Thermal and electrodynamic effects in melting current-carrying conductors

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(Received 9 October 1995; accepted for publication 21 March 1996)

This work studies dynamics of melting in current-carrying conductors. It is shown that during equilibrium melting, when the dynamics of a phase-transition front are determined by a heat balance, there exists a single-valued correlation between the rate of inductance change and a temperature at the phase-transition front. It is demonstrated that, although surface melting begins at temperatures less than the melting temperature of the current-free conductor, corrugation of a phase-transition front occurs at this temperature. It is shown that the nucleation rate of nuclei flattened in the direction normal to a conductor's axis is negligibly small. Current dynamics at the stage of melting are analyzed for the case of a fixed external voltage. It is demonstrated that there exists an instability stage at which an excitation of the electric current occurs. Conditions for the experimental observation of the predicted phenomena are discussed. © *1996 American Institute of Physics.* [S0021-8979(96)02313-4]

I. INTRODUCTION

In our previous studies[1,2] we demonstrated that first-order phase transitions in current-carrying conductors are essentially different from phase transitions in current-free conductors. Phase transitions in current-carrying conductors are accompanied by a number of new phenomena, e.g., shift of the critical temperature of a material and its dependence upon the magnitude of the electric current,[1,2] reduction of the melting temperature during surface melting,[3,4] occurrence of the temperature range where both phases are thermodynamically metastable and the material phase composition becomes heterogeneous.[1,2]

The cause of all these effects is that electric currents prevent those processes which reduce the inductance of the system. In order to support melting at the stage when the inductance of the system is reduced, it is necessary to overheat the phase transition front in comparison with its melting temperature in a current-free state. Similarly, at the stage of melting when the conductor's inductance increases, melting can occur in an underheated state.

Under high electric currents, the rate of inductance change $\dot{L}$ becomes comparable with the ohmic resistance R and the total damping resistance $\dot{L} + R$ is essentially different from R.

The goal of this study is to develop a zero-dimensional model which allows us to investigate thermal and electric phenomena in conductors at different stages of surface melting. Here, similar to our previous investigations,[1–4] the inductance effects caused by a hydrodynamic flow are neglected. Since, during melting, the density changes insignificantly, i.e., $\Delta\rho/\rho \sim 10^{-2}$, the induced flow velocity is negligibly small, and inductance effects caused by the induced flow can be neglected. All the inductance effects in this study, as in our previous investigations, are considered to be caused by motion of the front with a jump in electric conductivity. The velocity of this front is determined by the equations of thermodynamics and chemical kinetics which describe the phase transition. In our previous investigation,[4] we demonstrated that the contribution of the magnetostatic effects can also be neglected. All these simplifications allow us to apply a circuit formulation in order to describe the system rather than to use a continuum electromechanical formulation. We show that when a strong superheating of a melting front is feasible, a sharp increase in the magnitude of electric current due to a high rate of the decrease of inductance may occur at the final stage of an equilibrium melting.

Since a strong superheating is required at the final stage of surface melting, the phase-transition front becomes unstable with respect to a corrugation instability. Internal pan-shaped nuclei of a new phase which become supercritical at considerably lower superheat, provided that they are flattened in the direction of a conductor's axis, are formed at this stage of melting. In this study, we determine the minimum superheat which is required for corrugation of a melting front and for formation of internal nuclei in current-carrying conductors. A rate of formation of the nuclei flattened in the direction of a conductor's axis is determined.

The article is organized as follows. Thermodynamic aspects of the problem and zero-dimensional model of melting are presented in the second section. Detailed analysis of non-equilibrium melting for the case when the characteristic chemical and thermal relaxation times of a phase-transition front towards thermodynamic equilibrium are close, is presented. In the second section we also present a theory of equilibrium melting when the chemical relaxation of a phase-transition front proceeds much faster than its thermal relaxation.

Electrodynamic aspects of the problem are considered in the third section. An ''adiabatic'' model is developed which allows us to study the electrodynamics when the inductance changes. Direct relation between the superheat temperature and inductance change rate was determined in the framework of the adiabatic model.

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 0021-8979/96/80(1)/38/8/$10.00

In the fourth section of the article the minimum temperature required for internal melting is determined. We show that the superheat which is required for formation of the flattened pan-shaped nuclei is lower than that which is required for the nuclei with other shapes. However, the rate of formation of such flattened nuclei turns out to be negligibly small due to surface tension.

In the fifth section of this study we investigate current dynamics in melting conductors. It is demonstrated that there exist critical currents or voltages at which a sharp increase of the magnitude of electric current can occur at the final stage of surface melting.

II. HEAT BALANCE AND TEMPERATURE OF A MELTING FRONT IN CURRENT-CARRYING CONDUCTORS

A system of equations governing phase transitions in current-carrying conductors is formulated under the assumption that, without phase transition, the characteristic time of an electric current variation τ_I is much larger than the characteristic time of electric current diffusion τ_m, i.e., $\tau_I \gg \tau_m = (4\pi\sigma\rho_0^2)/c^2$, where σ is the electric conductivity, c is the speed of light, and ρ is the radius of the conductor. The latter inequality allows to neglect skin effects in a conductor.

Assume also that the characteristic spatial scale of a conductivity variation is the smallest characteristic length in the problem. Then the conductivity can be represented as a step function

$$\sigma = \sigma_S \theta_S + \sigma_L \theta_L, \tag{1}$$

where σ_S and σ_L are conductivities of solid and liquid phases, respectively. Functions θ_S and θ_L equal unity inside domains occupied by phases ''S'' and ''L,'' respectively, and vanish outside these domains.

Hereafter we use volume averaged variables f_i which are determined as follows:

$$\overline{f_i} = \frac{1}{V_i} \int_{V_i} f(\vec{r},t)\, dV, \tag{2}$$

where index i equals S or L.

The heat conduction equation with a Joule heat source and with standard boundary conditions[4,5] yields the heat balance equation during phase transition

$$(1-\beta)(c_S \dot{\overline{T}}_S - Q_S) + \beta(c_L \dot{\overline{T}}_L - Q_L) = \dot{\beta}(c_S \overline{T_S} - c_L \overline{T_L} - \lambda). \tag{3}$$

In Eq. (3) β and $(1-\beta)$ are volume fractions of phases L and S, respectively, T_S and T_L are temperatures averaged using Eq. (2), $Q_i = (j_1^2 v)/\sigma_i$, v is a specific volume which is assumed the same for both phases, c_S and c_L are specific heats of phases at constant pressure per unit mass of material, λ is a latent heat of phase transition and the dot denotes time derivative.

Since β varies monotonically with time, Eq. (3) can be rewritten as

$$\dot{\beta} = \frac{(1-\beta)Q_S(\beta) + \beta Q_L(\beta)}{\tilde{\lambda}(\beta)}, \tag{4}$$

where

$$\tilde{\lambda}(\beta) = \lambda + \frac{\partial}{\partial \beta}\left[(1-\beta)c_S \overline{T_S}(\beta) + \beta c_L \overline{T_L}(\beta)\right]. \tag{5}$$

When the temperature of the phase-transition front T_F remains constant during melting, the following condition can be imposed

$$\frac{\partial}{\partial \beta}\left[(1-\beta)c_S \overline{T_S}(\beta) + \beta c_L \overline{T_L}(\beta)\right] = 0. \tag{6}$$

When the temperature of the phase-transition front, T_F, is constant, condition (6) becomes an identity in the limit of an ideal thermal conductivity when $\overline{T_S} = \overline{T_L}$ and $c_S = c_L$. If the temperature of phase-transition front changes the condition (6) is unphysical. However, in the approximation of an infinitely high thermal conductivity, we can assume that

$$\overline{T_S} = \overline{T_L} = \overline{T_F} \tag{7}$$

and rewrite Equation (5) in the following form:

$$\tilde{\lambda} = \lambda + \frac{\partial}{\partial \beta}\, \overline{c}\, \overline{T_F}, \quad \overline{c} = c_S(1-\beta) + c_L \beta. \tag{8}$$

Equation (8) determines the renormalization (change) of the latent heat of phase transition caused by the change of the phase transition front temperature. In the case of infinitely high thermal conductivity which is considered here, this temperature coincides with the temperature of the whole conductor.

In a case of a nonequilibrium phase transition, the heat conduction or heat balance Eq. (4) must be supplemented with the macroscopic kinetic equation (see, e.g., Ref. 6, Chap. 12, Sec. 101)

$$\dot{\beta} \propto -\frac{\partial \Phi}{\partial \beta}.$$

The equation for the free energy $\Phi(\beta)$ of the conductor in the regime when the magnitude of electric current is constant, reads (see, e.g., Ref. 7, Chap. 4, Sec. 33)

$$\Phi(\beta) = \Phi_0(\beta) - \frac{L(\beta) I^2}{2},$$

where Φ_0 is the Gibbs potential of a current-free conductor, L is inductance and I is the total electric current. At this stage we neglect the contribution of the surface tension. Then the equation for relaxation of the phase-transition front reads

$$\dot{\beta} = \frac{u}{T_0}\left(\mu_S - \mu_L + \frac{I^2}{2N_0}\frac{\partial L}{\partial \beta}\right), \tag{9}$$

where N_0 is the number of moles in the relaxing system, $u = N_0/\tau$ is a relaxation rate ($[u]$=mol/s) which is a constant in the theory developed here, T_0 is the phase-transition temperature in a current-free conductor, and μ_S and μ_L are chemical potentials of the solid and liquid phases, respectively.

Hereafter we present $L(\beta)$ in the following form:

$$L(\beta)=L_0-\frac{l}{c^2}f(\beta,\kappa), \tag{10}$$

where the function $f(\beta,\kappa)$ is determined by the geometry of the conductor and the geometry of the melting zones, β is the fraction of volume melted, $\kappa=\sigma_L/\sigma_S$ is the ratio of conductivities at the phase-transition front, and L_0 is inductance of a homogeneous unlayered conductor.

As can be seen from Eq. (9), the equilibrium temperature of the phase-transition front is determined by the equation

$$\mu_S-\mu_L-p_m v\frac{\partial f}{\partial\beta}=0, \tag{11}$$

where $p_m=I^2/(p\rho_0^2c^2)$ is a magnetic pressure and ρ_0 is the radius of the conductor which is assumed here to be of a cylindrical shape. Linear extrapolation of Eq. (9) yields the equation for the instant temperature of a phase-transition front $T_F(\beta(t),t)$

$$T_F[\beta(t),t]=T_F^0(\beta)+\frac{T_0\dot{\beta}}{u\Delta S},$$

where $\Delta S=S_L-S_S$ is the difference of specific entropies in the liquid and solid states at temperature $T_F^0(\beta)$ which is determined from Eq. (11).

Hereafter we assume that the following conditions, which assure the fast chemical relaxation of a phase-transition front, are satisfied

$$\frac{T_0\dot{\beta}}{u\Delta S}\ll T_F^0(\beta),\quad \frac{\dot{\beta}}{u\Delta S}\ll\frac{p_m v}{\lambda_0},$$

where λ_0 is the latent heat of transition at the phase-transition temperature of a current-free conductor T_0. When the latter conditions are satisfied, $T_F(\beta(t),t)=T_F^0(\beta)$ and Eq. (11) yields

$$\frac{T_F^0(\beta)}{T_0}=1+\frac{p_m v}{\lambda_0}\frac{\partial f}{\partial\beta}. \tag{12}$$

Equation (12) allows one to express the equilibrium phase-transition temperature in a current-carrying conductor through the phase-transition front temperature in a current-free conductor T_0. In the following section we study current dynamics in a melting conductor. The main goal of this analysis is the investigation of the effect of the inductance change upon the current dynamics. In order to achieve this goal we must determine the inductance of a conductor with a conductivity jump at a moving front.

III. INDUCTANCE OF A HETEROGENOUS CONDUCTOR WITH A MOVING PHASE-TRANSITION FRONT AND EQUATION FOR CURRENT DYNAMICS

Equation for current evolution in a circuit with varying conductivity reads

$$L\dot{I}+I(R+\dot{L})=U, \tag{13}$$

where R is an ohmic resistance of a conductor undergoing phase transition, U is an effective electric driving force which can be expressed as an external electric driving force with ohmic resistance, capacity, and inductance, and L is an instant inductance of a conductor at a given location of the phase-transition front.

In the adiabatic approximation $\dot{L}$ can be determined through the rate of change of geometrical parameters characterizing a location of a phase-transition front inside a conductor. In order to validate such an approach consider a straight conductor with length l and radius ρ_0 which is subjected to a time-dependent voltage $U(t)$. In this case the circuit Eq. (13) is replaced by the following equation[8]:

$$\frac{U(t)}{l}=\frac{j(\rho_0)}{\sigma_0}+p\rho_0\left(\frac{\partial E}{\partial\rho}\right)_{\rho=\rho_0}, \tag{14}$$

where $\sigma(\rho_0)$ and $j(\rho_0)$ are an electric conductivity and electric current density at the conductor's surface, respectively, and p is a geometry factor. In the case of an exploding wire $p=\ln(b/\rho_0)$, where b is the radius of a metallic cylinder in which the electric explosion is performed (for details see Ref. 8).

Equation (14) provides a boundary condition to the problem of determining the distribution of electric current across the conductor $j(\rho,t)$ subjected to a time-dependent voltage. The first and the second term in this equation describe contributions due to the ohmic resistance of the conductor's surface and inductive voltage due to temporal variation of an external magnetic flux, respectively. The second term is determined by the rate of change of the total electric current in the conductor. Provided the magnitude of the total electric current I is given, motion of the phase-transition front does not affect the second term, i.e.,

$$\rho_0\left(p\,\frac{\partial E}{\partial\rho}\right)_{\rho=\rho_0}=L_0\dot{I}, \tag{15}$$

where L_0 is an inductance of a homogeneous conductor provided that the skin effect is small. The voltage associated with the inductance change due to the motion of the phase-transition front is given by the first term in Eq. (14). In order to obtain an explicit expression for this term we assume that the liquid phase is located near the surface and consider the average values of electric currents j_L and j_S [see formula (1)]. Then, assuming that these values are close to the magnitudes of the electric current at the surface and at the axis of the conductor, respectively, and using the equation $\nabla\times\mathbf{E}=-(1/c)(\partial\mathbf{H}/\partial t)$, we find that

$$\frac{\overline{j_L}}{\sigma_L}-\frac{\overline{j_S}}{\sigma_S}=\frac{1}{c}\int_0^{\rho_0}\frac{\partial H}{\partial t}\,d\rho\equiv\frac{1}{c}\frac{1}{l}\frac{\partial\Phi_{\text{in}}}{\partial t}. \tag{16}$$

The magnetic flux Φ_{in} is a function of $\overline{j_L}$, $\overline{j_S}$ and β, and it can be determined by solving the equation $\nabla\times\mathbf{H}=[(4\pi)/c]\,\mathbf{j}$. The function $\partial\Phi_{\text{in}}/\partial t$ must be calculated for the given value of the total electric current I. Therefore

$$\frac{\partial\Phi_{\text{in}}}{\partial t}=F\left(\overline{j_L},\overline{j_S},\frac{\partial\overline{j_L}}{\partial\beta},\frac{\partial\overline{j_S}}{\partial\beta},\overline{j},\beta(\beta)\right),$$

where the function F is determined by the geometry and

$$\overline{j} = \frac{I}{\pi r_0^2} = \overline{j_L}\beta + \overline{j_S}(1-\beta). \tag{17}$$

Using Eqs. (16) and (17) in order to eliminate $\overline{j_S}$, we arrive at the differential equation for j_L which has the following form:

$$F_L\left(\overline{j_L}, \frac{\partial \overline{j_L}}{\partial \beta}, \beta, \dot{\beta}(\beta), \overline{j}\right) = 0. \tag{18}$$

The explicit expression for F_L depends upon the geometry of the problem. Determining the solution of this equation $j_L = j_L[\beta, \dot{\beta}(\beta), j]$ and taking into account Eqs. (14) and (15) allows to derive an equation describing the time evolution of the electric current I.

However, there are some difficulties in accomplishing the above program. In this study we have chosen a simpler method which employs the adiabatic approximation and is elaborated in the following:

Since $\partial \Phi_{\text{in}}/\partial t \propto \dot{\beta}$, in the zeroth approximation in this parameter $\dot{\beta}$, Eq. (16) yields

$$\frac{\overline{j_L}}{\sigma_L} = \frac{\overline{j_S}}{\sigma_S} = \frac{\overline{j}}{\overline{\sigma}}, \quad \overline{\sigma} = \sigma_L \beta + \sigma_S(1-\beta). \tag{19}$$

Substituting Eqs. (15) and (19) into Eq. (14) we arrive at the equation for a circuit (13) with $L = L_0$ and $\dot{L} = 0$.

In the same approximation it is possible to determine $\Phi_{\text{in}} = \Phi_{\text{in}}(j, \beta, \kappa)$, where as in formula (10) $\kappa = \sigma_L/\sigma_S$. Since in this approximation Φ_{in} varies linearly with electric current I, we can define internal inductance L_{in} by the formula $\Phi_{\text{in}} = L_{\text{in}}(\kappa, \beta) I$. Substituting the latter formula into Eq. (16) and using the zeroth approximation for j_S/σ_S in the parameter $\dot{\beta}$ (19), we find that

$$\frac{\overline{j_L}}{\sigma_L} = \frac{\overline{j}}{\overline{\sigma}} + \frac{I\dot{L}_{\text{in}}}{l}. \tag{20}$$

Equations (14), (15), and (20) yield Eq. (13) ($L = L_{\text{in}} + L_0$). Since formally this approach is not self-consistent its accuracy cannot be evaluated. The more consistent analysis based upon solving Eq. (18) essentially does not simplify the solution of the problem in comparison with the analysis of the system with distributed parameters. Moreover, the latter approach considerably complicates the physical analysis since it does not allow to define inductance as a parameter depending only on the geometry of a conductor and on an electric conductivity distribution. Hereafter we use the above described adiabatic approximation which is often used in boundary value problems with moving boundaries[5] in spite of the lack of its formal justification.

Thus the inductance L is determined by Eq. (10). Then since $\dot{L} = (\partial L/\partial \beta)\dot{\beta}$ and using Eq. (4) allows to determine a relation between the rate of inductance change $\dot{L}$ and ohmic resistance R

$$\frac{\dot{L}}{R} = -\frac{p_m v}{\tilde{\lambda}} \frac{\partial f}{\partial \beta}. \tag{21}$$

Combining Eqs. (12) and (21) yields

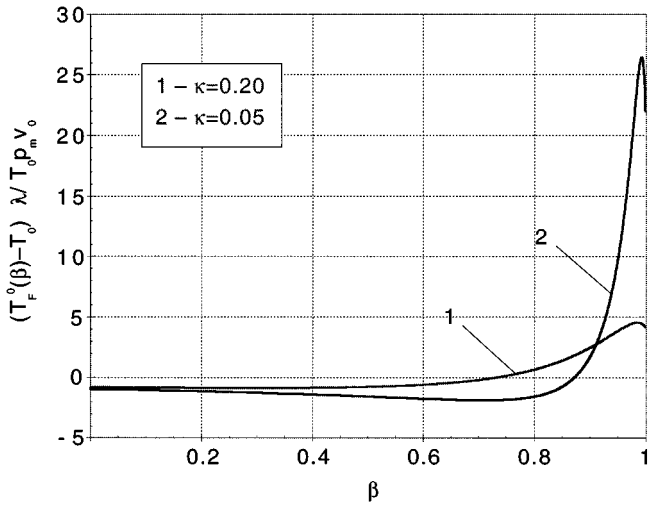

FIG. 1. Temperature at a phase-transition front vs volume fraction of a liquid phase.

$$\frac{\dot{L}}{R} = -\frac{\lambda_0}{\tilde{\lambda}} \frac{T_F(\beta) - T_0}{T_0}, \tag{22}$$

where $\tilde{\lambda}$ is determined by formula (5) and the magnitude of λ in the first approximation in a parameter $p_m v/\lambda_0$ is given by the following formula[2]:

$$\lambda = \lambda_0 + (c_L^0 - c_S^0) \frac{p_m v}{\lambda_0} T_0 \frac{\partial f}{\partial \beta}, \tag{23}$$

where c_L^0 and c_S^0 are specific heats of corresponding phases at temperature T_0.

Equation (22) implies that if inductance increases during melting, then the temperature at the phase-transition front $T_F(\beta) < T_0$, i.e., melting can occur at temperatures lower than the melting temperature of a current-free conductor. The latter phenomenon was discussed for a special case in,[3,4] where we determined an initial temperature of a surface melting $T_F(0)$ and showed that this temperature is lower than T_0.

The dependence of

$$\frac{\lambda_0[T_F(\beta) - T_0]}{T_0^2 p_m v_0} \equiv \frac{\partial f}{\partial \beta}$$

on β in the case of a cylindrical conductor with a concentric cylindrical phase-transition front is shown in Fig. 1. In this case the formula for $f(\beta)$ reads[3,4]

$$f(\beta) = \xi^2(1-\beta)[(1-\beta)\ln(1-\beta) + (1+a)\beta], \tag{24}$$

where $a = 1/(\kappa - 1)$ and $\xi = 1/(\beta + a)$. In a range $\kappa \ll 1 - \beta \ll 1$, $f(\beta) = \ln(1-\beta)$ and in a range $1 - \beta \ll \kappa \ll 1$,

$$f(\beta) = \left(\frac{1-\beta}{\kappa}\right)^2 \ln(1-\beta) - \frac{1-\beta}{\kappa} + 2\left(\frac{1-\beta}{\kappa}\right)^2.$$

Figure 1 and Eq. (12) show that when the conductivity changes strongly, i.e., $\kappa \ll 1$, high superheating of the phase-transition front is required in order to attain surface melting inside a conductor. However, strong superheating of the phase-transition front inside the conductor initiates an alternative process of the internal melting. In Ref. 3 we demonstrated that the internal melting is inhibited inside a conduc-

tor except for the case of formation of nuclei flattened in the direction normal to a conductor's axis. Therefore the pan-shaped nuclei could be formed if it were not for a surface tension that prevents the nucleus from flattening. Since the surface energy depends upon the nucleus shape, there exists an optimal shape and corresponding minimum superheating temperature for a given volume. In the next section we determine the optimal shape of nuclei which are supercritical for a given volume under the minimum superheat and calculate the time required for their formation.

IV. MINIMUM TEMPERATURE OF INTERNAL MELTING AND NUCLEATION RATE IN CURRENT-CARRYING CONDUCTORS

In order to determine a minimum temperature for internal melting, we can use a version of Eq. (11) which takes into account the work of the surface tension forces. Thus we obtain for the minimum overheat required for the internal melting

$$\frac{\Delta T}{T_0}=\frac{v(\tilde{p}_m+p_s)}{\lambda_0}, \tag{25}$$

where $\tilde{p}_m$ is an effective pressure caused by the variation of the magnetic flux during nucleus formation and p_s the additional pressure caused by surface tension.

Equation (25) determines the minimum temperature which is required for formation of an overcritical nucleus with a given volume. In the derivation of Eq. (25) we neglected the energy of elastic deformation due to formation of a liquid nucleus[9] since elastic pressure is negligibly small when compared with a characteristic pressure $p_m \sim 1$ kbar in electrically exploding conductors.[10] Without losing generality, we can solve this problem for the ellipsoid of revolution with an axis parallel to the axis of the conductor and use directly the results of our previous study.[3]

Assume that a nucleus is formed inside a solid phase with conductivity σ_S and volume fraction $1-\beta$ and that the electric conductivity of a liquid nucleus σ_a equals the conductivity of a liquid shell with a volume fraction β, i.e., $\sigma_a=\sigma_L$. Under these assumptions the formula for $\tilde{p}_m$ reads[3]

$$\tilde{p}_m=p_m\tilde{\xi}\Gamma_l(\beta), \tag{26}$$

where

$$\tilde{\xi}=\frac{1-\kappa}{2(\kappa\beta_z+1)}, \quad \beta_z=\frac{n_z}{1-n_z},$$

n_z is the depolarization factor of an ellipsoid of revolution which is an explicit function of its eccentricity ϵ.[7] The formula for function $\Gamma_l(\beta)$ reads

$$\Gamma_l(\beta)=\frac{2a}{a+\beta}\left[\ln\left(\frac{4z_a(l-z_a)}{\rho_0^2}\right)+1-\frac{\rho_a^2}{\rho_0^2}+\xi(\beta)\left(\beta\,\frac{\rho_a^2}{\rho_0^2}+(1-\beta)\ln(1-\beta)\right)\right], \tag{27}$$

where z_a and ρ_a are cylindrical coordinates of a center of mass of the nucleus.

Hereafter we assume that the corrugation of a phase-transition front corresponds to the formation of ellipsoids with coordinates $\rho_a=\rho_0(1-\beta)$. Thus Eqs. (25)–(27) can be used for determining the minimum temperature required for a corrugation of the phase-transition front and for estimation of the minimum temperature required for nucleus formation in the vicinity of a conductor's axis. For given values of electric current I, liquid phase volume fraction β corresponding to a surface melting and parameter κ, the magnitude of the superheating caused by pressure $\tilde{p}_m$ is determined by parameter $\tilde{\xi}(n_z)$, where $0\leqslant n_z\leqslant 1$. In the case when parameter $\kappa<1$ which is considered in this study, $\tilde{p}_m>0$ and attains its minimum value when $n_z\to 1$ and $\beta_z\kappa\gg 1$.

Since in a range $\beta_z\kappa<1$, $\tilde{p}_m$ is not sensitive with respect to the depolarization factor n_z, nucleus deformation does not reduce the required superheating. Provided that the volume of these nuclei is sufficiently large, so that $\tilde{p}_m>p_s$, the formula for the minimum overheat temperature reads[2,3]

$$\frac{\Delta T}{T_0}\cong\frac{\tilde{p}_m V}{\lambda_0}. \tag{28}$$

In systems with high magnitudes of electric current $\Delta T/T_0\cong 10^{-1}$. When the nuclei are strongly flattened in the direction of the conductor's axis $n_z\cong 1-[\pi/(2\epsilon)]$, where $\epsilon=[(r_\perp^2/r_\parallel^2)-1]^{1/2}\gg 1$ and $r_\parallel$ is the half length of an elliptical cross section and $r_\perp$ is the radius of its circular cross section. For these nuclei the formula for $\tilde{p}_m$ reads

$$\tilde{p}_m=\frac{\gamma_m}{\epsilon}, \quad \gamma_m=p_m\Gamma_l(\beta)\,\frac{1-\kappa}{\kappa}\,\frac{\pi}{4}. \tag{29}$$

Surface tension pressure prevents the unlimited increase of ϵ and corresponding temperature decrease. In a given volume V for $\epsilon\gg 1$, the formula for a surface tension pressure reads

$$p_s=\gamma_s\epsilon^{2/3}, \quad \gamma_s=\left(\frac{4\pi}{3}\right)^{1/3}\frac{\alpha}{V^{1/3}}, \tag{30}$$

where α is the coefficient of surface tension.

A minimum superheating occurs for an optimal nucleus shape ϵ^* which is determined by the following equation:

$$\left(\frac{\partial p_s}{\partial\epsilon}\right)_V+\left(\frac{\partial p_m}{\partial\epsilon}\right)_V=0.$$

The latter equation yields

$$\epsilon^*=\left(\frac{3}{2}\,\frac{\gamma_m}{\gamma_s}\right)^{3/5}. \tag{31}$$

Combining Eqs. (25), (29)–(31) we arrive at the following formula for the minimum overheat temperature:

$$\frac{\Delta T_{\min}}{T_0}=\frac{5\times 2^{1/2}}{6^{3/5}}\,\frac{v}{\lambda_0}\,\gamma_s^{3/5}\gamma_m^{2/5}. \tag{32}$$

The above analysis shows that deformation of the nuclei becomes an effective mechanism for phase-transition temperature reduction only if $\epsilon\kappa\gg 1$. According to the formula (31) the latter condition can be met only for large volumes, i.e.,

$$V^{1/3} \gg \frac{\alpha}{p_m \Gamma_l \kappa^{2/3}}.$$

From Eq. (32) it follows that $\Delta T_{\min} \propto T_0 V^{-1/5}$. Therefore the supercriticality of the nuclei with large volumes can be attained at negligibly low superheating. Obviously this does not imply that internal melting at low superheating occurs in reality. This mode of melting occurs under the condition that its characteristic time must be of the order of a characteristic time of a surface melting inside a conductor. In order to analyze this problem we estimate the characteristic time of internal melting as a function of a superheating taking into account the possibility of formation of flattened nuclei.

The probability of nucleus formation is given by the following formula:

$$P \propto \exp\left\{ -\frac{\Delta\Phi^*}{kT} \right\},$$

where $\Delta\Phi^*$ is the work of formation of a flattened nucleus with shape and size determined by Eq. (25) at a given superheating $\Delta T/T_0$. Independent of the nucleus shape, the work of its formation $\Delta\Phi^*$ can be represented as $\Delta\Phi^* = W^*/3$, where W^* is a surface energy of a critical nucleus. When a nucleus is an ellipsoid of revolution $W^* = \alpha S = \alpha F(\epsilon)(V^*)^{2/3}$, where V^* is the volume of a critical nucleus and $F(\epsilon)$ is a geometrical factor. Using (25), the surface energy of a critical nucleus W^* can be written as

$$W^* = \frac{4}{9}\,\alpha^3\,\frac{F^3(\epsilon)}{(\theta - \tilde{p}_m)}, \quad \theta = \frac{\Delta T}{T_0}\,\frac{\lambda}{v_0}.$$

Consider first a case when superheating is very small so that conditions for supercriticality are met for a large nucleus which can overcome the ponderomotive pressure $\tilde{p}_m$ by their own deformation. Since such nuclei are strongly flattened, $\epsilon \gg 1$,

$$F(\epsilon) = \frac{3}{2}\left(\frac{4\pi}{3}\right)^{1/3} \epsilon^{2/3} \quad \text{and} \quad W^* = \frac{2\pi\epsilon^4\alpha^3}{(\epsilon\theta - \gamma_m)^2}.$$

The latter formula implies that in the range $\epsilon \gg 1$ the minimum W^* is attained at $\epsilon = \epsilon_0 = 2\gamma_m/\theta$. Therefore the minimum magnitude of a surface energy W^* at a given superheating is given by the following formula:

$$W^*_{\min} = \frac{32\pi\alpha\gamma_m^2}{\theta^4} = 2W_0^*\,\frac{\gamma_m^2}{\theta^2}, \tag{33}$$

where $W_0^* = (16\pi\alpha^3)/\theta^2$ is the surface energy of a critical nucleus in a case of internal melting in a current-free conductor.

Since usually the magnitude W_0^*/T_0 is rather large (see Ref. 11, Chap. 7, § 3), the condition $\gamma_m/\theta \geqslant 1$ is sufficient to provide a zero nucleation rate.

Equation (28) shows that when $\gamma_m/\theta \ll 1$ the nucleus becomes supercritical already for $\epsilon \ll 1$. In this case the nucleation rate of the spherical nuclei is maximum, $F(\epsilon) = 3[(4\pi/3)]^{1/3}$ and $W^* = (16\pi\alpha^3)/(\theta - \tilde{p}_m)$.

Thus, although supercritical nuclei exist at any superheating, internal melting occurs only when $\theta > \tilde{p}_m$. Therefore the earlier obtained results[1–3] which did not take into account the flattening of the nuclei, are still valid when this flattening is taken into account. Since internal melting begins at a relatively strong superheating $\Delta T/T_0 \sim 10^{-1}$, it proceeds in an explosive regime similar to an explosive boiling of the superheated liquid.[12]

While compliance with the condition (28) is a necessary condition of an internal melting, corrugation of the surface of the formed melt occurs practically immediately upon reaching the melting temperature T_0. At this temperature, as can be seen from Eqs. (21), (22), and Fig. 1, the volume fraction of a liquid melt is quite high due to surface melting. Equations (31) and (32) determine the optimal parameters of the pan-shaped perturbations formed at the internal surface of the melt.

Obviously such a corrugation mechanism also causes the transversal stratification of phase composition and must be taken into account together with other mechanisms which cause a similar effect.[13,14]

V. PECULIARITIES OF ELECTRIC CURRENT DYNAMICS IN MELTING CONDUCTOR

Inductance variation during melting changes the current dynamics, and under certain conditions these changes can be significant. Inspection of Eq. (21) shows that $R + L$ can become negative. In the case of a cylindrical conductor this can occur inside a conductor in the vicinity of a point where $\partial f/\partial\beta$ is maximum. For small values of κ the magnitude of $\partial f/\partial\beta$ is large and the effect of ''negative'' damping is strong. Equation (22) shows that for attaining negative damping, the magnitude of superheating must be large, $(T_F - T_0)/T_F \geqslant 1$. Naturally, under these conditions the corrugation of the phase-transition front and internal melting may strongly inhibit this effect. Nevertheless, the effect of negative damping has certain qualitative peculiarities which warrant more detailed analysis.

Using formulas (4) and (21) Eq. (13) can be rewritten as follows:

$$L_0\dot{I} + R_0 I + R(\beta) I\left(1 - \frac{I^2}{I_*^2}\,\frac{\partial f}{\partial\beta}\right) = U, \tag{34}$$

where L_0 is inductance of a circuit with a melting conductor, R_0 is an ohmic resistance external with respect to a melting region, $R(\beta)$ is the electric resistance of a melting region associated with its inductance change according to Eq. (21) and $I_* = (\pi r_0^2 c^2 \tilde{\lambda})/v$.

Hereafter we neglect the inductance of a melting region and the dependence of the external resistance R_0 upon the electric current. Therefore the values L_0 and R_0 are assumed constant. We assume also that I_* does not depend upon β, i.e., that $\tilde{\lambda}(\beta) = \lambda_0$.

In the new notation Eq. (4) can be rewritten as

$$\dot{\beta} = \frac{4I^2}{\tau_m I_*^2}\,\frac{1}{1 - \beta(1-\kappa)}. \tag{35}$$

Combining Eqs. (34) and (35) yields

$$x^2\,\frac{\partial x}{\partial\beta} = m_0 - m_1 x + m_3 x^3, \quad x = \frac{I}{I_*}, \tag{36}$$

where

$$m_0=\frac{U}{4L_0}\frac{\tau_m}{I_*}[1-\beta(1-\kappa)]>0,$$

$$m_1=\frac{R_0}{4L_0}\tau_m[1-\beta(1-\kappa)]+\frac{l}{L_0c^2}>0,$$

$$m_3=\frac{l}{L_0c^2}\frac{\partial f}{\partial\beta},$$

and l is the length of a melting region. Current dynamics are determined by Eq. (36) and differ qualitatively depending not only upon the ratios of parameters

$$\frac{m_0}{m_1}=\frac{U}{I^*R_0}\frac{R_0}{R_0+R(\beta)},\quad \frac{m_3}{m_1}=\frac{\partial f}{\partial\beta}\frac{R(\beta)+R_0}{R(\beta)},\tag{37}$$

where $R(\beta)=R(0)[1-\beta(1-\kappa)]^{-1}$, but also upon the magnitude of electric current at the moment when melting begins $\beta=0$.

Therefore in order to elucidate the behavior of the current dynamics we restrict our analysis to the most general regularities.[15] Note first that with respect to a current dynamics $x(\beta)$, the interval $0\leqslant\beta\leqslant1$ can be divided into four characteristic subintervals with different stationary curves $x_s(\beta)$ whereby the rate of electric current variation $\partial x/\partial\beta=0$.

The first interval $0\leqslant\beta\leqslant\beta_1$ is determined by a condition $\partial f/\partial\beta<0$. At this interval the damping resistance increases and there exists only one stationary attracting curve $x_1(\beta)$, i.e., $\partial x(\beta)/\partial\beta<0$ when $x(\beta)>x_1(\beta)$ and $\partial x(\beta)/\partial\beta>0$ when $x(\beta)<x_1(\beta)$. The formula for $x_1(\beta)$ then reads

$$x_1(\beta)=\left(\frac{m_1}{3|m_3|}\right)^{1/2}y_1(d),$$

$$y_1(d)=\left[\frac{d}{2}+\left(\frac{d^2}{4}+1\right)^{1/2}\right]^{1/3}-\left[\left(\frac{d^2}{4}+1\right)^{1/2}-\frac{d}{2}\right]^{1/3},$$

where

$$d(\beta)=\frac{3^{3/2}m_0|m_3|^{1/2}}{m_1^{3/2}}=3^{3/2}\left(\frac{\partial f}{\partial\beta}\right)^{1/2}\frac{U}{R_0I_*}\frac{R_0R^{1/2}(\beta)}{(R(\beta)+R_0)^{3/2}}.$$

In this interval $\partial x_1(\beta)/\partial\beta<0$. The characteristic behavior of solution $x(\beta)$ in this region is determined by the following conditions. When sign $\partial x/\partial\beta=$sign $(\partial x_1/\partial\beta)$, the curves $x(\beta)$ and $x_1(\beta)$ do not intersect. In the opposite case, the curves $x(\beta)$ and $x_1(\beta)$ may intersect and derivative $\partial x/\partial\beta$ changes its sign. However, except for the latter very special situation, the characteristic behavior of electric current at this interval is to decrease without intersecting the curve $x_1(\beta)$.

In the vicinity of the point β_1 where $\partial f/\partial\beta=0$, with the accuracy of $\sim d^2$

$$x_1(\beta)=\frac{m_0(\beta)}{m_1(\beta)}.\tag{38}$$

The next interval $\beta_1<\beta<\beta_2$ is determined by conditions $\partial f/\partial\beta\geqslant0$ and $d\leqslant2$. At this interval there exist three stationary curves $\overline{x_1}(\beta)<\overline{x_2}(\beta)<\overline{x_3}(\beta)$ whereby at their intersection $\partial x/\partial\beta=0$. The curves $\overline{x_1}(\beta)$ and $\overline{x_3}(\beta)$ are repelling while the curve $\overline{x_2}(\beta)$ is an attractor. Formulas for the solutions $\overline{x_1}(\beta)$, $\overline{x_2}(\beta)$, and $\overline{x_3}(\beta)$ read

$$\overline{x_1}(\beta)=\left(\frac{m_1}{3m_3}\right)^{1/2}y_i(\beta),\quad i=1,2,3,$$

$$\overline{y_1}(\beta)=-(\cos\varphi+\sqrt{3}\sin\varphi),$$

$$\overline{y_2}(\beta)=\sqrt{3}\sin\varphi-\cos\varphi,$$

$$\overline{y_3}(\beta)=2\cos\varphi,$$

where $\varphi(\beta)=\frac{1}{3}\cos^{-1}(-d/2)$, $\pi/6\leqslant\varphi(\beta)\leqslant\pi/3$.

For small values of d with the accuracy of order $\sim d^2$

$$\overline{x_2}(\beta)=\frac{m_0(\beta)}{m_1(\beta)}$$

and it continues to decrease at the initial stage until $\partial f/\partial\beta$ remains so small that a negative contribution into a damping resistance caused by inductance decrease cannot compensate the increase of ohmic resistance caused by melting. At the final stage of melting $\overline{x_2}(\beta)$ increases. Thus, in contrast to $\overline{y_2}(\beta)$, which increases monotonically, $\overline{x_2}(\beta)$ has a minimum at the interval $\beta_1<\beta<\beta_2$.

The point β_2 where condition $d=2$ is satisfied is the critical point of the current dynamics. At this point β_2 the repelling curve $\overline{x_3}(\beta)$ and attracting curve $\overline{x_2}(\beta)$ merge into one neutrally stable stationary point. Thus in the interval $\beta_2<\beta<\beta_3$, where a point β_3 is determined by the same condition as a point β_2, i.e., $d(\beta_3)=2$, solution $x(\beta)$ always increases. At the fourth interval $\beta_3<\beta<1$, which exists if $d(1)<2$, the following condition is satisfied $\overline{x_2}(\beta_3)=\overline{x_3}(\beta_3)$. If $x(\beta_3)>\overline{x_2}(\beta_3)=\overline{x_3}(\beta_3)$, then solution $x(\beta)$ continues to increase until melting is completed since it is located beyond the repelling point.

If $x(\beta_3)<\overline{x_3}(\beta_3)$, dynamics are determined by the curve $\overline{x_2}(\beta)$. Since in this interval $\partial\overline{x_2}/\partial\beta<0$, then in the case when $x(\beta_3)<\overline{x_3}(\beta_3)$ an interval with current increase may occur at the final stage of melting.

Thus a condition $d>2$ is a critical condition for a melting process and when this condition is met during melting there occurs a stage at which the electric current increases. The latter condition determines the magnitude of the critical current or critical voltage. If these values are attained during melting, an instability stage arises whereby excitation of electric current occurs. Note that this increase of the electric current may be quite significant in a wide range of the initial conditions.

In the experiments with electrically exploding wires the amplitude of electric current attains the magnitude $I_0\sim U/[R_0+R_s(0)]\sim I_*/\sqrt{10}$. Therefore, in order to satisfy the condition $d>2$, a large difference between electric conductivities of solid and liquid phases is required, i.e., $\kappa\gg1$.

VI. CONCLUSIONS

In this study we performed a general analysis of thermoelectrodynamic peculiarities occurring during melting of current-carrying conductors.

It is shown that during equilibrium melting when the dynamics of a phase-transition front is determined by a heat balance, there exists a single-valued correlation between the rate of inductance change and a temperature at a phase-transition front. It is demonstrated that, although surface

melting begins at temperatures less than the melting temperature of a current-free conductor, corrugation of a phase-transition front begins at this temperature. It is shown that the nucleation rate of the nuclei flattened in the direction normal to a conductor's axis is negligibly small. It is demonstrated that there exists an instability stage whereby an excitation of the electric current occurs, and conditions for the experimental observation of this phenomenon are discussed.

The obtained results pose a number of problems which can be a subject of a separate study, e.g., determination of a characteristic size of the pan-shaped structures occurring due to corrugation of a phase-transition front, investigation of current dynamics, and stability using Maxwell equations, etc.

ACKNOWLEDGMENT

This work was partially supported by the Israel Ministry of Science and Arts.

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