Ponderomotive forces in liquid conductors with macroscopic solid inclusions

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This work studies the ponderomotive forces occurring in liquid conductors with macroscopic inclusions. It is shown that if the electric conductivity of inclusions is higher than the electric conductivity of the host medium then the inclusions are attracted to the axis of a conductor. In the opposite case the ponderomotive forces repel the inclusions towards the surface of a conductor. It is shown that if a certain relation between an amplitude and a frequency of an electric current is satisfied, the parametric resonance in the oscillations of the inclusions with electric conductivity higher than that in the bulk of the conductor can occur. The moments of the ponderomotive forces exerted on a particle of a nonspherical shape are determined. It is shown that the direction of the moment of a ponderomotive force alternates depending upon the sign of the difference between the electric conductivity of an inclusion and of the host medium.

The ponderomotive forces are the subject of many investigations.[1-3] The reason for this intensive interest stems both from the numerous applications of the phenomena occurring due to the ponderomotive forces and from the variety of mechanisms causing ponderomotive forces. Recently,[4,5] we studied the influence of the ponderomotive forces in the conductors with strong spatial dispersion of electric conductivity upon the phase transition of the first order. The mechanism of these effects is associated with the dependence of the inductivity of the conductor upon the location and geometry of the nuclei of the new phase with the value of electric conductivity different from that in the bulk whereby the effect of electric currents act to increase this inductivity.[6] In other words solid inclusions in liquid conductors are subjected to the ponderomotive forces which act both to change the location of the inclusions and their shape and volume. In the theory of phase transitions it is of interest to calculate the additional work performed during formation of the nuclei with electric conductivity different from that in the bulk of a conductor and to determine the dependence of this work upon the volume of the inclusions.

In this investigation we consider rigid particles and analyze the ponderomotive forces and their moments exerted on the particles which may cause their motion and rotation under certain conditions specified below. It should be emphasized that in this work we consider ponderomotive forces which are caused by the electric currents in a conductor and which are associated with the electric induction between a particle and a conductor. Apart from these ponderomotive forces there exist forces which arise due to electrostatic polarization at the interface between regions with different conductivity. This electrostatic polarization occurs due to condition $\operatorname{div}\mathbf{j}=0$ whereby the effective electric charge $\rho\propto E\nabla\sigma/\sigma$ is formed at the interface between the regions with different conductivities. However, if $r_0^2\sigma^2\gg c^2$, where σ is a conductivity of the surrounding medium, c is light velocity and r_0 is the characteristic size of the cross section of a conductor, the forces arising due to electrostatic polarization are considerably smaller than the ponderomotive forces caused by an electric current. In the following we assume that the above condition is met and neglect the forces of electrostatic origin.

According to the above formulation of the problem we must calculate the magnetic inductive part of the thermodynamic energy of the conductor. Under the fixed value of the total electric current this energy is determined by the expression:[6]

$$W=-\frac{1}{2c^2}\int\int\frac{d\mathbf{r}d\mathbf{r}}{|\mathbf{r}-\mathbf{r}'|}\mathbf{j}(\mathbf{r})\mathbf{j}(\mathbf{r}'). \tag{1}$$

The density of electric current $\mathbf{j}$ can be written as follows:

$$\mathbf{j}=\mathbf{j}_0+\delta\mathbf{j}, \tag{2}$$

where $\mathbf{j}_0$ is electric current in a homogeneous conductor and $\delta\mathbf{j}$ is a change of an electric current caused by inclusions.

Expressions (1) and (2) yield

$$W=W_0+\Delta W+\Delta W',$$

where W_0 is interaction energy of homogeneous currents $\mathbf{j}_0$, ΔW is the energy of interaction of the homogeneous current $\mathbf{j}_0$ and perturbation of the current $\delta\mathbf{j}$ and $\Delta W'$ is the energy of interaction of perturbations of the current $\delta\mathbf{j}$. Clearly the energy of interaction of homogeneous currents W_0 does not depend upon the geometrical parameters of inclusions and does not contribute to the forces and moments acting upon the inclusion. Under the specified further conditions the term $\Delta W'$ can be estimated as $\Delta W'\propto a^3/r_0^3\Delta W$, where a is a characteristic size of a nucleus. Since in this study we analyze the problem under the condition $a\ll r_0$, it is sufficient to take into account only the term ΔW

$$\Delta W=-\frac{1}{c^2}\int\int\frac{d\mathbf{r}\,d\mathbf{r}}{|\mathbf{r}-\mathbf{r}'|}\delta\mathbf{j}(\mathbf{r})\mathbf{j}_0(\mathbf{r}'). \tag{3}$$

Thus, the problem is reduced to calculation of electric current $\delta\mathbf{j}(\mathbf{r})$ for a given geometry and location of the inclusion in a conductor. Consider electric conductor of length ℓ

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and radius $\rho_0(\ell \gg \rho_0)$ with total electric current I and with homogeneous density of electric current $j_0 = I/\pi\rho_0^2$ prior to formation of a nucleus.

The problem can be solved in two steps. Assume first that the nucleus is of a spherical shape and is located at the distance d from the surface of a conductor which is considerably greater than the radius of a nucleus a. Then with the accuracy of terms $\propto a^3/d^3$ the distribution of the density of electric current $\delta\mathbf{j}$ can be described by the distribution in the homogeneous host medium with a spherical inclusion:[4,5]

$$\delta\mathbf{j} = -\mathbf{j}_0\xi_a\left[2\eta(a-|\mathbf{r}-\mathbf{r}_a|) + \frac{a^3}{|\mathbf{r}-\mathbf{r}_a|^3}(3\mu^2-1)\right.$$
$$\left.\times\,\eta(|\mathbf{r}-\mathbf{r}_a|-a)\right] - 3\mathbf{e}_\perp j_0\xi_a\mu\sqrt{1-\mu^2}\,\frac{a^3}{|\mathbf{r}-\mathbf{r}_a|^3}$$
$$\times\,\eta(|\mathbf{r}-\mathbf{r}_a|-a), \tag{4}$$

where $\mathbf{j}_0$ density of an electric current before the formation of a nucleus which is considered homogeneous over the cross section of a conductor, $\mathbf{r}_a$—radius vector of a center of mass of a nucleus, $\mathbf{e}_\perp$—unit vector in a plane normal to vector $\mathbf{j}_0$ and directed from a center of a nucleus and

$$\mu = \frac{(\mathbf{r}-\mathbf{r}_a)\mathbf{j}_0}{|\mathbf{r}_a-\mathbf{r}_a|j_0}, \quad \eta(x) = \begin{cases} 1, & x\geq 0 \\ 0, & x<0 \end{cases}, \quad \xi_a = \frac{\sigma_0-\sigma_a}{\sigma_a+2\sigma_0}. \tag{5}$$

Substitution of expression (4) into (3) yields for a spherical particle

$$\Delta W_a = \frac{2\xi_a}{c^2}\,\frac{I^2}{\pi\rho_0^2}\,V_a\left[\ln\left(\frac{4z_a(\ell-z_a)}{\rho_0^2}\right) + 1 - \frac{\rho_a^2}{\rho_0^2}\right], \tag{6}$$

where V_a is a nucleus volume, z_a and ρ_a are coordinates of the center of mass of a nucleus. Note that z_a and ρ_a are determined with the accuracy of terms $\propto a/\rho_0$. Since we analyze the dynamics of nucleation at characteristic spatial scale length of order $\rho_0 \gg a$ such accuracy is quite sufficient. Obviously it is assumed that the electric current in the conductor $I\neq 0$ in the domain $0\leq z<\ell$.

According to expressions (5) and (6) the particle is subjected to the ponderomotive force in the axial direction $F_z = -\partial W_a/\partial z_a$ and in the radial direction $F_\rho = -\partial W_a/\partial\rho_a$. The effect of these forces is determined by the sign of ξ_a. In the case of the conductivity of the host medium $\sigma_0<\sigma_a$, the ponderomotive forces cause the finite motion of particles with the equilibrium point in the center of a conductor $z_a = \ell/2$, $\rho_a = 0$. In the opposite case the particles are repelled from a conductor and the center of a conductor is the point of unstable equilibrium.

For a given value of electric current I, the characteristic frequency of motion of a particle in the direction transversal to the direction of electric current is determined by the following formula

$$\omega_\perp^* = \sqrt{\frac{2\xi_a}{\gamma\pi}}\,\frac{I}{\rho_0^2}\,\frac{1}{c}, \tag{7}$$

where γ is the density of the conductor.

It must be noted that the eigenfrequency $\omega_\perp^*$ depends upon the amplitude of electric current. Therefore the formula (7) determines the relation between the amplitude of electric current and its frequency $\omega_0 = n\omega_\perp^*$ at which there occurs the parametric excitation of the oscillations of solid inclusions when their conductivity is higher than that of the host medium.[7] The characteristic value of the reciprocal time scale of motion of a solid particle in the direction of electric current $\omega_\parallel^* = \omega_\perp^*\rho_0/\ell$.

So far we assumed that the inclusion is of a spherical shape. The transformation to other possible shapes of the particles can be achieved by the substitution of the coefficient ξ defined by expression (5) with the coefficient appropriate to a given shape of the particle. Thus, e.g., for the particle with the shape of the ellipsoid of rotation, with axis of rotation z in the direction of an electric current

$$\xi_a = \xi_\parallel = \frac{1}{2}\,\frac{(\sigma_0-\sigma_a)(1-n_z)}{\sigma_a n_z + \sigma_0(1-n_z)}, \tag{8}$$

where n_z is a geometric coefficient $0<n_z<1$ similar to the depolarization factor.[6]

Determine now, the moment of the ponderomotive force exerted on the inclusion of a nonspherical shape in a current-carrying conductor. Assume that the inclusion is an ellipsoid of rotation which can rotate in the plane parallel to the axis of a conductor. The angle of rotation of the particle ϕ is measured counterclockwise from direction parallel to the axis of a conductor. In this case the geometric factor ξ_a can be determined using the superposition principle.[6] Expanding $\delta\mathbf{j}$ into the two components parallel to the main axes of the ellipsoid and using the solution for each component we find

$$\xi_a = \xi(\phi) = \frac{\xi_\parallel+\xi_\perp}{2} + \frac{\xi_\parallel-\xi_\perp}{2}\cos(2\phi),$$

where $\xi_\parallel$ is determined by expression (8) and $\xi_\perp$ is determined by substitution $\xi_\parallel\to\xi_\perp$, $n_z\to 1-n_z/2$

$$\xi_\perp = \frac{1}{2}\,\frac{(\sigma_0-\sigma_a)(1+n_z)}{\sigma_a(1-n_z)+\sigma_0(1+n_z)}.$$

The magnetic moment $M_\rho = -\partial W_a/\partial\phi$ is determined by the following formula:

$$M_\rho = \sin(2\phi)p_m V_a\,\frac{(1-\kappa)\kappa(1-3n_z)}{[\kappa n_z+1-n_z][(1-n_z)\kappa+1+n_z]}$$
$$\times\left[\ln\frac{4z_a(\ell-z_a)}{\rho_0^2} + 1 - \frac{\rho_a^2}{\rho_0^2}\right],$$

where $\kappa = \sigma_a/\sigma_0$, $p_m = I^2/\pi\rho_0^2c^2$.

The characteristic frequency of rotation in inviscid medium can be determined as $\dot\phi\sim\sqrt{2M/\mathfrak{I}_\parallel}$ where $\mathfrak{I}_\parallel$ is a moment of inertia of the ellipsoid with respect to the axis normal to the axis of rotation.[7] Then $\dot\phi\sim\omega_\perp\rho_0/b$ where b is the length of the longer axis of the ellipsoid.

The mechanical efficiency can be defined as the ratio of mechanical work to the joule dissipation in the volume occupied by the particle (with electrical conductivity equal to the average conductivity of the system $\bar\sigma$) which performs mechanical work

$$\eta = \frac{\dot\phi\Delta W_a\bar\sigma}{j^2V_a} \sim I\,\frac{\rho_0}{b}\,\frac{\bar\sigma}{c^3}\sqrt{\frac{\pi\xi_a}{\gamma}}. \tag{9}$$

Thus, the mechanical efficiency is proportional to the dimension of a conductor, electric current and conductivity.

Determine the condition when the system is not overdamped even in the presence of viscosity. In order to obtain a simple estimate, use the ratio χ of the Stokes viscous friction force $F\approx 6\pi\mu b\omega_\perp\rho_a$ to the characteristic value of ponderomotive force $m\omega_\perp^2\rho_a$:

$$\chi\approx\frac{18}{4}\frac{c\rho_0^2\mu}{Ib^2}\sqrt{\frac{\pi}{2\xi_a\gamma}}\ll 1. \tag{10}$$

The characteristic value of viscosity of liquid metal conductors μ lies in the range from 0.01 to 0.1 g/cm s (see, e.g., Ref. 8). Thus if the characteristic size of the particle is not considerably smaller than the size of the conductor, e.g., $b\geqslant 0.1\rho_0$, for electric currents 1 kA, the motion of particle is not overdamped. Expressions (8) and (10) show that the condition that the motion is not overdamped considerably restrict the feasibility to increase efficiency η.

We have shown that ponderomotive forces occurring in the conductors with strong spatial dispersion of electric conductivity are determined by the ratio of the electric conductivity of the particle to the electric conductivity of the host medium. These forces can play a significant role in conductors with high values of electric currents. The derived formulas (7), (9), and (10) determine the parameters of the system when the effects of these forces are essential.

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