

Critical currents in normal conductors

Yu. Dolinsky and T. Elperin

The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, P.O. Box 653, Israel

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It is shown that in a normal conductor the value of a current that generates a magnetic pressure equal to the critical pressure may be considerably lower than it is generally considered. The lower value of the critical current occurs due to the shift of the whole curve of phase equilibrium in the presence of strong electric current. This shift arises due to the additional work performed against ponderomotive forces which prevents from the formation of the nucleus of a phase with a lower value of electric conductivity. In case of the Van der Waals model of the critical state the value of the critical current calculated taking into account the shift of the phase equilibrium curve is by factor 2–3 less than the critical current determined when this effect is neglected.

One of the most important characteristics of a normal conductor which determines its behavior under strong electric currents is the value of the critical current I_c^0 . Generally the critical current is defined as a current which generates the critical pressure p_c^0 in the center of the conductor. The relation between I_c^0 and p_c^0 is given by the following expression:

$$I_c^0 = kr_0 \sqrt{p_c^0} \quad (1)$$

where k is a constant dependent on the system of units, r_0 the radius of the conductor, p_c^0 the critical pressure which depends upon the material of the conductor.

In case of a metallic wire with critical pressure $p_c^0 = 10$ kbar and radius $r_0 = 0.1$ mm, the value of the critical current $I_c^0 = 17$ kA. In practice such currents can be encountered in exploding electrical conductors which are used in various fields of technology. Under these currents the phase transition of the conductor from a solid state to a liquid state or from a liquid state to a gaseous state cannot be realized as a phase transition of the first kind (Naturally we consider only such phase transitions of the first kind which have a critical point). It is generally accepted that materials under these conditions must sublime (we consider a case when an ionization energy is sufficiently high in comparison with the sublimation energy). Recently, it has been shown that a strong electric current changes not only a thermodynamic pressure in a conductor $p(v, T)$ (v is a specific volume, T the temperature) but also causes a shift of the whole curve of phase equilibrium.¹ This shift arises due to the additional work performed against ponderomotive forces which prevent the formation of the nucleus of a phase with lower value of electric conductivity.

We consider the adiabatic regime when electrodynamic and thermodynamic parameters of the system do not depend upon time explicitly. The time dependence of these parameters is determined by the time dependence of the size of the nucleus. Therefore the size of the nucleus can be viewed as an additional thermodynamic parameter, and the system with a persistent current can be described by the methods of equilibrium thermodynamics (see, e.g.,

Ref. 2, Chap. 4, Sec. 33). It can be shown¹ that the conditions for the adiabatic regime are met in a wide range of the parameters of exploding wires.

The condition of equilibrium of phases with the values of electrical conductivity σ_1 and σ_2 in the presence of electric current I is given by the following expression:¹

$$\mu_2 = \mu_1 + \tilde{p}v_1, \quad (2)$$

where $\tilde{p}$ is determined with adequate accuracy from the following expressions:

$$\tilde{p} = 4p_m \frac{\sigma_2 - \sigma_1}{\sigma_1 + 2\sigma_2}; \quad p_m = \frac{I^2}{k^2 r_0^2},$$

and μ_1 and μ_2 are the chemical potentials of phase 1 and phase 2. Similar expression for the change of the chemical potential of a dielectric with a small spherical inclusion in the presence of electric field was derived in Refs. 3 and 4.

Expression (2) was obtained under the assumption that the conductor is in the state of phase 2 and the nucleus is formed in the state of phase 1. The direction of the phase transition is essential since in the presence of electric current the phase equilibrium curves for the direct and inverse phase transitions do not coincide. The latter is caused by the fact that the work of the nucleus formation depends unsymmetrically upon the direction of the phase transition [see expression (2)]. Hereafter we consider a transition from the phase with a higher value of electric conductivity to the phase with a lower value of electric conductivity so that $\sigma_2 > \sigma_1$. Under these conditions the intersection of the new curve of phase equilibrium B defined by expression (2) and the spinodal line of the phase with the higher value of electric conductivity S determined by the expression $(\partial p / \partial v)_{v=v_2} = 0$ occurs at the point with temperature $T_c(I)$ and pressure $p_c(I)$ which are lower than the critical values (see Fig. 1).

The above arguments were employed to determine the critical current which can be found from the conditions

$$T_c(I) = T; \quad p_c(I) = p,$$

where T and p are the temperature and the pressure in the conductor. The latter equations determine two different

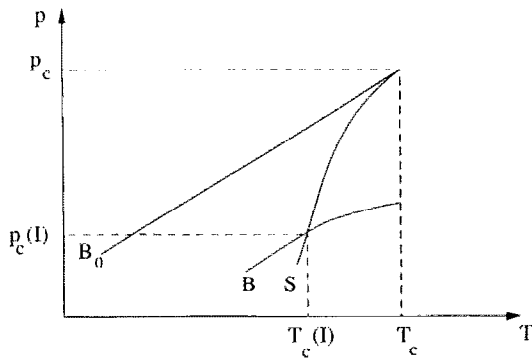


FIG. 1. B_0 the phase equilibrium curve $\mu_1 = \mu_2$ in the absence of electric current; B the phase equilibrium curve $\mu_2 = \mu_1 + \tilde{p}v_1$ in the presence of electric current; S , spinodal line ($\partial p / \partial v|_{T=0} = 0$).

critical currents $I_c(p)$ and $I_c(T)$ for heating and stretching, respectively. This problem was analyzed comprehensively in Refs. 3 and 4. Note in passing that the occurrence of different critical currents which depend upon temperature and pressure is not unique for the investigated problem. A similar situation is encountered also in the superconductors of the second kind. However, in accordance with the above arguments, it is expedient to redefine the critical current I_c^0 . Instead of expression (1), determining the critical current as the current which generates a magnetostatic pressure p_c^0 , define now a critical current I_c which generates a magnetostatic pressure $p_c(I)$ such that the new phase equilibrium curve intersects with the spinodal of a current-free conductor. The physical meaning of this critical current is similar to that of the critical current defined by expression (1). However, the magnitude of this current is essentially smaller since due to the shift of the phase equilibrium curve it intersects with the spinodal line of the current-free conductor at a lower pressure $p_c(I) < p_c^0$. Calculation of critical I_c is the purpose of the present investigation.

The critical current I_c is determined by the equation

$$p_c(I) = I^2 / k^2 r_0^2 \quad (3)$$

and by the equation determining $p_c(I)$.

The equation for determining $p_c(I)$ can be represented as

$$\int_{v_1(p)}^{v_2(p)} \{p - P[v, T_s(p)]\} dv' = \tilde{p}v_1(p), \quad (4)$$

where $v_1(p)$ is the specific volume of phase 1 on the spinodal line of phase 2; $v_2(p)$ the specific volume of phase 2 on its spinodal line; $T_s(p)$ the temperature as a function of pressure on the spinodal line of phase 2; $P(v, T)$ the equation of state.

Expressions (3) and (4) allow us to determine the value of the critical current provided that the equation of state and the equation of the spinodal line are known.

Determine the parameter $\gamma = I_c / I_c^0$ in case of the Van der Waals model of the critical state (see Ref. 5, Chap. 14, Sec. 152).

The equation of state is given by

$$\pi(\eta, \tau) = -b\tau - 2a\tau\eta + 4B\eta^3, \quad (5)$$

where $\pi = p/p_c^0 - 1$; $\eta = \rho/\rho_c^0 - 1$; $\tau = 1 - T/T_c^0$; ρ is density, and coefficients a, b, B are the parameters of the model (p_c^0, ρ_c^0, T_c^0 denote values at the critical point). Hereafter we consider the case $\eta \ll 1$, i.e., in the vicinity of the critical point. The equation of the spinodal line of phase 2 ($\rho_2 > \rho_1$) in these variables reads

$$\pi_s(\tau) = -b\tau - \frac{a^{3/2}}{B^{1/2}} \frac{4}{3 \times 6^{1/2}} \tau^{3/2}. \quad (6)$$

In this case it is convenient to rewrite Eq. (4) as follows:

$$\int_{\eta_1(\tau)}^{\eta_2(\tau)} [\pi_s(\tau) - \pi(\eta', \tau)] d\eta' = -\tilde{\pi}, \quad (7)$$

where $\pi_s(\tau)$ is determined by Eq. (6), $\pi(\eta, \tau)$ is determined by Eq. (5), $\tilde{\pi} = \tilde{p}/p_c^0$, $\eta_1(\tau)$ and $\eta_2(\tau)$ the densities of phase 1 and 2 at the spinodal line of phase 2 which are given by the following relations:

$$\eta_1 = -2\eta_2, \quad \eta_2(\tau) = \left(\frac{a\tau}{6B} \right)^{1/2}. \quad (8)$$

Equation (7) is equivalent to Eq. (4) and determines the critical temperature $\tau_c = \tau_c(\tilde{\pi})$ explicitly. Pressure $\pi_c(I) = p(I)/p_c^0 - 1$ is determined by substituting $\tau_c(I)$ into Eq. (6). Taking into account relations (6) and (8), expression (7) yields

$$\tau_c = \frac{2}{3^{1/2}} \frac{B^{1/2}}{a} \tilde{\pi}^{1/2}.$$

Then using formulae (3) and (6) we arrive at the following nonlinear algebraic equation for the parameter $\gamma = I_c / I_c^0$:

$$\frac{4}{3^{1/2}} \frac{bB^{1/2}}{a} \xi^{1/2} \gamma + \left(\frac{4}{3} \right)^{9/4} B^{1/4} \xi^{3/4} \gamma^{3/2} = 1 - \gamma^2, \quad (9)$$

where $\xi = (\sigma_2 - \sigma_1) / (\sigma_1 + 2\sigma_2)$.

Consider the case when $\sigma_2 \gg \sigma_1$ so that $\xi = 0.5$ and the values of the parameters a, b , and B assume to be equal to those of the Van der Waals gas ($a=2, b=4, B=3/8$). Then we obtain that $\gamma \approx 0.35$.

Therefore the value of the critical current calculated taking into account the shift of the phase equilibrium curve is by factor 2-3 less than the critical current determined when this effect is neglected. Note that the obtained results are based on expression (2), which was derived under the assumption that the formed nucleus is of a spherical shape. However, the same analysis performed with other simple shapes of the formed nucleus (e.g., cylindrical shape) showed that expression (2) is still valid with the accuracy of a coefficient of order 1.

The above analysis shows that the real critical current in the conductor may be considerably less than its value I_c^0 calculated on the basis of expression (1). The latter finding must be taken into consideration when investigating the processes in the presence of high-density electric currents.

The presented above approach employs only general thermodynamic relations. The consistent dynamic theory of nucleation and spinodal decomposition in current carrying conductors is a challenging problem which is beyond the scope of the present research.

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