Melting dynamics in current-carrying conductors and its effect on electrodynamic characteristics

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(Received 9 January 1995; accepted for publication 2 May 1995)

This work studies the dynamics of melting in current-carrying conductors. Formulae are derived which describe the dependence of temperature at the front of phase transition upon the distance from the axis of the conductor. The thermodynamic stability of a phase transition front is investigated. It is shown that due to strong variations of conductivity during melting the rate of change of conductivity is of the same order as an active resistance of a conductor. Clearly the magnitude of this effect depends upon the ratio of electric conductivities in liquid and solid phases. The effect is stronger when this ratio is lower. © *1995 American Institute of Physics.*

I. INTRODUCTION

Electric explosion of conductors was the subject of numerous experimental and theoretical investigations.[1–5] Various technological applications of this phenomenon are discussed in Refs. 1 and 3. Notably, explosion of the conductors is a limiting criterion in the design of safe current leads (cables) for large scale superconducting magnets.[6] In spite of the variety of approaches used to analyze and understand this phenomenon, some very important aspects which can control to the considerable extent the physical processes occurring in exploding wires were not investigated. Indeed, until recently,[7] the occurrence of regions with strongly different conductivities and changes of the inductivity of a conductor induced by motion of the boundaries separating these regions was completely ignored. However, at high magnitude of electric currents these inductivity variations have a strong effect upon the energy budget of a system and change the condition of the chemical equilibrium between phases[8–10] with different conductivities. It can be showed that if "2" denotes the formed phase and phase "1" is metastable, the condition of phase equilibrium reads

$$\mu_2(p+\tilde{p}_m,T)=\mu_1(p,T), \tag{1}$$

where μ is chemical potential of phases, p and T are pressure and temperature in the region of phase transition, and $\tilde{p}_m$ is ponderomotive pressure caused by inductivity changes during new phase formation.

The magnitude of $\tilde{p}_m$ depends upon the geometry of a conductor but in any case $\tilde{p}_m \propto I^2(\sigma_1-\sigma_2)$, where I is the total electric current, σ_1 and σ_2 are conductivities of phases.

Therefore, depending upon the sign of the difference of conductivities, the electric current causes either stabilization or destabilization of the existing phase. This effect occurs independently along with the regular effect of magnetic compression caused by renormalization of a thermostatic pressure in current-carrying conductor.

The thermostatic pressure is determined by the following relation:

$$p=p_{\text{out}}+p_m\left(1-\frac{\rho^2}{\rho_0^2}\right), \quad p_m=\frac{I^2}{\pi\rho_0^2c^2}, \tag{2}$$

where p_{out} is pressure at the conductor's surface, ρ_0 is radius of a conductor and c is the speed of light.

Using the regular condition of phase equilibrium[11] $\mu_1(p,T)=\mu_2(p,T)$ it is easy to show that the renormalization of pressure (2) results in the shift of the temperature of phase transition:

$$\Delta T_0=T_0\frac{(v_2^0-v_1^0)p_m}{\lambda_0},$$

where λ_0 is a latent heat of phase transition at the phase equilibrium curve $\lambda_0=T_0(S_2^0-S_1^0)$, T_0 is a temperature of phase transition (melting temperature of a current-free conductor), S_2^0,S_1^0 and v_2^0,v_1^0 are specific entropies and specific volumes of phases per unit mass, respectively.

It can be showed that the shift of phase transition temperature due to a phase equilibrium condition (1) reads[8]

$$\Delta T_0\propto\frac{T_0}{\lambda_0}v_2^0p_m.$$

In a case of melting $(v_1^0-v_2^0)/v_2^0\sim10^{-2}$ and contribution to the phase transition temperature shift arising because of the renormalization of a thermostatic pressure (2) is negligibly small and a static pressure can be considered equal to a pressure at the conductor's surface. In other cases, e.g., during vaporization of a conductor, the contribution to a phase transition temperature shift arising due to the renormalization of a thermostatic pressure is of the same order as a contribution due to the ponderomotive pressure. However the effect of renormalization of a thermostatic pressure does not depend upon the difference between electric conductivities of two phases and it amounts to the occurrence of an additional barrier preventing from the formation of a phase with a higher specific volume.

Motion of interphase boundaries causes the change of the electric current distribution in the conductor which results in the variation of the magnetic flux. Thus a new contribution to the voltage drop in the conductor appears during its melting. Although the magnitude of this contribution is

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proportional to the electric current, it is determined not by Ohmic resistance R but by the rate of inductivity change $\dot{L}$. Notably, depending upon the relation between conductivities of solid and liquid phases, this additional contribution can be of the same order as a voltage drop due to the Ohmic resistance.

The goal of this investigation is to analyze the dynamics of conductor's melting taking into account the effects of the ponderomotive pressure and to study the phenomena which were not studied in our previous works.[8–10] Although in this work we use the results of our previous study,[10] the problems investigated here were not addressed before.

The paper is organized as follows. In Sec. II we develop a mathematical model describing the dynamics of conductor's melting for a cylindrical phase transition front. Since the complete solution of this problem can be achieved only numerically we present only the general analysis of the problem and determine the maximum attainable temperature difference between liquid and solid phases and estimate the maximum and minimum melting rates.

In Sec. III of the paper we determine the dependence of an equilibrium temperature at the phase transition front T_F^0 upon the distance from the conductor's axis. We demonstrate the occurrence of the relatively large difference between the initial equilibrium temperature $T_F^0(\rho_0)$ and its final value $T_F^0(0)$. Here we analyze also the thermodynamic stability of a melting front depending on its location in the conductor.

In Sec. IV we analyze the effect of inductivity change upon the electrodynamic characteristics and determine the ratio of the rate of change of inductivity to the Ohmic resistance of a conductor. It is demonstrated that the ratio of the rate of change of inductivity to the Ohmic resistance of a conductor is very sensitive to the ratio of conductivities of solid and liquid phases due to inductivity change caused by the motion of phase transition front.

II. MATHEMATICAL MODEL OF MELTING IN CURRENT-CARRYING CONDUCTORS

In this study similar to our previous investigations[8,9] we consider a long linear cylindrical conductor with a small radius such that skin effects are negligibly small.

The derivation of the mathematical model of melting in current-carrying conductors is based on the equation for free energy of the conductor $\Phi(p,T)$. Under the condition of the prescribed total current I, the equation for the free energy of a conductor reads[12]

$$\Phi(p,T)=\Phi_0(p,T)-\frac{LI^2}{2}, \tag{3}$$

where $\Phi_0(p,T)$ is a free energy of a current-free conductor and L is its inductivity. Inductivity of a conductor $L=L_0+\Delta L$, where L_0 is an inductivity of a homogeneous conductor and ΔL is an inductivity change due to stratification of a conductor during phase transition. The formula for ΔL for the two-layer conductor consisting of a cylindrical core with radius ρ_F and conductivity σ_S, and a cylindrical shell with a width $\rho_0-\rho_F$ and conductivity σ_L, was derived in our previous work[10] After simple algebra this formula yields:

$$\Delta L=-\frac{\ell}{c^2}f(\beta),$$
$$f(\beta)=\xi^2(1-\beta)[(1-\beta)\ln(1-\beta)+(1+a)\beta], \tag{4}$$

where $\beta=1-\rho_F^2/\rho_0^2$, $\xi=1/(\beta+a)$, $a=1/(\kappa-1)$, $\kappa=\sigma_L/\sigma_S$, "L" and "S" denote liquid and solid phases, respectively, and ℓ is a conductor's length.

In the framework of the macroscopic kinetics the following relation is valid for the velocity of a melting front $\dot{\beta}$:[13]

$$\dot{\beta}\propto-\frac{\partial\Phi}{\partial\beta}.$$

The free energy $\Phi(\beta)$ can be written as follows:

$$\Phi(\beta)=\bar{\mu}(\beta)N_0,$$

where $\bar{\mu}(\beta)=\mu_S(\beta)(1-\beta)+\mu_L(\beta)\beta+p_m v_0 f(\beta)$ is a chemical potential per one mole and N_0 is number of moles in a conductor and v_0 is a specific volume which is assumed to be the same for liquid and solid phases. Using the latter relations the expression for the velocity of a melting front can be rewritten as

$$\dot{\beta}=\frac{u}{T_0}\left(\mu_S-\mu_L-p_m v_0\frac{\partial f}{\partial\beta}\right), \tag{5}$$

where $u=N_0/\tau$ is a relaxation rate ($[u]$=moles/s) which is a phenomenological constant in our theory.

Equation (5) must be supplemented with a heat conduction equation with distributed heat sources:

$$\frac{\partial T_i}{\partial t}=\chi_i\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial T_i}{\partial\rho}\right)+\frac{Q_i}{c_i} \tag{6}$$

and equation of heat balance at solid-liquid interface:

$$c_S\chi_S\left.\frac{\partial T}{\partial\rho}\right|_{\rho_F-\epsilon}-c_L\chi_L\left.\frac{\partial T}{\partial\rho}\right|_{\rho_F+\epsilon}=\dot{\rho}_F\lambda,$$
$$\dot{\rho}_F=-\frac{1}{2}\rho_0\frac{\dot{\beta}}{(1-\beta)^{1/2}}, \tag{7}$$

where index i equals S or L, χ_i and c_i are temperature conductivity and specific heat per one mole of phase i, respectively, Q_i is a rate of release of Joule heat per one mole in phase i, $Q_i=j_i^2 v_0/\sigma_i$, where j_i is electric current density. In a case of a cylindrical melting front which is considered in this study, Q_i can be written as follows:

$$Q_i=\frac{\sigma_i\sigma_S}{\bar{\sigma}^2}\frac{4p_m v_0}{\tau_m}, \tag{8}$$

where $\tau_m=4\pi\rho_0^2\sigma_S/c^2$ and $\bar{\sigma}=\sigma_S[(1-\beta)+\kappa\beta]$.

Note that λ equals to the difference between enthalpies of the two phases at the phase equilibrium curve (1) and it differs from λ_0, the latent heat of the phase transition calculated at the phase equilibrium curve of a current-free conductor:

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$$\lambda-\lambda_0=(c_L-c_S)p_m v_0\frac{T_0}{\lambda_0}\frac{\partial f}{\partial\beta}. \tag{9}$$

Equation (9) can be derived from Eq. (5) at $\dot{\beta}=0$ similar to that in Ref. 8.

Equations (5)–(7) must be supplemented with the boundary conditions which determine the heat exchange between a conductor and a surrounding medium and with the condition of temperature continuity at the liquid-solid interface:

$$T_L[r_F(t),t]=T_S[r_F(t),t]=T_F[\beta(t),t]. \tag{10}$$

An equation for the temperature at the phase transition front $T_F[\beta(t),t]$ can be derived directly from Eq. (5) by expanding chemical potential into power series in the vicinity of the point $T_F^0(\beta)$, which is determined by

$$\mu_S^0-\mu_L^0-p_m v_0\frac{\partial f}{\partial\beta}=0, \tag{11}$$

where μ_S^0 and μ_L^0 are chemical potentials at temperature $T_F^0(\beta)$.

Then Eqs. (5) and (11) yield

$$T_F[\beta(t),t]=T_F^0(\beta)+\frac{T_0\dot{\beta}}{u\Delta S}, \tag{12}$$

where ΔS is the difference between specific entropies of liquid and solid phases at temperature $T_F^0(\beta)$,

$$\Delta S=\frac{\lambda_0}{T_0}+(c_L^0-c_S^0)\frac{p_m v_0}{\lambda_0}\frac{\partial f}{\partial\beta}.$$

Heat exchange with a surrounding medium and radiation heat losses can be neglected due to very short duration of the process. Indeed, the characteristic time of electric explosion of conductors is of order 10^{-6} s. Therefore Eqs. (6), (7), (10), and (12) must be supplemented with the following conditions:

$$\left(\frac{\partial T}{\partial\rho}\right)_{\rho=\rho_0}=\left(\frac{\partial T}{\partial\rho}\right)_{\rho=0}=0. \tag{13}$$

A system of Eqs. (6), (7), (10), (12), and (13) completely describes melting of current-carrying conductors and takes into account the renormalization of chemical potential of the formed phase caused by ponderomotive forces. This renormalization results in the dependence of the equilibrium temperature at phase transition front upon the distance from the conductor's axis (or upon the fraction of a conductor's volume occupied by a liquid phase β) and also in the dependence of all the parameters in the theory upon the magnitude of electric current.

In some cases the Stephan problem which is formulated above admits self-similar solutions.[14] However the Stephan problem with cylindrical symmetry and distributed heat sources is not amenable to the similarity transformations. A numerical solution of this problem neglecting the effects caused by inductivity variations was presented in Ref. 2 where melting of thin metallic foils was investigated in cases of low and high heat release rates.

At this stage we assume that a behavior of the phase transition front determined in Ref. 2 is qualitatively valid also in the case of a thin cylindrical wire. Dynamics of melting depends upon the relation between two characteristic times, $\tau_{T,i}=\rho_0^2/\chi_i$ and $\tau_{Q,i}=\lambda/Q_i$. When $\tau_{T,i}\ll\tau_{Q,i}$, the temperature change across the wire cross section is small, i.e., $c_i\Delta T\ll\lambda$. In the opposite case (high energy release rate), $\tau_{T,i}\gg\tau_{Q,i}$, the temperature gradient across the wire is very high. For further analysis we use the energy balance equation. Introduce the effective temperatures:

$$T_S^*=\frac{2}{\rho_F^2}\int_0^{\rho_F(t)}T(\rho',t)\rho'd\rho' \quad \text{and}$$

$$T_L^*=\frac{2}{\rho_0^2-\rho_F^2}\int_{\rho_F(t)}^{\rho_0}T(\rho',t)\rho'd\rho'.$$

Multiplying Eq. (7) by ρ' and integrating from 0 to $\rho_F(t)$ and from $\rho_F(t)$ to ρ_0 and using the boundary conditions (13) we find:

$$(1-\beta)(c_S\dot{T}_S^*-Q_S)+\beta(c_L\dot{T}_L^*-Q_L)$$
$$=\dot{\beta}(c_S T_S^*-c_L T_L^*-\lambda). \tag{14}$$

Equation (14) is exact and does not depend upon the regime of melting and upon the temperature change at the phase transition front. Note that although there is no mass flux in the system, Eq. (14) includes a convective term $\propto\dot{\beta}(c_S T_S^*-c_L T_L^*)$ which is associated with the motion of the boundary between the two phases.

Since rate of temperature increase during melting is limited by the energy release rate, i.e., $c_S\dot{T}_S^*\leqslant Q_S$ and $c_L\dot{T}_L^*\leqslant Q_L$, and since $\dot{\beta}\geqslant 0$, the following condition determines the maximum temperature difference between the liquid and solid phases:

$$c_S T_S^*-c_L T_L^*\leqslant\lambda. \tag{15}$$

Inequality (15) determines the maximum temperature difference between the liquid and solid phases at all regimes of energy release and poses restriction only upon the temperature of a solid phase.

According to the model developed in Ref. 2 the temperature variation of a solid phase has an extremum. At the initial stage the solid phase temperature increases faster than that of a liquid phase because of a small value of the interphase surface area per unit mass of solid phase. Temperature of a liquid phase increases relatively slowly since the interphase surface area per unit mass of liquid phase is high. This situation is preserved until the temperature $T_S^*(\beta)$ reaches its maximum value. Determine now the velocity of a phase transition front at point $\dot{T}_S^*(\beta)=0$. The above analysis shows that at this point the Eq. (14) can be written as

$$-(1-\beta)Q_S\approx\dot{\beta}(c_S T_S^*-c_L T_L^*-\lambda). \tag{16}$$

According to numerical data presented in Ref. 2, the maximum temperature difference in a thin foil for $c_S=c_L=\bar{c}$ is

$$T_S^*-T_L^*\sim 0.9\frac{\lambda}{\bar{c}}. \tag{17}$$

Then the formula for the maximum rate of melting reads

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$$\dot{\beta}_m \sim \frac{40 p_m v_0}{\tau_m \lambda}. \tag{18}$$

For characteristic values of electric currents attained during explosion of a conductor, $p_m v_0/\lambda \sim 0.1$. Indeed, for the metals like Al, Cu, and W and parameters ($\pi\rho_0^2=0.14$ mm, $I_{\max}=20$ kA) employed in experiments, [15] this parameter equals to 0.27, 0.15, and 0.08, respectively. The characteristic value of τ_m for metals with high conductivity (e.g., Cu or Al) under the experimental conditions presented above, $\tau_m \sim 4\times10^{-8}$ s and, therefore, the maximum rate of melting $\dot{\beta}_m \sim 10^8\ \mathrm{s}^{-1}$. Such high value of $\dot{\beta}_m$ is associated with very high temperatures difference (17) which is not attained in the experiments for several reasons. The main reason is that volumetric melting occurs at such high temperature differences, causing formation of liquid zones which absorb Joule energy. The second reason is the corrugated instability of the phase transition front which increases the front surface area per unit mass of conductors and thus causes cooling.

It is most likely that the phenomenon of a conductor's melting occurs as follows (see also Ref. 16). During melting the conductor does not acquire high temperatures and melting is determined by the energy budget. The temperature of a conductor assumes the minimum value which can support the maximum possible number of melting zones. Temperature differences, with respect to the melting front remain low in each of these zones and, therefore, $\dot{T}_S=\dot{T}_F^0$. Assuming that specific heat capacities of solid and liquid phases are equal, i.e., $c_S=c_L=\bar{c}$, Eq. (14) yields the formula for melting rate at the phase transition front:

$$\dot{\beta}=\frac{(1-\beta)Q_S+\beta Q_L}{\lambda+(\partial T_F^0/\partial\beta)\bar{c}}. \tag{19}$$

Note that in the derivation of Eq. (19) the contribution of the volumetric melting into the volume fraction of a liquid phase was neglected. As can be seen from the further analysis this does not change the obtained results. The more detailed analysis of formula (19) shows that it provides the estimate of a melting rate close to the minimum value. Therefore in order not to overestimate the effect of renormalization of effective current damping coefficient which is proportional to $\dot{\beta}$, we use precisely this formula for the calculation of the damping coefficient in Sec. IV of this paper.

Thus the supplementary effect of electric current on melting is effectively reduced to the renormalization of the latent heat of phase transition λ, $\tilde{\lambda}\dot{\beta}=(1-\beta)Q_S+\beta Q_L$:

$$\tilde{\lambda}=\lambda+\bar{c}\,\frac{\partial T_F^0}{\partial\beta} \tag{20}$$

where λ is determined by Eq. (9).

III. EQUILIBRIUM TEMPERATURE OF MELTING FRONT AND RELAXATION PHENOMENA IN CURRENT-CARRYING CONDUCTORS

Formula (12) shows that if

$$\dot{\beta} \ll \frac{u\lambda_0}{T_0}, \tag{21}$$

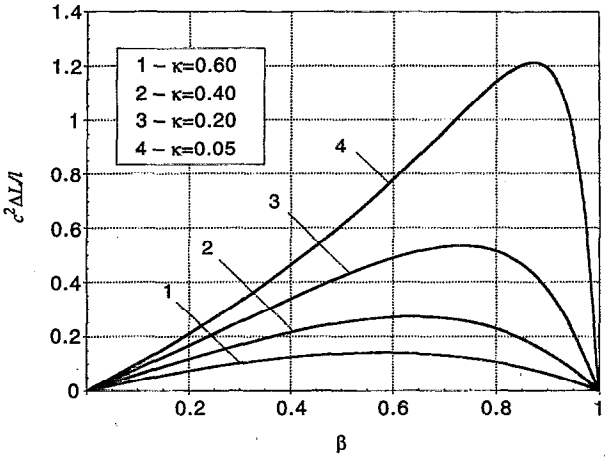

FIG. 1. Inductivity vs volume fraction of a liquid phase.

then melting front is in an equilibrium state and temperature at the front is determined by Eq. (11). In the linear approximation Eq. (11) yields

$$T_F^0(\beta)=T_0\left(1+\frac{p_m v_0}{\lambda}\frac{\partial f}{\partial\beta}\right).$$

Figures 1–3 shows the dependence of inductivity change per unit length of conductor $c^2\Delta L/\ell=-f(\beta)$ and temperature at a phase transition front in an equilibrium state $(T_F^0(\beta)-T_0)/T_0(\lambda/(p_m v_0))$ as functions of volume fraction of a liquid phase β. As can be seen from these plots the behavior of the system is very sensitive to the magnitude of parameter $\kappa=\sigma_L/\sigma_S$. Expanding $f(\beta)$ into power series

$$f(\beta)=\frac{\beta}{a}-\frac{(3+2a)\beta^2}{a^2}+0(\beta^3), \quad \beta \ll 1$$

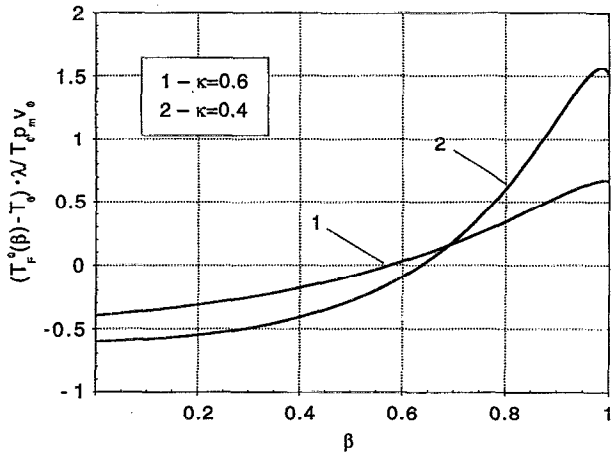

FIG. 2. Temperature at a phase transition front vs volume fraction of a liquid phase.

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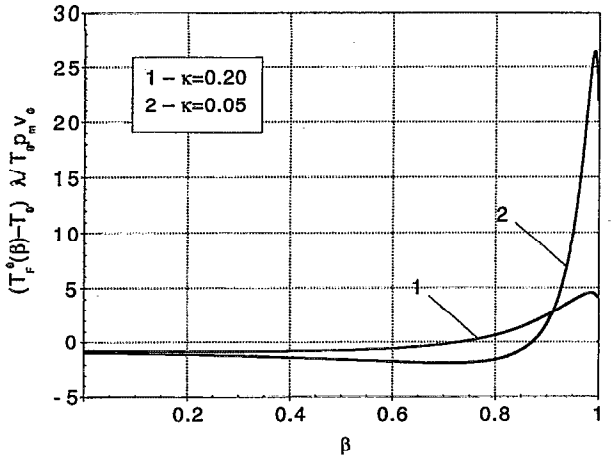

FIG. 3. Temperature at a phase transition front vs volume fraction of a liquid phase.

$$f(\beta)=\frac{y}{1+a}+\frac{1}{(1+a)^2}\,y^2\ln y+\frac{y^2}{(1+a)^2}+0(y^3),$$

$$y=1-\beta\ll\kappa.$$

It can be seen that temperature at the melting front decreases very slowly if $\kappa<\frac{1}{3}$. When $\kappa>\frac{1}{3}$ temperature at the melting front increases although melting begins at a temperature which is less than a melting temperature of a current-free conductor.

It is interesting to note that there is a relatively strong temperature difference between a melting point at the surface, $\Delta T/T_0=-p_m v_0/\lambda(1-\kappa)$ and a melting temperature at the axis of a conductor ($\beta=1$), $\Delta T/T_0=(1-\kappa)p_m v_0/\kappa\lambda$.

There exists also a possibility of occurrence of a very sharp maximum at $1-\beta\approx e^{-5/2}$. At this point, $\Delta T=2T_0(p_m v_0/\lambda)((\kappa-1)/\kappa)^2 e^{-5/2}\approx 0.16T_0(p_m v_0/\lambda)\times((\kappa-1)/\kappa)^2$. For $\kappa\sim 0.1$ the shift of the equilibrium temperature of phase transition front is of the order of the melting temperature T_0 (see Fig. 3). The presented plots show that depending upon the parameter κ, the considerable fraction of the conductor is melted before the temperate of the conductor reaches the melting point. When the temperature reaches T_0 the structure of a phase transition front changes since at temperatures $T>T_0$ new melted regions are formed due to the feasibility of an internal melting. Nucleation temperature inside a partially melted conductor, ΔT_L^{ins}, is determined by the following formula (for details see Ref. 10):

$$\frac{\Delta T_L^{\rm ins}}{T_0}=\frac{p_m v_0}{\lambda}\,\frac{2\sigma_S}{\bar{\sigma}}\,\tilde{\xi}_a\Gamma_L. \tag{22}$$

Assuming that conductivity of a nucleus $\sigma_a=\sigma_L$, the geometrical coefficient $\tilde{\xi}_a$ in formula (22) for an ellipsoidal nucleus with the axis parallel to that of the conductor is given by the following formulae:

$$\tilde{\xi}_a=\frac{1}{2}\,\frac{1-\kappa}{\kappa\beta_z+1},\quad \beta_z=\frac{n_z}{1-n_z}, \tag{23}$$

where n_z is a depolarization factor (see Ref. 10) and

$$\Gamma_L=\ln\frac{4z_a(\ell-z_a)}{\rho_0^2}+1-(1-\beta\xi)\,\frac{\rho_a^2}{\rho_0^2}+(1-\beta)\xi\times\ln(1-\beta), \tag{24}$$

where ρ_a and z_a are cylindrical coordinates of the nucleus ($0<\rho_a<\rho_F$).

Formulae (22)–(24) show that in both cases, for $n_z\ll 1$ and for $n_z\to 1$, $\Delta T_L^{\rm ins}/T_0>0$. The optimal shape of the growing nucleus is that of the nucleus flattened in the direction of the conductor's axis where $\beta_z\to\infty$.

Thus a conductor can be melted to a high extent before its temperature reaches the melting point T_0 whereby the volumetric melting is inhibited. At some temperature, slightly higher than the melting temperature, the volumetric melting begins whereby the optimal shape of the melted region is the flattened along the conductor's axis liquid melt zones. Such special geometry of these zones is determined by maximum increase of resistance and minimum decrease of inductivity of the conductor during their formation.

In conclusion consider the stability of a melting front for a given temperature. According to Equation (5) for the spontaneous relaxation of a fraction of a liquid phase β at a given temperature T can be written as follows:

$$\dot{\beta}=-\frac{(\beta-\beta_0)}{\tau},\quad \frac{1}{\tau}=\frac{u}{T_0}\,p_m v_0\,\frac{\partial^2 f}{\partial\beta^2}\bigg|_{\beta=\beta_0}. \tag{25}$$

According to Eq. (25) the condition for a thermodynamic stability of a melting front is $\partial^2 f/\partial\beta^2>0$. It can be seen from Figs. 2 and 3 that a melting front is stable until the region where the front temperature attains its maximum value.

Thus, while in current-free conductors the phase transition front remains in a state of a neutral equilibrium, in current-carrying conductors to every value of temperature there corresponds a particular location of a melting front. Temperature increase is required in order to shift a location of a melting front towards the conductor's axis. Such a situation holds until the front temperature reaches its maximum. Starting from this point, phase transition proceeds in a re-

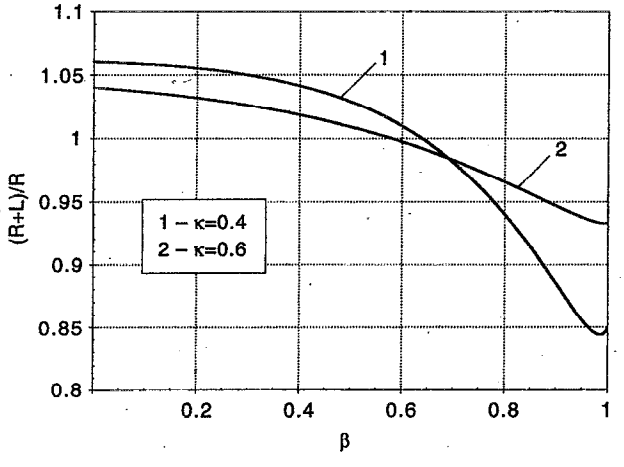

FIG. 4. $(R+\dot{L})/R$ vs volume fraction of a liquid phase.

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gime of a "spontaneous" melting since beyond this point location of a melting front is unstable. As will be shown further a "spontaneous" melting regime has some very interesting features: it is precisely in this regime that there occurs a voltage drop $\Delta U = I\dot{L} < 0$ which may compensate Ohmic voltage drop $\Delta U = IR$. However, high overheating is required in order to reach point ρ_m where phase transition front temperature is maximum. Therefore internal melting starts before the surface melting front reaches the point ρ_m. In this case among all the possible patterns of melting there occur those patterns which correspond to a maximum value of $\dot{L}$, i.e., patterns with minimum compensation of the voltage $\Delta U = IR$.

IV. ELECTRODYNAMIC CHARACTERISTICS OF EXPLODING WIRES IN EVAPORATION AND MELTING REGIMES

The main information on a thermodynamic state of a conductor during electric explosion is gained via analysis of its current/voltage characteristics. Current/voltage characteristics allows to determine the conductor's resistance which is a function of its thermodynamic state. In a case when there occurs a complicated melting pattern the determination of electric conductivity is fairly complicated problem. In the case of a cylindrical melting front which is considered in this study, the electric field strength **E** is spatially homogeneous. Then the total resistance is determined by the following formula:

$$R = \frac{\ell}{\pi \rho_0^2 \sigma_S (1 - \beta + \kappa\beta)}. \tag{26}$$

However, motion of a melting front changes the inductivity of a conductor, and electric current dynamics is governed by the following equation:

$$L\dot{I} + I(R + \dot{L}) + \int_0^t \frac{I(t')}{C}\, dt' = U, \tag{27}$$

where C is electric capacity and U is voltage. Inspection of Eq. (27) shows that the current damping resistance is determined by a sum of the Ohmic resistance R and $\dot{L}$.

According to Eq. (4), $\dot{L} = -\ell/c^2 \partial f/\partial\beta\dot{\beta}$, and using Eqs. (8) and (19) we find

$$\frac{\dot{L}}{R} = -\frac{p_m v_0}{\tilde{\lambda}} \frac{\partial f}{\partial \beta}, \quad \tilde{\lambda} = \lambda + \frac{\partial T_F^0}{\partial \beta} c_S. \tag{28}$$

The value of the effect of the change of the damping resistance relative to the Ohmic resistance is determined by the ratio $(R+\dot{L})/R$. The plots of the function $(R+\dot{L})/R = [1 - (p_m v_0/\tilde{\lambda}\partial f/\partial\beta)]$ for $p_m v_0/\tilde{\lambda} = 0.1$ and $\tilde{\lambda} = \lambda$ are shown in Figs. 4 and 5. As can be seen from these plots, the ratio $(R+\dot{L})/R$ has a minimum with the width depending on the relation between conductivities of solid and liquid phases, σ_S and σ_L. A remarkable property of Eq. (28) is that for a fixed value of electric current, the magnitude of the effect does not depend explicitly on the conductivity of both phases but only upon the ratio of conductivities

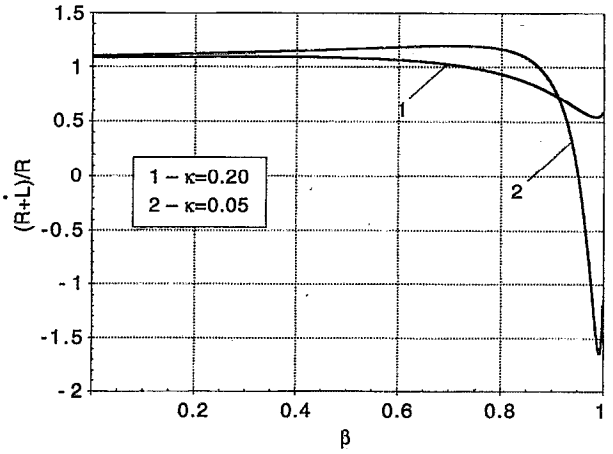

FIG. 5. $(R+\dot{L})/R$ vs volume fraction of a liquid phase.

$\kappa = \sigma_L/\sigma_S$: the less is this ratio the stronger is the effect. Therefore, this effect can occur not only during melting but also during boiling of a conductor.

Comparison of the obtained results with the experimental data requires both the detailed calculation of the electrodynamic system used in experiment and to conduct the experiment so that processing of the experimental results is performed taking into account the discussed above effects. Nevertheless it seems quite feasible that the occurrence of the intervals with decreasing resistance which was observed in experiments reported in Refs. 3 and 4 is associated with the analyzed above stages of melting and boiling and is caused by inductivity drop at the final stages of melting or boiling.

V. CONCLUSIONS

The analysis performed in this study shows that inductivity change at the stage of melting has a strong effect not only on the dynamics of phase transition but also directly on the current/voltage characteristics of a conductor. Notably, the obtained results are also valid in a case of conductor's boiling. This phenomenon is of high importance in the investigation of electric explosion of wires and foils. Sensitivity of the discussed effects to the geometrical pattern of the new phase formed during phase transition warrants further investigation of the discussed here problems.

ACKNOWLEDGMENTS

This work was partially supported by Israel Ministry of Science and Technology.

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