Theory of Elasticity ME 6102 **National University of Singapore**

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Assignment # 2

1. Given the following stress tensor in the coordinate system (x_1, x_2, x_3):

$$\tau_{ij} = \begin{pmatrix} 2.74 & -2.12 & 4.07 \\ -2.12 & 3.69 & -3.02 \\ 4.07 & -3.02 & 5.05 \end{pmatrix} \cdot 10^3 \, KPa$$

find the stress tensor τ'_{ij} in a new coordinate system which is rotated by 30° around axis x_3 with respect to the original system.

2. The state of stress at a point is known to be:

$$\tau_{ij} = \begin{pmatrix} 100 & 200 & -200 \\ 200 & 300 & 400 \\ -200 & 400 & -100 \end{pmatrix} KPa$$

(a) Calculate the components of the hydrostatic and the deviatoric stress tensors at the point.

(b) Evaluate the invariants of the above hydrostatic and deviatoric stress tensors.

(c) Determine the three principal stresses and their direction at this point.

(d) What is the maximum shear stress at this point and on what plane does it act?

3. Determine the normal component σ and the tangential component s of the traction vector on the plane $5x + 3y + 4z = 1$ for the stress tensor given in problem 2.

4. Write a computer program (in any language) which evaluates the principal stresses, the principal directions and the value of the stress tensor components in a coordinate system rotated with respect to the original one. The input to the program should be:

   (a) Six stress components in the original coordinate system (x, y, z)
   (b) The nine direction cosines of the rotated coordinate system (x^1, y^1, z^1)

   The output of the program should be:

   (a) The principal stresses and their directions (including a graphic representation).
   (b) The maximum shear stress and the plane upon which it acts.
   (c) The hydrostatic and deviatoric stress tensors and their invariants.
   (d) The new stress tensor in a system rotated with respect to the original coordinate system, (based on the direction cosines matrix).

   Check your program by applying it to problem 2.