Fracture Mechanics ME 6104 **National University of Singapore**

Prof. M. Perl **Dept. of Mechanical Engineering**

Assignment # 4

Problem 1

An X-ray of a machine part that operates under uniaxial tension revealed an internal crack, of almost circular shape, on a plane perpendicular to the applied load. The radius of the crack ia a_0=2mm. Such a crack is commonly referred to as a Penny-shaped crack and its stress intensity factor is given by $K_I = \frac{\pi}{2}\sigma_\infty \sqrt{\pi a}$. The applied load has a cycle ratio of R=0 and its amplitude is σ_{max}. The part is made of a steel alloy with the following properties:

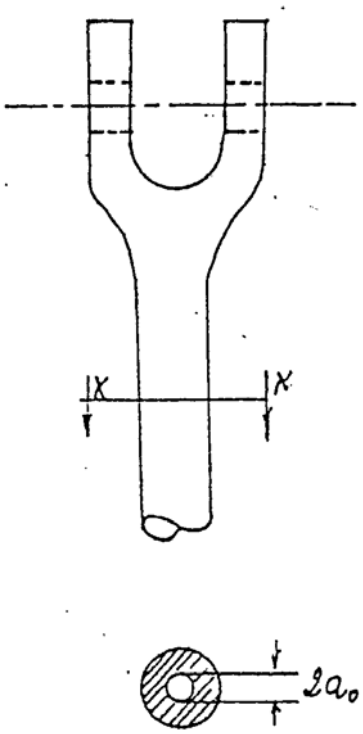


Fig. 1

toughness $K_{Ic} = 60\ MPa\sqrt{m}$

fatigue threshold $\Delta K_{th} = 5\ MPa\sqrt{m}$

Paris constants n_p=4, A_p=$10^{-12}\ MN^{-4}m^2$

(a) What is the maximum allowable stress that will not extend the crack?

(b) What is the maximum allowable stress for which the fatigue life will be at least N=10^4 cycles? (Assume fracture occurs before the crack reaches the outer diameter of the part).

(c) It is common to amend Paris' law by including in it the fatigue threshold:

$$\frac{da}{dN} = A_p\left[(\Delta K_I)^{n_P} - (\Delta K_{th})^{n_P}\right]$$

What is the effect of this correction in this particular case? Draw a general conclusion.

Problem 2

To determine the fatigue properties of HY-TUF steel, the following data was obtained from crack growth tests at constant amplitude:

Crack length a [in]	Experiment #1 $\Delta\sigma = 16 ksi; \quad R = 0$ Number of cycles	Experiment #2 $\Delta\sigma = 10 ksi; \quad R = 0.5$ Number of cycles
0.1	0	0
0.105	1100	2000
.	.	.
.	.	.
.	.	.
1.5	i	j
1.55	i+100	j+170

The tests were performed on a rectangular plate of width W=20" with a center-crack of length 2a. Assume that

$$K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

(a) Establish the rate diagram using two data points for each *R*-value; assume straight lines between data points.

(b) Determine Walker's equation using the above experimental data.

(c) Determine the parameters of Forman's equation assuming the above material has a toughness of $K_{Ic} = 80 ksi\sqrt{in}$.

Hint: First, replot the data as $\{[(1-R)K_{Ic} - \Delta K_I]\, da/dN\}$ versus ΔK_I.

Problem 3

The main beam of a press machine, made of steel, is described in the Fig.2a. A through the thickness notch of depth d=5mm with a rounded tip r=1mm is located at the bottom of the beam as described in Fig. 2b. A cyclic load is applied to the beam alternating between P_{min}=0 and P_{max}=1MN/m (pay attention to the units!).

(a) Will a fatigue crack develop from the notch under these condition?

In a routine inspection it was found that a fatigue crack of length a=1mm (see Fig. 2c) developed at the notch root. $K_I(a/b, L/b)$ for this configuration is given in Fig. 5.5 of the lecture notes.

(b) Determine the critical crack length a_{cr} (to the nearest 0.5mm). Explain your assumptions and each step in calculations.

(c) Determine the remaining fatigue life of the beam. Explain your assumptions.

(d) Estimate the accuracy of your solution.

Beam data

b= 50mm

t= 20mm

L=200mm

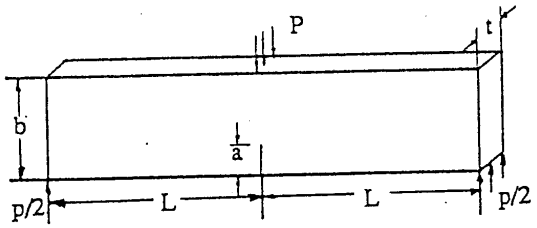


Fig. 2a

Material data

$\sigma_{yp} = 900\,MPa$

$K_{Ic} = 55\,MPa\sqrt{m}$

$E = 200\,GPa$

$\frac{da}{dN} = 2.5x10^{-10}(\Delta K)^{2.5}\ m/cycle$

for $[\Delta K] = MPa\sqrt{m}$

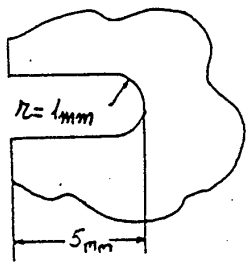


Fig. 2b

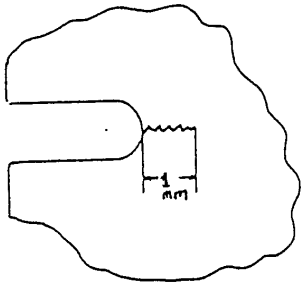


Fig. 2c