Theory of Elasticity ME 6102 **National University of Singapore**

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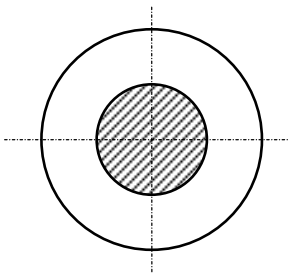
Assignment # 7

Prior to solving the problems in this assignment, please read in any Elasticity book such as Cook and Young [1] the chapters regarding the elastic solution for *"Thick-Walled Cylinders"* and for *"Shrink Fits"*.

[1] Cook, D. R. and Young, W.C., "Advanced Mechanics of Material," Pearson Education, 1998.

1. Reduce the elastic solution for a thick-walled cylinder under internal pressure P_i, to that for a "thin-walled" vessel:
   (a) Show the reduction for the stress components $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$.
   (b) Plot the maximum difference between the stress components $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$, in a thick-walled cylinder and a thin one, as a function of the radii ratio η defined as $\eta=b/a-1$, (a and b are the inner and the outer radii of the cylinder respectively).
   (c) Plot the maximum effective stress (based on von-Mises yield criterion) as a function of η. Which point in the cylinder is the most critical?

2. A long hollow cylinder (under plane strain conditions) of inner radius a and outer radius 2a is shrunk onto an infinitely rigid solid cylinder of radius a (no interference). Pressure P_o is applied on the outer cylinder.



   (a) Determine the stress field and the displacements in the outer cylinder.
   (b) What pressure does the outer cylinder exert on the inner one?
   (c) Determine the principal stresses in the two cylinders.
   (d) Using Tresca's yield criterion, determine the maximum allowable external pressure P_o if : $\mu = \lambda/2; \nu = 0.3; \tau_{al} = 2400 Kg/cm^2; a = 10 cm$.

3. A cylinder of inner radius *a* and outer radius *b* is shrunk onto a solid cylinder of outer radius a+δ (δ << a). The two cylinders are made of the same material ($\nu = 0.25; \lambda = \mu$) and are under plane strain conditions.

   (a) In the absence of external loading, what is the largest value of δ so that the maximum shear stress does not exceed a given value of τ_{al}.

   (b) After shrinking, a uniform radial tensile stress S is applied to the outer surface of the cylinder. Write the proper boundary conditions and determine the *minimum* value of S that will cause separation of the two cylinders.

   Hint: assume that $\tau_{rr}(a+\delta) \approx \tau_{rr}(a)$; $u_r(a+\delta) \approx u_r(a)$