Theory of Elasticity ME 6102 **National University of Singapore**

Prof. Mordechai Perl **Dept. of Mechanical Engineering**

Assignment # 3

1. Given the following stress tensor:

$$\tau_{ij} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} MPa$$

Find the three direction cosines of the unit vector $\hat{n}$ which is perpendicular to the particular plane on which all three traction components T_i are nil.

2. The state of stress for a given body is described by the tensor

$$\tau_{ij} = \begin{pmatrix} 3x_1x_2 & -x_2^{\,2} & 0 \\ -x_2^{\,2} & 0 & 400 \\ 0 & 400 & 0 \end{pmatrix} MPa$$

(a) What *body force* vector is required for the body to be in equilibrium?

(b) Determine the normal and tangential stress components and the traction vector components acting on the plane $x + y + z = 6$ at point (5,1,0). Select the normal to the plane so that its n_x is positive.

3. The following stress field is given [KPa]

$$\tau_{11} = 80x_1^3 + x_2 \qquad \tau_{12} = 1000 + 100x_2^2$$
$$\tau_{22} = 100x_1^3 + 1600 \qquad \tau_{23} = 0$$
$$\tau_{33} = 90x_2^2 + 100x_3^3 \qquad \tau_{13} = x_1 x_3^3 + 100x_1^2 x_2$$

(a) What distribution of *body forces* is required to maintain equilibrium ?

(b) If this body is immersed in a liquid pressurized to 17 MPa, determine the principal stresses at point (1,2,0) of this body.

4. Write the boundary conditions on the planes: AB, CD, AC, and BD. (Note that traction **So** is constant along all boundaries while traction **Qo** is linearly distributed along the upper and the lower boundaries only).

