Theory of Elasticity ME 6102 **National University of Singapore**

Prof. Mordechai Perl **Dept. of Mechanical Engineering**

Assignment #4

1. The following displacement field is given:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} x_1^3 + x_1 x_2^2 + x_3^2 \\ x_3^2 - x_2^3 \\ 2x_1 x_2 x_3 - x_1^3 \end{Bmatrix} \cdot 10^{-4}\, cm$$

(a) Determine the strain tensor at point (1,0,1).

(b) Determine the rotation tensor at the same point.

(c) Determine the relative change of length of a short linear element passing through the same point whose direction is given by $\hat{n} = 1/\sqrt{3}\hat{e}_1 + \sqrt{2/3}\hat{e}_2$.

(d) Determine the change in volume at the same point.

2. The following displacement field is given:

$$u_1 = -\frac{M}{EI} x_1 x_2 + a_{1j} x_j + b_1$$
$$u_2 = \frac{M}{2EI}\left[x_1^2 + \nu\left(x_2^2 - x_1^2\right)\right] + a_{2j} x_j + b_2$$
$$u_3 = \frac{\nu M}{EI} x_2 x_3 + a_{3j} x_j + b_3$$

where: $M, E, I, \nu, a_{ij}, b_i$ are constants.

(a) Determine the strain and the rotation tensors.

(b) Determine the relative change in volume.

3. Given the following strain field:

$$\begin{aligned}
\varepsilon_{11} &= (1-\nu)\left(ax_2^2 - bx_1^2\right) - \nu\left(ax_1^2 - bx_2^2\right) \\
\varepsilon_{12} &= -2bx_1x_2 \\
\varepsilon_{22} &= -\nu\left(ax_2^2 - bx_1^2\right) + (1-\nu)\left(ax_1^2 - bx_2^2\right) \\
\varepsilon_{3j} &= \varepsilon_{j3} = 0
\end{aligned}$$

where: ν, a, b are constants,

determine for which values of a and b the strain field is compatible.

4. Given the following strain field:

$$\varepsilon_{ij} = \begin{pmatrix} 4x_1x_2 & x_1^2 & x_2x_3 \\ x_1^2 & 0 & x_1x_3 \\ x_2x_3 & x_1x_3 & 2x_1x_2 \end{pmatrix}$$

(a) Are the compatibility requirements satisfied by this strain field?

(b) Is question (a), in this particular case, redundant? Explain.