coordinate.

Eq. (34) must be supplemented with boundary conditions. As before we denote the external phase by $i=0$ and internal phase by $i=1$. In a case of a cylindrical interphase boundary we find that:

$$c_1 \chi_1 \frac{\partial T}{\partial \rho} \Big|_{\rho_F - \varepsilon} - c_0 \chi_0 \frac{\partial T}{\partial \rho} \Big|_{\rho_F + \varepsilon} = \dot{\rho}_F \lambda, \quad (35)$$

where ρ_F is a radius of the interface boundary, λ is a latent heat of a phase transition, and

$$T_1(\rho) \Big|_{\rho=\rho_F} = T_0(\rho) \Big|_{\rho=\rho_F} = T_0. \quad (36)$$

Eq. (35) describes energy balance at the interphase boundary while Eq. (36) corresponds to the case of a phase transition under condition of chemical equilibrium.

In the case of a non-equilibrium phase transition Eqs. (34) and (35) must be supplemented with an equation of chemical kinetics, instead of Eq. (36). Neglecting the mechanical effects and the effects of mass transfer equation of chemical kinetics can be written as follows:

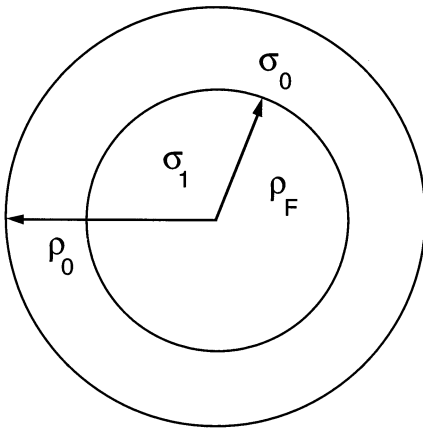


Fig. 5. Configuration of a conductor undergoing surface phase transition.

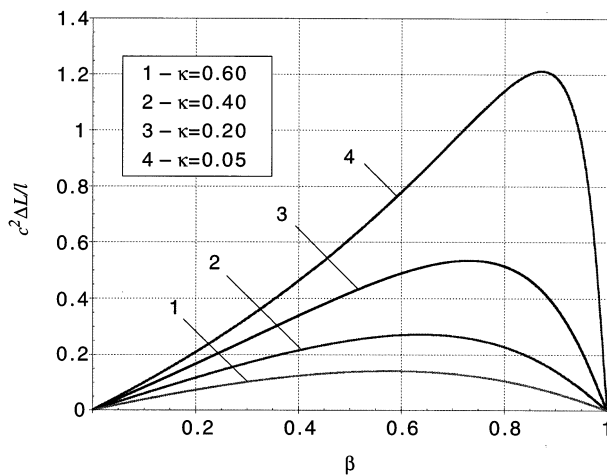


Fig. 6. Variation of the inductance of a melting current-carrying conductor vs. β .

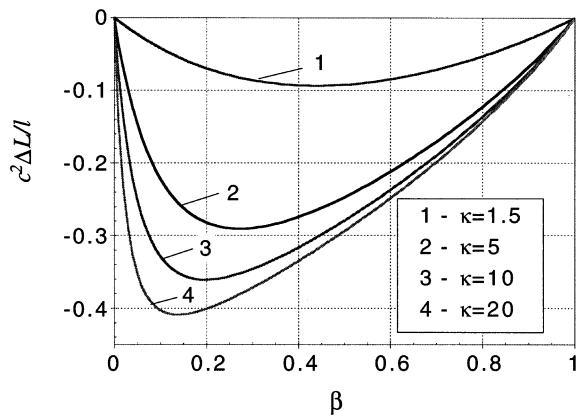


Fig. 7. Variation of the inductance of a crystallizing current-carrying conductor vs. β .

$$\dot{\beta} = \frac{u}{T_0} \left(\mu_1 - \mu_0 - p_m v \frac{\partial f}{\partial \beta} \right), \quad (37)$$

where $u = N_0/\tau$ is a relaxation rate ($u = \text{mol s}^{-1}$) which

is a phenomenological constant in our theory, v is a specific volume which is considered the same for both phases, $\beta = 1 - \rho_F^2/\rho_0^2$ and ρ_0 is a radius of a cylindrical conductor (see Fig. 5). Function $f(\beta)$ is found by determining an inductance of the conductor $L = L(\beta)$ (for details see Refs. [2,3]):

$$L(\beta) = L_0 - \frac{\ell}{c^2} f(\beta),$$

$$f(\beta) = \xi^2(1 - \beta)[(1 - \beta) \ln(1 - \beta) + (1 + a)\beta],$$

where $\xi = 1/(\beta + \alpha)$, $a = 1/(\kappa - 1)$, $\kappa = \sigma_0/\sigma_1$, and L_0 is an inductance of a homogeneous conductor.

Condition of a chemical equilibrium $\dot{\beta} = 0$ determines the temperature at the interphase boundary T_F^0 which depends upon the location of a phase front, i.e. $T_F^0 = T_F^0(\beta)$. Linearizing Eq. (37) and using condition $\dot{\beta} = 0$ we find that:

$$T_F^0 = T_0 \left(1 \pm \frac{p_m v_0}{\lambda} \frac{\partial f}{\partial \beta} \right).$$

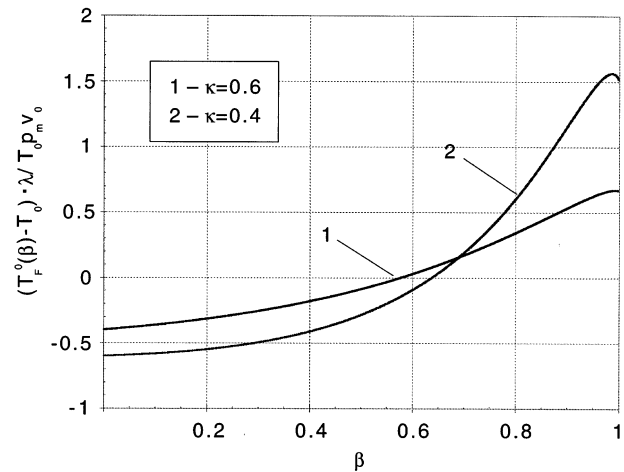


Fig. 8. Temperature shift at a phase transition front (melting) vs. volume fraction of a high temperature phase.

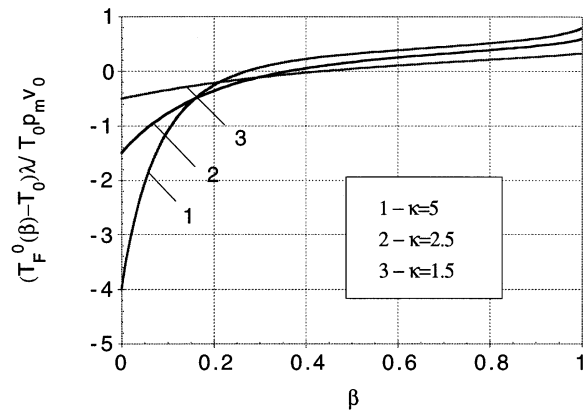


Fig. 9. Temperature shift at a phase transition front (crystallization) vs. volume fraction of a low temperature phase.

In Figs. 6–9 we show variation of the inductance of the conductors for the cases of melting and crystallization, respectively. As can be seen from these plots at the initial stage the inductance increases during melting and decreases during crystallization. Naturally, we assume that electric conductivity of a solid phase σ_s is higher than that of a liquid phase σ_L . Inductance increase at the initial stage of melting corresponds to the reduction of the melting temperature while an inductance decrease during crystallization corresponds to the reduction of the crystallization temperature.

Thus at the initial stage of phase transition ponderomotive forces prevent surface crystallization and promote surface melting. When a phase transition front penetrates deep into the conductor the situation changes and becomes essentially the same as in the case of a volumetric phase transition which was considered above. In the latter case ponderomotive forces in current-carrying conductors promote the formation of a phase with a higher electric conductivity.

Now let us discuss the systems where the above results can be applied.

Note that our investigation was initiated by experiments with electrically exploding wires where very small particles of different material were formed. In these experiments the density of electric current was of the order of 10^7 A sm^{-2} [10], and the resulting magnetic pressure was of the order of $p_m \sim 1 \text{ GPa}$. Clearly, under these high magnetic pressures the considered above effects are quite significant. Indeed, the relative shift of the fusion temperatures for Al, Cu, W is of the order of $\Delta T/T_0 \sim 0.1$. The value of the relative shift of the latent heat of phase transition $\Delta\lambda/\lambda_0$ is of the same order. Using the concept of the existence of the region of simultaneous metastability of phases allowed us to explain the formation of small particles with radii of the order of $r \sim 10^{-6} \text{ cm}$ in exploding wires [11].

Recently a number of studies [12–14] reported the strong influence of the relatively small electric current on the structural change in different alloys. The physical mechanism which was discussed in Refs. [12,13], namely electric migration, cannot explain the observed phenomena, employing physically reasonable assumptions about the magnitudes of the parameters of the system. In one of these investigations [13] it was showed that the applied strain $\varepsilon \sim 10^{-3}$ causes the effect similar to that caused by an electric current with the amplitude $I \sim 1 \text{ A}$ and electric current density $j \sim 10^3 \text{ A sm}^{-2} \div 10^4 \text{ A sm}^{-2}$. The magnetic pressure for the sizes of specimens which were used in this study can be estimated as $p_m \sim 10 \text{ Pa}$. However the strain $\varepsilon \sim 10^{-3}$ results in considerably larger pressure $P \sim 10^2 \text{ MPa}$ in these materials with Young's modulus $M \sim 10^2 \text{ GPa}$.

One of the possible reasons for this discrepancy is that in our studies we analyzed a simple geometry of the electric current which does not take into account the internal strong fields localized near the boundaries of the grains. These fields can change considerably the electric currents near the grains. Analysis of the latter effects is the subject of our ongoing investigation.

5. Conclusions

We may summarize the above analysis as follows. Ponderomotive forces in current-carrying conductors cause the following effects: (1) Splitting of the curve of phase equilibrium into two different curves for the direct and the inverse phase transitions. (2) Occurrence of the domain of simultaneous metastability of both phases where the direct and the inverse phase transitions occur simultaneously. (3) Effect of ponderomotive forces on the dynamics of phase transitions is different for volumetric and surface phase transitions.

In the case of a volumetric phase transition ponderomotive forces prevent the formation of the high-temperature phase with lower electric conductivity and promote formation of the low-temperature phase with higher electric conductivity.

In the case of a surface phase transition at the initial stage ponderomotive forces prevent the formation of the low temperature phase with higher electric conductivity and promote formation of a high temperature phase with lower electric conductivity.

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