

Effect of a finite thickness transition layer between media with different permittivities on charged particle localization in an external electric field

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In this study we investigate the effect of a finite thickness transition layer on the character of charged particle localization at the boundary separating domains with different electric permittivities. It is shown that the effective potential at the location of the particle can be determined using perturbation theory, and the conditions for the validity of the theory are discussed. We discuss various effects that can be observed experimentally in the behavior of charged particles located near the boundary.

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I. INTRODUCTION

The behavior of charged particles near boundaries between regions with different permittivities is of interest in view of numerous applications. Examples of such applications include the effect of charge on the dynamics of capillary phenomena,^{1–3} the effect of electrode coating on the electric breakdown, the nature of double layers,⁴ the ionization of gases and liquids in the presence of additives,⁵ etc. Investigations of these phenomena have been conducted for a long time, and there exists a vast number of publications related to studies of various aspects of these phenomena (see, e.g., Refs. 4–7).

One of the basic problems in the theory of electrodynamics of heterogeneous media is associated with spatial variation of the electric properties of a medium, described in this study in terms of its permittivity ϵ . Inhomogeneity of the electric properties complicates calculations of the spatial distribution of the electric potential even for rarefied media. Many results for the classical electrodynamics of heterogeneous media are obtained assuming that the media consist of homogeneous domains with different permittivities. More complicated media can be treated using averaging techniques.⁸

A model of a piecewise homogeneous medium allows the determination of the electric field caused by a point charge, at least in the case of simple geometries.^{8,9} However, since in these models the thickness of the transition layer between the media is assumed to be zero, the potential $\varphi(\mathbf{r}, \mathbf{a})$ caused by the particle located at point $\mathbf{a}$ has a discontinuity when the particle crosses the boundary between the two media as shown below. This discontinuity restricts the study of various phenomena in the vicinity of the boundary using classical electrodynamics.

In this study we investigate the effect of the finite thickness of a transition layer between regions of dielectric materials with different permittivities ϵ_1 and ϵ_2 on charged particles localization in an external electric field in the vicinity

of the interface. We show that when the finite thickness of the transition layer is taken into account, the effective potential of repulsion from the surface for a charged particle located in the region with $\epsilon_1 > \epsilon_2$ has a finite maximum. Similarly, the effective potential of attraction to the surface for a charged particle located in the region $\epsilon_2 < \epsilon_1$ has a finite minimum. An external electric field changes the distribution of the effective potential. This change of the effective potential depends upon the direction of the force applied by the external field. Thus if the direction of the force acting upon the electric charge due to the external field coincides with the direction of the decrease of the electric permeability, then along with the potential well caused by self-localization of the charged particle by a polarization mechanism an additional potential well is formed. This additional potential well is associated with the compensation of the repulsive force acting at the particle in the region with higher ϵ by the external field.¹⁰ We will consider the nature of both potential wells. Hereafter the first potential well is called a polarization well (PW) and the second potential well is called a compensation well (CW). In the case when a force acting on the particles due to the external field is directed in the direction of the increasing permittivity, a CW is not formed and a PW is deformed.

II. MATHEMATICAL MODEL

Consider a heterogeneous medium with spatially dependent permittivity $\epsilon(\mathbf{r})$. The potential (electrostatic) energy of free electric charge with a density $\gamma_0(\mathbf{r})$ is (see Refs. 8 and 9 and the Appendix):

$$W = \frac{1}{2} \int \gamma_0(\mathbf{r}) \varphi(\mathbf{r}) d^3\mathbf{r}, \quad (1)$$

where the potential $\varphi(\mathbf{r})$ is determined by the equation:

$$\nabla(\epsilon \nabla \varphi) = -4\pi \gamma_0(\mathbf{r}). \quad (2)$$

When functions $\gamma_0(\mathbf{r})$ and $\epsilon(\mathbf{r})$ are known, determining the potential $\varphi(\mathbf{r})$ reduces to a solution of a linear boundary value problem. The most widely used method for the solution of Eq. (2) employs the approximation of piecewise ho-

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homogeneous media (see, e.g., Refs. 8 and 9). This approximation assumes zero thickness of the transition layer. At the boundaries separating two media the conditions of continuity of the potential φ and of the normal component of the electric displacement D_n are imposed.

For a point charge $\gamma_0(\mathbf{r}) = e\delta(\mathbf{r} - \mathbf{a})$ the solution $\varphi(\mathbf{r}, \mathbf{a})$ of Poisson equation (2) for a piecewise homogeneous medium is well known (see Ref. 8, Chap. 2, Sec. 7):

$$\begin{aligned}\varphi(\mathbf{r}, \mathbf{a}) &= \frac{e}{\varepsilon_1 |\mathbf{r} - \mathbf{a}|} + \frac{e(\varepsilon_1 - \varepsilon_2)}{\varepsilon_1(\varepsilon_1 + \varepsilon_2) |\mathbf{r} - \hat{\sigma}\mathbf{a}|}, \quad z < 0, \\ \varphi(\mathbf{r}, \mathbf{a}) &= \frac{2e\varepsilon_2}{(\varepsilon_1 + \varepsilon_2) |\mathbf{r} - \mathbf{a}|}, \quad z \geq a.\end{aligned}\quad (3)$$

Here $\hat{\sigma}\mathbf{a}$ denotes a vector obtained after mirror reflection of vector $\mathbf{a}$ with respect to a plane $z=0$, i.e., if $\mathbf{a} = (a_x, a_y, a_z)$, then $\hat{\sigma}\mathbf{a} = (a_x, a_y, -a_z)$. Equation (3) describes a potential caused by an electric charge located in a region with electric permittivity ε_1 . When a charge is located in a region with electric permittivity ε_2 , the potential $\varphi(\mathbf{r}, \mathbf{a})$ is given by Eq. (3) with a replacement $\varepsilon_1 \rightarrow \varepsilon_2$. Thus when a particle crosses a boundary between two media with different electric permittivities $a_z = 0$, the electric potential $\varphi(\mathbf{r}, \mathbf{a})$ has a discontinuity at every point $\mathbf{r}$.

Therefore a model with piecewise homogeneous media can be applied only when a particle is located far from the boundary and does not allow the study of phenomena associated with particle localization near the boundary. One of the methods that can be used to remove the singularity is to introduce a finite thickness transition layer where permittivity $\varepsilon(z)$ depends on position z relative to the boundary. Introduction of the transition layer qualitatively changes the behavior of the potential and allows us to describe the renormalization of an ionization coefficient at the surface of a particle, renormalization of the coefficient of surface tension caused by surface charges, spontaneous formation of double layers in heterogeneous fluids, etc. In this study we do not analyze all these problems which warrant a separate investigation and consider only the effect of a finite thickness transition layer on the resonance response of particles localized at the boundary.

Since we study a situation when a particle is located near the boundary we can consider a plane boundary and use a one-dimensional model. Let ε_1 and ε_2 be the magnitudes of $\varepsilon(z)$ at $z \rightarrow -\infty$ and $z \rightarrow \infty$, respectively. Assume that $\varepsilon(z)$ is a monotonically decreasing function and at the boundary $z = 0$, $\varepsilon(0) = (\varepsilon_1 + \varepsilon_2)/2$ (see also Fig. 1). Thus

$$\varepsilon(z) = \frac{\varepsilon_1 + \varepsilon_2}{2} - \frac{\varepsilon_1 - \varepsilon_2}{2} f\left(\frac{z}{\delta}\right), \quad (4)$$

where $f(0) = 0$, $f(\pm\infty) = \pm 1$, and δ is a width of the transition layer.

For further analysis let us represent the solution of Eq. (2), $\varphi(\mathbf{r})$, as

$$\varphi(\mathbf{r}) = \varphi_0(\mathbf{r}) + \varphi_1(\mathbf{r}), \quad (5)$$

where $\varphi_0(\mathbf{r})$ is determined by the following equation

$$\varepsilon(\mathbf{r}) \nabla^2 \varphi_0(\mathbf{r}) = -4\pi \gamma_0(\mathbf{r}). \quad (6)$$

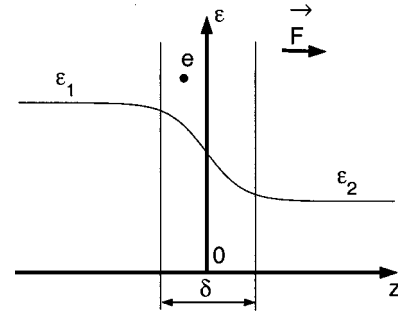


FIG. 1. Spatial distribution of permittivity in the transition layer and direction of the force $\mathbf{F} = e\mathbf{E}$ ($\mathbf{E}$ is the strength of the electric field inside the capacitor). The plane $z=0$ corresponds to the boundary between two media in the model with the infinitely thin transition layer.

The solution of Eq. (6) reads

$$\varphi_0(\mathbf{r}) = \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \frac{\gamma_0(\mathbf{r}')}{\varepsilon(\mathbf{r}')} d^3\mathbf{r}'. \quad (7)$$

Using Eqs. (2), (5), and (6) we arrive at the following integral equation for function $\varphi_1(\mathbf{r})$:

$$\begin{aligned}\varphi_1(\mathbf{r}) &= \frac{1}{4\pi} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \hat{p}' [\varphi_1(\mathbf{r}') + \varphi_0(\mathbf{r}')] d^3\mathbf{r}', \\ \hat{p} &= \left(\frac{\nabla \varepsilon}{\varepsilon} \cdot \nabla \right).\end{aligned}\quad (8)$$

Formally, Eq. (8) allows us to represent the solution $\varphi_1(\mathbf{r})$ as an expansion into powers of the operator $\hat{p}$:

$$\begin{aligned}\varphi_1(\mathbf{r}) &= \frac{1}{4\pi} \int \frac{d^3\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \left[\hat{p}' + \int \frac{d^3\mathbf{r}''}{|\mathbf{r} - \mathbf{r}''|} \hat{p}'' \right. \\ &\quad \left. \times \int \frac{d^3\mathbf{r}'''}{|\mathbf{r}'' - \mathbf{r}'''} \hat{p}''' + \dots \right] \varphi_0(\mathbf{r}').\end{aligned}\quad (9)$$

Convergence of expansion (9) requires that starting from some n th term in the series, $|\varphi_1^{n-1}(\mathbf{r})| \gg |\varphi_1^n(\mathbf{r})|$. Whether this condition is satisfied depends on the behavior of functions $\nabla \varepsilon(\mathbf{r})/\varepsilon$ and $\varphi_0(\mathbf{r})$. Hereafter we consider an isolated charge, $\gamma_0(\mathbf{r}) = e\delta(\mathbf{r} - \mathbf{a})$. Then from Eq. (7) we find that $\varphi_0(\mathbf{r}) = e/[\varepsilon(\mathbf{a})|\mathbf{r} - \mathbf{a}|]$. Since $\varphi_0(\mathbf{r}) \rightarrow 0$ when $|\mathbf{r} - \mathbf{a}| \rightarrow \infty$, the perturbation theory is valid only in the range of sufficiently small values $|\mathbf{r} - \mathbf{a}|$. The size of this domain is determined by the behavior of function $\nabla \varepsilon/\varepsilon$. In this study we investigate a potential caused by a polarization charge $\bar{\rho}(\mathbf{r})$ (see Ref. 8, Chap. 2, Sec. 7)

$$\bar{\rho}(\mathbf{r}) = \frac{\hat{p}}{4\pi} \varphi(\mathbf{r})$$

at the location of the free charge $\mathbf{a}$. When $\mathbf{r} \rightarrow \mathbf{a}$, $\varphi_0(\mathbf{r}) \rightarrow \infty$ and $|\varphi_1^1(\mathbf{r})| \ll |\varphi_0(\mathbf{r})|$. Clearly the latter condition does not imply the convergence of the series (9). However, one can consider $\varphi_1^1(\mathbf{r})$ as the first asymptotic correction to the potential $\varphi_0(\mathbf{r})$ caused by the polarization charge. Using Eq. (9) we obtain the following expression for this asymptotic correction at $\mathbf{r} = \mathbf{a}$, $\varphi_1^1(\mathbf{a})$:

$$\varphi_1(\mathbf{a}) = \frac{1}{4\pi} \int \frac{1}{|\mathbf{r}' - \mathbf{a}|} \hat{p} \varphi_0(\mathbf{r}') d^3\mathbf{r}'. \quad (10)$$

Hereafter we omit the upper index, i.e., $\varphi_1^1(\mathbf{a}) \equiv \varphi_1(\mathbf{a})$. Compare now the magnitude $\varphi_1(\mathbf{a})$ obtained from Eq. (10) with the value determined by the exact solution of the problem using a model with a piecewise homogeneous medium. Since in this model $\nabla \varepsilon - (\varepsilon_1 - \varepsilon_2) \delta(z) \mathbf{k}$, where $\mathbf{k}$ is a unit vector, Eq. (10) yields

$$\varphi_1(a) = -\frac{e\alpha(1-\alpha)}{a\varepsilon_2(2-\alpha)[2-\alpha(1+\text{sgn}(a))]}, \quad (11)$$

where $a \equiv a_z$ is a component of vector $\mathbf{a}$ in the direction of vector $\mathbf{k}$, and $|a|$ is the distance of the particle from the boundary, $\alpha = (\varepsilon_1 - \varepsilon_2)/\varepsilon_1$. The function $\varphi_1(a)$ can be also written as

$$\varphi_1(a) = -\frac{e\alpha}{2a(2-\alpha)\varepsilon(a)}.$$

It is readily seen that the latter expression is the second term in the first of Eqs. (3) ($z < 0$) at $\mathbf{r} = \mathbf{a}$. Thus, already in the first order of perturbation theory, it is possible to determine the force acting on the charged point particle in the vicinity of the boundary between two media taking into account polarization of the medium caused by the particle. Similarly one can calculate the value $\varphi_1(a)$ in the case of a spherical boundary. In the latter case as in the case with the plane boundary (10) the value $\varphi_1(a)$ coincides with the value obtained using the exact solution of the problem.⁹

Substituting $\gamma_0(\mathbf{r}) = e\delta(\mathbf{r} - \mathbf{a})$ into Eq. (1) and using Eqs. (5) and (10) and dropping out the diverging terms as it is done in classical theory of fields (see Ref. 11, Chap. 5, Sec. 27), we find that $W = (e/2)\varphi_1(a)$. Substituting $\varphi_1(a)$ given by expression (11) we obtain

$$W(a) = \frac{e^2(\varepsilon_2 - \varepsilon_1)}{4\varepsilon_1(\varepsilon_2 + \varepsilon_1)a}, \quad a < 0,$$

$$W(a) = \frac{e^2(\varepsilon_2 - \varepsilon_1)}{4\varepsilon_2(\varepsilon_2 + \varepsilon_1)a}, \quad a > 0. \quad (12)$$

An analysis of Eq. (12) shows that for small a the model with a piecewise homogeneous medium yields unphysical results, e.g., infinite acceleration at the boundary and the impossibility of extracting charged particles from the regions with large permittivity ε . Thus in order to eliminate these inconsistencies it is necessary to take into account the finite thickness of the transition layer.

Consider a plane boundary, $\nabla \varepsilon = \mathbf{k}|\nabla \varepsilon|$, where as before $\mathbf{k}$ is a unit vector in the direction of the z axis. Integrating Eq. (10) over the radial coordinate we arrive at the following expression:

$$\varphi_1(a) = -\frac{e}{4\varepsilon(a)} \int_{-\infty}^{\infty} \frac{dz}{z-a} \left[\frac{\partial}{\partial z} \ln \varepsilon(z) \right]. \quad (13)$$

Equation (13) is valid for the arbitrary function $\varepsilon(z)$.

In order to calculate the integral in expression (13) it is necessary to define the contour of integration, taking into account that a pole is located on the axis of integration. Since potential $\varphi_1(a)$ is a real function, the integral in Eq. (13) is understood as a principal value.

In this study we adopt the model given by Eq. (4) with $f(z) = \tanh(z/\delta)$ for describing the dependence of $\varepsilon(z)$ on position z , taking into account the finite thickness of the transition layer:

$$\varepsilon(z) = \frac{\varepsilon_1 + \varepsilon_2}{2} - \frac{\varepsilon_1 - \varepsilon_2}{2} \tanh\left(\frac{z}{\delta}\right),$$

where δ is the characteristic width of the transition layer. The particular choice of function $f(z)$ is not important provided that it satisfies the conditions formulated for the function given by Eq. (4).

Using the dimensionless variable $u = z/\delta$ and the parameter $\alpha = 1 - \varepsilon_2/\varepsilon_1$ which varies in the range $0 < \alpha < 1$ ($\varepsilon_1 > \varepsilon_2$), $W(a)$ can be expressed as follows:

$$W(a) = \frac{e^2\alpha(1-\alpha)}{4\delta\varepsilon_2[2-\alpha(1+\tanh a^*)]} \Phi(a^*), \quad (14)$$

where

$$\Phi(a^*) = \int_0^\infty \sum_{\sigma=\pm 1} \frac{\sigma du}{u \cosh^2(a^* + \sigma u) [2 - \alpha(1 + \tanh(a^* + \sigma u))]} \quad (15)$$

and $a^* = a/\delta$. Formulas (14) and (15) can be used for calculating the potential and its derivatives.

Before discussing the results of numerical calculations, consider the asymptotic behavior of the potential at finite a and $\delta \rightarrow 0$ and finite δ and $a \rightarrow \infty$. The above equations show that these two limits do not coincide. Using Eq. (13) for $\delta \rightarrow 0$ and finite a we arrive at formulas (11) and (12) while for $a \rightarrow \infty$ and finite δ this formula yields

$$W(a) = -\frac{e^2 \log(1-\alpha)}{8\varepsilon(a)a}. \quad (16)$$

Thus limiting values of the potential at finite a and $\delta \rightarrow 0$ and finite δ and $a \rightarrow \infty$ do not coincide. These values are different because the functional dependence of the potential $W = W(a, \delta, \alpha)$ is not determined only by the ratio a/δ , i.e., $W(a, \delta, \alpha) \neq W(a/\delta, \alpha)$.

In this study instead of potential $W(a)$ we use potential $W^*(a)$ which recovers potential (12) for $a \rightarrow \infty$ and finite δ . The latter requirement determines the potential uniquely:

$$W^*(a) = -\frac{2\alpha}{(2-\alpha)\log(1-\alpha)} W(a). \quad (17)$$

This choice of the potential $W^*(a)$ is done because of the following reasons. As can be seen from Eq. (16) the potential $W(a)$ has a logarithmic singularity at $\alpha \rightarrow 1$, i.e., $W(a) \propto \log \varepsilon_2/\varepsilon_1$. At this stage it is not clear whether this singularity has a physical origin or if it is caused by the inapplicability of the perturbation theory for $\varepsilon_2/\varepsilon_1 \ll 1$. On the other hand the main goal of this study is to investigate electric resonance in dielectrics where $\alpha \ll 1$ and $W(a) \approx W^*(a)$. In order to avoid using a potential singular at $\alpha \rightarrow 1$ and to compare the results with those obtained in the previous studies hereafter we will use the potential $W^*(a)$. In Fig. 2

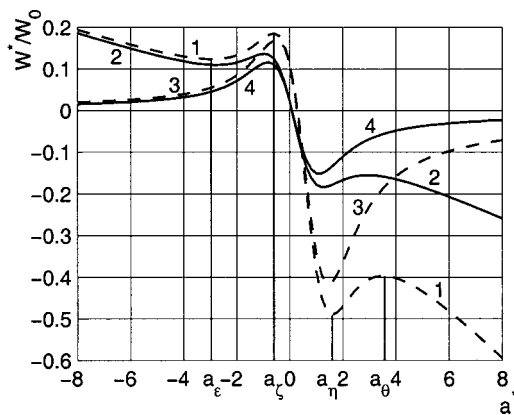


FIG. 2. Dependence of the normalized potential energy, W^*/W_0 , of a charged particle with and without external electric field vs dimensionless distance, a/δ , from the interphase boundary. Curve 1: $\alpha=0.3$, $\tilde{E}=0.1\alpha$; curve 2: $\alpha=0.7$, $\tilde{E}=0.1\alpha$; curve 3: $\alpha=0.3$, $\tilde{E}=0$; and curve 4: $\alpha=0.7$, $\tilde{E}=0$, $\tilde{E}=E_0 4 \delta^2 \epsilon_2 / e$.

(curves 3 and 4) we show the behavior of the potential $W^*(a)/W_0$ for several values of α ($W_0 = e^2/4\delta\epsilon_2$). Clearly the behavior of the potential vs parameter α is essentially different in the vicinity of the potential barrier and the potential well. In order to elucidate the results we define the function $W_1 = W^*(\alpha, a^* - a_{\max}^*)/W_{\max}^*(\alpha)$, where $W_{\max}^*(\alpha)$ is the maximum value of the function in the vicinity of the potential barrier and $a_{\max}^*$ is the location where the function, $W^*(\alpha, a)$, attains its maximum value. In Fig. 3 we show the behavior of the function $W_1(\alpha, a^* - a_{\max}^*)$ in the range $0 < \alpha < 0.9$. Curves 1 and 2 correspond to $\alpha=0.1$ and $\alpha=0.9$, respectively. Curves corresponding to different values of the parameter α in this interval are located between these two curves and do not intersect.

As can be seen from Fig. 3 the behavior of function W_1 is universal while $a_{\max}^*$ changes strongly with a change in the parameter α and shifts towards the boundary (see Fig. 5). In Fig. 6 we show the dependence of the maximum value of the barrier $W_{\max}^*/W_0$ vs parameter α .

Similarly, in order to describe the behavior of the function $W^*(a)$ in the vicinity of the potential well, it is conve-

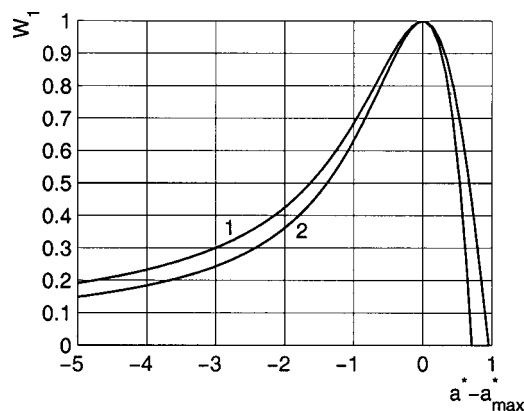


FIG. 3. Dependence of the normalized potential barrier vs dimensionless distance from the apex of the barrier. Curve 1: $\alpha=0.1$ and curve 2: $\alpha=0.9$.

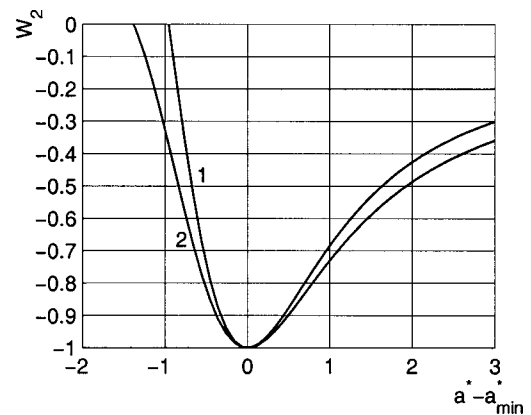


FIG. 4. Dependence of the normalized potential well vs dimensionless distance from the bottom of the well. Curve 1: $\alpha=0.1$ and curve 2: $\alpha=0.9$.

nient to introduce the function $W_2 = W^*(\alpha, a^* - a_{\min}^*)/W_{\min}^*(\alpha)$, where $W_{\min}^*(\alpha)$ is the depth of the potential well and $a_{\min}^*$ is the location of the minimum of the well. Results of the calculation of these parameters are displayed in Figs. 4–6. Figure 6 shows that the depth of the potential well, $W_{\min}^*(\alpha)$, is considerably larger than the height of the potential barrier, $W_{\max}^*(\alpha)$. For transition layers with widths of the order of several Bohr radii, the depth of the potential well changes by several electron volts.

The physical essence of the results obtained is quite clear. In order to penetrate into the region with higher ϵ , a charged particle must have a kinetic energy $mv^2/2 > e^2 W_{\max}^*(\alpha)/4\delta$. A particle must acquire the same energy in order to exit in the opposite direction. Without accounting for the finite thickness of the transition layer it is not clear how a particle can exit from a medium with a larger magnitude of ϵ .

In this study we use a point charge approximation which is valid only under certain restrictions. Usually this model can be applied when the spatial scale of the problem is much larger than the distance between atoms.

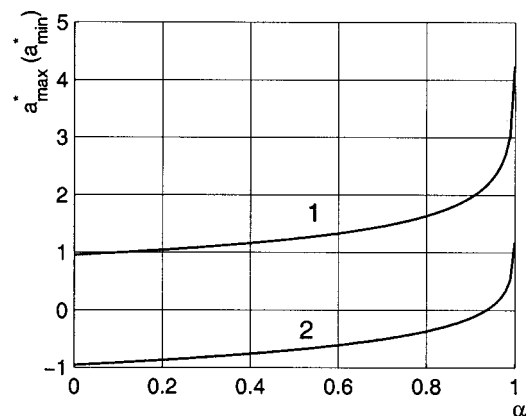


FIG. 5. Dependence of normalized by δ location of the maximum $a_{\max}^*$ (curve 1) and minimum $a_{\min}^*$ (curve 2) vs parameter α .

III. ELECTRIC RESONANCE OF THE LOCALIZED CHARGED PARTICLE

The above results show that a charged particle can be localized in the region with $\epsilon_2 < \epsilon_1$ at distances of the order of δ from the boundary where ϵ assumes its average value $\epsilon = (\epsilon_1 + \epsilon_2)/2$. In the presence of a potential well with a finite depth, the spectrum of localized particles is different from that in the case when particles localize in the potential $\varphi(a) \propto 1/a$. The spectrum in the latter case was calculated previously.¹⁰ Near the bottom of the potential well which is determined by expressions (14) and (15) the spectrum of the particles is similar to that of the harmonic oscillator, i.e., $\Delta_\omega = (n + 1/2)\omega$, where $n = 0, 1, \dots$, $\omega^2 = (\partial^2 \varphi / \partial a^2) / m$, m is a mass of the particle, and the derivative is calculated at the location of the extremum. The characteristic frequency can be estimated as $\omega_0^2 = e^2 / 8m \delta^3 \epsilon_2$.

A transition layer with a finite thickness has an even more profound effect when an external field is applied to the system. Consider a capacitor filled with a dielectric with permittivity distribution $\epsilon(z)$ given by formula (4) with $f(z) = \tanh(z)$, where the z axis is normal to the plates of the capacitor. A capacitor produces an additional electric field acting on a particle. This additional field is determined by the following equation:

$$\frac{\partial}{\partial z} [\epsilon(z)E(z)] = 0, \quad E(z)|_{z \rightarrow \infty} = E_0. \quad (18)$$

Thus the potential of the electric field at the location of the particle can be represented as a sum of two potentials. The first potential is $W^*(a)/e$, where $W^*(a)$ is determined by formulas (14), (15), (17), and an additional potential $\varphi(a)$,

$$\varphi(a) = - \int_0^a \frac{\epsilon_2 E_0 dz'}{\epsilon(z')}. \quad (19)$$

We assume that the distance between the plates of the capacitor $d \gg \delta \sim a$. Then Eq. (18) and condition $\epsilon(z)|_{z \rightarrow \infty} = \epsilon_2$ yield solution (19) for the potential $\varphi(a)$.

We consider the case when the force acting on the particle due to the external field is in the direction of decreasing ϵ , i.e., $\mathbf{F} \propto -\nabla \epsilon$. In this case the spatial behavior of the potential energy is similar to that shown in Fig. 2 (curves 1 and 2). As can be seen from this figure, particles located in the region with $\epsilon_1 > \epsilon_2$ are localized in the potential well. This potential well is associated with the compensation of the repulsive force acting on the particle in the region with higher ϵ by the external field. The particle oscillation frequency depends on the position of the equilibrium point which, in turn, depends upon the amplitude of the external field. The dependence of the oscillation frequency of the localized particle on the amplitude of the external field can be used to determine the mass of the particle by observing the electric resonance. Such an experiment, suggested in Ref. 10, was reported in Refs. 12 and 13. Both the theoretical model¹⁰ and the analysis of experiments in Refs. 12 and 13 employed the infinitely thin transition layer model.

As noted above, the model with piecewise homogeneous media yields physically reasonable results only when the equilibrium position $a_0 \gg \delta$. However, as can be seen from

further analysis, $a_0 \propto \sqrt{\alpha/E_0}$. Therefore at high external field amplitudes E_0 or small α , the model with a piecewise homogeneous medium is not valid, yielding an unphysical dependence of ω on E_0 and on α . We will show that when the finite thickness of the transition layer is taken into account, this dependence is physically correct for all values of E_0 and α .

The principal difference of the model with a finite length transitional layer from the model with $\delta = 0$ is that there are four extrema points, a_ϵ , a_ξ , a_η , and a_θ (see Fig. 2, curves 1 and 2) while in the model with $\delta = 0$ there are only two extrema points, a_ϵ and a_θ . The points a_ξ and a_η are due to the finite length of the transition layer. The locations of these points depend weakly on the amplitude of the external field and $a_\xi, a_\eta \sim \delta$. On the other hand, points a_ϵ and a_θ are caused by the external field and their locations are determined by the amplitude of the field. The points a_ϵ and a_η correspond to the stable equilibrium while the points a_ξ and a_θ correspond to the unstable equilibrium. Let us show that the points a_ξ and a_η always lie in the domain $a \sim \delta$ and depend weakly on the magnitude of the applied external field, while the points, a_ϵ and a_θ are caused by the external field and depend upon its strength.

Indeed, assume that equilibrium points are located far from the boundary $a \gg \delta$ and use asymptotic formulas (11) and (12) for the potential $W^*(a)$. Then a condition for the mechanical equilibrium,

$$\frac{\partial W^*}{\partial a} - eE(a) = 0$$

yields

$$a_0^2 = \frac{e\alpha}{4(2-\alpha)} \frac{1}{E_0}. \quad (20)$$

Equation (20) has only two roots corresponding to a_ϵ and a_θ . Therefore the points a_ξ and a_η cannot lie in the domain $a \gg \delta$ for any strength of the external field. The negative root of Eq. (20) corresponds to the stable equilibrium point, a_ϵ , with the frequency given by

$$\omega_\epsilon^2 = \frac{4\sqrt{e}}{m} \sqrt{\frac{2-\alpha}{\alpha}} \frac{1-\alpha}{\epsilon_2} E_0^{3/2}. \quad (21)$$

The second positive root of Eq. (20) corresponds to the unstable equilibrium point a_θ with $\omega_\theta^2 = -\omega_\epsilon^2$. Expression (21) becomes nonphysical, i.e., $\omega^2 \rightarrow \infty$ when $\alpha \rightarrow 0$ ($\epsilon_1 \rightarrow \epsilon_2$).

The reason is that the model with a piecewise homogeneous medium is not valid for small α when according to Eq. (20) $a_0 \rightarrow 0$. This implies the existence of the characteristic length scale δ such that expression (21) is only valid for $a \gg \delta$. In the theory developed in this study the characteristic length scale δ is the thickness of the transition layer.

Thus the range of validity of expressions (20) and (21) is given by $a \gg \delta$ or

$$E_0 \ll \frac{e}{\delta^2} \frac{\alpha}{4(2-\alpha)}. \quad (22)$$

When the external field is increased so that condition (22) is violated, the frequency ω_ϵ tends to its maximum value

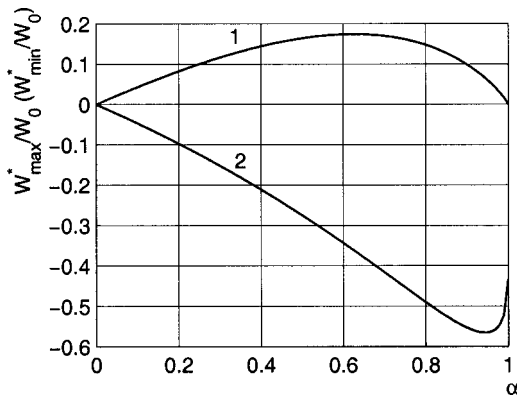


FIG. 6. Dependence of normalized by δ location of the maximum $W_{\max}^*/W_0$ (curve 1) and minimum $W_{\min}^*/W_0$ (curve 2) vs parameter α .

$\omega_{\max}(\alpha)$. The dependence $\omega_{\epsilon}^2(E_0)$ is shown in Fig. 7 for a given value of α . Although determining this dependence analytically is quite involved, general behavior is described in the following. When the external field increases, frequency attains its maximum value, $\omega_{\max}$, which varies with α nonmonotonically (Fig. 8). If the field is increased further, eigenfrequency decreases, and when $E_0 = E_c$, the locations of the minima a_{ϵ} and a_{ξ} coincide. The condition $a_{\epsilon} = a_{\xi}$ determines the critical strength of the external field E_c whereby the frequency $\omega_{\epsilon} = 0$ and the charged particle delocalizes. Thus at a given frequency, ω , there are two values of the strength of the external field, E_{ω}^1 and E_{ω}^2 , for which the electric resonance can be observed. If $\omega_{\epsilon} \ll \omega_{\epsilon}^{\max}$, the amplitudes of the electric field are significantly different and $E_{\omega}^1 \ll E_{\omega}^2$. The field E_{ω}^1 can be determined using expression (19):

$$E_{\omega}^1 = \frac{\omega^{4/3}}{f}, \quad f = \left[\frac{4(1-\alpha)}{m\epsilon_2} \right]^{2/3} \left[\frac{e(2-\alpha)}{\alpha} \right]^{1/3}.$$

These results show that it is of interest to investigate the localized particles in the range of subcritical fields $E \leq E_c$. For particles with a given charge in this range there are two resonance peaks at a given frequency. It should be noted that conditions for observation of the resonance for a particle localized in the vicinity of the boundary inside a medium with the larger ϵ are no less favorable than for particles localized far from the boundary, i.e., for $a \gg \delta$.

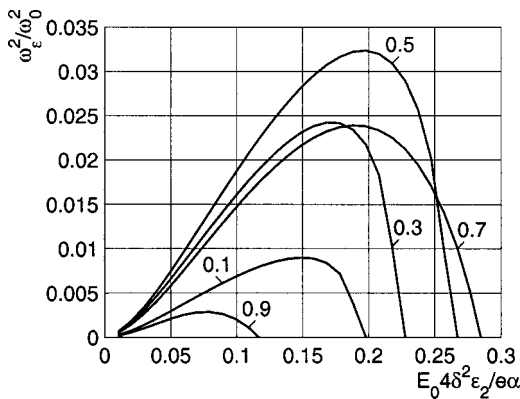


FIG. 7. Dependence of the normalized ω_{ϵ}^2 vs normalized amplitude of the external electric field $\tilde{E} = E_0 4 \delta^2 \epsilon_2 / e$.

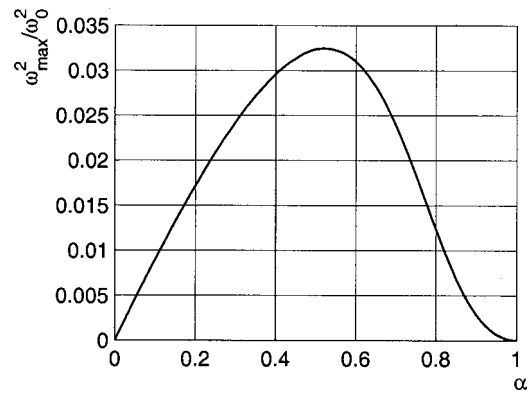


FIG. 8. Dependence of the normalized $\omega_{\max}^2$ vs parameter α .

To the best of our knowledge electric resonance of particles localized in the CW (the region in the vicinity of a_{ϵ} in Fig. 2) was investigated only in Refs. 12 and 13. These experiments studied localization of helium ions at the boundary separating gas and liquid media. Electric resonance was observed at a magnitude of electric field of the order of $E \sim 10^2$ V/cm. Since for such fields particles localize far from the boundary, the corrections for the finite thickness transition layer are small. Note that low temperature conditions are needed in order to observe localization of particles in the CW far from the boundary since the frequency is low (of the order of 10^2 MHz). Much stronger fields are required in order to observe resonance peaks $E_{1\omega}$ and $E_{2\omega}$ simultaneously. These experiments can be performed under the room temperature conditions. The required strength of an electric field scales as $E \sim \alpha e / [4(2-\alpha)\delta^2]$. Since the transition layer width is usually of the order of several interatom distances it can be easily seen that simultaneous observation of both peaks requires subcritical fields which are of the order of the electric breakdown field.

IV. CONCLUSIONS

In this study we considered localization of a point charge particle at the boundary separating domains with different electric permittivities. The effect of the finite thickness transition layer on point charge localization at the boundary between two media with different permittivities has been investigated. In spite of the large number of publications concerning interaction of charged particles with surfaces, the effect of the finite thickness transition layer was not analyzed despite its physical importance. Indeed, the finite thickness transition layer directly explains the finite value of energy required for emission or penetration of charged particles into matter without using approximate models (see, e.g., Ref. 14, p. 70). We investigated the shape of the polarization potential well as a function of electric permittivities ϵ_1 and ϵ_2 and studied electric resonance occurring at near critical magnitudes of electric field.

Clearly, in real systems there exist additional mechanisms causing localization of charged particles, e.g., interaction with defects, surface phonons, etc., which are discussed in the literature (see Refs. 1, 3, 5, and 10). Nevertheless, investigation of the corrections due to a finite thickness tran-

sition layer is of interest by itself since the thickness of the transition layer is the real physical parameter which can be calculated from the first principles and provides information about important properties of materials, e.g., interaction between molecules (see, e.g., Ref. 15). It must be noted that we considered only the case of a point charge. The case with continuous distribution of free charges is more difficult and can be considered only using a model of a piecewise homogeneous medium.^{16,17}

It is shown that the effective potential at the location of the particle can be determined using perturbation theory, and the conditions for the validity of the perturbation theory are discussed. The developed theory implies corrections in the conditions for electric resonance in a large electric field for a particle localized in the vicinity of the boundary inside the medium with a larger permittivity. We have discussed the conditions required for the experimental confirmation of these corrections.

APPENDIX: DERIVATION OF EQ. (1)

In a locally isotropic medium electric displacement $\mathbf{D}$, permittivity ϵ and a strength of the electric field $\mathbf{E}$ are determined by the following equations:

$$\nabla \cdot \mathbf{D} = 4\pi\gamma_0(\mathbf{r}), \quad \mathbf{D} = \epsilon\mathbf{E}, \quad \nabla \times \mathbf{E} = 0. \quad (\text{A1})$$

From the last equation we find that $\mathbf{E} = -\nabla\varphi$. Let us define the energy of the electromagnetic field as a functional, W , such for a given distribution of free charges $\gamma_0(\mathbf{r})$ and without free charges at infinity, the condition for extremum of this functional, $\delta W/\delta\varphi = 0$, implies the first equation in Eq. (A1). The condition for absence of charges at infinity reads:

$$\int \nabla \cdot (\varphi \mathbf{D}) d\mathbf{r} = \int (\varphi \mathbf{D}) ds = 0, \quad (\text{A2})$$

where integration in the last integral is performed over the infinitely far located surface. Using the second equation in Eq. (A1) and condition (A2) it can be easily shown that functional W can be written as follows:

$$W = \int \gamma_0(\mathbf{r}) \varphi(\mathbf{r}) d\mathbf{r} - \int \frac{\mathbf{E} \cdot \mathbf{D}}{8\pi} d\mathbf{r}. \quad (\text{A3})$$

Indeed,

$$\delta W = \int \gamma_0(\mathbf{r}) \delta\varphi(\mathbf{r}) d\mathbf{r} - \int \frac{\delta\mathbf{E} \cdot \mathbf{D}}{8\pi} d\mathbf{r} - \int \frac{\mathbf{E} \cdot \delta\mathbf{D}}{8\pi} d\mathbf{r},$$

and using Eq. (A2) in the case of a linear medium when $\epsilon(\mathbf{r})$ does not depend on $\mathbf{E}$, we obtain

$$\int \frac{\delta\mathbf{E} \cdot \mathbf{D}}{8\pi} d\mathbf{r} = \int \frac{\delta\varphi \nabla \cdot \mathbf{D}}{8\pi} d\mathbf{r}.$$

Then the second equation in Eq. (A1) for the case of a linear material yields

$$\int \frac{\mathbf{E} \cdot \delta\mathbf{D}}{8\pi} d\mathbf{r} = \int \frac{\mathbf{D} \cdot \delta\mathbf{E}}{8\pi} d\mathbf{r}.$$

Using the condition for extremum $\delta W/\delta\varphi = 0$ we obtain

$$\nabla \cdot \mathbf{D} = 4\pi\gamma_0(\mathbf{r}). \quad (\text{A4})$$

Using Eqs. (A2) and (A4) we can write the expression for energy W as

$$W = \frac{1}{2} \int \gamma_0(\mathbf{r}) \varphi(\mathbf{r}) d\mathbf{r} = \int \frac{\mathbf{E} \cdot \mathbf{D}}{8\pi} d\mathbf{r}. \quad (\text{A5})$$

The first equation in Eq. (A5) coincides with Eq. (1).

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