

Blast waves in dusty gases

BY O. IGRA, T. ELPERIN AND G. BEN-DOR

The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel

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The conservation equations for the flow field developed behind a spherical blast wave propagating into a dusty medium (gas seeded with small uniformly distributed solid particles) are formulated and solved numerically by using the random choice method. The solution was carried out for the following three cases:

(1) the dust is uniformly distributed outside the exploding spherical diaphragm;

(2) the dust is uniformly distributed inside the exploding spherical diaphragm;

(3) the dust is uniformly distributed inside a spherical layer located outside the exploding spherical diaphragm.

The solutions obtained were compared with a similar pure-gas case. It was found that the dust presence weakens the blast wave, i.e. the gas velocity, temperature and pressure immediately behind the blast-wave front were lower than those obtained in a similar pure-gas case. The presence of dust changed the flow field behind the blast wave. The typical blast-wave pressure signature (i.e. a monotonic reduction in the pressure after the jump across the blast-wave front) changed to a different shape. Now the pressure increases after the blast-wave front until it reaches a maximum value followed by a monotonic pressure reduction. The maximum pressure is attained between the blast-wave front and the contact surface. Higher values of total pressure are obtained in the dusty-gas case. The initial uniform spatial distribution of the dust particles changed into a bell-shaped pattern with a pronounced peak. The development of the sharp maximum in the dust spatial-density distribution might be of interest in assessing the effects of atmospheric nuclear explosions.

INTRODUCTION

Spherical blast waves are common occurrences in the Earth's atmosphere. They result from a sudden release of a relatively large amount of energy; typical examples are lightnings and chemical or nuclear explosions. When the generated blast wave propagates into a pure gas, the evaluation of the post-shock flow properties is relatively simple and well known (see Brode 1955; Glass & Hall 1957). This is not the case when the gaseous phase contains small solid particles (dust). The latter is a more realistic situation because the Earth's atmosphere contains many small dust particles, especially when the explosion-generated blast wave propagates over a sandy desert terrain. There are a few numerical schemes suitable for evaluating the flow field behind the blast wave. The first numerical

solution to the problem of a spherical blast wave propagating into a pure gas was obtained via a finite-difference scheme with an artificial viscosity suggested firstly by von Neumann & Richtmyer (1950). Recently, Chorin (1976) introduced the random-choice method for numerical solution of gas-dynamics problems. This method was employed by Miura & Glass (1982) for analysis of the flow field resulting from the non-stationary propagation of a normal shock wave into a dusty gas (the case of a dusty shock tube). This is a relatively simple problem because of its plane symmetry, and it has a limited physical importance because normal shock waves are hard to find in Nature. The random-choice method is especially suitable for treating flow discontinuities (such as shock waves and contact surfaces) where conventional methods often fail. Another numerical method that can be used for solving problems of gas dynamics involving propagation and interaction of shock waves is the method of characteristics. Recently, Higashino (1983) has applied the method of characteristics for the analysis of blast waves in a dusty gas. However, the method of characteristics can be applied to problems where the number of discontinuities is small. The purpose of the present paper is to consider several examples involving spherical blast waves propagating into a gas-dust suspension using the random choice method. The blast wave was generated by the rupture of the spherical diaphragm (of radius r , see figure 1a) inside which the pressure was higher than the ambient pressure. Three different cases were considered:

- case 1: the dust is uniformly distributed outside the diaphragm;
- case 2: the dust is uniformly distributed inside the diaphragm;
- case 3: the dust is uniformly distributed inside a spherical layer located outside the diaphragm.

The obtained results were compared with a similar pure gas case.

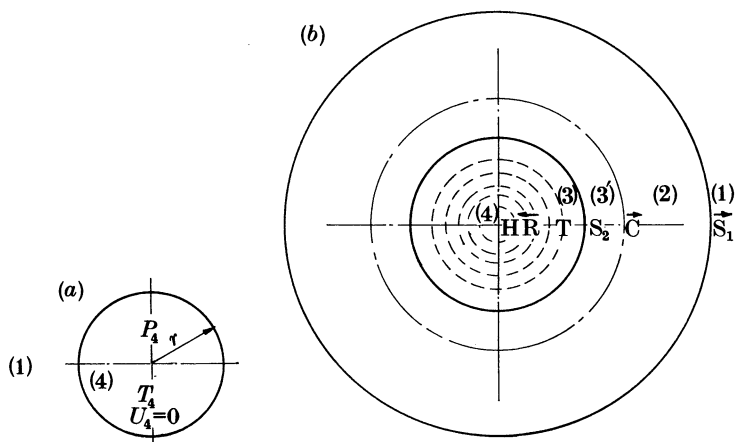


FIGURE 1. Schematic description of the considered flow field. (a) Before the diaphragm rupture, $t = 0$. $P_1 \ll P_4 (= 20P_1)$; $u_1 = 0$; $T_1 = T_4$; $r = 20$ cm. (b) After the diaphragm rupture, $t > 0$. Abbreviations: R, rarefaction wave; H, head of the rarefaction wave; T, tail of the rarefaction wave; S_2 secondary shock wave; C, contact surface; S_1 , primary shock wave.

THEORETICAL BACKGROUND

The analysis is based on the assumption that the number density of the solid particles is sufficiently large to consider the solid phase as a continuum. Therefore the minimal dimension in the flow region, l , which can be described on the basis of this approach must satisfy the inequality

$$d \ll l \ll L,$$

where d is the diameter of the solid particle and L is a characteristic length scale of the considered flow field. The value l can be viewed as a minimal size of a spatial mesh that can be allowed in numerical calculations involving a finite-difference approximation of the problem. The dusty gas is assumed to be composed of a thermally perfect gas and inert solid particles of spherical form and uniform size. The volume occupied by the particles is negligibly small because the density of the solid particles is several orders of magnitude larger than that of the gas and the dust mass loading ratio used in the present solution is low (*ca.* 1).

The gas and the particles interact with each other through the drag force, D and the heat transfer rate, Q . The experimental correlations for D and Q used in the calculation were adopted from Rudinger (1980).

$$\begin{aligned} D &= 0.125\pi d^2 \rho (u-v) |u-v| (0.48 + 28Re^{-0.85}), \\ Q &= \pi d \mu c_p Pr^{-1} (T - \Theta) (2.0 + 0.6Pr^{\frac{1}{3}} Re^{\frac{1}{2}}), \end{aligned} \quad (1)$$

where ρ , u , T are the gas density, velocity and temperature, respectively, v and Θ are velocity and temperature of the solid particles, μ is the gas viscosity, c_p is the gas specific heat capacity at constant pressure, Pr is the Prandtl number and Re is the Reynolds number based on the diameter, d , of the solid particles and the slip velocity, i.e. $Re = \rho |u-v| d / \mu$. As can be seen from the above expression for the drag force D , the gravity force, the buoyancy force, the Basset force and the force acting on the particle because of pressure gradients are neglected. These assumptions are quite accurate when the solid-gas density ratio is large and the pressure gradients are not high, except for the narrow regions in the vicinity of the shock front (see Rudinger 1980, ch. 2). In the present analysis the gas was considered to be a continuous medium with a molecular mean free path much smaller than the size of the solid particles. This assumption is perfectly valid for solid-particle sizes of the order of 10^{-5} m (the size adopted for the present work) and the gas parameters used in the present solution. The compressibility and kinetic effects on the drag coefficient and heat-transfer rate, i.e. the dependence of D and Q upon the flow Mach number and the Knudsen number were neglected (see Rudinger 1980, p. 12). The expression for the heat-transfer rate Q implies that only convective heat transfer is taken into account. Certainly, for high temperatures radiative heat transfer becomes important. It was shown by Boothroyd (1971) and Igra & Ben-Dor (1980) that for the temperature range considered in this study the contribution of radiative heat transfer is negligibly small. The temperature distribution inside the dust particle was assumed to be

uniform. This is a reasonable assumption because the ratio of the thermal conductivities of the gas and the solid particles is small (see Rudinger 1980, p. 17).

It was assumed that the concentration of the solid particles was sufficiently low so that the effects of the thermal, mechanical and hydrodynamical interactions between the solid particles could be ignored. Under these assumptions the spherically symmetric, non-stationary flow of gas-particle mixture is governed by the following system of partial differential equations expressing the conservation of mass, momentum and energy of the gaseous and the solid phases (see Rudinger 1980, pp. 87–89)

$$\frac{\partial \rho}{\partial t} + \frac{1}{x^2} \frac{\partial}{\partial x} (x^2 \rho u) = 0, \quad (2)$$

$$\frac{\partial}{\partial t} (\rho u) + \frac{1}{x^2} \frac{\partial}{\partial x} [x^2 (\rho u^2 + p)] = -\frac{\sigma}{m} D, \quad (3)$$

$$\frac{\partial}{\partial t} [\rho (c_v T + 0.5 u^2)] + \frac{1}{x^2} \frac{\partial}{\partial x} [x^2 \rho u (c_p T + 0.5 u^2)] = -\frac{\sigma}{m} (v D + Q), \quad (4)$$

$$\frac{\partial \sigma}{\partial t} + \frac{1}{x^2} \frac{\partial}{\partial x} (x^2 \sigma v) = 0, \quad (5)$$

$$\frac{\partial}{\partial t} (\sigma v) + \frac{1}{x^2} \frac{\partial}{\partial x} (x^2 \sigma v^2) = \frac{\sigma}{m} D, \quad (6)$$

$$\frac{\partial}{\partial t} [\sigma (c_m \Theta + 0.5 v^2)] + \frac{1}{x^2} \frac{\partial}{\partial x} [x^2 \sigma v (c_m \Theta + 0.5 v^2)] = \frac{\sigma}{m} (v D + Q), \quad (7)$$

where σ is the mass spatial density of solid particles, p is the gas pressure, m is the mass of the solid particle, c_m is the specific heat capacity of the solid phase, c_v is the specific heat capacity of the gaseous phase at constant volume and x is a radial coordinate.

It is important to note that the pressure gradient in the momentum balance, equations (3) and (6), is assigned solely to the gaseous phase; the solid-phase contribution to the pressure gradient is assumed to be zero. This assumption is valid when the volume fraction occupied by the solid particles is negligibly small. Only under this assumption are the conservation equations describing the motion of a gas–solid particle mixture hyperbolic (see Stewart & Wendroff 1984). In a case when these equations are coupled also through a volume fraction of the solid phase the resulting system of partial differential equations becomes of a mixed type. This causes mathematical and numerical complications, because the initial-value problem for the system of equations (2)–(7) is ill-posed and is unstable with respect to short-wave perturbations. In such a case the system of conservation equations may be transformed to become hyperbolic by introducing a pressure term due to the solid particles. However, this approach is very involved, and to the best of our knowledge it was not, as yet, implemented for calculating gas–solid particle flows.

The system of partial hyperbolic nonlinear differential equations (2)–(7) was

solved numerically by using the modified random-choice method with operator splitting technique. This technique is based on splitting one time step into two stages. In the first stage, the homogeneous part of the equations is solved by using the random-choice method, which is of the first order of accuracy in time. In the second stage, the non-homogeneous part is solved by using the Euler method. More accurate methods are not needed here because the random-choice method is of the first order of accuracy only. The random-choice method was developed by Glimm (1965) as a part of a constructive existence proof for equations of hyperbolic type, and was used for solving gas-dynamics problems by Chorin (1976). Since then, the random-choice method has been used extensively in solving various problems of gas-dynamics; see, for example, Sod (1977), Colella (1982), Miura & Glass (1982) and Saito & Glass (1984). In the following, an outline of this numerical scheme is given.

The system of partial differential equations (2)–(7) can be written in a vector form as

$$U_t + F(U)_x = -W(U),$$

where the vector U represents the dependent variables (ρ, u, p, T, σ, v and Θ) and the vector W includes the inhomogeneous and non-conservative terms. Strictly speaking, the random choice method is capable of solving the hyperbolic equations in the conservative and homogeneous form only, i.e.

$$U_t + F(U)_x = 0.$$

This system of equations is solved in the first stage of the operator-splitting technique. The results are used in the second stage when the system of ordinary differential equations $U_t = -W(U)$ is solved. This is done by using the Euler method, i.e. $U(t + \tau) = U(t) - W(U)\tau$.

The random-choice method is essentially a two-step method that performs numerical solution with a staggered spatial grid. In performing the numerical solution, the time is divided into intervals, τ , and the space coordinate into increments h .

Let U_i^n approximate $U(ih, n\tau)$ and $U_{i+\frac{1}{2}}^{n+\frac{1}{2}}$ approximate $U[(i+\frac{1}{2})h, (n+\frac{1}{2})\tau]$. For finding the solution $U_{i+\frac{1}{2}}^{n+\frac{1}{2}}$ at the next time level we consider the system of homogeneous equations with piecewise-constant initial data

$$U(x, n\tau) = \begin{cases} U_{i+1}^n & \text{if } x > (i+\frac{1}{2})h \\ U_i^n & \text{if } x < (i+\frac{1}{2})h. \end{cases}$$

Thus we obtain a sequence of Riemann problems, i.e. initial value problems for hyperbolic partial differential equations with a step-like initial data.

If the value of the time step satisfies the Courant–Friedrichs–Lewy condition, i.e. $\tau < h/(2|u| + a)$, where a is the local speed of sound, then the waves generated while solving each of the Riemann problems do not interact. The solution of the Riemann problems $U(x, t)$ can be combined into a solution at the next time level of the form

$$U_{i+\frac{1}{2}}^{n+\frac{1}{2}} = U[(i+\frac{1}{2} + \xi)h, (n+\frac{1}{2})\tau],$$

where ξ are uniformly distributed numbers in the interval $[-\frac{1}{2}, \frac{1}{2}]$. A similar procedure can be applied to obtain U_i^{n+1} at the second step. The performance of the method depends upon the availability of fast and exact Riemann solvers. Detailed description of the random choice method can be found in Chorin (1976) and Colella (1982).

The main advantage of the random-choice method over other numerical schemes is that it allows high resolution of shock waves and contact discontinuities, whereas in other finite-difference methods they are usually smeared over a few mesh points as a consequence of artificial viscosity and truncation error of the scheme. The random-choice method uses the exact solution of the self-similar Riemann problem. The Riemann problem is solved repeatedly between each pair of neighbouring spatial mesh points. The successive positions of the discontinuities (shock waves and contact surfaces) between these mesh points are sampled with the help of the uniformly distributed sequences of Van der Corput as suggested by Glimm *et al.* (1980) and Colella (1982) (except for the points at the boundary of the numerical mesh where the uniformly distributed numbers were set to be equal to 0.5 to obtain the zero velocity and the true value of the pressure). Elperin & Igra (1986) showed that the implementation of uniformly distributed sequences considerably reduces the numerical noise and thus improves the quality of the results. The solution of the Riemann problem was found by using the Godunov iterative scheme described in Chorin (1976). For the reader's convenience, in the following we summarize the two stages of the operator splitting technique described earlier.

The solution of equations (2)–(7) is obtained by solving alternately in each time step two systems of differential equations. The system solved at the first stage is derived from (2)–(7) by omitting the inhomogeneous and non-conservative terms to yield

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0, \quad (8)$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p) = 0, \quad (9)$$

$$\frac{\partial}{\partial t}[\rho(c_v T + 0.5u^2)] + \frac{\partial}{\partial x}[\rho u(c_p T + 0.5u^2)] = 0, \quad (10)$$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial}{\partial x}(\sigma v) = 0, \quad (11)$$

$$\frac{\partial(\sigma v)}{\partial t} + \frac{\partial}{\partial x}(\sigma v^2) = 0, \quad (12)$$

$$\frac{\partial}{\partial t}[\sigma(c_m \Theta + 0.5v^2)] + \frac{\partial}{\partial x}[\sigma v(c_m \Theta + 0.5v^2)] = 0. \quad (13)$$

Equations (8)–(13) were solved by using the random-choice method. The second

system of ordinary differential equations is derived from (2)–(7) by omitting the spatial derivatives, i.e.

$$\frac{\partial \rho}{\partial t} + \frac{2}{x} \rho u = 0, \quad (14)$$

$$\frac{\partial}{\partial t} \rho u + \frac{2}{x} (\rho u^2 + p) + \frac{\sigma}{m} D = 0, \quad (15)$$

$$\frac{\partial}{\partial t} [\rho(c_v T + 0.5u^2)] + \frac{2}{x} \rho u(c_p T + 0.5u^2) + \frac{\sigma}{m} (vD + Q) = 0, \quad (16)$$

$$\frac{\partial \sigma}{\partial t} + \frac{2}{x} \sigma v = 0, \quad (17)$$

$$\frac{\partial \sigma v}{\partial t} + \frac{2}{x} \sigma v^2 - \frac{\sigma}{m} D = 0, \quad (18)$$

$$\frac{\partial}{\partial t} [\sigma(c_m \Theta + 0.5v^2)] + \frac{2}{x} \sigma v(c_m \Theta + 0.5v^2) - \frac{\sigma}{m} (vD + Q) = 0. \quad (19)$$

Equations (14)–(19) were solved by using Euler's method with the initial data obtained from the random-choice solution of (8)–(13). This implementation of the operator-splitting technique allows us to find the solution of (2)–(7) at each time step. All computations were performed with the reflection boundary condition for the gaseous phase and transmissive boundary condition for the solid phase. The Riemann problem for the solid phase, i.e. (5)–(7) was solved analytically, by using a method suggested by Miura & Glass (1982). This method assumes a linear distribution of flow variables of the solid phase between adjacent mesh points. This approach allows us to preserve the first-order accuracy of the random-choice method and to avoid difficulties arising as a result of the multivalued solutions of (5)–(7) with step-like initial data.

The thermodynamic and dynamic properties appearing in (2)–(7) were presented in a non-dimensional form as follows:

$$\begin{aligned} \bar{p} &= p/p^*; \quad \bar{\rho} = \rho/\rho^*; \quad \bar{\sigma} = \sigma/\sigma^*; \quad \bar{u} = \sqrt{\gamma}u/a^*; \\ \bar{v} &= \sqrt{\gamma}v/a^*; \quad \bar{\Theta} = \Theta/T^*; \quad \bar{T} = T/T^*; \\ \bar{t} &= a^*t/(\sqrt{\gamma}L); \quad \bar{x} = x/L; \end{aligned}$$

where the asterisk corresponds to flow properties ahead of the blast wave, a is the speed of sound and $\gamma = c_p/c_v$.

All calculations were performed with 750 mesh points. The size of the time steps were regulated by the Courant–Friedrichs–Lewy condition. The average running time was *ca.* 900 s CPU (central processing unit) on the CDC CYBER 180-840 computer.

RESULTS AND DISCUSSION

Equations (2)–(7) were solved numerically by using the random-choice method. The results shown subsequently are presented in two groups. In the first (figures 2–5), the dusty flow field is compared with a similar pure gas case at a given time.

In the second (figures 6–26), the temporal evolution of the radial distributions of the suspension properties is examined.

Case 1. Dust is uniformly distributed outside the spherical diaphragm

The spatial distributions of the gas velocity at a given time ($\bar{t} = 0.158$), are shown in figure 2. Before the diaphragm rupture the entire flow field is at rest, i.e. $\bar{u} = 0$ (see figure 1*a*). Upon the passage of the spherical blast wave at time $\bar{t} > 0$ the gas experiences a sudden increase in its velocity. The gas velocity further increases until it reaches the tail of the rarefaction wave T . Through the rarefaction wave the gas velocity decreases down to its initial value of $\bar{u}_4 = 0$. Comparing the results obtained for the dusty gas case with those of the pure gas case reveals the following.

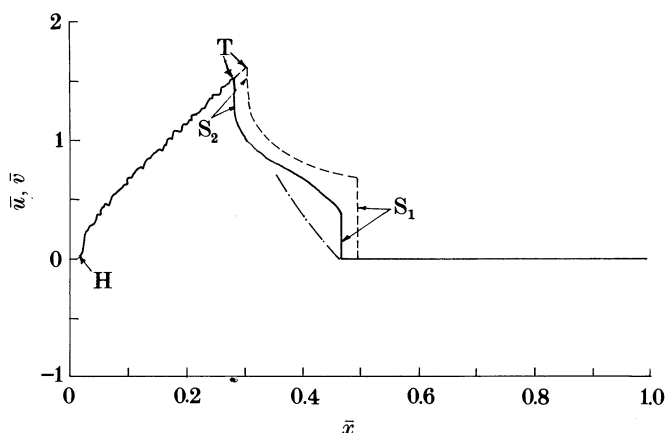


FIGURE 2. Velocity distributions at $\bar{t} = 0.158$. Lines: - - - - -, pure gas ($\bar{u}$); —, dusty gas ($\bar{u}$); - · - · -, dust particles ($\bar{v}$).

- (i) The dust presence causes a delay in the propagation of the blast wave, S_1 .
- (ii) A weaker blast wave is observed in the suspension. As a result, the gas velocity immediately behind the blast wave is always lower in the dusty-gas case.

(iii) As expected, the dust particles cross the blast-wave front unaffected. The reason is that their diameter is much larger than the width of the blast wave, which is of the order of a few mean free paths of the gas molecules. After crossing the blast-wave front the dust particles are accelerated by the drag force acting on them. Because the drag force is proportional to the velocity difference between the two phases (see (1)), they are accelerated until equal velocities are attained by the gas and the solid particles. The curve describing the dust velocity is terminated at the contact surface because the mass spatial density of the dust particles behind it is negligibly small. The contact surface is not visible in figure 2 because equal velocities exist on both of its sides.

The radial temperature distribution is shown in figure 3 for $\bar{t} = 0.158$. The temperature jump through the primary blast wave S_1 is clearly visible in this

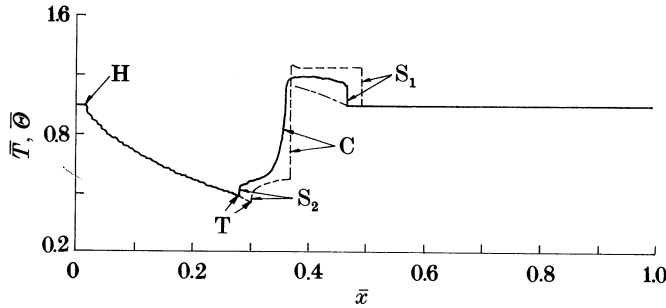


FIGURE 3. Temperature distributions at $\bar{t} = 0.158$. Lines: -----, pure gas ($\bar{T}$); —, dusty gas ($\bar{T}$); - · - · -, dust particles ($\bar{\Theta}$).

figure. Unlike the previous case (figure 2), here the contact surface is clearly noticeable. The reason is that there is a very large temperature decrease through the contact surface. It should be noted that unlike the case of a pure gas, in the dusty-gas case the contact surface changes into a contact region because some dust penetrates through it. The formation of a secondary shock wave, just in front of the rarefaction wave tail, is also noticeable in figure 3. Again a delay in the shock-wave propagation, caused by the dust presence, is clearly noticeable. It should also be noted that a higher temperature jump across the primary blast wave S_1 is obtained in the pure-gas case. The dust particles cross the blast wave unaffected and are heated relatively slowly behind it via heat convection from the hot gas environment. Because part of the thermal energy of the gas is consumed while heating the dust particles, the gas temperature after the blast wave is lower in the case of gas-particle suspension. Again the curve describing the dust temperature terminates at the contact surface because the dust concentration behind it is negligibly small.

The pressure distribution behind the blast wave is shown in figure 4 (again for

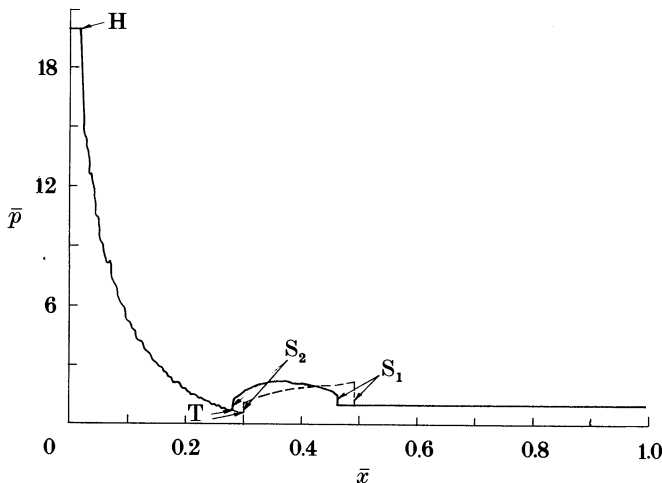


FIGURE 4. Pressure distributions at $\bar{t} = 0.158$. Lines: -----, pure gas; —, dusty gas.

$\bar{t} = 0.158$). The blast wave S_1 and the secondary shock wave S_2 are clearly visible and both are delayed in comparison with the pure-gas case. The contact surface is not visible in figure 4 because equal pressures exist on both of its sides. The pressure jump across the blast wave S_1 is significantly higher in the pure-gas case. The dust presence completely changes the pressure signature behind the blast wave. In the pure-gas case the pressure monotonically decreases behind the blast-wave front. In the dusty-gas case it increases up to a maximum value and then decreases. This is of interest for assessing the destructive effects of blast waves because in the dusty-gas case the shock impact is lower although the absolute value of the pressure behind a shock wave can be higher than that obtained in a similar pure-gas case.

The destructive effects of blast waves to structures can be judged by evaluating the total pressure behind the blast wave. In figure 5 the total pressure distribution, at $\bar{t} = 0.158$, is shown for the pure-gas case (broken line) and the dusty-gas case (solid line); in both cases $p_t = p + 0.5\rho u^2$. Because the dust is accelerated by the gas, as was shown in figure 2, it is reasonable to add the dust momentum to the total pressure. The dashed-dotted line in figure 5 shows the spatial distribution of the total pressure, $p_t = p + 0.5\rho u^2 + 0.5\sigma v^2$. It is clearly visible from figure 5 that although the largest pressure jump through the shock-wave front is associated with the pure-gas case, the maximum total pressure behind the dusty blast wave is much higher than that obtained in the pure-gas case. The difference is 60% for $\bar{t} = 0.158$ and it will be higher at later times. In the pure-gas case, the highest value of p_t is obtained immediately behind the contact surface whereas in the dusty-gas case it is obtained further downstream of the contact surface. Because the destructive effect of blast waves is a combination of the peak pressure and the suddenness with which it is applied, it is difficult to make a categorical statement about the change of the magnitude of the destructive power associated with the dust presence.

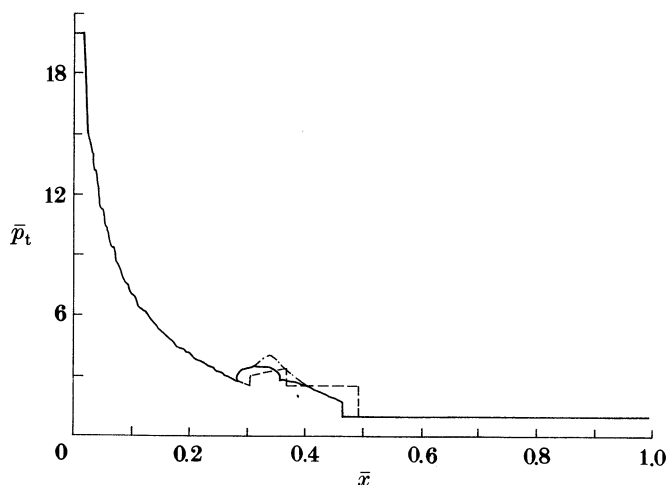


FIGURE 5. Total pressure distributions at $\bar{t} = 0.158$. Lines: - - - -, pure gas; —, dusty gas; - · - · -, $\bar{p}_t = p + 0.5\rho u^2 + 0.5\sigma v^2$.

The previously observed delay in the primary blast wave is also visible in figure 5. The contact surface is clearly visible for the pure-gas case because there are large changes in the gas density through the contact surface. It is not seen so clearly in the dusty-gas case (solid line) because the changes in density are smaller now as a result of the presence of the dust.

Thus the dust presence significantly affects the flow field behind the blast wave. It causes a delay in the wave propagation and reduces the blast wave strength. The maximum temperature in region 2 is lower in the dusty case, whereas the pressure (and total pressure) will reach higher values than those obtained in a similar pure-gas case. Rakib *et al.* (1987) reported similar findings in their numerical solution, by using the finite-difference scheme described in Brode (1955).

In the second set of numerical results the time dependency of the suspension properties is shown for the following conditions: $p_4/p_1 = 20$, $T_4/T_1 = 1$, $r = 0.2$ m, dust mass-loading ratio = 1, dust diameter $d = 10^{-5}$ m, dust density $\rho_s = 2.5 \times 10^3$ kg m $^{-3}$ and dust specific heat capacity $c_m = 800$ J kg $^{-1}$ K $^{-1}$.

In figures 6–11 results obtained for the case when the dust is uniformly distributed outside the spherical diaphragm of radius $r = 0.2$ m are shown. The spatial distributions of the gas velocity at four different times are shown in figure 6. This figure exhibits the expected behaviour of a decaying spherical blast wave. In figure 6 the reduction in the blast-wave strength is manifested as a reduction in the velocity jump through the shock front, S_1 . When time increases, the rarefaction wave penetrates deeper into the high-pressure zone. Once the rarefaction-wave head reaches the centre of the spherical diaphragm it reflects and starts moving outward (not shown in the figure because the reflected rarefaction wave is very weak).

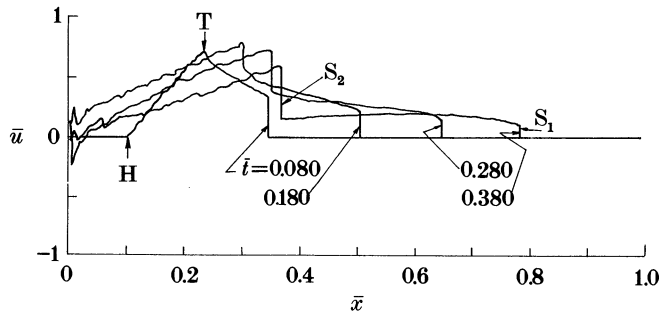


FIGURE 6. Flow velocity distributions at different times.

To match between the pressures existing behind the primary blast wave, S_1 and the rarefaction-wave tail T, a secondary shock wave is developed. It is hardly noticeable at time $\bar{t} = 0.08$; however, it is clearly visible as a well-formed shock wave at later times. For example, the velocity jump across the secondary shock wave at $\bar{t} = 0.380$ (see figure 6 at $\bar{x} = 0.37$) is 3.6 times larger than the jump through S_1 at the same time ($\bar{t} = 0.380$).

The contact surface is not visible in figure 6 because the velocities are equal on

both of its sides. It is clearly noticeable in figure 7 where the temperature field is shown at four different times after the diaphragm rupture. As noticed earlier, as time goes on the primary blast wave weakens because the initial energy is distributed over ever-increasing mass of the suspension. The largest temperature change is observed across the contact region (see figure 7). A further temperature jump occurs across the secondary shock wave S_2 . Similar to the velocity curves, here too the secondary shock wave is hardly noticeable at early times ($\bar{t} \leq 0.080$). It becomes noticeable and grows in strength at later times (see figure 7).

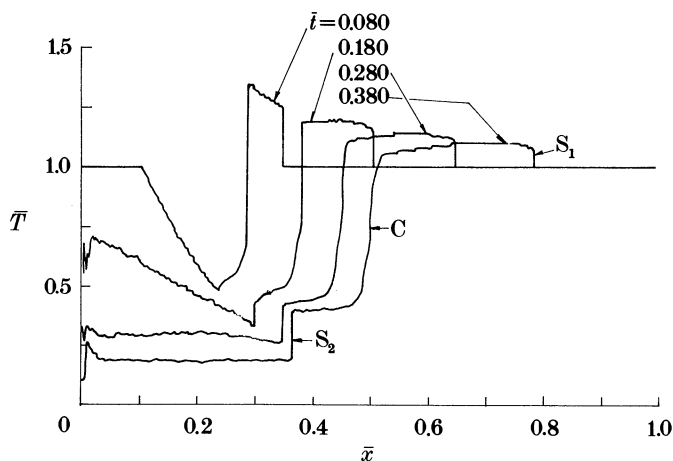


FIGURE 7. Gas temperature distributions at different times.

The spatial pressure distribution is shown in figure 8. Here again the weakening of the primary shock wave S_1 with increasing time and the formation and strengthening of the secondary shock wave, S_2 , are clearly visible. The contact surface is not seen in figure 8 because equal pressures exist on both of its sides.

Before the diaphragm rupture the dust was uniformly distributed outside the diaphragm where the dust loading ratio was equal to one. The effect of the blast-wave propagation into the suspension on the dust spatial-density is shown in figure 9. Immediately behind the blast wave front there is a pronounced increase in the dust concentration. The dust concentration reaches a maximum in proximity of the contact surface; thereafter, a very sharp decline, down to zero, is visible. It is because of this behaviour that in the following figures (figures 10 and 11), where the dust velocity and temperature are shown, the plots terminate in the vicinity of the contact region. Comparing figure 10, where the dust velocity is shown, with figure 6 reveals that immediately behind the blast wave, S_1 , the dust acceleration starts and its velocity increases. Should enough time be allowed, a kinematic equilibrium could have been reached.

The spatial distribution of the dust temperature is shown in figure 11. Behind the blast wave, S_1 , the dust is heated by the gas. The heat transfer continues until the two phases reach the same temperature. Because of the limited extent of region 2 there is not enough time for the suspension to reach a thermal equilibrium.

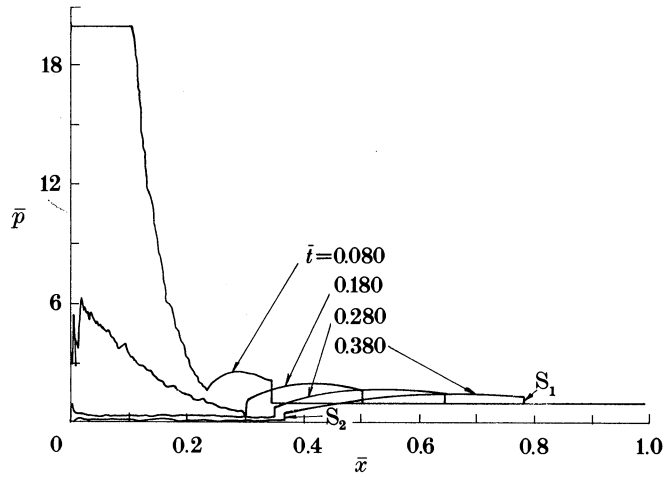


FIGURE 8. Pressure distributions at different times.

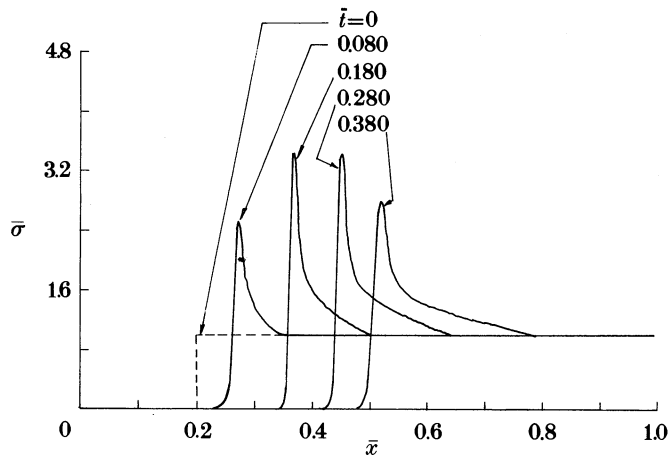


FIGURE 9. Dust spatial-density distributions at different times.

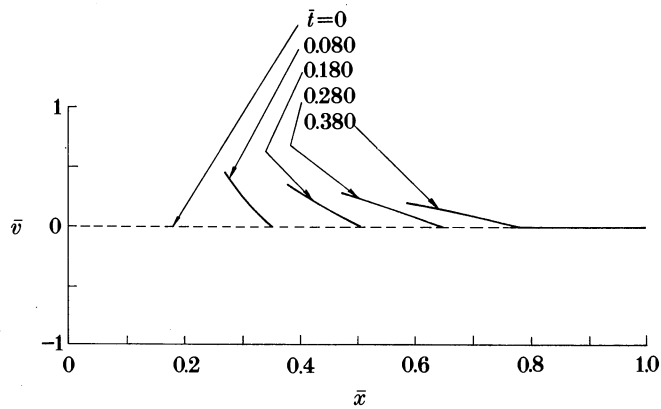


FIGURE 10. Dust velocity distributions at different times.

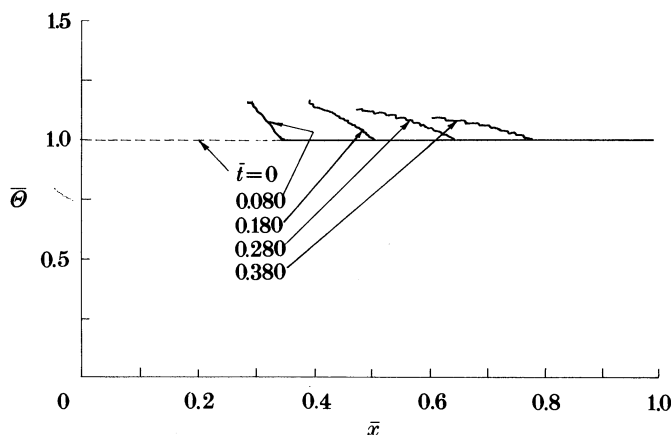


FIGURE 11. Dust temperature distributions at different times.

In summary, the numerical solution indicates the existence of the following wave system: a primary blast wave S_1 , which weakens as it propagates outwards; a contact region, behind which the dust density is negligibly small; a secondary shock wave S_2 , which becomes noticeable shortly after the diaphragm rupture and strengthens with time; and a rarefaction wave travelling towards the explosion centre. With increasing time the dust temperature and velocity approach those of the gaseous phase. There is a sharp increase in the dust concentration behind the blast wave. This increase in the dust concentration, up to a value four times higher than the initial dust concentration must be taken into account in assessing the effects of atmospheric nuclear explosions.

Case 2. Dust is uniformly distributed inside the spherical diaphragm

In figures 12–18, the case where the dust was initially confined to the space inside the exploding spherical diaphragm of radius $r = 0.2$ m is shown. Analysis of the suspension properties behind the primary blast wave S_1 (in region 2, see figure 1b) i.e. the spatial distributions of the gas velocity (figure 12), gas temperature (figure 13) and pressure (figure 14) show that they are identical to those obtained in a similar pure-gas case. This should be anticipated because the primary blast wave S_1 propagates into a pure gas. As a result, the blast wave S_1 has a step-like shape and it weakens with time. Comparing figures 12–14 with figures 6–8 shows that a stronger blast wave occurs when the blast wave propagates into a pure gas. A direct comparison with the results shown in figures 2–4 is not possible because these results correspond to a time $\bar{t} = 0.158$.

In figure 15 the dust spatial density is shown. At $\bar{t} = 0$ the dust was confined in the space inside the spherical diaphragm. After the diaphragm rupture the wave system shown schematically in figure 1b develops, and the dust is carried outwards by the gas flow. When time increases the dust becomes confined inside a thin spherical layer whose radius increases with time. The peak in the dust spatial density is positioned between the secondary shock wave S_2 and the contact surface, closer to the contact surface. This explains why the flow between

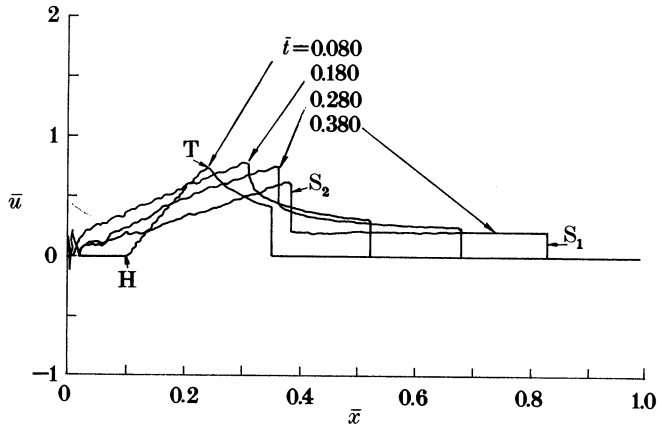


FIGURE 12. Flow velocity distributions at different times.

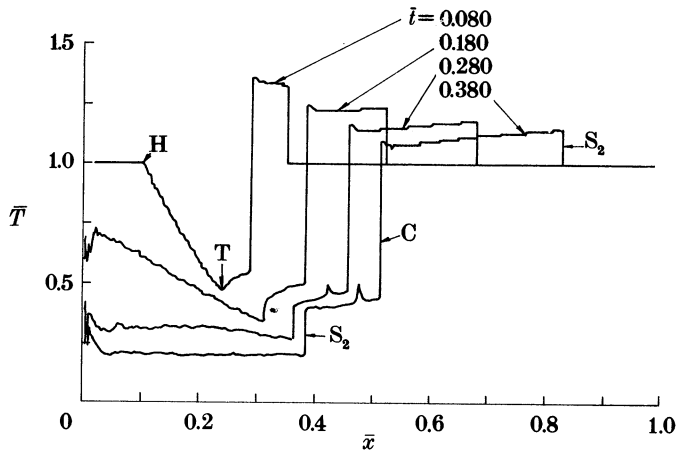


FIGURE 13. Gas temperature distributions at different times.

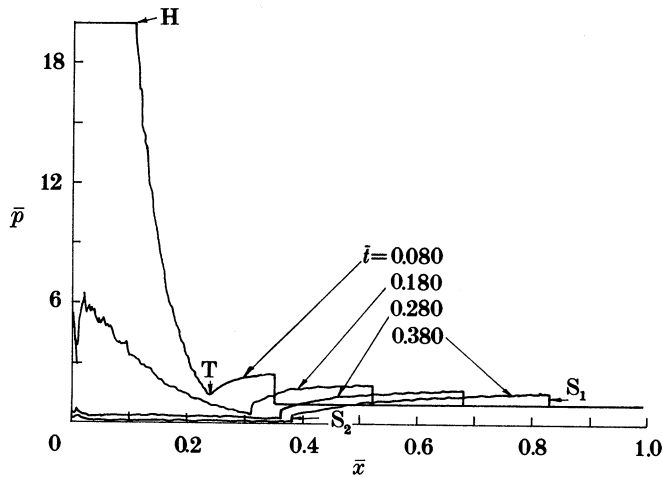


FIGURE 14. Pressure distributions at different times.

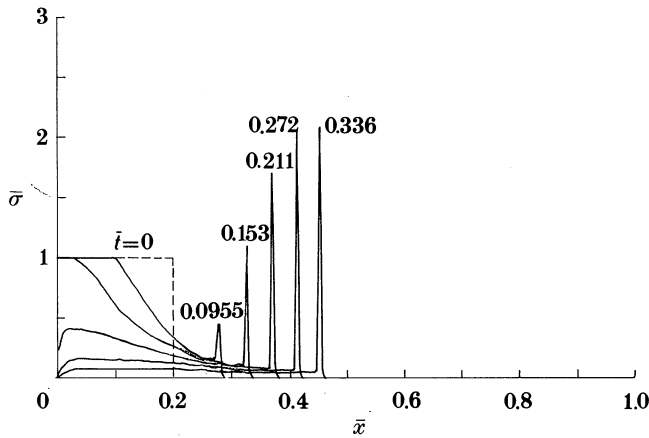


FIGURE 15. Dust spatial-density distributions at early times, $0 < \bar{t} < 0.336$.

S_1 and the contact surface is identical to that in the case of a pure gas. At early times ($\bar{t} < 0.336$) the post-blast gas velocity is relatively high, and therefore a large mass of dust is swept outwards by the gas flow. However, at later times the gas velocity reduces and the volume of the spherical layer inside which the dust is contained increases. These two effects result in a decrease in the peak of the dust spatial density (see figures 15 and 16).

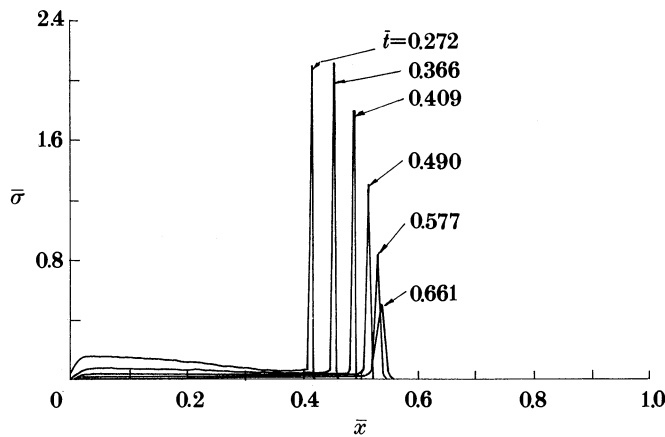


FIGURE 16. Dust spatial-density distributions at late times, $0.272 < \bar{t} < 0.661$.

The dust velocity distributions at various times are shown in figure 17. Comparing figures 12 and 17 demonstrates the process of the dust acceleration by the gas flow. In figure 17 the velocity of the solid particles is not plotted in the flow regions where the dust spatial density is negligibly small (the four arrows mark the position of the primary blast wave at the corresponding times). This explains the termination of the curves of the velocity of the solid particles shown in this figure. In the present case where the dust is initially confined to the space inside the

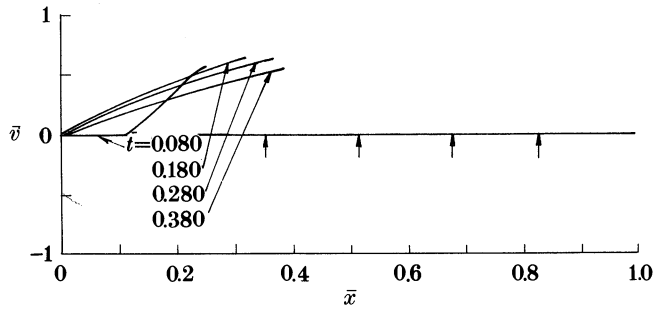


FIGURE 17. Dust velocity distributions at different times.

exploding spherical diaphragm, the resulting flow in region 2 is identical to that obtained in a similar pure-gas case, and after the diaphragm rupture a sharp peak in the dust spatial density distribution develops. This peak initially grows in magnitude and at later times decreases. The transient peak in the dust spatial density is of importance in assessing the effects of atmospheric nuclear explosions.

Case 3. Dust is uniformly distributed in a spherical shell located outside the diaphragm

The geometry of the considered flow field before the diaphragm rupture is shown schematically in figure 18. The velocity distribution for five different times after the diaphragm rupture are shown in figure 19. Until the blast wave S_1 reaches the dusty-gas layer the velocity distribution behind S_1 is identical to that obtained for a similar pure-gas case. The blast wave S_1 reached the dusty-gas layer at $\bar{t} = 0.078$ after the diaphragm rupture. Until that time the flow velocity monotonically

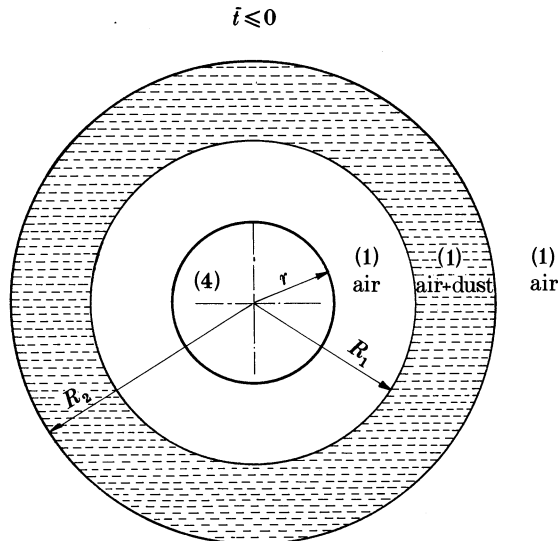


FIGURE 18. Schematic description of the flow field at $\bar{t} = 0$. Abbreviations: r , the diaphragm radius; $r = 0.2$ m; R_1 and R_2 radii of the dust layer; $R_1 = 0.35$ m; $R_2 = 0.55$ m.

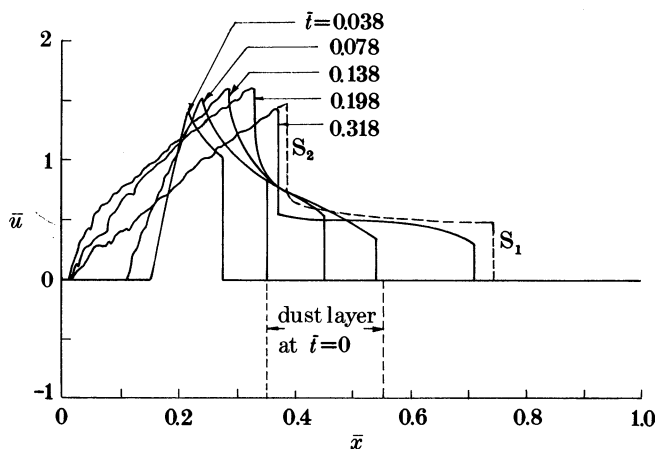


FIGURE 19. Flow velocity distributions at different times. Lines: —, dusty gas; ----, pure gas.

increases between S_1 and the tail of the rarefaction wave (see also figure 2, for the pure-gas case). In the dusty-gas case (at time $\bar{t} > 0.078$) a maximum in the gas velocity is obtained between S_1 and S_2 . This maximum becomes more pronounced at later times. Lower gas velocities are obtained in the dusty-gas case as compared with a similar pure-gas case (see figure 19 for $\bar{t} = 0.318$). The lower gas velocities in the dusty-gas case are the result of momentum transfer from the gas to the dust. Like in the previous cases, here too the development and the strengthening of a secondary shock wave S_2 is clearly visible (see figure 19).

The temperature distributions in the gaseous phase for five different times are shown in figure 20. In the pure-gas case ($\bar{t} \leq 0.078$ in figure 20 and the broken line in figure 3) a practically uniform temperature was observed between S_1 and the contact surface. After the blast wave S_1 reached the dust layer a non-uniform temperature distribution developed in region 2 (see figure 20 for $\bar{t} > 0.078$).

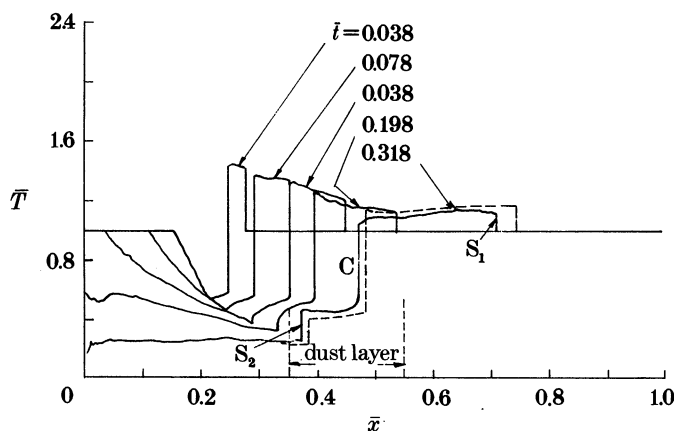


FIGURE 20. Gas temperature distributions at different times. Lines: —, dusty gas; ----, pure gas.

As was noticed earlier, the temperature change through the contact surface is larger than that obtained while crossing the blast waves S_1 and/or S_2 . As time goes on the blast wave S_1 weakens and the temperature immediately behind it decreases. Lower gas temperatures are obtained behind a blast wave in a dusty gas compared with a similar pure-gas case. The reduction in the temperature results from the heat transfer from the gas to the solid particles.

The spatial pressure distributions are shown in figure 21. For times $\bar{t} \leq 0.078$ the typical blast-wave pressure signature is observed, i.e. a sharp pressure jump through the blast-wave front followed by a monotonic decrease in pressure until the tail of the rarefaction wave. At later times, $\bar{t} > 0.078$, the dust presence changes the picture. Now the pressure behind the blast wave increases first, and only later it decreases down to the value appropriate to S_2 . This pressure signature is quite different from the typical blast-wave pressure signature.

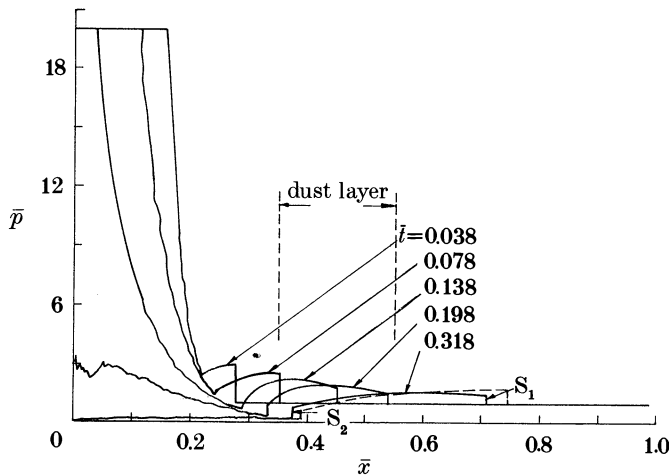


FIGURE 21. Pressure distributions at different times.
Lines: —, dusty gas; - - - - -, pure gas.

The dust spatial density at $\bar{t} = 0$ is shown by a broken line in figure 22. The interaction of the blast wave with the dust layer results in a change in the dust spatial-density distribution from the original uniform pattern to a bell-shaped distribution with a sharp maximum. At early times, $\bar{t} < 0.318$, a transition from a uniform to a bell-shaped distribution develops via non-symmetrical modes. However, at later times the dust spatial-density distribution transformed into a more symmetrical bell-shaped distribution, and the dust continues to move outwards. Because of this radial drift the peak in the dust spatial density distribution reduces with increasing time.

The dust velocity and temperature distributions for three different times are shown in figures 23 and 24 respectively. The curves are similar to those shown in figures 10 and 11. The dust particles cross the shock front S_1 unaffected. Behind the shock front the dust particles are accelerated and heated. The curves depicting the dust velocity (figure 23) and dust temperature (figure 24) terminate at the point where the dust spatial density is negligibly small. This occurs before the dust

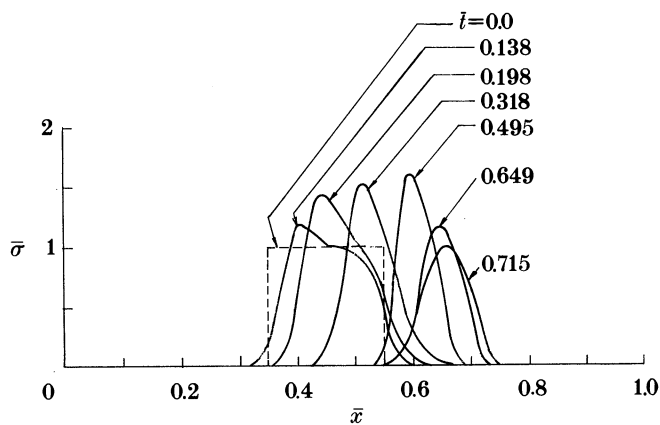


FIGURE 22. Dust spatial-density distributions at different times.

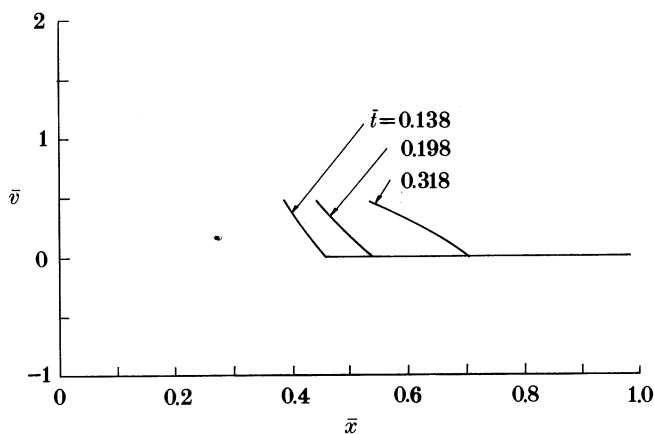


FIGURE 23. Dust velocity distributions at different times.

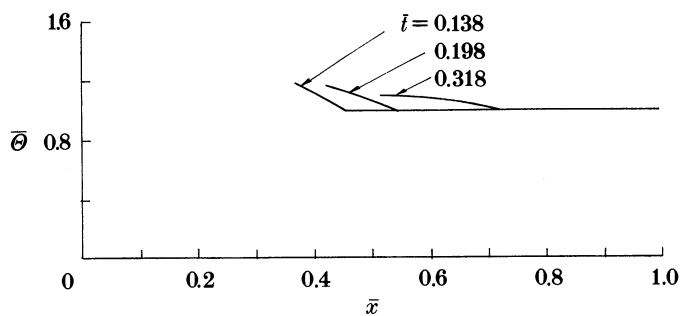


FIGURE 24. Dust temperature distributions at different times.

reaches the gas velocity u and temperature T because the time available for momentum and heat transfer in region 2 is not sufficient to reach equilibrium. The plots of the dust velocity and temperature distributions are presented for times after the blast wave has reached the dust layer at $t \approx 0.078$.

When the dusty-gas layer was located immediately behind the exploding spherical diaphragm (i.e. $R_1 = r = 0.2$ m and $R_2 = 0.4$ m in figure 18), the results obtained for the suspension properties are similar to those shown in figures 19–24. Therefore, only two examples will be shown here. The spatial distributions of gas temperature are shown in figure 25. Comparing these results with those obtained for the case of uniform dust spatial distribution outside the diaphragm (figure 6) shows that a stronger blast wave is obtained in the present case (figure 25). The results presented in figure 25 are similar to those shown in figure 20. A detailed comparison is not possible because the two plots correspond to different times.

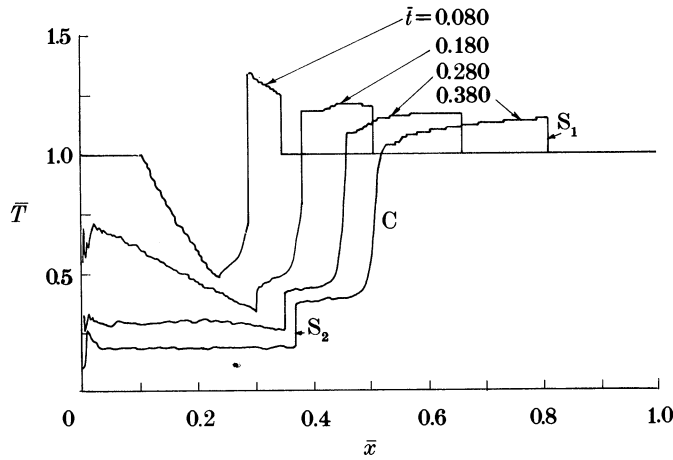


FIGURE 25. Gas temperature distributions at different times.

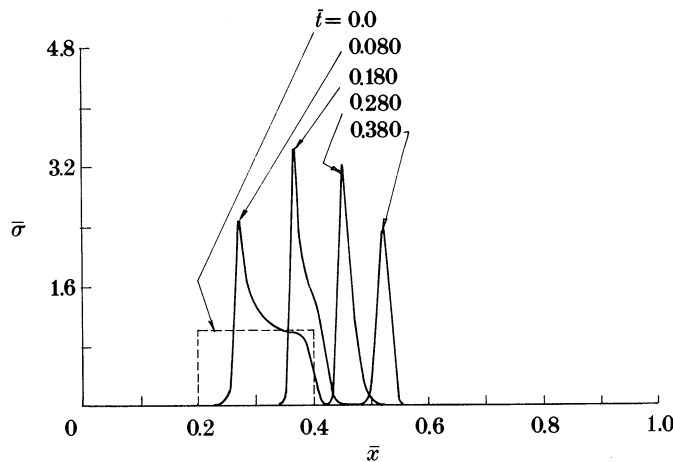


FIGURE 26. Dust spatial-density distributions at different times.

When the dusty-gas layer was placed immediately behind the exploding diaphragm, the dust spatial distribution behaved as shown in figure 26. Comparing the results shown in figure 26 with those shown in figure 22 (for the dusty-gas layer of figure 18) reveals that the dust moves outwards much faster when the dusty-gas layer was close to the diaphragm. In both cases the transition from a uniform to a bell-shaped dust spatial-density distribution was observed.

CONCLUSIONS

The numerical results presented here for the propagation of spherical blast waves into a dust-gas suspension indicate the following.

1. The dust presence causes a delay in the blast-wave propagation and reduces its strength in comparison with a similar pure-gas case. The reduction in the strength is manifested by lower values for the flow properties (p , u and T) obtained immediately behind the blast front in the dusty-gas case.

2. Although lower static pressures are obtained immediately behind the blast-wave front in the dusty-gas case, a higher post-blast wave pressure is obtained in the dusty-gas case. In the pure-gas case, the maximum pressure is obtained immediately behind the blast front. Thereafter, the static pressure monotonically decreases. In the dusty-gas case, the static pressure increases beyond the pressure jump obtained upon crossing the blast-wave front. After attaining a maximum value (which can exceed the frozen pressure jump of the pure-gas case) the static pressure declines.

3. The destructive effect of blast waves can be assessed by inspecting the post-blast wave total pressure. It was shown that significantly higher total pressures are obtained in the dusty-gas case. Because the destructive effect is often a combination of the peak pressure and the suddenness with which it is applied, it is plausible that the dust presence increases the destructive potential of blast waves.

4. In all analysed cases the dust was initially uniformly distributed. The passage of a blast wave changes this uniform dust spatial distribution. It was transformed to a bell-shaped distribution with a sharp peak. The peak value of the dust spatial density is significantly higher than the initial value. The development of this sharp maximum in the dust spatial-density distribution can be of interest in assessing the effects of atmospheric nuclear explosions.

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