

BRIEF COMMUNICATIONS

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Downstream pressure induced hysteresis in the regular $\leftrightarrow$ Mach reflection transition in steady flows

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The regular $\leftrightarrow$ Mach reflection transition in steady flows was investigated numerically using the W-modified Godunov's scheme. In addition to the incident-shock-wave-angle-induced hysteresis, which was discovered a few years ago and reconfirmed in the present study, a new downstream-pressure-induced hysteresis was found to exist. © 1997 American Institute of Physics. [S1070-6631(97)02810-9]

As indicated in Ref. 1, two shock-wave-reflection configurations are possible in steady flows, regular reflection (RR) and Mach reflection (MR). The RR consists of two shock waves; the incident shock wave, i , and the reflected shock wave, r . They meet at the reflection point, R , which is located on the reflecting surface. The flow states are (0) ahead of i , (1) behind it, and (2) behind r . The angle of incidence, ϕ_1 , of a regular reflection is sufficiently small so that the streamline deflection, θ_1 , caused by the incident shock wave, i , can be canceled by the opposite streamline deflection, θ_2 , caused by the reflected shock wave, r . Therefore, the boundary condition of an RR is $\theta_1 - \theta_2 = 0$. The MR consists of three shock waves: the incident shock wave, i , the reflected shock wave, r , and the Mach stem, m , and one slipstream, s . They all meet at a single point, the triple point, T . The Mach stem, m , is usually a curved shock wave which is perpendicular to the line of symmetry at the reflection point R . The flow states are (0) ahead of i and m , (1) behind i , (2) behind r , and (3) behind m . Unlike the case of an RR, where the net deflection of the streamline is zero, in the case of an MR the net deflection of the streamline is nonzero, in general, and the streamlines behind the triple point are directed towards the line of symmetry. Since the streamlines on both sides of the slipstream must be parallel, the boundary condition of an MR is $\theta_1 - \theta_2 = \theta_3$.

Two extreme criteria, ϕ_1^N —the von Neumann criterion and ϕ_1^D —the detachment criterion,¹ at which the RR $\leftrightarrow$ MR transition can occur, are known. Theoretically, RR is not possible for $\phi_1 > \phi_1^D$ and MR is not possible for $\phi_1 < \phi_1^N$. In the range $\phi_1^N \leq \phi_1 \leq \phi_1^D$, both RR and MR are theoretically possible. For this reason this range is known as the dual solution domain.

The existence of the dual solution domain led, in the early 1980s, to the hypothesis that a hysteresis can exist in

the RR $\leftrightarrow$ MR transition.² An experimental study³ failed to record the hysteresis phenomenon. The fact that no experimental evidence of this hypothesis was reported was attributed to the belief that the RR is unstable in the dual solution domain.

Based on the principle of minimum entropy production it was shown analytically⁴ in the mid-1990s that the RR wave configuration is stable in the dual solution domain. Soon after, the hysteresis phenomenon in the RR $\leftrightarrow$ MR transition was recorded experimentally for the first time.⁵ While the MR $\rightarrow$ RR transition was recorded to occur at the von Neumann condition, ϕ_1^N , the RR $\rightarrow$ MR transition was found to take place at about 2° below the detachment criterion, ϕ_1^D . It is important to note that in a recent study⁶ it was claimed that the above mentioned hysteresis could be observed only with models having small aspect ratios. Then the existence of both RR and MR wave configurations in the dual solution domain [for the same flow Mach numbers, M_0 , and reflecting wedge angles, θ_w , but different distances from the line of symmetry] was demonstrated numerically using an FCT based algorithm.⁷ Following these studies numerical simulations based on the DSMC method^{8–10} and the TVD algorithm^{9,11} demonstrated the above mentioned hysteresis phenomenon, for identical initial conditions, for the first time.

In order to better understand the shock wave reflection, in general, and the hysteresis process, in particular, a detailed numerical study was conducted. The numerical code is based on the W-modified Godunov's scheme. It is an explicit monotone scheme of second order accuracy in space and time. The scheme uses additive corrections to fluxes in the governing equations, with subsequent employment of the first order accurate Godunov scheme to the modified system of equations. As proven by Vasiliev¹² such a procedure pro-

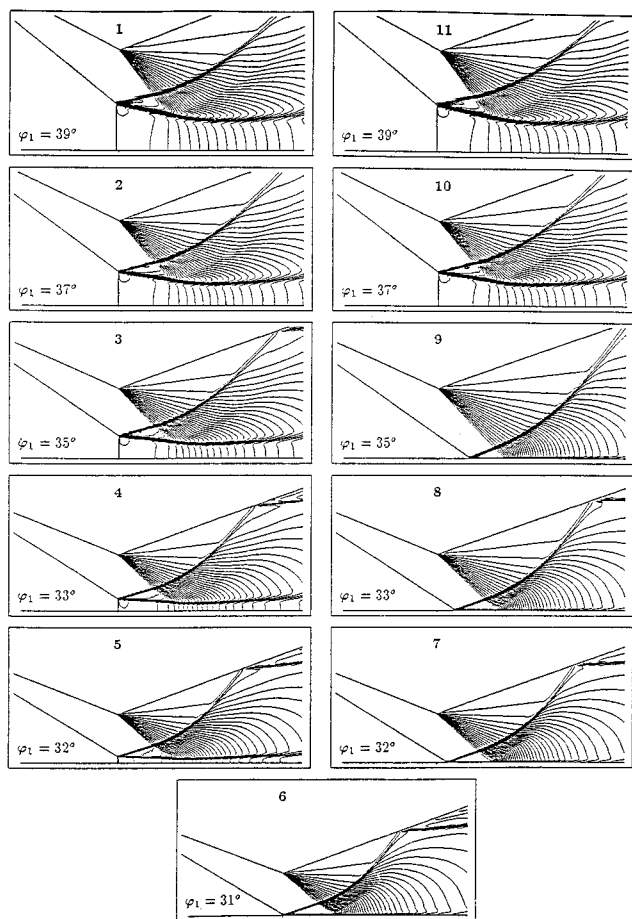


FIG. 1. Density contours for $M_0=4.96$ and 11 different values of ϕ_1 illustrating the incident-shock-wave-angle-induced hysteresis in the RR $\leftrightarrow$ MR transition. Grid 100×60 .

vided a second order accurate in space and time solution of the governing equations. The scheme was used with an adaptive curvilinear grid which moved with the Mach stem. The grid consisted of 70×160 cells, where initially the 70 cells were assigned along the Mach stem. The computation was initiated by setting up an oblique shock wave over the reflecting wedge (appropriate to the given flow Mach number, M_0 , and wedge angle, θ_w) and a straight Mach stem, normal to the line of symmetry, at the minimum cross section. The stationary solutions were calculated using relaxation techniques. After a few thousands time steps a satisfactory convergence was obtained. The computation was continued for 200–300 additional time steps with gradual linear change of the reflecting wedge angle until the new value of the wedge angle was obtained. The initial conditions for each case were the solution of the previous case.

The above mentioned incident-shock-wave-angle-induced hysteresis was reconfirmed. The numerical results for $M_0=4.96$ and 11 different values of incident-shock-wave-angles, ϕ_1 , which follow the sequence $39^\circ, 37^\circ, 35^\circ, 33^\circ, 32^\circ, 31^\circ, 32^\circ, 33^\circ, 35^\circ, 37^\circ$, and 39° are shown in Fig. 1, respectively. The numerical MR $\rightarrow$ RR transition angle was found to be 31.1° , in good agreement with the theoretical value $\phi_1^N=30.9^\circ$. For the reversed RR $\rightarrow$ MR transition the numerical value was found to be 36.5° in comparison to the theoretical value $\phi_1^D=39.3^\circ$. It should be noted here that

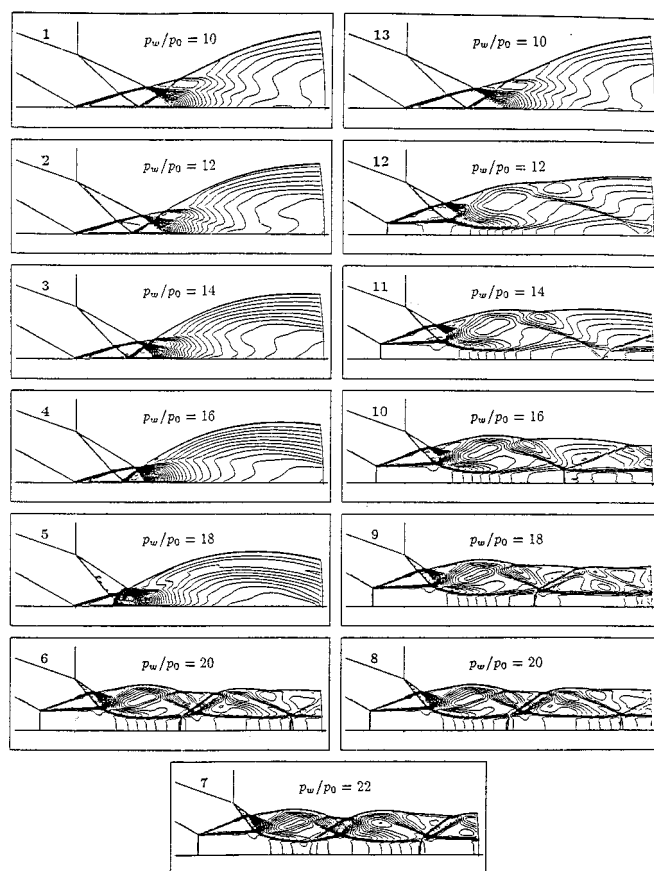


FIG. 2. Density contours for $M_0=4.96$ and 13 different values of p_w/p_0 illustrating the downstream-pressure-induced hysteresis in the RR $\leftrightarrow$ MR transition. $\phi_1=29.5^\circ$, Grid 100×60 .

both numerical values, i.e., 31.1° for the MR $\rightarrow$ RR transition and 36.5° for the RR $\rightarrow$ MR transition are in good agreement with the respective experimental values reported in Ref. 5, i.e., 30.9° and 37.3° . It should also be noted here that Ref. 6 suggests that owing to the three dimensionality of the experiments, this agreement is fortuitous.

In addition to the above described hysteresis, a hysteresis based on a completely different mechanism, i.e., a downstream-pressure-induced hysteresis, was discovered. The numerical results for $M_0=4.96$, $\phi_1=29.5^\circ$, and 13 dif-

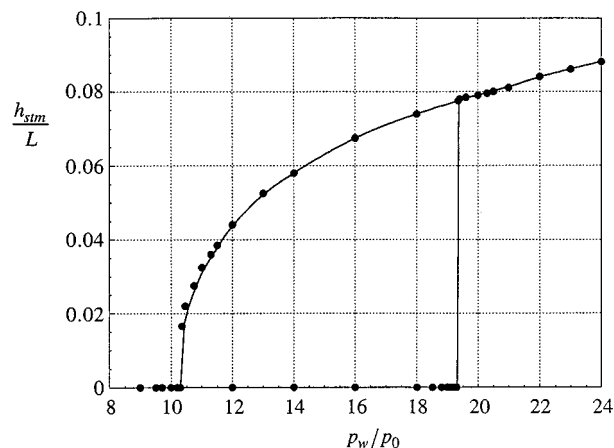


FIG. 3. The actual hysteresis loop in the $(h_{stm}/L, p_w/p_0)$ plane for $M_0=4.96$ and $\phi_1=29.5^\circ$.

ferent values of relative downstream pressure ratios, p_w/p_0 , which follow the sequence 10, 12, 14, 16, 18, 20, 22, 20, 18, 16, 14, 12, and 10 are shown in Fig. 2, respectively. The sequence clearly illustrates a downstream-pressure-induced hysteresis in the RR $\leftrightarrow$ MR transition. The RR $\rightarrow$ MR transition occurs between $p_w/p_0=18$ and 20 at $p_w/p_0=19.5$ (see Fig. 3), and the reversed MR $\rightarrow$ RR transition occurs between $p_w/p_0=12$ and 10 at $p_w/p_0=10.3$ (see Fig. 3). The hysteresis loop in the $(h_{\text{stm}}/L, p_w/p_0)$ plane, where h_{stm} is the Mach stem height and L is the length of the reflecting wedge surface, is shown in Fig. 3.

It should be noted here that for the combination of $M_0=4.96$ and $\phi_1=29.5^\circ < \phi_1^N=30.8^\circ$ which is out of the dual solution domain, only a regular reflection is theoretically possible provided the flow field is free of downstream pressure influence. However, as shown by the present calculations, when the downstream pressure is sufficiently high, a stable Mach reflection can exist for $\phi_1 < \phi_1^N$. To the best of the authors' knowledge, the downstream-pressure-induced hysteresis has never been reported before. An analytical study aimed at better understanding the dependence of the reflection phenomenon on the downstream pressure is currently in progress.

ACKNOWLEDGMENT

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