

Review article

Critical comments to results of investigations of drop
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Abstract

During the last decade numerous studies considered collisions of inertial particles in turbulent flows. A magnitude of the turbulence-induced collision rate enhancement factor reported in these studies ranges from a few percent to several hundred. The authors of the majority of the studies apply their results to explanation of rain formation in atmospheric clouds. At the same time many of these investigations were performed under the conditions quite different from those encountered in real clouds. For instance, in most analytical and direct numerical simulations (DNS) the effect of gravity-induced differential drop sedimentation was neglected. Using the collision enhancement factors obtained in these studies for cloud modeling may lead to unrealistic cloud evolution and impair research in cloud physics. In this study we present an analysis of the applicability of the results obtained in different recent studies (mainly DNS simulations) to actual clouds. We discuss the progress reached in the topic as well as unsolved problems.

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Keywords: Collisions of inertial particles in turbulent flows; Direct numerical simulations; Turbulent clouds; Rain formation

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1. Introduction

Theoretical evaluations (Jonas, 1996) and in situ measurements (Brennguier and Chaumat, 2001; Chaumat and Brennguier, 2001) in clouds indicate that droplet spectrum width grows with height and raindrops form faster than it follows from the classical equations of diffusion growth and the stochastic collision equation, in which the collision kernel is calculated under pure gravity conditions. The most plausible mechanism explaining the droplet spectrum broadening and acceleration of raindrop formation is the effect of cloud turbulence. Clouds are known as zones of enhanced turbulence, with the turbulent kinetic energy dissipation rate ε varying from $10 \text{ cm}^2 \text{ s}^{-3}$ in stratiform clouds to $1000 \text{ cm}^2 \text{ s}^{-3}$ in strong Cb clouds (Mazin et al., 1989; Pinsky and Khain, 2003; Pinsky et al., 2000a; Siebert et al., 2006).

The role of cloud turbulence in the acceleration of cloud droplet collisions has attracted the attention of cloud physicists for a long time. The first theoretical attempts to evaluate different aspects of turbulent effects on collisions of cloud droplets were performed by Arenberg (1939), Saffman and Turner (1956), Ivanovsky and Mazin (1960), Almeida (1976, 1979), Manton (1977), Grover and Pruppacher (1985), Reuter et al. (1988) and others. Different turbulent mechanisms were analyzed, and the conclusions reached were quite different. In some studies (e.g. Ivanovsky and Mazin 1960) the effect of turbulence on droplet collision rate was found much weaker than the effect of gravity, while Almeida reported a dramatic increase in the collision rate. The results obtained in the majority of these studies were successively criticized both from the point of view of the methods used in the investigations, and the validity of turbulent flow description (e.g., Pruppacher and Klett, 1997). The results obtained in the course of further theoretical and laboratory investigations, as well as in situ observations were overviewed by Pinsky et al. (2000a), Khain et al. (2000), Vaillancourt and Yau (2000), Shaw (2003), Riemer and Wexler (2005). It seems that the fact that turbulence enhances the rate of particle collisions can be considered as being established.

In parallel to investigations of turbulent effects on droplet collisions in clouds, numerous studies considered motion and collisions of inertial particles in turbulent flows. It is conceivable to suggest that many of these studies were motivated by investigations of turbulence effects in clouds.

It is generally believed now that turbulence affects the collision rate of inertial particles through three major mechanisms: a) increase in the relative particle velocity (or increase in the swept volume); this effect is also known as the turbulent transport effect; b) formation of concentration inhomogeneities (particle clustering or effect of preferential concentration), and c) turbulence effect on the hydrodynamic drop interaction (HDI) that increases the collision efficiency.

Several main methods of investigation of the turbulent effects are used: analytical studies considering droplet motion in an idealized turbulent flow, direct numerical simulations (DNS),¹ utilization of statistical models of a turbulent flow and hybrid methods. During the last decade collisions of inertial particles in turbulent flows were considered in numerous studies.

Because of the large number of these studies in this survey we cannot review or even refer here to all of them. Notably, a wide range of the turbulence-induced collision rate enhancement was reported in these studies: from a few percent to the factor as high as several hundred.

In many of these studies (even those published in engineering and fluid mechanics literature) the authors apply their results to explanation of rain formation in atmospheric clouds (or at least, they mention droplet collisions in clouds as relevant to their investigations). At the same time many investigations have been performed under the conditions quite different from those encountered in real clouds. Therefore, a careful

¹ The great advantage of DNS is that they simulate real turbulent flows obeying the Navier–Stokes equations. DNS have proved to be a powerful tool in investigation of both turbulent flows themselves and motion and collisions of inertial particles in turbulent flows. However, the applicability of some results obtained in DNS to cloud conditions is questionable, as it will be shown in the paper.

analysis is required concerning the applicability of the particular result to real clouds.

The typical discrepancies between the conditions assumed in most of the studies and the conditions encountered in real clouds are the following:

a) Many studies considered particle behavior inside monodisperse suspensions with high concentration of particles. High droplet number densities were assumed in simulations of cloud droplets with the radii below $\sim 20 \mu\text{m}$, as well as for droplets with the radii larger than $20 \mu\text{m}$, including small rain drops with the $40\text{--}70 \mu\text{m}$ radii. The high concentration of small rain drops assumed in these studies corresponds to the liquid water content in the range from 20 to 1000 gm^{-3} . On the contrary, real clouds are characterized by a very low concentration of droplets having, however, a wide range of sizes. The majority of cloud droplets have the radii below $20 \mu\text{m}$. Droplet number density ranges from 50 cm^{-3} in maritime clouds to 1000 cm^{-3} in very continental clouds, so that the mean separation distance between droplets usually exceeds 1 mm . Concentration of $40\text{--}70 \mu\text{m}$ -radii drops is often less than one per cm^3 . The mean separation distance between such drops usually exceeds 1 cm . The maximum liquid water contents are observed in convective clouds and they do not exceed $4\text{--}5 \text{ gm}^{-3}$, so that the mass loading is relatively low, of the order of $1\text{--}5 \times 10^{-3}$.

b) In most of theoretical studies gravity-induced sedimentation was neglected. At the same time the range of droplet sedimentation velocities in clouds is quite wide, being proportional to the square of the droplet radius. Clearly, neglecting gravity-induced sedimentation is not valid for raindrops where the role of gravitation may be dominating.

c) Most theoretical studies and laboratory experiments (e.g., experiments by Fessler et al., 1994; Vohl et al., 1999) were performed in turbulent flows characterized by the Taylor microscale Reynolds numbers $Re_\lambda < 10^2$, while Re_λ ranges from $\sim 5 \cdot 10^3$ in stratiform clouds to $\sim 2 \cdot 10^4$ in strong deep convective clouds (according to the evaluations performed by Pinsky et al., 2006).

d) It is common practice to use a frozen turbulent flow field in DNS studies of particle motion and interaction. Time evolving flow was used in a limited number of studies (e.g., Zhou et al., 1998; Franklin et al., 2005, 2007; Wang et al., 2005b). We would like to stress that the utilization of a frozen flow leads not only to the quantitative differences in the results but can also yield a qualitatively wrong representation of turbulent effects on the droplet motion and interaction. It is well known (e.g., Maxey, 1987) that the motion of droplets in a turbulent flow is determined by the *Lagrangian* accel-

eration of the flow. In a frozen flow the Lagrangian acceleration $\frac{dW_i}{dt}$ is actually replaced by the *inertial* acceleration $W_j \frac{\partial W_i}{\partial x_j}$. However, it is known (Monin and Yaglom, 1975; Pinsky et al., 2000b) that the Lagrangian acceleration is the sum of the inertial acceleration and temporal acceleration $\frac{\partial W_i}{\partial t}$ which are mutually compensated. The compensation is especially pronounced in high Re_λ flows. The Lagrangian and inertial accelerations actually represent different turbulent characteristics with different statistical properties. For instance, according to the Kolmogorov 1941 theory the Lagrangian acceleration does not depend on Re_λ , while the inertial acceleration scales as $l^{1/3}$ (where l is the spatial scale). Recent studies indicate an increase of the Lagrangian acceleration with Re_λ (e.g., Hill, 2002), i.e., with the increase of the spatial scale, as $l^{1/6}$. In more detail the differences in the statistics of these accelerations are discussed by Pinsky et al. (2000b). Zhou et al. (1998), Franklin et al. (2005) found that the utilization of a frozen flow overestimates the relative velocities and the clustering effect by several tens of percent. The fact that the overestimation was not very high can be attributed to the small sizes of the DNS computational volumes and to small Re_λ used. Under real cloud conditions these differences should be much larger. Errors caused by the utilization of the frozen turbulence in the estimation of droplet clustering will be discussed below as well.

Application of these results obtained under the conditions quite different from those in real clouds for

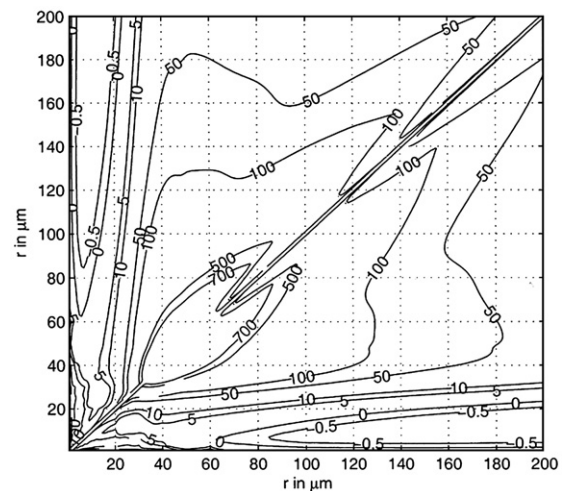


Fig. 1. Dependence of the ratio $(K_{\text{turb}} - K_{\text{grav}})/K_{\text{grav}}$ on droplet size after Riemer and Wexler (2005). K_{turb} is the collision kernel in a turbulent flow, and K_{grav} is the collision kernel in still air. One can see that within a wide range of the droplet sizes the enhancement factor exceeds 50 and reaches 700 in the maximum for $\sim 50 \mu\text{m}$ -radii drops.

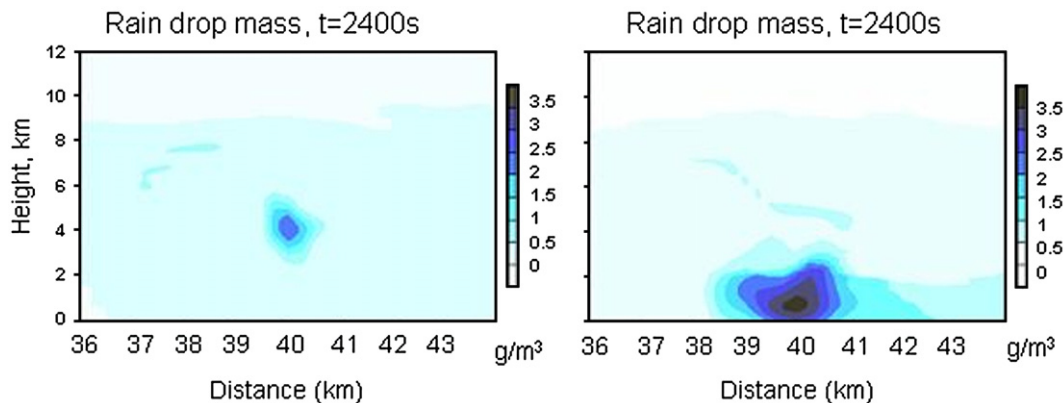


Fig. 2. Fields of the rain water mass content obtained in simulations of a deep continental cloud typical of Texas summertime at $t=40$ min. The field shown at the left panel was obtained using the turbulence-induced collision enhancement factor taken from Pinsky and Khain (1997b, 2004). The field shown at the right panel was obtained using the turbulence-induced collision enhancement factor after Riemer and Wexler (2005). In the first case a small amount raindrops form at 4 km level and do not fall on the surface. In the second case a very intense warm rain at the surface is seen, which is a non-realistic feature for very continental clouds.

the investigation of cloud evolution may lead to a wrong assessment of the role of turbulence in clouds and may hinder the correct understanding of cloud physics. As an example of such situation, we present the results of simulations of the development of a deep continental cloud typical of Texas during summertime (Rosenfeld and Woodley, 2000; Khain et al., 2001, 2004). Vertical velocities in such clouds often exceed 20–25 m/s, the cloud top heights are 12–14 km. The turbulence in these clouds is quite strong, with the turbulent kinetic energy dissipation rate $\sim 10^3 \text{ cm}^2 \text{ s}^{-3}$. Droplet concentration in such clouds is about 10^3 cm^{-3} , the mean droplet radius is about 6–7 μm . The droplets ascend within strong updrafts to high levels, where they give rise to the formation a large amount of ice crystals, small graupel and snow that spread over large areas in the cloud anvils and sublimate. As a result, these clouds produce as a rule very little precipitation at the surface, mainly due to cloud ice melting, and do not produce warm rain (when raindrops fall to the surface without freezing).

Two simulations were performed using the Hebrew University cloud model with spectral (bin) microphysics (Khain et al., 2004). These simulations differ by the collision enhancement factors indicating the increase in the collision kernel. In the first simulation (run 1) the collision enhancement factor was taken within the range 1–5 for cloud droplets, as it was evaluated by Pinsky and Khain (2004). For larger drops the enhancement factor was assumed to be equal 1.2 (Pinsky and Khain 1997b). In the second simulation (run 2) the collision enhancement factor was calculated using the results presented by Riemer and Wexler (2005) for the dissipation rate of $\varepsilon=300 \text{ cm}^2 \text{ s}^{-3}$ (Fig. 1), who derived

their parameterization of turbulent effects based on the DNS results obtained by Zhou et al. (2001) and Wang et al. (1998, 2000) and some other studies.

Fig. 2 shows the rain water mass content obtained in run 1 (left panel) and run 2 (right panel). In case the collision enhancement factor is adopted after Riemer and Wexler (2005), precipitation at the surface starts 7 min after the cloud formation, precipitation rate rapidly attains the value of the order of $\sim 40 \text{ mm/h}$. Such heavy precipitation is caused both by large liquid rain drops (warm rain) and by melted frozen drops. Such rain development is even faster than in Hawaiian clouds, which are the most maritime clouds in the world (which however do not contain much ice) and is unlikely to be observed in continental clouds. At the same time, no surface precipitation was recorded after 40 min in run 1. In agreement with the observations, the evolution of the cloud in run 1 is typical of highly continental clouds, without warm rain at the surface. Consequently, utilization of the collision enhancement factor presented by Riemer and Wexler (2005) leads to a non-realistic prediction of warm and ice cloud microphysics.²

² Wang et al. (2006a) presented a critical analysis of the results obtained by Riemer and Wexler (2005). As was shown by Riemer and Wexler (2006), accounting for some corrections proposed by Wang et al. (2006a) led to a relatively weak deceleration of DSD evolution. The deceleration was too small to provide realistic cloud development. Some other comments remained unanswered owing to the complexity of the problem. However, in this discussion the main cause of the error in the paper by Riemer and Wexler (2005) was not mentioned. We mean the application of the DNS results to collisions of drop pairs containing large cloud droplets or small raindrops.

In this study we critically evaluate the applicability of the results obtained using different methods to actual clouds. We discuss the turbulent effects on a) turbulence-induced relative droplet velocity; b) droplet concentration fluctuations (droplet clustering) and c) the hydrodynamic interaction of droplets. Finally, we will discuss some problems related to parameterization of collision kernel for purposes of cloud modeling.

2. Turbulence-induced relative velocities between droplets (turbulent transport effect)

An important characteristic of a droplet inertial response to a turbulent flow is the Stokes number, $St = \varepsilon^{1/2} \nu^{-1/2} \tau$ (ε is the turbulent kinetic energy dissipation rate, τ is the drop momentum relaxation time (Stokes time), ν is the air viscosity), which is usually below 0.3 for cloud droplets. Small ~ 50 μm -radii rain drops (or drizzle particles) are characterized by $St \sim 0.5$ –1.0.

Since the behavior of cloud droplets and small rain drops in turbulent flows is quite different, as well as the levels of understanding their behavior, we will consider them separately.

In many studies published in fluid mechanics journals the behavior of inertial particles in a turbulent flow is compared to that of non-inertial particles. It is common practice in the state-of-the-art studies in Cloud Physics (both theoretical and numerical) to calculate droplet collisions in clouds under pure gravity conditions (still air assumption). Therefore, in order to evaluate effect of

turbulence on droplet collisions, it is necessary to compare the collision rate in a turbulent flow with that under still air conditions.

2.1. Cloud droplets

The formation of relative velocities between droplets in turbulent flows was investigated in numerous analytical and DNS studies. In most DNS studies the effect of differential sedimentation was neglected. The relative velocities between droplets in a turbulent flow in these studies are normalized by the standard air velocity fluctuations, or compared with those in the zero-inertia case. The results of these studies do not allow one to evaluate the collision enhancement factor caused by the turbulent transport effect as compared to that in still air.

There are a few studies where the gravitation sedimentation is taken into account and the results allow such comparison. Saffman and Turner (1956), Pinsky and Khain (1997a,b); Wang et al. (1998) and Dodin and Elperin (2002) presented analytical formulas accounting for the sedimentation effect. This effect was taken into account by Pinsky et al. (2006) using a statistical model of a turbulent flow and by Franklin et al. (2005, 2007) in DNS simulations.

Fig. 3 presents the comparison of the results obtained in some of these studies. The difference in the results does not exceed 10–15%, and it can be attributed to different definitions of collision kernels (the cylindrical vs. spherical formulation) and to the Gaussian/non-

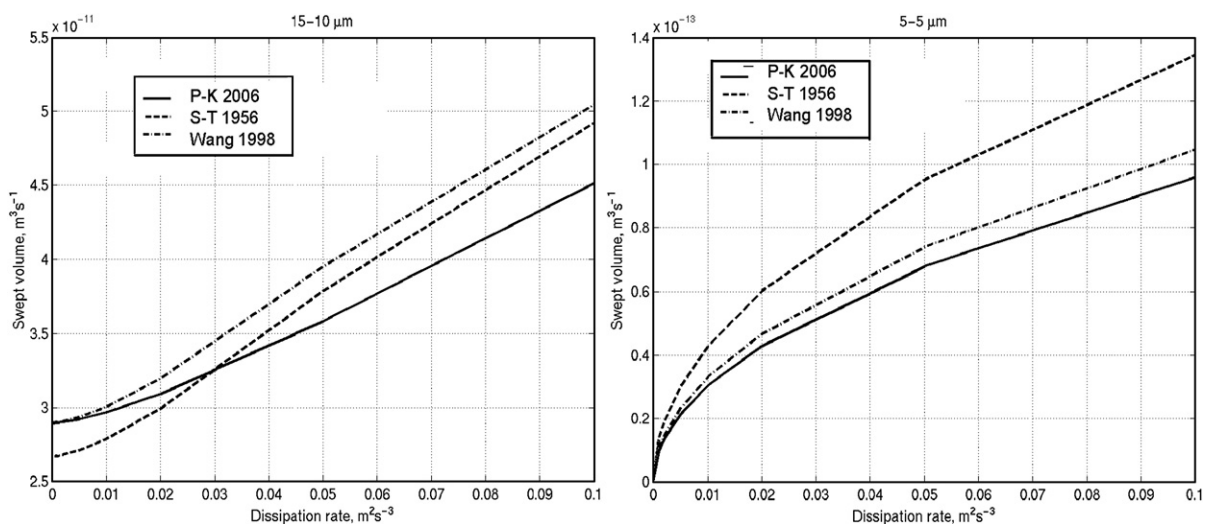


Fig. 3. Dependence of the turbulence-induced swept volumes (which are proportional to the absolute relative velocities between droplets) on the dissipation rate according to the results of Saffman and Turner (1956), Wang et al. (1998) and Pinsky et al. (2006) for 10 μm and 15 μm –radii droplet pair (left); and 5 μm –5 μm radii droplet pair (right). $Re_\lambda = 2.10^4$. Results of Dodin and Elperin (2002) coincide with those of Wang et al. (1998).

Gaussian assumptions for velocity distributions. The results obtained by [Dodin and Elperin \(2002\)](#) agree well with those obtained by [Wang et al. \(1998\)](#). Fig. 4 shows the dependence of the mean normalized swept volume on the turbulent kinetic energy dissipation rate ε for the 15–10 μm -radii droplet pair at different Re_λ obtained by [Pinsky et al. \(2006\)](#). As can be seen from Fig. 4, an increase in the relative velocities between cloud droplets does not exceed 60% even under very intense turbulence. These results agree fairly well with those obtained in DNS by [Franklin et al. \(2005\)](#), who evaluated the increase in the relative velocity as compared to the difference in gravity-induced terminal velocities by the factor of 1.2–1.4 for the 10–15 μm -radii droplet pair. Smaller enhancement factors obtained by [Franklin et al. \(2005\)](#) as compared to those obtained by [Pinsky et al. \(2006\)](#) can be attributed to small Re_λ used in the DNS.

Therefore, the magnitude of the collision rate enhancement factor for cloud droplets caused by turbulent transport effect can be evaluated in the range from a few to several tens of percent depending on the turbulence intensity and droplet size.

2.2. Small rain drops (40–70 μm radii)

Motion of small rain drops in a turbulent flow differs significantly from that of air parcels because of the large inertia and large gravity-induced sedimentation velocities. Small rain drops respond to turbulent vortices of scales larger than those for cloud droplets. Fig. 5 shows

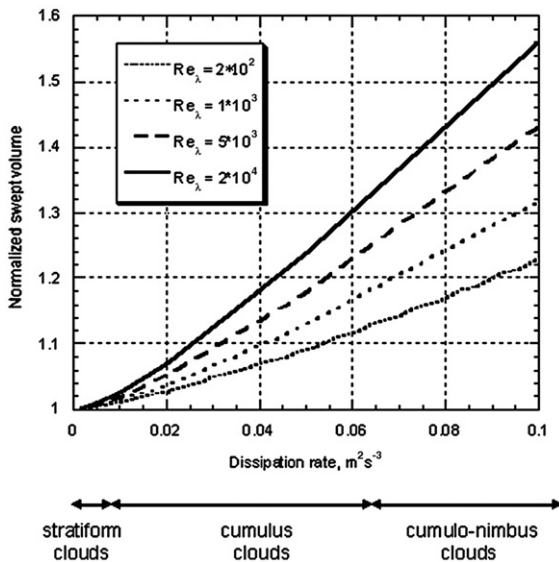


Fig. 4. Dependence of the mean normalized swept volume on turbulent dissipation rate ε for the 15–10 μm -radii droplet pair at different Re_λ (after [Pinsky et al., \(2006\)](#)).

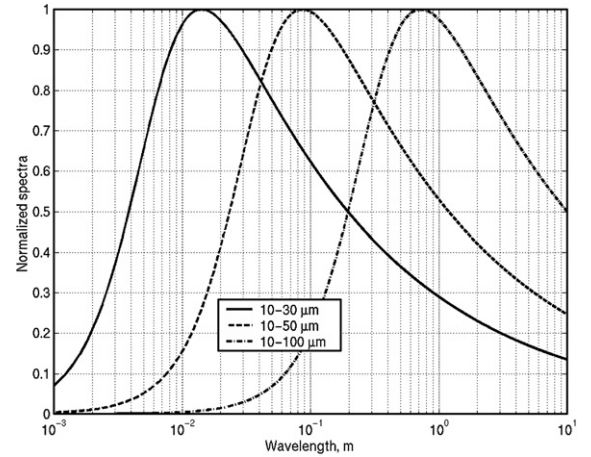


Fig. 5. Relative velocity spectra (normalized by their maxima) for 10 μm –30 μm , 10 μm –50 μm and 10 μm –100 μm -radii drop pairs (after [Khain and Pinsky, 1995](#)).

the spectra of relative velocities for three drop pairs of 10 μm –30 μm , 10 μm –50 μm and 10 μm –100 μm drop radii. The calculations are performed following [Khain and Pinsky \(1995\)](#) assuming Kolmogorov's spectral density in the inertial subrange and taking gravity effects into account. Clearly the shape of the spectrum is determined by the larger drop in a drop pair. The maxima of these spectra are located at the wavelengths of 1.5 cm, 8 cm and 70 cm, respectively. These maxima represent the characteristic sizes of the turbulent vortices that affect the relative velocity between the drops in these pairs. The relative motion between drops in the droplet pairs containing small rain drops is affected by turbulent vortices with the scales from several to a few tens of centimeters. Since the linear scales of the computation areas in most DNS models are of the order of several cm, these models are incapable to describe these vortices adequately, and, consequently, they are unable to describe the motion of small rain drops reliably. These circumstances indicate significant difficulties encountered with the theoretical analysis and DNS in investigating transport effect related to large cloud droplets and small rain drops.

Some theoretical evidence of the “unusual” behavior of small rain drops can be elucidated by the following considerations. [Pinsky et al. \(2006\)](#) showed that the equation for turbulence-induced particle velocity deviation along the particle track V'_i can be written as follows:

$$\frac{dV'_i}{dt} = -V'_j \left(\frac{1}{\tau} \delta_{ij} + S_{i,j}(x_i, t) \right) - (A_i(x_i, t) + V_i S_{i,3}(x_i, t)), \quad (1)$$

where $i, j = 1, 2, 3$; $V_i' = V_i - W_i(x_i, t) - V_i \delta_{3i}$ is the relative fluctuating particle velocity, V_i and W_i are particle and air flow velocities, respectively; V_i is the gravity-induced sedimentation velocity, $A_i(x_i, t)$ is the Lagrangian acceleration of the turbulent flow in the particle location, $S_{ij}(x_i, t)$ is the turbulent shear tensor.

Eq. (1) is a simplified motion equation obtained when neglecting the “added mass” force due to involving the surrounding fluid into motion and the Basset “history” force due to diffusion of vorticity from an accelerating particle, as well as the terms resulting from the stress field of the fluid flow acting on the particle. The applicability of these simplifications is discussed by Shaw (2003).

The character of the solution of Eq. (1) depends on the eigenvalues of the tensor $(\frac{1}{\tau} \delta_{ij} + S_{ij})$. In case all real parts of the eigenvalues are positive, the velocity V_i' tends to the quasi-stationary value. In this case droplets adjust to the surrounding flow during the time intervals which are much shorter than the characteristic times of the turbulent shear and Lagrangian acceleration. If the stationary case is realized, the increase in the swept volume is comparatively small (Pinsky et al., 2006). In case when the real part of any eigenvalue is negative, the local relative velocity grows exponentially, which implies that the velocity of droplets will continuously deviate from the surrounding air velocity. The latter case will be referred to as a non-stationary (or unstable) one. The condition for the exponential growth is

$$\lambda_{\min} < -1/\tau, \quad (2)$$

where $\lambda_{\min}$ is the minimum real part of the eigenvalue of $(\frac{1}{\tau} \delta_{ij} + S_{ij})$. To illustrate the comparable fractions of stable and unstable solutions, the condition (2) has been validated for a large number of realizations of the turbulent flow characterized by certain pairs of velocity shear and Lagrangian acceleration. In order to generate the components of turbulent shears we used a statistical model of the turbulent shears developed by Pinsky et al. (2004). Fig. 6 shows a fraction of the non-stationary (unstable) cases for drops with different sizes under turbulent conditions typical of three types of atmospheric clouds: stratiform ($Re_\lambda = 5 \cdot 10^3$, $\varepsilon = 0.001 \text{ m}^2 \text{ s}^{-3}$), cumulus ($Re_\lambda = 2 \cdot 10^4$, $\varepsilon = 0.02 \text{ m}^2 \text{ s}^{-3}$) and cumulo-nimbus ($Re_\lambda = 2 \cdot 10^4$, $\varepsilon = 0.1 \text{ m}^2 \text{ s}^{-3}$). Inspection of Fig. 6 shows that the motion of cloud droplets with the radii below $\sim 20 \mu\text{m}$ satisfies the quasi-stationary condition for clouds of all types. At the same time, the velocities of droplets with the radii exceeding $40 \mu\text{m}$ tend to deviate greatly from the air flow velocity in all cloud types.

Note that Fig. 6 shows the local growth of the relative velocities of small rain droplets during a small periods of

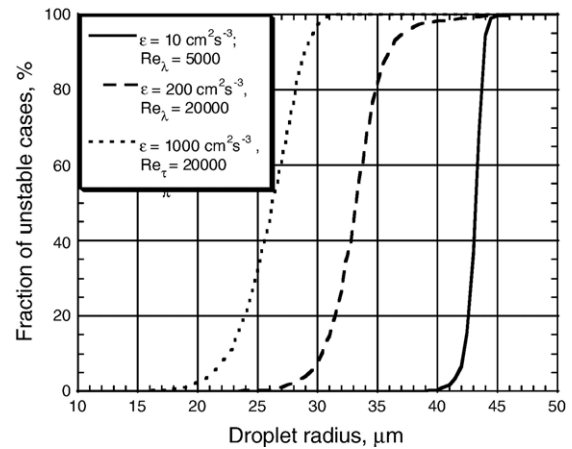


Fig. 6. Dependence of the fraction of non-stationary (unstable) cases vs. droplets size under turbulent conditions typical of stratiform, cumulus and cumulo-nimbus clouds.

time (or in small spatial scales), where shear and acceleration can be assumed nearly constant. Rapid sedimentation of the drops must destroy such “resonance” response of small rain drops to local turbulent vortices.

Nevertheless, the analysis presented above shows that the motion of large cloud droplets and small raindrops represent a difficult theoretical problem, which is still unsolved.

An attempt to overcome the difficulties has been made by Pinsky and Khain (1997a,b) who performed the matching of two asymptotic regimes of droplet motion when $V_i \ll u'$ and $V_i \gg u'$, where u' is the characteristic turbulent velocity. The results reported in these studies indicate a rapid decrease in the turbulence-induced relative drop velocity for drop radii $a > 30 \mu\text{m}$. The conclusions reached in these theoretical studies are supported by the results obtained using the simplified turbulent velocity field model (Pinsky and Khain, 1996).

We believe that the magnitude of the enhancement factor obtained in these studies for cloud droplets is overestimated because simplifying assumptions (e.g. flow stationarity) were imposed on the turbulent velocity field. Nevertheless, these results (e.g., destruction of the droplet “resonance” response) indicate a dramatic influence of the sedimentation velocity on the behavior of small rain drops.

It should be mentioned that the relative velocities between the drops with large St were calculated in numerous DNS studies (e.g., Wang et al., 2000; Zhou et al., 2001).³

³ Note that the works of Wang et al. and Zhou et al. did not address specifically the rain formation problem. Their neglect of sedimentation is possibly appropriate for many problems of engineering interest. Here we discuss, however, the validity of the application of the results to droplet collisions in clouds.

Numerous parameterization formulas for the calculation of swept volumes were proposed. Unfortunately, in the DNS studies gravitational sedimentation was neglected. Besides, the small computational volumes used in DNS do not allow describing the vortexes which affect the motion of small rain droplets. *Therefore, the results obtained in DNS can hardly be applied to cloud conditions.*

Taking into account some limitations of the analytical approach and the numerical simulations, further investigations are required to obtain a more accurate quantitative (and, qualitative) evaluation of the transport effect for small rain drops.

3. Droplet clustering

While droplet clustering is being considered, two main questions arise: a) does small scale clustering occur in real clouds? and b) if small-scale clustering does occur in clouds what can be its effect on droplet collision rate in clouds?

The existence of small-scale droplet clustering in real clouds has been questioned for some period of time. Pinsky and Khain (2001, 2003) and Kostinski and Shaw (2001) found centimeter scale droplet concentration fluctuations caused by the turbulent-inertia effect analysing in situ measurements in clouds. Small-scale

fluctuations of liquid water content were found by Gerber et al. (2001). Fig. 7 shows the results obtained by Pinsky and Khain (2003) in the statistical analysis of long series of drop arrival times in ~ 60 cumulus clouds. This study indicates that droplet clustering does occur in clouds and the rate of the clustering was found to increase with St , in compliance with the theoretical predictions. One can also see the increase in the clustering rate with the decrease in the spatial scale.

The mechanism of inertial particle accumulation in a randomly oriented cellular flow was formulated by Maxey and Corrsin (1986) and Maxey (1987). Maxey (1987) showed that the particle velocity field turns out to be compressible even in non-compressible fluid flow.

The field of inertial particles can be characterized by the divergence of the particle velocity flux that for small particle size can be written as $-\tau \frac{\partial W_i}{\partial x_j} \frac{\partial W_j}{\partial x_i}$. Maxey (1987) asserted that “The departure of the average particle-settling velocity from the still-fluid value is determined by the accumulated bias of the particle trajectory towards the regions of high strain rate or low vorticity.... In certain situations, as in the cellular-flow-field computations of Maxey and Corrsin (1986), this bias may lead to singular accumulations of particles in narrowly confined regions. This is unlikely to occur in a turbulent flow though, because of the limited spatial and temporal correlations of turbulence.” Nevertheless,

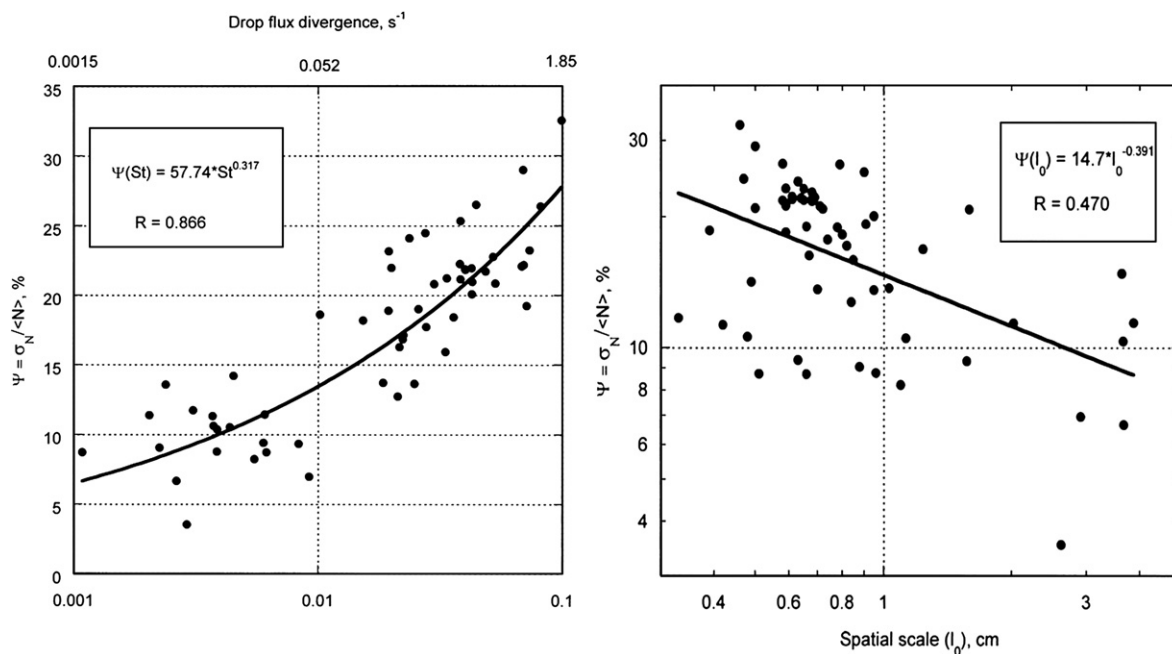


Fig. 7. Normalized fluctuations of droplet concentration in clouds as the function of Stokes number (left) and on spatial scales (right) (after Pinsky and Khain 2003).

Elperin et al. (1996) demonstrated that the inertial mechanism suggested by Maxey can be used to describe the formation of small-scale particle concentrations in turbulent flows. The particles inside the turbulent eddies are carried out to the boundary regions between the eddies by the inertial forces. This mechanism of the preferential concentration acts at all scales of turbulence, increasing toward small scales.

Analytical investigations of particle clustering were first conducted by Elperin et al. (1995, 1996) who showed that the clustering can be described by the dynamics of the two-point second-order correlation function (particle number density fluctuations) and higher order correlation functions of the random particle number density field. Later on the spherically symmetric second-order correlation function (radial correlation function) was used in many DNS and statistical studies for the analysis of particle distribution in turbulent flows (e.g., Sundaram and Collins, 1997; Reade and Collins, 2000; Wang et al., 2000; Zhou et al., 2001; Chun and Koch, 2005; Chun et al., 2005). Elperin et al. (1996) suggested the mechanism for the particle clustering which is associated with the exponential growth of the two-point second-order correlation function of the particle number density. The divergence of the particle velocity field also results in the appearance of the source term in the equation for the two-point second-order correlation function of the particle number density, $I_1 = 2\tau_k \langle \text{div } V'(\mathbf{x}) \text{div } V'(\mathbf{y}) \rangle N(\mathbf{x})N(\mathbf{y})$, which can also cause formation of particle inhomogeneities (e.g., Elperin et al., 1995, 1996; Pinsky and Khain, 1997a,b; Balkovsky et al., 2001). Here $N(\mathbf{x})$ is the mean number density of particles at the point $\mathbf{x}$, V' are the particle velocity fluctuations, $\text{div } V'(\mathbf{x})$ is the divergence of the particle velocity determined at the point $\mathbf{x}$ and τ_k is the Kolmogorov time scale of the turbulent velocity field. Elperin et al. (1996, 2002) identified two different types of the particle clustering: (i) the “source” clustering (caused by the source term I_1) and (ii) the clustering instability (the exponential growth of the second-order correlation function of particle number density). These two types of the particle clustering depend on the particle size. The critical size of a particle (which separates these two different types of the particle clustering) is of the order of $a_{\text{cr}} \approx 10 \mu\text{m}$ for typical atmospheric turbulent flows. This corresponds to the critical Stokes number $St_{\text{cr}} = 2/9$. For $a > a_{\text{cr}}$, the clustering instability, i.e., exponential growth of the second-order correlation function of the particle number density is the dominant mechanism for particle clustering (Elperin et al., 1996, 2000, 2002, 2007). The clustering instability results in a spontaneous breakdown of the homogeneous spatial

distribution of particles suspended in turbulent flow. It must be noted that for the simplest model of the turbulent velocity field, i.e., for the delta-correlated in time random velocity field, the clustering instability does not occur (Elperin et al., 2000). However, as it was shown by Elperin et al. (2002, 2007) accounting for a finite correlation time of the fluid velocity field results in the clustering instability of inertial particles.

In several studies (e.g., Bec, 2003; Falkovich and Pumir, 2004; Marshak et al., 2005) a fractal interpretation of the particle structures formed as a result of particle clustering is given.

Effects of droplet clustering on the collision rate between droplets has been investigated in numerous DNS where the collision enhancement factor is usually characterized by functions G_{11} (for monodisperse suspensions) and by function G_{12} (bi-disperse suspensions) introduced by Reade and Collins (2000) and since then widely used in many studies (e.g., Wang et al., 2000; Zhou et al., 2001; Chun and Koch, 2005; Chun et al., 2005). In DNS $G_{11}(R)$ and $G_{12}(R)$ are computed as (Wang et al., 2005b):

$$G_{11}(R) = \frac{N_{\text{pair}}/V_s}{N^2/V_b}; G_{12}(R) = \frac{N_{\text{pair}}/V_s}{N_1 N_2/V_b}, \quad (3)$$

where N_{pair} is the total number of pairs detected with separation distance falling in a spherical shell of the inner radius equal to $R - \delta_1$ and the outer radius equal to $R + \delta_2$. Here δ_1 and δ_2 are small fractions of R ; V_s is the volume of the spherical shell, N_1 and N_2 are the total numbers of droplets of where N_1 and N_2 are the number densities of droplets of radii a_1 and a_2 , respectively; V_b

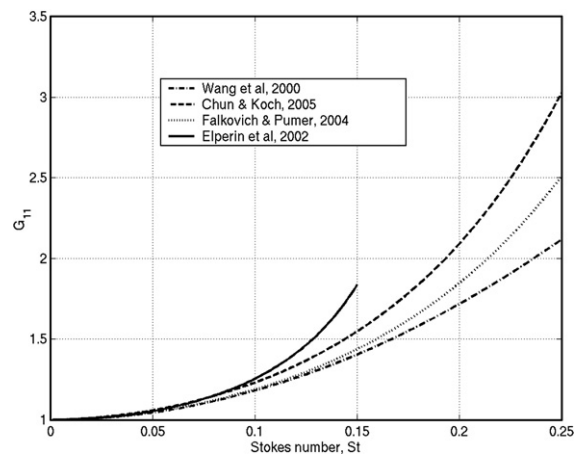


Fig. 8. Dependencies of G_{11} on St obtained by different authors (Wang et al., 2000; Chun and Koch, 2005; Falkovich and Pumir, 2004; Elperin et al., 2002).

is the volume of the computational domain. In order to characterize droplet collisions, functions G_{11} and G_{12} are determined at the separation distances $R=2a$ for $G_{11}(R)$ and $R=a_1+a_2$ for $G_{12}(R)$ (i.e., upon contact between drops) in monodisperse and bi-disperse suspensions, respectively. These functions are widely interpreted from the statistical point of view as follows (e.g. Zhou et al., 2001):

$$G_{11}(R) \approx \frac{\langle N^2 \rangle}{\langle N \rangle^2}; G_{12}(R) \approx \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}, \quad (4)$$

Note that no enhancement ($G_{11}=1$) takes place only when the random concentration field satisfies $\langle N^2 \rangle = \langle N \rangle^2$ that is the concentration is uniform throughout space. This, of course, implies Poisson number fluctuations in

any given volume so that the enhancement ($G_{11}-1$) is over the Poisson fluctuations.

3.1. Cloud droplets

Fig. 8 shows the dependence of G_{11} on St obtained by different authors using theoretical analysis and DNS. Clearly the difference between the curves is small for $St < 0.1$. The differences between the results can be attributed to the differences in the approaches and employment of small Re_λ in DNS. In general, G_{11} is smaller than 1.3 for the droplets with $St < 0.25$ but remains larger than the values obtained from analysis of measurements (Fig. 7).

Fig. 9 compares the function G_{12} for bi-disperse suspensions of small droplets obtained by Zhou et al.

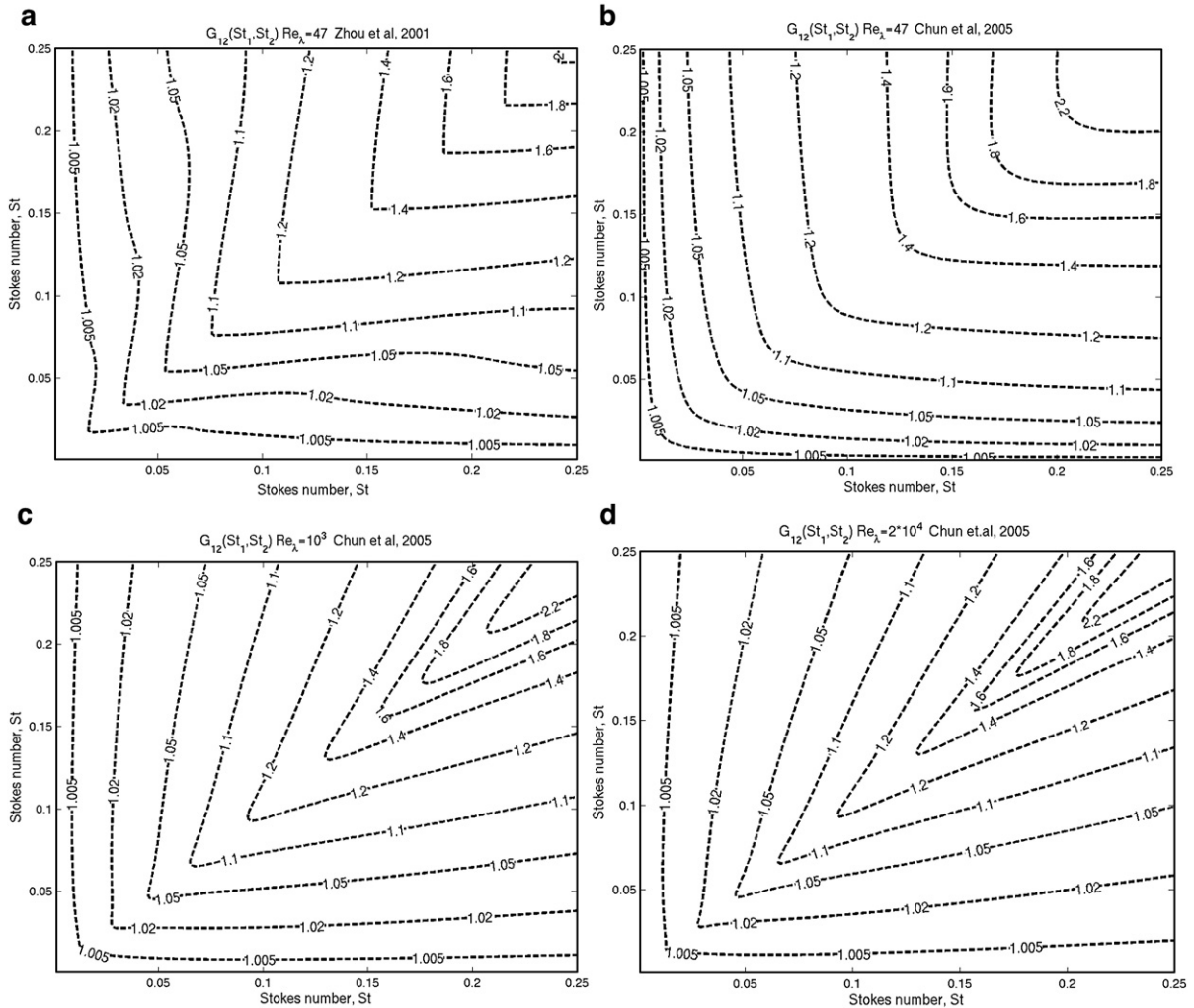


Fig. 9. a–d. The function G_{12} for bi-disperse suspensions of small droplets. (a) Zhou et al. (2001), $Re_\lambda=47$; (b) Chun et al. (2005b), $Re_\lambda=47$; (c) Chun et al. (2005b), $Re_\lambda=10^3$; (d) Chun et al. (2005b), $Re_\lambda=2 \times 10^4$.

(2001) in DNS (Fig. 9a), and the theoretical results of Chun et al. (2005) (Fig. 9b) calculated for the same values of $Re_\lambda=47$. Clearly, there is a very good agreement between these results. In the model by Chun et al. (2005) function G_{12} depends on the Lagrangian acceleration that, in its turn, depends on Re_λ . Fig. 9c shows function G_{12} in case when the Lagrangian accelerations measured by La Porta et al. (2001), Voth et al. (2002) for $Re_\lambda=10^3$ were used. According to La Porta et al. (2001) further increase in Re_λ does not increase the fluctuations of Lagrangian

acceleration, and, consequently, does not change G_{12} . According to the parameterization proposed by Hill (2002) fluctuations of the Lagrangian acceleration continue increasing with Re_λ also for $Re_\lambda>10^3$. Function G_{12} calculated using the parameterization proposed by Hill for $Re_\lambda=2\cdot 10^4$ is shown in Fig. 9d. It should be noted that the increase of Re_λ leads to a strong decrease in G_{12} for droplets with different sizes. This effect is attributed to the fact that the Lagrangian acceleration tends to destroy the spatial correlation of concentration fluctuations of droplets with different

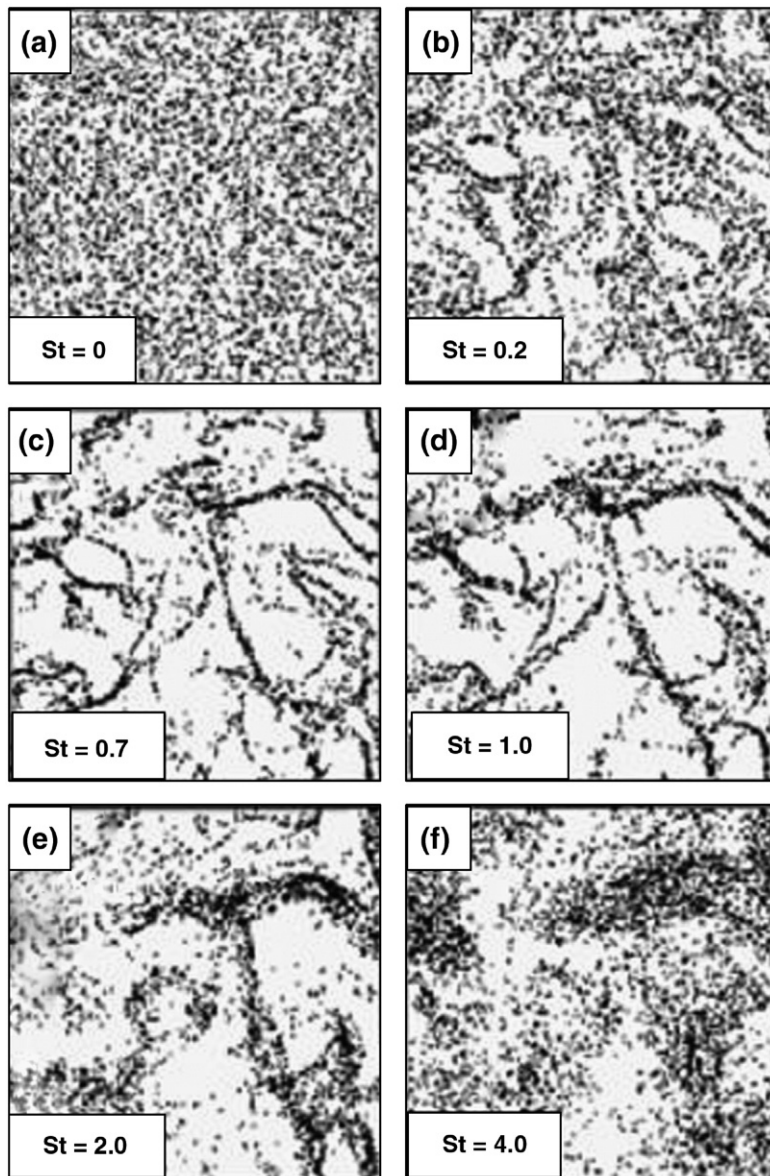


Fig. 10. Cross-sections of ghost-particle spatial distributions for (a) $St=0.0$; (b) $St=0.2$; (c) $St=0.7$; (d) $St=1.0$; (e) $St=2.0$; and (f) $St=4.0$; dots indicate particle center locations (after Reade and Collins, 2000).

sizes. Chun et al. (2005) showed that accelerations tended to destroy clustering at very small scales even in case of droplets of similar size.

The collision enhancement factor is maximum for droplets with similar size $St \sim 0.25$ ($G_{12} \sim 2$). When $St > 0.1$ the difference between droplets sizes causes a steep decrease of the function G_{12} because of the growth of the Lagrangian acceleration. Note that these results were obtained neglecting droplet sedimentation. Accounting for the differential drop sedimentation may further decrease the clustering rate. The necessity considering the droplet sedimentation was stressed earlier by Grabowski and Vaillancourt (1999). Therefore, the values of G_{12} function must be recalculated to take into account the effect of sedimentation.⁴ Accounting for the differential sedimentation is especially important for droplets with $St > 0.1$.

3.2. Small raindrops

Wang et al. (2000), Reade and Collins (2000), Zhou et al. (2001), Elperin et al. (2002), Falkovich et al. (2002) reported a dramatic increase in G_{11} and G_{12} for $St > 0.3$ (see Figs. 1 and 10). Elperin et al. (2002, 2007) distinguished between two modes of particle clustering, weak clustering and strong clustering. The latter mode is characterized by sharp increase of the clustering rate and begins at the critical value of the Stokes number, $St_{cr} = 2/9$.

DNS simulations (e.g. Reade and Collins, 2000) indicate that the zones with the enhanced concentration represent narrow elongated structures whose width depends on St (Fig. 10).

For $St \sim 0.7$ –1 the width of the regions with enhanced concentration of particles is less than the Kolmogorov microscale, so that function G_{12} increases by one-two orders of magnitude with the decrease of the scale from the Kolmogorov microscale down to the droplet size scale. Spatial redistribution of droplets and formation of these regions with enhanced concentration is often interpreted as the formation of fractal structures (Falkovich and Pumir, 2004; Marshak et al., 2005).

We believe, however, that two main factors that have not been taken into account in these studies should dramatically decrease clustering of small rain drops, as well as its effect on the rain drops growth in real clouds. These factors are: the significant difference in the gravity-induced sedimentation velocities and the smallness of these drops concentration.

3.2.1. Effect of differential drop sedimentation

Fig. 11 illustrates the sedimentation-induced spatial separation of two zones with preferential concentration in initially well mixed bi-disperse suspension. Spatial decorrelation is especially significant if one of the species is represented by small rain drops with significant (several ten cm/s) sedimentation velocity.

Another illustration of the effect of differential sedimentation on droplet clustering is shown in Fig. 12, where the trajectories of droplets with the radii of 10 μm , 30 μm and 50 μm are plotted. These trajectories were calculated using the approximate turbulence model (Pinsky and Khain, 1996) in which turbulence was frozen. This figure is shown only for pictorial illustration of effect of spatial decorrelation of droplet locations due to differential sedimentation velocities. One can see that in the presence of gravity, the angle between the trajectories of droplets increases with difference in droplet sizes. The increase of the angles reduces the volume of the regions where droplets of different sizes can be located simultaneously.

Since different drops respond to the vortices of different sizes (Fig. 5), the structure and locations of the enhanced concentration regions formed by droplets of different sizes should be quite different and well separated (simple evaluations indicate that the distance between the zones with preferential concentration of cloud and small rain drops can easily exceed several centimeters or even tens of centimeters. Note that rain drops grow largely by collisions with cloud droplets. Taking into account the significant difference in sedimentation velocities, we conclude that the locations of the regions with the enhanced concentration of cloud droplets and rain drops should be completely uncorrelated.

Here we return to the comments concerning the evaluations of the clustering effect by the DNS performed

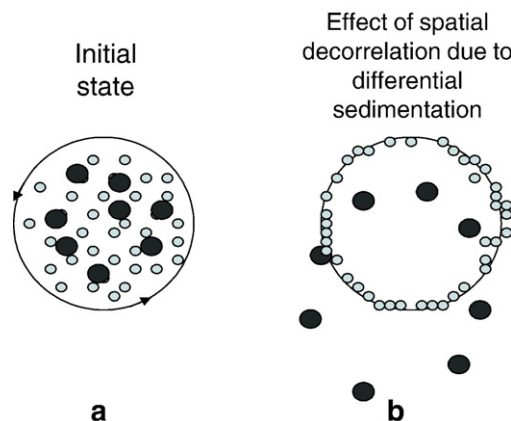


Fig. 11. Schematic illustration of the effect of differential drop sedimentation.

⁴ During the revision of the paper a study by Franklin et al. (2007) has been published where a decrease in G_{12} was found due to differential sedimentation for droplet-collectors with the radii exceeding 20 μm –30 μm .

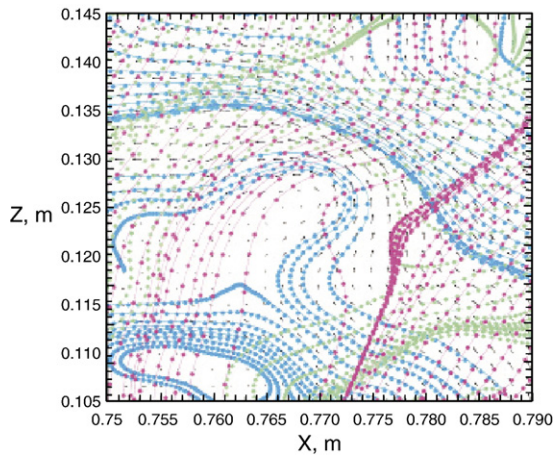


Fig. 12. Trajectories of droplets with radii of 10 (ascending trajectories from right to the left), 30 (nearly horizontal trajectories) and 50 (sharply descending trajectories from right to left) for the area $4\text{ cm} \times 4\text{ cm}$. These trajectories were calculated using the approximate turbulence model (Pinsky and Khain, 1996). (For better identification of the trajectories, the reader is referred to the web version of this article, where tracks of these droplets are marked blue, green and red, respectively.)

when neglecting the droplet sedimentation and using frozen turbulent flows. If turbulence is frozen, all vortices have an infinitely long life time, including those of the Kolmogorov microscale. In these DNS non-settling droplets of all sizes have an infinite long time to attain the zones of potential “preferential concentration”. In time dependent flows falling droplets (drops) spend only a limited time period within the zones of negative drop velocity flux divergence. Respectively, the droplets will often not be able to reach the location of “potential” clustering. Therefore, two simplifications taken together lead to a dramatic overestimation of the clustering rate.

Therefore, the gravitation sedimentation must dramatically decrease the effect of turbulence-induced droplet clustering on the growth of small (as well as large) raindrops.

3.2.2. Possible effects of the drop number density smallness

As it was mentioned above, the concentration of the $40\text{--}70\text{ }\mu\text{m}$ radii drops in clouds is small, so that the mean distance between such drops usually exceeds $\sim 1\text{ cm}$. Small rain drops of similar sizes may be separated by the distance easily exceeding 10 cm . In this connection the following question arises: does the collision rate enhancement caused by clustering of these drops depend on their concentration or is it equal to that found in DNS under high drop concentration? Note that according to Eq. (3) the clustering rate must not depend on drop concentration.

We speculate, however, that at very low concentration the situation may change. Fig. 13 shows the regions with enhanced concentration of droplets characterized by $St=0.7$ obtained in DNS performed by Reade and Collins (2000). One can see that the characteristic scale of the zones with the enhanced droplet concentration is about one cm. This value agrees well with the characteristic scale of the droplet velocity flux divergence determining the regions of droplet convergence or divergence within a turbulent flow (e.g. Pinsky and Khain, 1997c; Pinsky et al., 1999a). This implies that droplets located at separation distances exceeding $\sim 1\text{ cm}$ are located within different, non-correlated (in the sense of the magnitudes of turbulent shears and acceleration) zones in the turbulent flow. Fig. 13 shows the probable location of drops within a turbulent flow, when their concentration is very low, so that the mean separation distance is of the order of a few centimeters. These droplets belong to different clusters (different attractors) and move independently within statistically uncorrelated regions in a turbulent flow. If the droplet concentration is high, there are many droplet pairs available within any vortex (any attractor). We suspect that in case of very low concentration a mechanism responsible for the appearance of at least one droplet pair within one vortex must be taken into account. In other words, when the drop concentration is very small the discrete nature of the droplet flux may play an important role. One can speculate that effects of clustering must be decreased significantly for droplet concentrations below $0.1\text{--}0.3\text{ cm}^{-3}$.

We mention here another set of studies (e.g., Bec, 2003; Falkovich and Pumir, 2004; Marshak et al., 2005; Goto and Vassilicos, 2006), according to which droplets

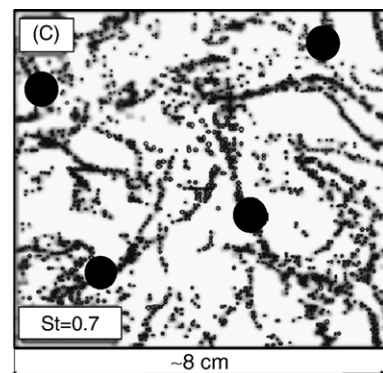


Fig. 13. Regions with enhanced concentration of droplets characterized by $St=0.7$ in DNS performed by Reade and Collins (2000). Large black circles indicate the probable location of large cloud droplets with concentration typical of real clouds. Droplets are located in regions of different attractors (different clusters).

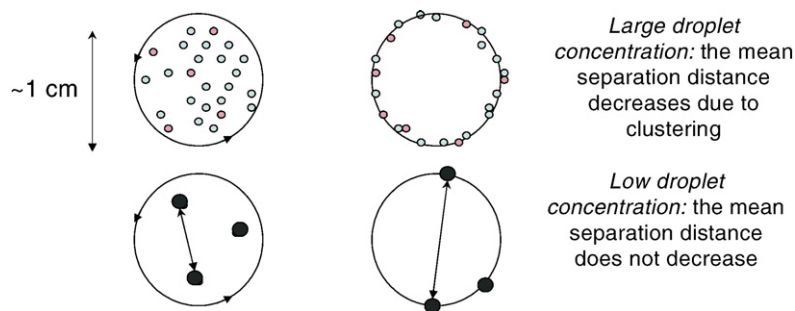


Fig. 14. Illustration of the effect of smallness of drop concentration on the mean distance between droplets in case when the droplets of similar size belong to the same region of preferential droplet concentration.

tend to form fractal structures in a turbulent flow. We believe that the structure formed by particles at $St=0.7$ shown in Fig. 10 (after Reade and Collins, 2000) can be considered as an example of such fractal structures. In this case droplets do not tend to approach each other, but rather to be redistributed within the domains characterized by fractal dimension α (as schematically shown in Fig. 14). Since the probability of the droplet collisions is related to the mean separation distance between the colliding droplets within a corresponding volume, the following question should be asked: does the mean separation distance between the droplets always decrease as a result of droplet accumulation in the regions with the preferential concentration? The decrease of the separation distance between the droplets can be characterized by the factor $G=(Nl^3)^{(1/\alpha-1/3)}$, where N is the droplet concentration in the volume V that belongs to one turbulent element (cluster), l is the linear scale of the vortex volume (chosen as ~ 1 cm). In case the fractal dimension $\alpha=3$ (i.e., usual 3-D case), factor G does not depend on droplet concentration which agrees with definitions (Eq. (3)). According to Falkovich and Pumir (2004) the fractal dimension α is the function of St . Table 1 presents the results of the evaluations when the parameters are chosen close to those typical of actual clouds.

Table 1 shows that the decrease in the characteristic separation distance occurs for 20 μm -radius droplets (16%). The separation distance between 50 μm -radius droplets practically does not change. Hence, according to these evaluations the low concentration of drops with $St \sim 1$ renders the clustering effect “apparent”. Turbu-

lence-induced droplet spatial redistribution does not lead to the actual decrease of the separation distances between droplets in the clusters. According to this argument, the clustering effect becomes negligible already at drop concentrations of the order of a few drops per cm^3 . Note that the fractal droplet structure was obtained in frozen turbulent flows neglecting drop sedimentation. Supposedly, these factors being taken into account would destroy the fractal structures.

To our knowledge, there are no DNS in which particle concentration was as small as concentration of large cloud droplets or small raindrops. Besides, as it was discussed above, large drops respond to turbulent vortices of larger spatial (up to meters) and time scales. Their behavior requires special analysis. Apparently, this problem cannot be solved using state-of-the art DNS because of too small computational volumes.

In summary, the results obtained theoretically or numerically neglecting the differential sedimentation and the smallness of raindrop concentration are not applicable to real clouds. It is conceivable that the effect of clustering of such drops on collision rates in real clouds is much weaker (if any) than that reported in many studies.

4. Effects of turbulence on hydrodynamic droplet interaction

4.1. Cloud droplets

There are a few studies that take into account the turbulence effect on the hydrodynamic droplet interaction (HDI) (Koziol and Leighton, 1996; Pinsky et al., 1999b, in press-a; Pinsky and Khain, 2004; Franklin et al., 2004; Wang et al., 2005b).

The small number of such studies is surprising because the gravity values of the collision efficiencies are small (0.001–0.1) leaving it to turbulence to increase the collision efficiency and the collision rate. All studies

Table 1

Droplet radius r , μm	10	20	50
St	0.1	0.3	0.7
N , cm^{-3}	300	10	1
α	2.9	2.5	1.4
G	1.07	1.16	1

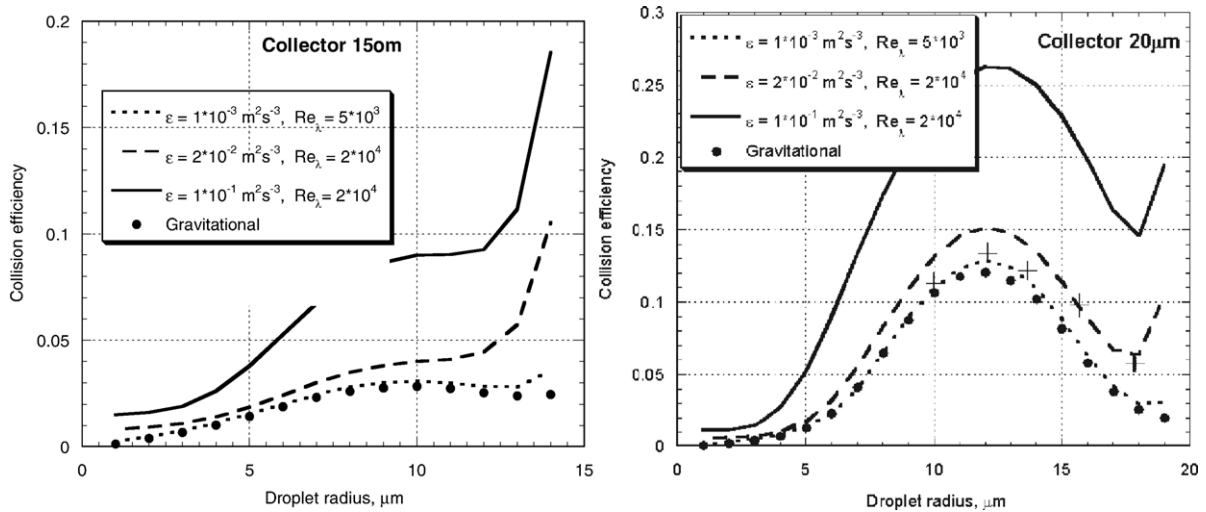


Fig. 15. Collision efficiencies between 15- μm (left) and 20- μm radii (right) collectors with smaller droplets under different dissipation rates and Re_λ (after Pinsky et al., in press-a). The results obtained by Wang et al. (2005a,b) for $\epsilon = 100 \text{ cm}^2 \text{ s}^{-3}$ are marked by crosses on the right panel.

employed the version of a superposition method (Wang et al., 2005a; Pinsky et al., 1999b, in press-a) to calculate hydrodynamic interaction between approaching droplets. Pinsky et al. (1999b) calculated collision efficiencies between droplets moving within a turbulent flow field using an approximate turbulence model where distribution of velocities was assumed Gaussian. Pinsky and Khain (2004) calculated the collision efficiencies and collision kernels in a turbulent flow with high Re_λ typical of atmospheric clouds. However, in their study only the Lagrangian accelerations were taken into account, while the effects of turbulent shear were disregarded. Franklin et al. (2004, 2007); and Wang et al. (2005b) calculated collision efficiencies between droplets of several sizes using the velocity field generated by DNS models.

Recently, Pinsky et al. (in press-a) used a statistical turbulence model (Pinsky et al. 2006) that yields acceleration and shear fields similar to those measured under high Re_λ . They calculated collision efficiencies within the whole range of the cloud droplet sizes under different turbulent intensities typical of clouds of different type. Comparison of the values of collision efficiencies obtained by Pinsky et al., in press-a,b calculated under the conditions comparatively close to those used by Wang et al. (2005b) ($\epsilon = 100 \text{ cm}^2 \text{ s}^{-3}$, the 20 μm radius drop collector) shows a reasonably good agreement between the results.

Fig. 15 shows the collision efficiencies for 15 μm - (left) and 20 μm -radii (right) collectors and smaller droplets for different dissipation rates and Re_λ according to the results obtained by Pinsky et al. (in press-a) and

the pure gravity values of the collision efficiencies. An inspection of Fig. 15 shows that strong turbulence significantly increases the collision efficiencies between the cloud droplets, and an especially significant increase in the collision efficiency occurs for droplets of similar sizes.

Fig. 16 shows the dependence of the averaged normalized collision kernel for the 10 μm - and 20 μm -radii droplet pair on the turbulent kinetic energy dissipation rate ϵ for different Re_λ .

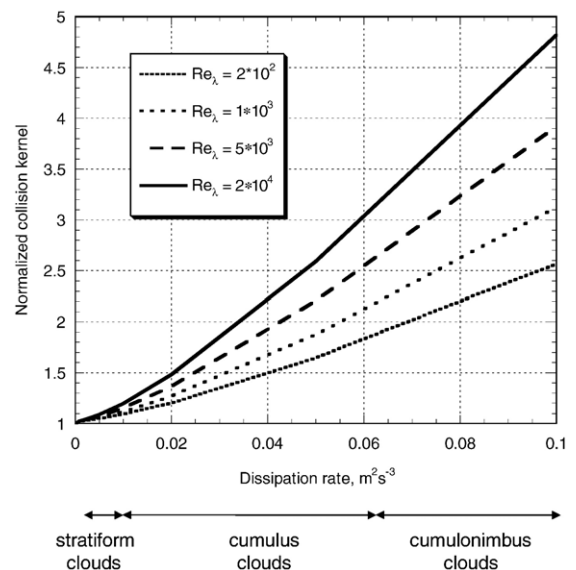


Fig. 16. Dependence of the averaged normalized collision kernel for the 10 μm - and 20 μm -radii droplet pair vs. dissipation rate ϵ under different Re_λ (after Pinsky et al., in press-a).

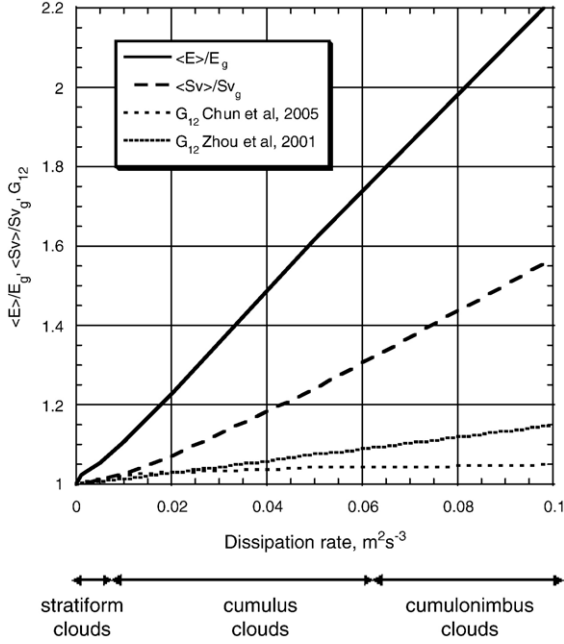


Fig. 17. Dependences of function $G_{12}(a_1+a_2)$, normalized swept volume and normalized collision efficiency vs. dissipation rate for the 10 μm - and 20 μm - radii droplet pair. The collision efficiency increases faster than other quantities. The clustering effect is evaluated using the results of Zhou et al. (2001) and Chun et al. (2005).

While the swept volume enhancement factor was found to be 1.6 for very strong turbulence intensity (see Fig. 4), the collision kernel increases by the factor as large as 4.8. Therefore, the effect of turbulence on the hydrodynamic droplet interaction appears to be the main mechanism by means of which turbulence increases the rate of cloud droplet collisions. Note that the turbulence-induced enhancement factor for the 10 μm –20 μm -radii droplet pair is not the maximum one. The increase of the collision kernel is more pronounced for droplet pairs containing droplets of similar sizes or for pairs where one of the droplets has the radius smaller than $\sim 3 \mu\text{m}$. An inspection of Fig. 16 shows that the collision kernel increases with the increase of ε or Re_λ . Therefore, accounting for the effect of Re_λ is as important as accounting for the effect of the turbulence dissipation rate ε .

Fig. 17 compares the contributions of different turbulence-induced mechanisms to the collision rate enhancement. The figure depicts the dependence of the collision efficiency, swept volume and function G_{12} enhancements on the dissipation rate for the 10 μm –20 μm radii droplet pair. The value of Re_λ was set equal to $2 \cdot 10^4$. The droplet concentration fluctuations caused by the turbulence-inertia mechanism were evaluated using the results of Zhou et al. (2001) ($Re_\lambda=47$) and Chun et al. (2005) ($Re_\lambda=1000$).

Inspection of Fig. 17 shows that the increase of the collision efficiency is the major factor by means of which turbulence increases the collision rate between cloud droplets. According to Chun et al. (2005) the effect of clustering for large dissipation rate turned out to be less than that obtained by Zhou et al. (2001), supposedly because of a stronger spatial decorrelation of droplets of different size within a flow with higher Lagrangian accelerations.

4.2. Small rain drops

Collision efficiencies between small rain droplets and most of the cloud drops are close to 1 even in the pure gravity case (Pruppacher and Klett, 1997). Consequently, turbulence cannot increase further the collision efficiency for droplets of these sizes. The question remains, therefore, how turbulence influences the collision efficiency between large drops and the smallest 1–5 μm droplets. Laboratory experiment (Vohl et al., 1999) indicates that turbulence increases the collision rate between small droplets and small raindrops by ~ 10 –15% as compared to the pure gravitational case. These values provide the upper limit of the increase in the collision efficiency. However, it is to be noted that a) in the experiments the drop collector size varied from 60 μm to about 200 μm , i.e., drop collectors were characterized by $St > 1$; b) in this experiment Re_λ was much smaller than in the atmosphere. *More numerical, theoretical and laboratory investigations are required to elucidate the role of turbulence in the HDI of drops with $St \sim 1$ and cloud droplets.*

5. Parameterization of turbulent effects in the stochastic equation of collisions

5.1. Averaging and correlations

In numerous theoretical and modeling investigations the evolution of droplet size distributions is calculated using the stochastic collision equation in the following form (Pruppacher and Klett, 1997):

$$\begin{aligned} \frac{dN}{dt} = & \frac{1}{2} \int_0^m N(m')N(m-m')S_v(m-m',m') \\ & \times E(m-m',m')dm' \\ & - N(m) \int_0^\infty N(m')S_v(m,m')E(m,m')dm'. \end{aligned} \quad (5)$$

Here S_v is the swept volume between the colliding droplets; E is the collision efficiency indicating a decrease in the collision rate due to HDI. The swept volume is defined as the influx of the relative velocity vector

through the spherical surface with the radius $a_1 + a_2$ (see, e.g., Pinsky et al., 2006). This equation is usually used in Cloud Physics to describe collisions within large air volumes with linear scales in the range from several tens to a few hundred meters. Respectively, a proper averaging of the terms representing the product $N_1 N_2 S_v E$ (each of the values is determined at small time and spatial scales) should be performed. The usual practice of the state-of-the-art numerical models is to neglect the correlation between these values, i.e., it is assumed that $\langle N_1 N_2 S_v E \rangle = \langle N_1 \rangle \langle N_2 \rangle \langle S_v \rangle \langle E \rangle$ (in most cases pure gravitational values of the swept volume and collision efficiencies are used). In numerous studies dedicated to the effects of clustering (e.g., Zhou et al., 2001), where hydrodynamic interaction is neglected ($E=1$), the increase of the collision rate due to the clustering effect is evaluated using the averaging approximation whereby $\langle N_1 N_2 S_v \rangle = \langle N_1 \rangle \langle N_2 \rangle \langle S_v \rangle G_{12}$.

Note first that to get statistically significant results, the averaged volume must contain many drops. Since the concentration of large cloud droplets and small rain drops is low, the required volumes exceed several m^3 . It is especially true, if one takes into account that large drops respond to turbulent structures with the scales up to several meters. The enhancement factors obtained as a result of many realizations on small scales are not equivalent to enhancement factors to be applied at larger scales. Hence, some caution is required when applying the DNS results to large volumes.

Besides, since all the terms in the product $N_1 N_2 S_v E$ depend on the Lagrangian accelerations and turbulent shear, they are statistically dependent. For instance, as it was mentioned above, while the increase in the Lagrangian accelerations attenuates the droplet clustering effect (decrease of G_{12}), it increases the collision efficiency. Pinsky et al. (in press-a) showed that there was a positive correlation between S_v and E under strong turbulent conditions when the Lagrangian accelerations are high. Therefore, there are positive and negative correlations between the functions in the product. Strictly speaking, the product $N_1 N_2 S_v E$ should be averaged as a whole; i.e., the value of $\langle N_1 N_2 S_v E \rangle$ should be calculated using the same approach that takes into account differential drop sedimentation.

We would like to draw attention to the following important subject related to utilization of Eq. (5) for calculation of collisions in a turbulent medium. Pinsky et al. (in press-a) show that turbulent collision kernels are random variables with the probability density functions (PDF) having long tails. The tails become more pronounced with an increase in the turbulent intensity. Employing mean turbulent kernels (increased by

some factor as compared to those in pure gravity case) actually leads to the loss of important information concerning the PDFs of the collision kernels. The reciprocal of the collision kernel estimates the time between successive collisions. The presence of the tails shows, therefore, the existence of a comparatively large number of “lucky” droplets experiencing collisions several times faster than other droplets even under spatially uniform droplet concentration. These “lucky” droplets turn out to be large enough to trigger enhanced collisions during a shorter time than it follows from the stochastic collision equation, in which only the mean values of the collision kernels are used. The clustering effect also fosters the formation of “lucky” droplets. The effect of the stochastic nature of the collision kernels on the DSD evolution is the subject of further investigations. Note that Kostinski and Shaw (2005) showed that the “lucky” fraction of “the fastest 10^{-6} droplets required for warm rain formation; the 10^{-9} fraction needed for patchy drizzle formation, or the 10^{-12} fraction of lonely drops occasionally falling from seemingly thin clouds”.

In spite of the fact that some useful approaches to averaging (Eq. (5)) are proposed by Wang et al. (2006b), the problem of proper averaging the stochastic collision equation (Eq. (5)) using the simple and accurate enough approaches is still waiting for its solution.

6. Conclusions

The relevance of the results obtained in numerous studies of turbulent effects on droplet collisions in turbulent flows to real clouds is analyzed. It is shown that in many studies the conditions under which the turbulence-induced collision rate enhancement was estimated are completely different from those in real clouds. The main limitations of the majority of theoretical and DNS studies is neglecting the effect of differential sedimentation of droplets caused by gravity, as well as using much higher concentrations of large drops and small rain drops as compared with those in real clouds. Neglecting these effects dramatically overestimates the droplet clustering effect on the collision rate, especially for small raindrops characterized by $St \sim 0.3-1$. In addition, employing a frozen turbulent flow in many DNS leads to the replacement of *Lagrangian* acceleration by the *inertial* acceleration. Since these accelerations have different statistical properties, direct application of these results to real clouds is questionable.

During the past few years a significant progress in the understanding of turbulence effects on collisions of cloud droplets has been achieved. The results obtained till now indicate that the turbulence-induced collision

rate enhancement factor for cloud droplets varies within the range from a few percent for stratiform clouds to a factor of ~ 5 –10 in strong cumulus clouds (e.g., Pinsky et al., in press-a,b). Pinsky et al. (in press-b) presented high resolution tables of collision efficiencies and collision kernels for cloud droplets under different turbulent intensities typical of real clouds. These tables can replace the gravitation tables in advanced cloud models. Such a replacement would be an important step toward the more realistic representation of cloud evolution. Recent simulations of droplet size distribution evolution under different turbulent intensities performed by Pinsky et al. (in press-b) and Wang and Grabowski (2007) showed that turbulence significantly accelerates the rain drop formation. Note that the simulations have been carried out using a) mean values of turbulent collision kernels, and b) mean values of the dissipation rate. At the same time the collision kernels in a turbulent flow have PDFs with elongated tails, indicating the opportunity of the formation of large “lucky” droplets experiencing collisions several times faster than other droplets. Besides, the dissipation rates experience high variability in clouds, so that in stratiform cloud the maximal values can reach 1000 under the mean value of 10 (Siebert et al., 2006). It is possible that the dissipation rate in some zones of cumulus clouds can significantly exceed 1000.

Thus, additional efforts are required to find methods for including full stochasticity in the mathematical treatments of coalescence. It is very important also to perform measurements of variability of the turbulence intensity in clouds of different types and to take it into account in the calculation of turbulence effects on precipitation formation.

The effect of turbulence on collisions of droplet pairs containing the largest cloud droplets and small raindrops remains largely unknown. More studies are required to understand the motion and collisions of these drops within turbulent clouds. These studies should take into account the specific features of these drops in clouds: high sedimentation velocity and a very low concentration. Note that the problem of collisions with particles characterized by $St \sim 1$ is especially important for ice cloud microphysics, because many ice particles can be characterized by these values of Stokes number.

The problem of averaging the stochastic collision equation with the purpose of parameterization of turbulence effects in cloud models remains largely unsolved. This situation is partially related to the fact that the effects of turbulence on clustering, on relative velocity and on collision efficiency are not independent but are positively or negatively correlated. A proper

averaging of the stochastic coagulation equation should be performed using the approach that takes into account the effects of differential drop sedimentation.

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