Theory of Elasticity ME 6102 **National University of Singapore**

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Assignment # 1

1. Expand the following expressions and simplify them as much as possible (*i,j=1,2,3*):

   (a) $\bar{a} \cdot \bar{b} = a_i b_i$ (scalar product)

   (b) $u_i = a_{ij} x_j + b_i$

   (c) $\delta_{ij} t_{ij}$ (δ_{ij} is Kronecker's delta)

   (d) $m_i = e_{ijk} x_j t_k$ (vector product, where e_{ijk} is the permutation tensor)

   (e) $b_{ij} x_i x_j$ (where $b_{ij} = -b_{ji}$ for $i \neq j$)

   (f) $\delta_{im} \delta_{mj} b_{ij}$

2. Prove the following identities:

   (a) $\delta_{ii} = 3$ (b) $\delta_{ij} b_j = b_i$ (c) $\delta_{ij} b_{jk} = b_{ik}$

   (d) $\delta_{ij} b_i b_j = b_i b_i = b_j b_j$ (e) $e_{ijk} b_j b_k = 0$

3. Write the following expressions using the summation convention:

   (a) $(x_1)^2 + (x_2)^2 + (x_3)^2 + \ldots + (x_n)^2$ (b) X_i^2

   (c) $g_{12} g_{11} + g_{22} g_{12} + g_{32} g_{13}$ (d) $\dfrac{d\bar{x}_k}{dt} = \dfrac{\partial \bar{x}_k}{\partial x_1}\dfrac{dx_1}{dt} + \dfrac{\partial \bar{x}_k}{\partial x_2}\dfrac{dx_2}{dt} + \ldots + \dfrac{\partial \bar{x}_k}{\partial x_n}\dfrac{dx_n}{dt}$

4. Write out in full:

   (a) $f_{,ii}$ (b) $t_{k,k} - \delta_{ij} t_{i,j}$ c) $h_{,iijj}$

5. Given the tensor τ_{ij} and the transformation tensor β_{ij}, find the components of the new tensor τ'_{12}, τ'_{22} in the new coordinate system; where:

$$\tau_{ij} = \begin{pmatrix} 3 & 0 & 1.5 \\ 0 & 7.5 & 0 \\ 1.5 & 0 & 0 \end{pmatrix}; \qquad \beta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 \\ 0 & -1/2 & \sqrt{3}/2 \end{pmatrix}$$